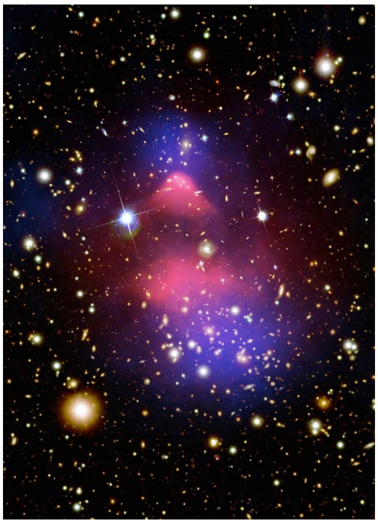
ALBERT TARANTOLA

1949-2009

Klaus Mosegaard, Niels Bohr Institute, University of Copenhagen

Skience 2020

The legacy of general relativity



- Degree in theoretical astrophysics 1976 on the evolution of galactic clusters
- General relativity

Inverse theory = The theory of measurement

- Around 1980 Albert turned his interests towards inverse problems, a topic of importance and profound philosophical implications.

Inverse Theory in the late 1970s

- **Regularization methods** (Levenberg, 1944; Tikhonov, 1963; Marquardt, 1970)

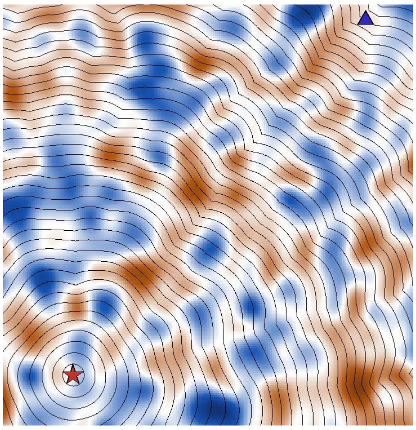
$$\mathbf{m}_{est} = (\mathbf{G}^T\mathbf{G} + \varepsilon^2\mathbf{I})^{-1}\mathbf{G}^T\mathbf{d}_{\mathbf{obs}}$$

- **The method of Backus and Gilbert** (Backus and Gilbert, 1967, 1970)

$$\mathbf{m}_{BG} = \mathbf{H}\mathbf{d}_{\mathbf{obs}}$$

where $\mathbf{H}$ jointly minimizes model variance and resolution.

Many inverse problems in geoscience are (sometimes highly) non-linear!



Wavefront-, ray or streamline locations depend on the properties of the medium

- Seismology: rays paths depend on wave propagation velocities
- Flow problems: Streamlines depend on permeabilities

Courtesy of Sambridge et al. 2011

Albert and the grand scheme

- An ambitious vision for data integration
- A passionate dedication to the probabilistic view
- An objectivist's view: rationality, consistency, extension of logic.

Bayesian Inversion

$$f(\mathbf{m}|\mathbf{d}) \equiv \frac{f(\mathbf{d}|\mathbf{m})f(\mathbf{m})}{f(\mathbf{d})}$$

posterior

likelihood

prior

Bayesian Inversion – an inconsistent method!

$$f(\mathbf{m}|\mathbf{d}) \equiv \frac{f(\mathbf{d}|\mathbf{m})f(\mathbf{m})}{f(\mathbf{d})}$$

posterior

likelihood

prior

Information Theory Formulation

Inverse Problems = Quest for Information (1982)

Forward density

prior

posterior

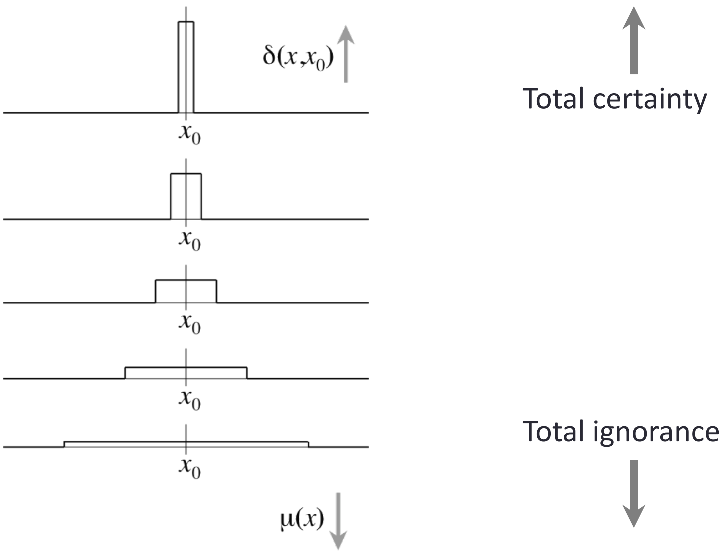
$$\sigma(\mathbf{d},\mathbf{m}) = \frac{\rho(\mathbf{d},\mathbf{m})\theta(\mathbf{d},\mathbf{m})}{\mu(\mathbf{d},\mathbf{m})}$$

