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An Efficient and Flexible Camera Calibration Technique for Large-Scale Vision Measurement Based on Precise Two-Axis Rotary Table

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Abstract: A novel camera calibration technique is presented in the field of large-scale vision measurement, which employs a precise two-axis rotary table and a single stationary optical reference point. By moving the camera at different angular positions of the rotary table, and simultaneously taking photos of the stationary optical point in front of it, a precise angular control field is established for calibration. This technique brings such advantages as: there is no need to know the location relationship among the camera, the reference point and the rotary table previously; the whole course of calibration is processed by computer program automatically; and this method adapts to different imaging systems and measuring tasks easily by adjusting the rotating range of the rotary table and the distance from the reference point to the camera, so that the calibration space and the measurement space coincide with each other. Calibration experiments verify that the proposed method has high accuracy.

Keywords: large-scale vision measurement; camera calibration; precise rotary table; angular control field

基于精密二轴转台的大尺寸测量相机高效灵活标定方法

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摘 要: 针对大尺寸视觉测量领域的相机标定问题, 研究了一种使用精密二轴转台和单个固定不动的光学参考点进行相机标定的方法。通过精密二轴转台带动相机做二维转动并拍摄前方光学参考点, 从而构建精密的角度基准控制场并实现标定。该方法有如下优点: 无需调整或已知参考光点与相机和转台之间的位置关系; 测量全程由程序控制自动高效完成; 同时, 针对不同的测量任务和测量相机, 测量空间和测量距离可通过调整转台的转动幅度以及控制参考点到相机的距离实现任意灵活配置, 从而保证标定控制场和相机测量视场相一致。标定实验验证了本文提出的标定方法具有较高的标定精度。

关键词: 大尺寸视觉测量; 相机标定; 精密转台; 角度控制场

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Large-scale metrology has been developed in recent years on precision measurement within the measurement

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range from a few meters to tens of meters, which in size is between the traditional precision measurement and engineering surveying. It is the tendency that the traditional precision measurement from the conventional size to large size and that for engineering surveying to be more precise^[1]. Vision based measurement method is an increasingly popular method in large-scale metrology. By identifying key features of a target from the image of a camera, the relative position and orientation can be inferred when the 3D location of these features on the target are known (monocular system), or when the position of these features from different view of points is known (multi-camera or monocular systems)^[2]. A typical application is in the shield machine guidance. An electro-optical system has been carried out in measuring the relative pose between the cutting shield and the gripper shield, the distance between which is up to 5 m^[3]. To achieve high measurement accuracy, the measuring camera must be calibrated precisely in advance.

Camera calibration has been studied deeply in the field of computer vision and traditional precision measurement and has resulted in a number of calibration methods^[4-5]. Most of these calibration methods are based on control targets or virtual targets whose positions or coordinate information are known. Zhang Zhengyou's method^[6] typically uses a flat pattern with multiple positions as the calibration benchmark. However, it is difficult to obtain large size precision targets during the application of large-size direction measurement, and even if it is possible to fabricate such a target or to construct a virtual target, it is difficult to adapt to the measuring camera's field of view and the measurement space flexibly. Naturally, research has been carried out in using targets of limited sizes, moving it in a number of locations, and covering the measurement space to achieve calibration. Although this method has the capability of filling the camera measurement space flexibly, the lack of contact among the various positions cannot offer enough constraints, thus limits the calibration accuracy, and the operation process is usually very complex^[7-8]. In the field of surveying and mapping, the interior orientation elements of the aerial and satellite mapping cameras working at high altitudes are generally calibrated by the method of exact measuring angle in the laboratory using precise rotary table, collimator and star tester^[9-10]. These methods require precise adjustment a-

mong the collimator's optical axis, the rotary table and the camera, which is very inconvenient. In the field of large-scale vision measurement and photogrammetry, the self-calibration method is often adopted to realize camera calibration^[11]. Although the method is very flexible, the computational complexity is great, the computing robustness of the equations is poor, and the calibration accuracy is usually not as good as the laboratory optical calibration methods. In addition, the calibration method independent of the imaging model does not use the traditional camera model but a similar exhaustion technique to map each measurement angle on the camera imaging surface. Although the calibration precision of this method is high, the workload is huge^[12].

