Improvements in Genetic Algorithms

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***Abstract*—This paper presents an exhaustive study of the Simple Genetic Algorithm (SGA), Steady State Genetic Algorithm (SSGA) and Replacement Genetic Algorithm (RGA). The performance of each method is analyzed in relation to several operators types of crossover, selection and mutation, as well as in relation to the probabilities of crossover and mutation with and without dynamic change of its values during the optimization process. In addition, the space reduction of the design variables and global elitism are analyzed. All GAs are effective when used with its best operations and values of parameters. For each GA, both sets of best operation types and parameters are found. The dynamic change of crossover and mutation probabilities, the space reduction and the global elitism during the evolution process show that great improvement can be achieved for all GA types. These GAs are applied to TEAM benchmark problem 22.**

***Index Terms*—Electromagnetics, genetic algorithms, optimization.**

I. Introduction

THE USE of a genetic algorithm (GA) requires the choice of a set of genetic operations between many possibilities [1]. For example, the crossover operation with two cut points, mutation bit by bit, and selection based on the roulette well. This choice can be effective for one type of GA but worse for another. In addition, the number of generations and the population size, crossover and mutation probabilities are values that must be given to initialize the optimization process. All these parameters have great influence on the GA performance. What is the best choice is an important question. To answer this question, an exhaustive investigation is performed for the SGA, SSGA and RGA [2], [3], using three analytical test functions. Finally, the best GA's are applied to TEAM benchmark problem 22 [4].

II. Methodology

To attain the purpose of this paper the following methodology was employed. Each GA, with both sets of parameters and operation types, is executed 3×30 times for each function. Each parameter is analyzed separately keeping all others fixed. In this analysis, we choose a group of selection, crossover and mutation methods, associated with three additional procedures: *i)* dynamic change of the mutation (pm) and crossover (pc) probabilities; *ii)* reduction of the optimization space and *iii)* global elitism. These methods and procedures *i)* and *iii)* are specified in Table I.

TABLE I
Genetic Operations and Others Procedures

Selection [5]	• Roulette well (RO) • Deterministic sampling (DS) • Stochastic remainder sampling (SRS)
Crossover	• One cut point by genotype (1 CP) • Two cut points by genotype (2CP) • One cut point by chromosome (1CPC) • Uniform crossover (UC) [1]
Mutation	• Bit by bit (BB) • One mutation by chromosome at a random position (BC) • One mutation by individual at a random position (BI)
Dynamic change of pm and pc	• Based on a linear interpolation (LI) • Based on a measure of genetic diversity exterior to some limits (EL)
Elitism	• Simple elitism • Global elitism

TABLE II
Analytical Test Functions

Rastrigin:	$ff(x) = 10 * n + \sum_{i=1}^{n} [x_i^2 - 10 * \cos(2 * \pi * x_i)]$
Peaks:	$ff(x) = 3*(1-x_1)^2 * e^{-x_1^2-(x_2+1)^2} - \left(10*(\frac{x_1}{5} - x_1^3 - x_2^5) * e^{-x_1^2-x_2^2} + \frac{1}{3} * e^{-(x_1+1)^2-x_2^2}\right)$
Degree:	$ff(x) = \sum_{i=1}^{n} abs(int(x_i))$

The analytical test functions were chosen with different characteristics and degrees of difficulties. Table II gives their mathematical expressions.

The number of variables, n, for the Rastrigin and Degree functions was made equal to 2. The variable bounds for the three functions were respectively taken as $\mathrm{x_{Ri}} \in [-5.12; 5.12]$; $\mathrm{x_{Pi}} \in [-3.0; 3.0]$ and $\mathrm{x_{Di}} \in [-20.0; 20.0]$. The global minimum for these functions are respectively: $\mathrm{x_R}^* = [0.0; 0.0]$; $\mathrm{x_P}^* = [0.0094; 1.5814]$ and $|\mathrm{x_{iD}}^*| < 0.5; \mathrm{i} = 1, 2$. To measure the success for these functions, the following criteria of convergence were used: *i)* $\|\mathrm{x_R}\|_2 < 0.02^{1/2}$; *ii)* $\|\mathrm{x_D}\|_\infty < 0.5$ and *iii)* $\|\mathrm{x_P} - \mathrm{x_P}^*\|_2 < 0.02^{1/2}$, respectively. Each GA was started with the same basic parameters and genetic operations as given in Table III.

The values in Table III were chosen with some preliminary tests. The mutation probability was fixed at 0.025 for mutation bit by bit, 0.5 for one mutation by chromosome and 1.0 for one mutation by genotype or individual. In all these cases, each individual has more than 50% of chance to have one bit changed by this operation.

