A Proposal to Research Cutting Plane Tree Algorithm and Compare it with other Algorithms

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November 11, 2014

1 Introduction

A mixed-integer linear program (MILP) is a mathematical program with linear constraints in which a specified subset of the variables are required to take on integer values. let a standard form polyhedron be

$$\mathcal{P} = \{x \in \mathbb{R}^n | Ax = b, x \geq 0\}, \tag{1}$$

where $A \in \mathbb{Q}^{m \times n}, b \in \mathbb{Q}^m$. Without loss of generality, we assume that the variables indexed 1 through $p \leq n$ are the integer variables. If we denote $\mathcal{P}' = \mathcal{P} \cap \mathbb{Z}^p \times \mathbb{R}^{n-p}$, the mixed-integer linear programming problem is then to compute the optimal value

$$z^{IP} = \max_{x \in \mathcal{P}'} c^T x, \tag{2}$$

where $c \in \mathbb{Q}^n$ is a vector that defines the objective function.
We state some common strategies to find a solution of the general MILP

- Lower bounding method: A method for determining a lower bound on the objective function value of an optimal solution to a given subproblem.
- Upper bounding method: A method for determining an upper bound on the optimal solution value.
- Branching method: A procedure for partitioning a subproblem to obtain two or more children.
- Search strategy: A procedure for determining the search order.

2 Theoretical framework and research plan

In this project, we follow the ideas in [1] and [2] to further implement and research the cutting plane tree(CPT) algorithm. In [1], a finitely convergent convex hull tree algorithm was developed, which is used to obtain the convex hull of general MILP and, moreover, construct a linear program that has the same optimal solution as the associate MILP. After combining this methods with standard notion of sequential cutting planes, they derive the CPT algorithm, which shows to converge to an integral optimal solution in finitely many iterations.

To implement the CPT algorithm, one should generate multi-term disjunctive cuts (seems hard..)

After going into the details of the ideas, we will compare the effects of it with other preliminary algorithms. By test different problems under variety noncommercial or commercial software in [3] and the CPT algorithm, we aim to know the efficiency of the algorithm in [1].

References

[1] Binyuan Chen, Simge Küçükyavuz, and Suvrajeet Sen. Finite disjunctive programming characterizations for general mixed-integer linear programs. *Operations research*, 59(1):202–210, 2011.

[2] Binyuan Chen, Simge Küçükyavuz, and Suvrajeet Sen. A computational study of the cutting plane tree algorithm for general mixed-integer linear programs. *Operations Research Letters*, 40(1):15–19, 2012.

[3] Jeffrey T Linderoth and Ted K Ralphs. Noncommercial software for mixed-integer linear programming. *Integer programming: theory and practice*, 3:253–303, 2005.