Nowadays, the technique of precise rotary table has been greatly developed. Since the essential function of a measuring camera is to obtain angles in two directions, the measuring camera can be calibrated by precise biaxial rotary table which provides the angle reference of two directions. As a result, the angular control field provided by the rotary table is very flexible compared with the control field of known positions or coordinates because the width and depth of the calibration space can be flexibly arranged. Based on the above research methods, to achieve calibration, this paper applies a precise two-axis rotary table on which the measuring camera is fixed, and a single optical beacon which is fixed in front of them. During the calibration process, the camera is doing two-dimensional rotational motion on the rotary table, capturing the image of the beacon at the same time. There is no need to know or adjust the position relationship among the light source, the rotary table and the measuring camera during the calibration process. And the calibration process can be done by the program automatically without human intervention. Last but not the least, the rotation range and the distance between the reference beacon and the camera can be arranged arbitrarily according to the application requirements.

1 Camera model

The camera imaging model is to establish the relationship between the point in the world coordinate system and that in the camera image plane. Assume that the points in the world coordinate system ($O_w X_w Y_w Z_w$) are

noted as $\mathbf{p}_w = [x_w \ y_w \ z_w]^T$, the corresponding points under the camera coordinate system ($O_c X_c Y_c Z_c$) are noted as $\mathbf{p}_c = [x_c \ y_c \ z_c]^T$, and the image points in the pixel coordinate system are (u, v) without considering distortion conditions, and according to the perspective projection model, we have

$$\begin{aligned} z_c \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} &= \begin{bmatrix} a_x & 0 & u_0 & 0 \\ 0 & a_y & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \\ &= \begin{bmatrix} a_x & 0 & u_0 & 0 \\ 0 & a_y & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R}_{wc} & \mathbf{t}_{wc} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \quad (1) \end{aligned}$$

where a_x is the normalized focal length on the u -axis and a_y is the normalized focal length on the v -axis; ($u_0 \ v_0$) is the coordinates of the main point on the image plane in the pixel coordinate system; the rotation matrix \mathbf{R}_{wc} and the translation vector \mathbf{t}_{wc} represent the transformation relationship between the world coordinate system and the camera coordinate system.

The above model is based on the ideal pinhole imaging model. For the actual visual imaging system, due to the influence of factors such as optical system processing and assembly error, it is necessary to compensate them by introducing additional parameters in the model, usually including distortion parameters (k_1, k_2, k_3), tangential distortion parameters (p_1, p_2), affine and nonorthogonal deformation parameters (b_1, b_2)^[2]. The distortion that causes the feature point imaging offset can be formulated as:

$$\begin{cases} \Delta x = x_n^2 k_1 + x_n^4 k_2 + x_n^6 k_3 + (r^2 + 2x_n^2) p_1 + \\ \quad 2x_n y_n p_2 + x_n b_1 + y_n b_2 \\ \Delta y = y_n^2 k_1 + y_n^4 k_2 + y_n^6 k_3 + 2x_n y_n p_1 + \\ \quad (r^2 + 2y_n^2) p_2 + y_n b_1 \end{cases} \quad (2)$$

where $r = \sqrt{x_n^2 + y_n^2}$ is the distance from the feature image point to the main point in the normalized image coordinate system. Thus, the pixel projection on the camera plane in the world coordinate system can be corrected as:

$$\begin{cases} u_d = u_0 + a_x(x_n + \Delta x) \\ v_d = v_0 + a_y(y_n + \Delta y) \end{cases} \quad (3)$$

The model described above is a complete camera im-

aging model in which the camera's intrinsic parameters $\mathbf{x}_{ins} = [a_x \ a_y \ \mu_0 \ \nu_0 \ k_1 \ k_2 \ k_3 \ p_1 \ p_2 \ b_1 \ b_2]^T$ requires precise calibration.

2 Calibration principle based on precise two-axis rotary table

This paper introduces a precise two-axis rotary table, rotating the camera in two dimensions to establish the benchmark of angles. Fig. 1 shows the calibration configuration diagram. The two-axis rotary table includes three parts, namely the fixed base, the outer frame and the inner frame. The outer frame rotates around the horizontal axis relative to the fixed base, the inner frame rotates around the vertical axis relative to the outer frame, and the intersection of the vertical and horizontal axes is the rotation center of the rotary table. The camera is mounted on the inner frame of the rotary table with its viewing field facing forward. Meanwhile, in front of the rotary table, an optical reference point is fixed in the measurement space of the camera. According to the actual application of the camera system, LED light spots or concentric circles can be used as the reference points. A tooling should be designed according to the size of the camera and the rotary table to make the camera optical center as close as possible to the rotation center of the rotary table.