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TABLE III
GA'S INITIAL PARAMETERS AND PROCEDURES

Parameter/Operation	SGA	SSGA	RGA
Population size	20	20	20
Chromosome length	20	20	20
Generation number	(*)	(*)	(*)
Mutation probability	0.025	0.025	0.025
Crossover probability	0.75	0.75	0.75
NcalM – Rastrigin (**)	1000	1000	1000
NcalM – Peaks (**)	600	600	600
NcalM - Degree - (**)	400	400	400
N_Sel (***)	100%	80% and 50%	10% (2 individuals)
Selection type	SRS	SRS	SRS
Mutation type	BB	BB	BB
Crossover type	1CP	1CP	1CP
Elitism	simple	simple	Simple
Dynamic adaptation	without	without	without
Interval reduction	without	without	without

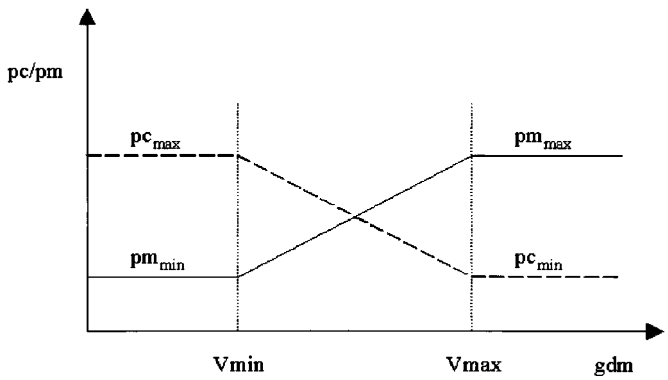

Fig. 1. Linear interpolation between bounds.

A. Description of the Used Additional Procedures

Elitism: The SGA works quite well with the *simple elitism* procedure. In this case, the best individual at generation k (the father) is maintained in the next generation $k+1$ if its child has a performance inferior than its father. Without the elitism, the best results can be lost during the selection, mutation and crossover operations. The SGA, SSGA and RGA were initialized with *simple elitism*. In the case of *global elitism*, each individual in the population of generation $k+1$ can replace its father of generation k, if it has a performance superior than him. In this case, at a generation $k+1$, the individuals are better than the individuals at generation k.

Dynamic Adaptation of Crossover and Mutation Probabilities: The goals with adaptive probabilities of crossover and mutation are to maintain the genetic diversity in the population and prevent the GA to converge prematurely to local minima. To measure the genetic diversity (gdm), we use the ratio between the mean and the maximum values of the fitness function at each generation, in which case $0 \leq gdm \leq 1$. When the value of gdm is one, this indicates that all individuals have the same genetic code, that is, they represent the same point in the optimization space. In this case, the genetic diversity is the smallest. To avoid premature convergence, the probabilities of mutation (pm) and crossover (pc) must be changed in such a way to introduce new genetic characteristics and to reduce the loss of genetic material. So, pm must be augmented and pc reduced. In the other case, if $gdm \ll 1$ means that there are a lot of genetic diversity introduced by the mutation operation. To avoid a basically random search, pm must be reduced and pc augmented [6].

TABLE IV
PARAMETERS FOR EL AND LI ADAPTATIONS

Parameter	EL	LI
Vmin	0.005	0.0
Vmax	0.15	1.0
pc_{min}	0.5	0.5
pc_{max}	1.0	1.0
pm_{min}	0.001	0.025
pm_{max}	0.25	0.25
Km	1.1	-
Kc	1.1	-

Two different procedures to perform the dynamic adaptation of probabilities pm and pc were tested. The first one, the LI procedure, is schematically represented in Fig. 1.

The second procedure, denoted by EL, can be understood by the following pseudo-code:

if $gdm > V_{\max}\{$
$p_m = k_m * p_m;$
$p_c = p_c / k_c;$
}
else if $m_{dg} < V_{\min}\{$
$p_m = p_m / k_m;$
$p_c = k_c * p_c;$
}
if $p_m > p_{m_{\max}}$, $p_m = p_{m_{\max}}$;
if $p_m < p_{m_{\min}}$, $p_m = p_{m_{\min}}$;
if $p_c > p_{c_{\max}}$, $p_c = p_{c_{\max}}$;
if $p_c < p_{c_{\min}}$, $p_c = p_{c_{\min}}$;

The values of all parameters in both procedures were chosen after preliminary tests and are given in Table IV.