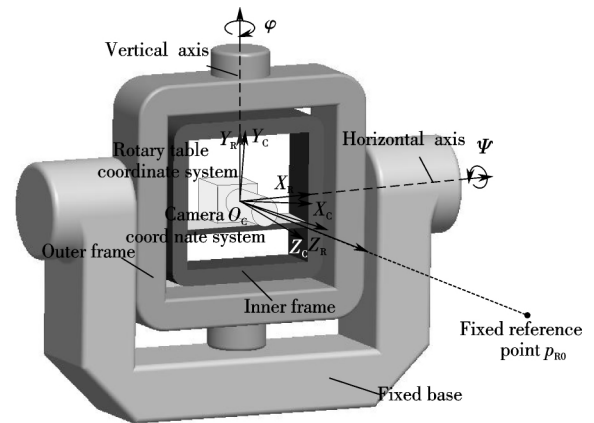


Fig. 1 Calibration setup and coordinates definition

In addition to the camera coordinate system ($O_c X_c Y_c Z_c$), the rotary table coordinate system ($O_R X_R Y_R Z_R$) should be defined to achieve calibration. The rotary table coordinate system is built and fixed on the inner frame of

the rotary table, and also rotates on it, as shown in Fig. 1. The origin of the rotary table coordinate system is established at the center of the rotary table, the $O_R X_R$ axis coincides with the inner frame of the rotary table zero, and the $O_R Y_R$ axis coincides with the outer frame of the rotary table zero. When the camera is mounted on the rotary table, the three-axis direction of the camera coordinate system is closer to the rotary table coordinate system.

The transformation relationship between the rotary table coordinate system and the camera coordinate system is represented by rotation matrix \mathbf{R}_{RC} and translation vector \mathbf{t}_{RC} , and the coordinates of the reference point in the rotary table coordinate system at zero position are $\mathbf{p}_{R0} = [x_{R0}, y_{R0}, z_{R0}]^T$. During the calibration process, when the outer frame and inner frame of the rotary table are rotated by angles ψ_i and φ_i respectively, the coordinates of the reference point in the rotary table coordinate system has become

$$\begin{bmatrix} x_{Ri} \\ y_{Ri} \\ z_{Ri} \end{bmatrix} = \mathbf{R}_i \begin{bmatrix} x_{R0} \\ y_{R0} \\ z_{R0} \end{bmatrix} \quad (4)$$

where

$$\mathbf{R}_i = \begin{bmatrix} \cos \varphi_i & 0 & -\sin \varphi_i \\ 0 & 1 & 0 \\ \sin \varphi_i & 0 & \cos \varphi_i \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi_i & \sin \psi_i \\ 0 & -\sin \psi_i & \cos \psi_i \end{bmatrix} \quad (5)$$

which represents the rotation matrix of the rotary table coordinate system relative to the zero position. The coordinates of the reference point in the camera coordinate system are

$$\begin{bmatrix} x_{Ci} \\ y_{Ci} \\ z_{Ci} \end{bmatrix} = \mathbf{R}_{RC} \mathbf{R}_i \begin{bmatrix} x_{R0} \\ y_{R0} \\ z_{R0} \end{bmatrix} + \mathbf{t}_{RC} \quad (6)$$

And according to the camera imaging model,

$$z_{Ci} \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & 0 & u_0 \\ 0 & a_y & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{Ci} \\ y_{Ci} \\ z_{Ci} \end{bmatrix} = \begin{bmatrix} a_x & 0 & u_0 \\ 0 & a_y & v_0 \\ 0 & 0 & 1 \end{bmatrix} \left(\mathbf{R}_{RC} \mathbf{R}_i \begin{bmatrix} x_{R0} \\ y_{R0} \\ z_{R0} \end{bmatrix} + \mathbf{t}_{RC} \right) \quad (7)$$

where (u_i, v_i) is the pixel position which the reference point projects on the imaging plane. By using the precise

tooling, the optical center of the camera and the rotation center of the rotary table are almost the same, therefore the value of \mathbf{t}_{RC} can be treated as 0. Since the measurement camera can be regarded as a two-dimensional angle measuring unit, Eq. (7) actually only has two degrees of freedom, and only two degrees of freedom of the point position \mathbf{p}_{R0} has been involved in the calculation of equations. Therefore, the solution of Eq. (7) needs to be given a constraint. From a geometric perspective, the projection of the calibration reference point in the camera coordinate system is equivalent to a vector direction, so \mathbf{p}_{R0} can be added with a scale constraint and become a unit vector as

$$\|\mathbf{p}_{R0}\| = 1 \quad (8)$$

The series equations above establish the transformation from the reference light vector to the camera pixel plane during the rotary table rotation process. The unknown variables contained in the camera calibration model of intrinsic and extrinsic parameters are $(\mathbf{p}_{R0}, \mathbf{R}_{RC}, \mathbf{x}_{ins})$. Assuming that (\hat{u}_i, \hat{v}_i) is the true position detected by the camera at i -th position of the rotary table, it should ideally be equal to the projection point of the optical reference point on the image plane. Therefore, a minimized objective function is established as follows:

$$J(\mathbf{p}_{R0}, \mathbf{R}_{RC}, \mathbf{x}_{ins}) = \sum_{i=1}^n [\hat{u}_i - u_i]^2 + [\hat{v}_i - v_i]^2 + M(\|\mathbf{p}_{R0}\| - 1)^2 \quad (9)$$

where M is the penalty factor and is generally set to a large value. To solve the unit orthogonal constraint problem of the rotation matrix \mathbf{R}_{RC} , it can be decomposed into three Euler angles. Solving the objective function is a nonlinear optimization problem, which can be solved by the Levenberg-Marquardt algorithm^[13].

To obtain the global optimal solution as much as possible, the optimization method needs to be given a reasonable initial value. The initial value of the intrinsic parameter \mathbf{x}_{ins} can be given by the ideal value of the imaging system, where the initial value of the main point (u_0, v_0) is set to half of the total number of pixels, the normalized focal length (a_x, a_y) is the quotient of the lens' design distance divided by the camera pixel size, and all the distortion parameters are set to be 0. The initial values of the external parameters $(\mathbf{p}_{R0}, \mathbf{R}_{RC})$ can be given by the approximate relationship among the camera, the rotary table and the optical reference point. Since the axes of the

camera and the rotary table coordinate system are nearly parallel, the initial position of the posture matrix R_{RC} can be set as a unit matrix. And since the reference point is approximately in front of Z -axis, the initial value of vector p_{R0} can be set to be $[0 \ 0 \ 1]^T$.

The calibration model shows that based on the two-dimensional angle measuring principle of the camera and normalization, the constraint relationship among the camera model parameters can be established.

3 Experimental verification

This calibration experiment is conducted for a class of practical camera in large-scale measurement^[3]. The camera resolution is $2\,048 \times 2\,048$ pixels, the size of unit pixel is $0.005\,5\text{ mm} \times 0.005\,5\text{ mm}$, the lens focal length is 25 mm , and the measuring depth of the imaging system is $2.5\text{—}5.0\text{ m}$. The experimental diagram is shown in Fig. 2, and the measurement accuracy of the precise rotary table used is less than 1 arcsec . A precise tooling is designed according to the size of the camera and the rotary table to make sure that the center of the camera coordinate system and the rotary table coordinate system are coincident, and a LED light spot is fixed in front of the rotary table as the reference point. The center position of the LED light spot on the image plane is obtained by using the weighted centroid method^[14].

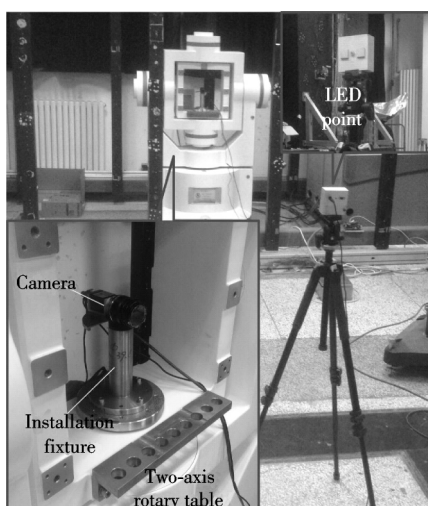


Fig. 2 Calibration experiment for large-scale measurement camera

The calibration is processed as follows. First, the

LED light spot is placed at the middle measurement distance from the imaging system, that is, 3.75 m away from the camera for calibration. Based on the measuring field of the camera, both horizontal and vertical directions of the rotary table are rotated from -12° to 12° at steps of 1° , resulting in $625 (25 \times 25)$ calibration positions which almost cover the entire measuring field of camera. As shown in Fig. 3, the recorded image of the light spot at each angle position is composite. The whole calibration process is conducted by the program automatically without any human intervention.