Procedure to Reduce the Space of Design Variables: The technique used is very simple. The idea is to reduce the interval of the design variables to augment the precision of the final results and facilitate the search toward the global minimum. In all cases where the reduction was performed, it was made when the counter of generations reached the integer nearest of Nbgen/3. Before the reduction, the best result is saved and it is introduced at generation equal to the integer nearest of 2*Nbgen/3, where Nbgen is the maximum number of generations.

III. RESULTS

Analytical Problems: The results are schematically shown in Table V. It is shown clearly that all SGA, SSGA and RGA implemented work better with mutation bit by bit, with the EL dynamic adaptation of pm and pc, with the space reduction and with global elitism (GE). Basically, the difference is the type of selection and crossover operations. The SGA works better with the deterministic sampling (DS) and with one cut point by chromosome (1CPC). The SSGA, on the other hand, gives better results with roulette well (RO) as selection in both SGGA50 and SSGA80 (respectively 50% and 80% of individuals are selected in each generation to perform the genetic operations). In

TABLE V
MAIN RESULTS FOR THE ANALYTICAL TESTS

	Nbgen	Function	Sel.	Crossover	Mut.	Adap.	Red.	Elitism
SGA	50	Rastrigin	DS	2CP	BB	EL	Ok	GE
	20	Degree	DS	2CP/1CPC	BB	EL	Ok	GE
	30	Peaks	DS	1CPC	BB	EL	Ok	GE
Best operations for SGA			*DS*	*1CPC*	*BB*	*EL*	*Ok*	*GE*
SSGA50	98	Rastrigin	RO/SRS	2CP	BB	EL	Ok	GE
	38	Degree	DS	1CP	BB	EL	Ok	GE
	58	Peaks	SRS	1CPC	BB	EL	Ok	GE
Best operations for SSGA50			*RO*	*2CP*	*BB*	*EL*	*Ok*	*GE*
SSGA80	62	Rastrigin	RO	1CP/1CPC	BB	EL	Ok	GE
	25	Degree	RO	1CP/2CP	BB	EL	Ok	GE
	37	Peaks	DS	1CPC	BB	EL	Ok	GE
Best operations for SSGA80			*RO*	*1CPC*	*BB*	*EL*	*Ok*	*GE*
RGA	491	Rastrigin	RO	1CP	BB	EL	Ok	GE
	191	Degree	RO	1CP	BB	EL	Ok	GE
	291	Peaks	RO	1CP	BB	EL	Ok	GE
Best operations for RGA			*RO*	*1CP*	*BB*	*EL*	*Ok*	*GE*

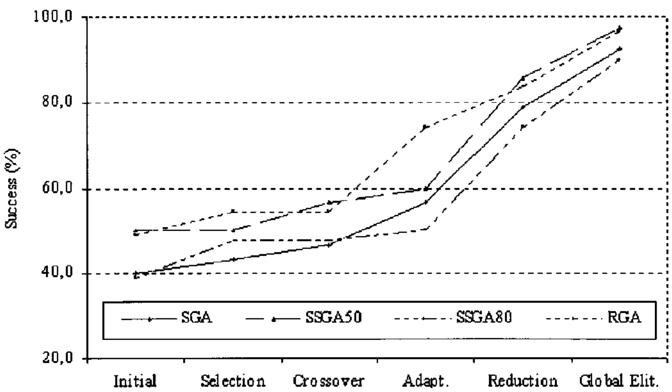

Fig. 2. Evolution of SGA, SSGA and RGA—rastrigin function.

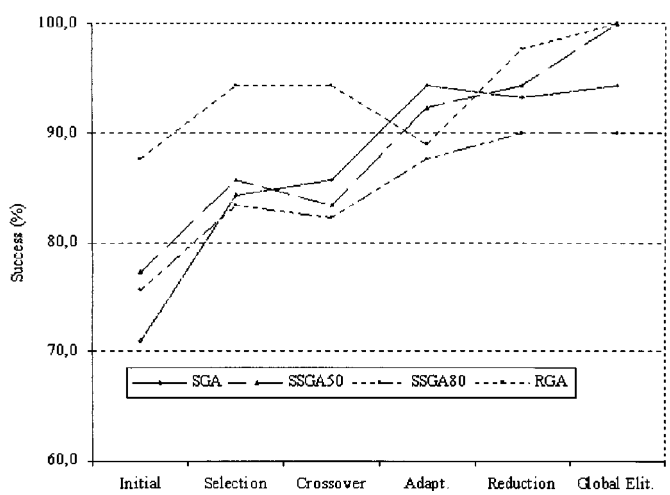

Fig. 3. Evolution of SGA, SSGA and RGA—peaks function.

addition, the crossover operations with two cut points (2CP) and one cut point (1CPC) by chromosome give better results for the SSGA50 and SSGA80 respectively.