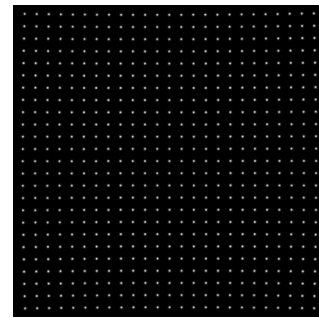


Fig. 3 Composite image of the light spot at all the angle positions

The calibration method can be evaluated by the standard deviation of the calibrated parameters after multiple calibrating. Therefore, all the collected data is divided into 4 sets that contain 169, 156, 156 and 144 calibration points respectively by alternating sampling in two directions. And their calibration results are shown in Tab. 1.

From the experimental results in Tab. 1, in which the unit of the coordinates of the primary point and RMS are pixels, it can be seen that the standard deviations of the camera intrinsic parameters are very small, verifying the reliability of the calibration method proposed. Meanwhile, the root mean square (RMS) of the pixel residual error is about 0.06 pixels , and its corresponding angular measuring error is 2.7 arcsec , indicating that the method is feasible.

In addition, since the measuring field of view of the camera in practical application is from 2.5 m to 5.0 m , to verify the effects of the calibration distance on the camera intrinsic parameters, the LED light spots are placed at 2.5 m and 5.0 m from the camera respectively to do the calibration experiment, and their results are compared with those at 3.75 m . The results are shown in Tab. 2

Tab.1 Camera calibration result at 3.75 m

Group	a_x	a_y	x_0	y_0	k_1	k_2	k_3	$p_1/10^{-5}$	$p_2/10^{-4}$	$b_1/10^{-3}$	$b_2/10^{-5}$	Pixel residual RMS
1	4 562.08	4 562.93	988.52	1 024.87	-0.311 9	-0.072 5	1.424 6	4.330	2.0	3.7	4.18	0.059 4
2	4 562.08	4 562.96	989.73	1 024.71	-0.315 2	0.023 5	0.705 1	0.453	2.0	3.7	5.10	0.071 3
3	4 562.09	4 562.95	987.95	1 026.72	-0.312 9	-0.056 5	1.383 8	7.430	2.0	3.7	4.71	0.063 3
4	4 562.18	4 562.99	989.42	1 021.74	-0.318 2	0.097 9	0.138 3	2.140	2.0	3.7	3.71	0.063 5
Average value	4 562.11	4 562.96	988.90	1 024.51	-0.314 5	-0.001 9	0.912 9	3.590	2.0	3.7	4.42	0.064 4
Standard deviation	0.04	0.02	0.71	1.78	0.002 4	0.068 1	0.530 7	2.610	0.18	1.23	0.526	0.004 3

(where the unit of the coordinates of the primary point and RMS are pixels).

It can be seen in Tab. 2 that the RMS errors of the pixel residuals at three calibration distances are all about 0.06 pixels, indicating that the method proposed has high measurement accuracy. In addition, the standard deviation at different distances is significantly greater than that at the same distance, especially the normalized focal length, whose standard deviation increases by two orders

of magnitude. This means that the calibration distance has a large effect on the calibration accuracy, especially the normalized focal length. Therefore, it is reasonable to place a number of reference points at different depths of the camera's measuring space so that the calibration space and the measurement space coincide with each other, and the entire measurement space of the intrinsic parameters can be taken into account.

Tab.2 Camera calibration result at difference distances

Distance	a_x	a_y	x_0	y_0	k_1	k_2	k_3	$p_1/10^{-5}$	$p_2/10^{-4}$	$b_1/10^{-3}$	$b_2/10^{-5}$	Pixel residual RMS
2.50 m	4 565.21	4 566.02	985.01	1 027.29	-0.313 2	-0.063 3	1.365 4	4.82	4	4.4	-1.29	0.056 1
3.75 m	4 562.09	4 562.94	988.52	1 024.88	-0.313 5	-0.030 7	1.131 7	4.21	2	3.7	4.61	0.065 6
5.00 m	4 560.39	4 561.14	986.73	1 025.27	-0.310 7	-0.063 9	1.344 7	4.35	3	3.3	0.444	0.062 3
Average value	4 562.56	4 563.37	986.75	1 025.81	-0.312 4	-0.052 6	1.280 6	4.46	3	3.8	1.26	0.061 3
Standard deviation	2.00	2.01	1.43	1.06	0.001 2	0.015 5	0.105 7	0.258	0.701	0.4	2.48	0.003 9

4 Conclusion

In this paper, a camera calibration technique is proposed in the field of large-scale vision measurement by using a precise two-axis rotary table and a single stationary optical reference point. There is no need to know the location relationship among the camera, the reference point and the rotary table previously, and the whole course of calibration is processed by the computer program automatically. This method adapts to different imaging systems and measuring tasks easily by adjusting the rotating range of the rotary table and the distance from the reference point to the camera.

The calibration results of experiments show that the standard deviations of the intrinsic parameters are very small, indicating that the calibration method proposed has

high reliability. At the same time, the RMS of the pixel residual error is about 0.06 pixels, and its corresponding angle error is 2.7 arcsec, showing that the method has high calibration accuracy. In addition, the experiment also shows that the calibration results vary at different distances where the reference points from the camera are significantly different, indicating that placing multiple reference points at different depths of the camera's measuring space is more reasonable, so that the calibration space and the measurement space can coincide with each other.

Future research will analyze the influence of the installation errors of the camera and other factors on calibration accuracy mathematically.

References:

- [1] Schmitt R H, Peterek M, Morse E, et al. Advances in large-scale metrology: Review and future trends [J]. *CIRP*

- Annals - Manufacturing Technology* , 2016 , 65(2) : 643-665.
- [2] Zhu Jigui , Yu Zhijing. *The Principle of Vision Metrology* [M]. Beijing: China Machine Press , 2012(in Chinese) .
- [3] Gao Yang , Lin Jiarui , He Feiyan , et al. A monocular vision and inclinometer combined system for 6DOF measurement of double shield TBM [J]. *Sensors & Actuators A: Physical* , 2016 , 249: 155-162.
- [4] Zhang Z , Lu H , Wang B. Overview of camera calibration for machine vision [J]. *Intelligent Control and Automation* , 2015: 743-746.
- [5] Luhmann T , Fraser C , Maas H G. Sensor modelling and camera calibration for close-range photogrammetry [J]. *ISPRS Journal of Photogrammetry & Remote Sensing* , 2016 , 115: 37-46.
- [6] Zhang Zhengyou. A flexible new technique for camera calibration [J]. *IEEE Transactions on Pattern Analysis & Machine Intelligence* , 2000 , 22(11) : 1330-1334.
- [7] Sun Junhua , Liu Zhen , Zhang Guangjun , et al. Camera calibration based on flexible 3D target [J]. *Acta Optica Sinica* , 2009 , 29(12) : 3433-3439.
- [8] Yang Bowen , Zhang Liyan , Ye Nan , et al. Camera calibration technique of wide-area vision measurement [J]. *Acta Optica Sinica* , 2012 , 32(9) : 0915001 (in Chinese) .
- [9] Wang Zhihe , Shu Rong , He Zhiping , et al. New method of CCD camera calibration based on collimator [J]. *Journal of Infrared and Millimeter Waves* , 2007 , 26(6) : 465-468 (in Chinese) .
- [10] Yuan Guoqin , Ding Yalin , Hui Shouwen , et al. Grouped approach algorithm for mapping camera calibration based on method of exact measuring angle [J]. *Acta Optica Sinica* , 2012 , 32(1) : 0112005 (in Chinese) .
- [11] Fraser C S. Automatic camera calibration in close range photogrammetry [J]. *Photogrammetric Engineering & Remote Sensing* , 2013 , 79(4) : 381-388.
- [12] Zhu Jigui , Zou Jian , Lin Jiarui. Camera calibration method independent of imaging model [J]. *Journal of Tianjin University: Science and Technology* , 2013 , 46(6) : 477-481(in Chinese) .
- [13] Moré J J. The Levenberg-Marquardt algorithm: Implementation and theory [J]. *Lecture Notes in Mathematics* , 1978 , 630: 105-116.
- [14] Rufno G , Accardo D. Enhancement of the centroiding algorithm for star tracker measure refinement [J]. *Acta Astronaut* , 2003 , 53: 13-47.

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