The evolution of all GAs for the Rastrigin, Peaks and Degree functions are plotted in Figs. 2–4. Clearly, we can see that the behavior of the curves shows an increasing success when better operations are used in place of the initial ones. The initial point of these graphs represents the mean value obtained for the success number with three groups of thirty executions (3×30) and all GAs configured with the parameter values of Table III. The

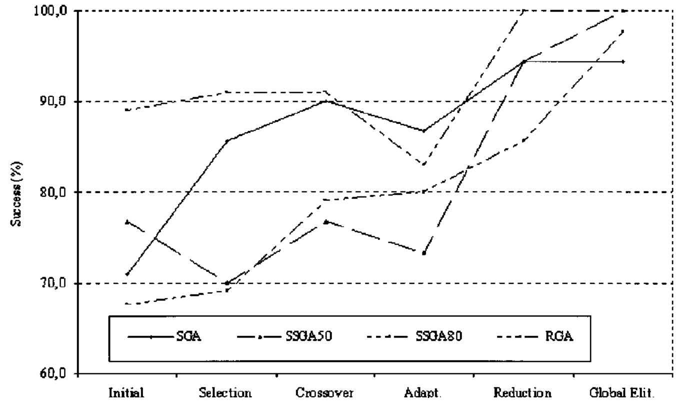

Fig. 4. Evolution of SGA, SSGA and RGA—degree function.

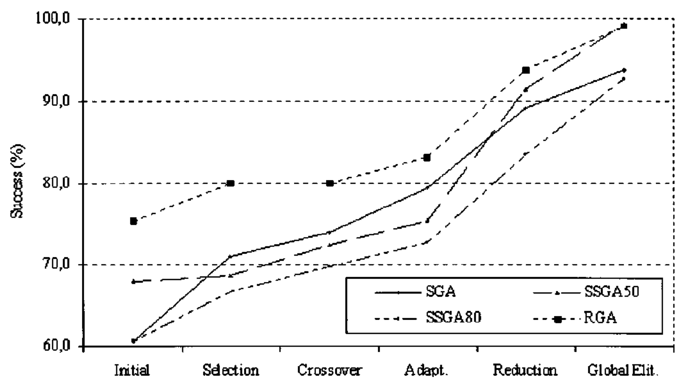

Fig. 5. Evolution of SGA, SSGA and RGA.

next point for each GA was obtained with the same number of evaluations, but with its better selection type. The other points were obtained in a similar way.

In Fig. 5 a summary of the results obtained for the three analytical test functions is given. Each point in this graph was obtained taking the sum of the number of success for each test function given in Figs. 2–4 and dividing it by three. It is clear that the three final operations, adaptability of pm and pc, interval reduction and global elitism are very important to improve all GAs.

Optimization in Electromagnetics: The TEAM Benchmark Problem 22 was chosen to show the application of the best GAs found in the previous study. This problem consists of determining the optimum design parameters of a superconducting magnetic energy storage device (SMES) [4], as given in Fig. 6. The device is composed of two axisymmetric concentric coils, with current densities J_1 and J_2, which are in opposite direction. The aim is to minimize an objective function with two terms: *i)* the first one considers the stray field; *ii)* the second one takes into account the stored energy related to a prescribed value. The constraint conditions are the bounds in the design variables and the quench physical condition that guarantees superconductivity. This problem was solved considering three design variables, continuous case (see Table VI).

Mathematically, the optimization problem can be stated as:

min

$$OF = \frac{B_{stray}^2}{B_{normal}^2} + \frac{|Energy - E_{ref}|}{E_{ref}} \tag{1}$$

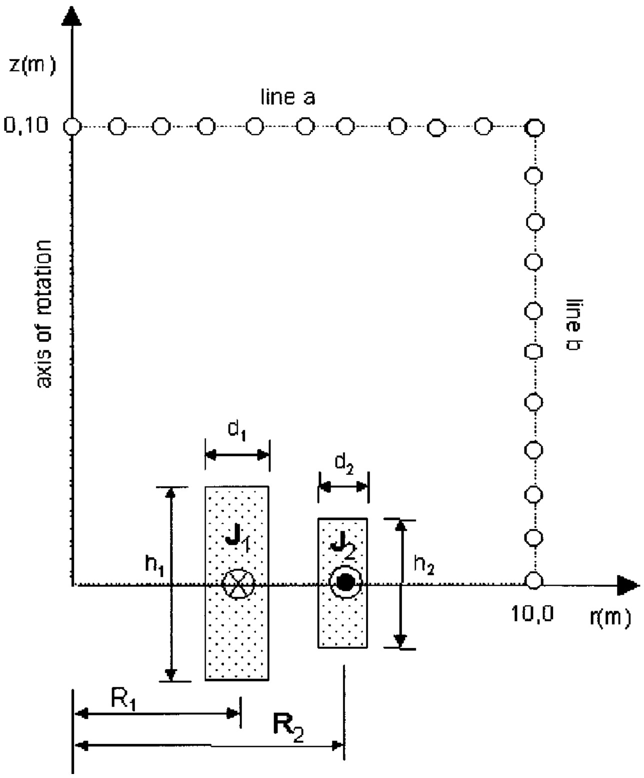

Fig. 6. SMES basic geometry.

TABLE VI
GEOMETRICAL CONSTRAINTS AND FIXED PARAMETERS

	R_1	R_2	$h_1/2$	$h_2/2$	d_1	d_2	J_1	J_2
	m	m	m	m	m	m	MA/m²	A/mm²
Min	-	2.6	-	0.204	-	0.1	-	-
Max	-	3.4	-	1.1	-	0.4	-	-
fixed	2.0	-	0.8	-	0.27	-	22.5	- 22.5

subject to

$$|J| = (-6.4|B_{\max}| + 54.0) \quad \mathrm{A/mm}^2 \tag{2}$$

where $B_{normal} = 3.0 * 10^{-3}(T)$ and $E_{ref} = 180 * 10^6(J)$. The two terms in the objective function OF account for the aims given in *i)* and *ii)*, respectively. The constraint, (2), is used to assure superconductivity. The stray field B_{stray} is evaluated along 22 points, eleven points on each line "a" and "b." The stray field is evaluated with the equation

$$B_{stray}^2 = \frac{\sum_{i=1}^{22} |B_{stray_i}|^2}{22}. \tag{3}$$

The best solution—BS for this problem is known; see the last line in Table VII. In our simulations, this best solution gives a $B_{\max} = 4.73$, which do not satisfy the quench condition given in (2) for a fixed current density equal to 22.5 A/mm^2. So, to compare ours results we simulated this problem without this constraint. The fitness function based on the above optimization problem, without the quench condition was stated as:

$$\max FF = \frac{1}{f_1 + f_2}, \tag{4}$$

TABLE VII
RESULTS FOR THE THREE-PARAMETERS CASE

Method	Mesh	OF(1)	R2	h2/2	d2	Energy	B_{stray}^2
SGA	0.75	0.1661	3.20880	0.22120	0.38830	1.73E+08	1.13E-06
SSGA50	0.75	0.1756	3.23840	0.28095	0.30590	1.73E+08	1.22E-06
SSGA80	0.75	0.1742	3.13940	0.23600	0.38210	1.69E+08	9.96E-07
RGA	0.75	0.1679	3.25570	0.21885	0.38200	1.74E+08	1.23E-06
BS(*)	0.75	0.1806	3.0800	0.23900	0.3940	1.66E+08	9.24E-07

TABLE VIII
RESULTS WITH A MESH FACTOR EQUAL TO 8

Method	Mesh	OF(1)	Energy	B_{stray}^2
SGA	8	0.1306	1.82E+08	1.06E-06
SSGA50	8	0.1533	1.84E+08	1.19E-06
SSGA80	8	0.1074	1.80E+08	9.43E-07
RGA	8	0.1520	1.84E+08	1.15E-06
BS	8	0.1148	1.77E+08	8.77E-07
BS	4order	0.088	1.80E+08	7.88E-07

in which f_1 and f_2 are the first and second term of (1) respectively.

The results in Table VII were obtained with the best GAs, see Table V, coupled with a finite element code, which use a mesh of triangular elements of first order. The mesh factor used, Mesh $= 0.75$ (about 1000 nodes and 1840 elements), gives a coarse mesh. This choice was made to limit the simulation time.

The results in Table VIII are for the same parameters R2, h2/2 and d2, as given in Table VII, simulated with a very fine mesh (around 8800 nodes and 17 000 elements). In addition, the last line presents the BS simulated with another FEM code with triangular elements of forth order. We can see that these results are very close.

IV. CONCLUSION

This paper shows how it is possible to improve the SGA, SSGA and RGA methods. The results reveal that the dynamic adaptability of crossover and mutation probabilities, the reduction space and the global elitism enhance the performance of all GAs. The improved GAs applied to the SMES problem show that their application in electromagnetic problems are reliable.

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