Null Information Density

Two Formulations Compared

	Classical Bayes	Tarantola-Valette
Spaces	Separate $\mathcal{D}$ and $\mathcal{M}$	Joint $\mathcal{D}\times\mathcal{M}$
Information	(Realizations of) $\mathbf{d}$ and $\mathbf{m}$	Distributions over $\mathcal{D}\times\mathcal{M}$
Forward Relation	$\mathbf{d} = g(\mathbf{m})$	$\theta(\mathbf{d},\mathbf{m})$
Conditionals	YES	NO

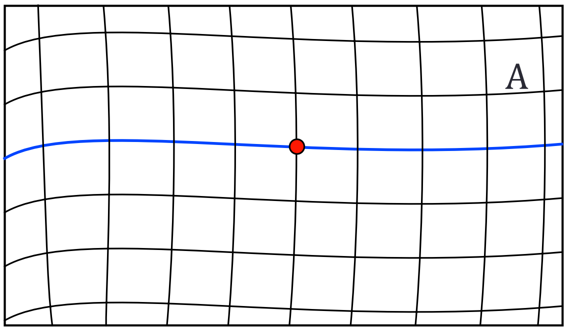
From ignorance to certainty..



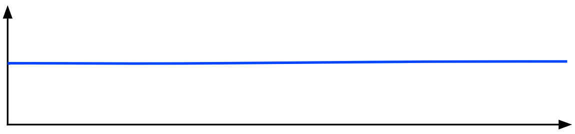
The Borel Paradox

The Borel Paradox

- Near-Cartesian reference frame
- Equal volumes have equal probabilities

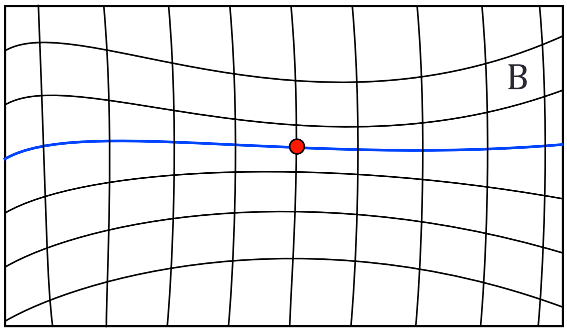


- Conditional probability density is constant



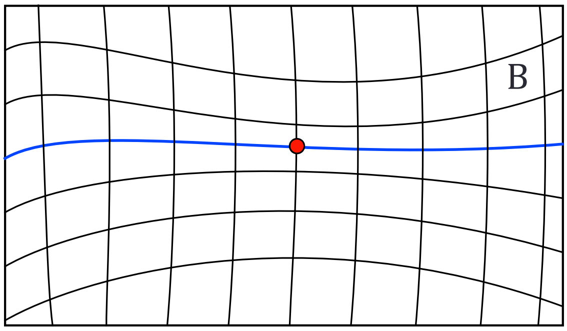
The Borel Paradox

- $f(x, y)\ dx\ dy$ is the probability between $x + dx$ and $y + dy$
- $f(x, y)$ must be smaller in B to ensure that "equal volumes have equal probabilities"

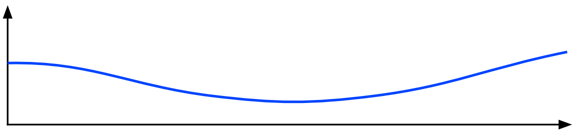


The Borel Paradox

- $f(x, y)\, dx\, dy$ is the probability between $x + dx$ and $y + dy$
- $f(x, y)$ must be smaller in B to ensure that "equal volumes have equal probabilities"



- Conditional probability density is **not** constant



Albert's conclusion on conditional probability densities

- Conditional probability densities are **inconsistent**, because different analysts may arrive at different (conflicting) results.

Albert's conclusion on conditional probability densities

- Conditional probability densities are **inconsistent**, because different analysts may arrive at different (conflicting) results.

Borel's paradox disappears if $f(\mathbf{x})$ is replaced with

$$g(\mathbf{x}) = f(\mathbf{x})/\mu(\mathbf{x})$$

where $\mu(\mathbf{x})$ is the *null information* pdf (a nonzero *volume density*)

Generalized nonlinear inverse problems solved using the least-squares criterion (1982)

Albert pointed out that

$$\max_{\mathbf{m}} \ \sigma(\mathbf{m})$$

is not invariant under **re-parametrization**

An **invariant** result is obtained by maximizing

$$\max_{\mathbf{m}} \ \frac{\sigma(\mathbf{m})}{\mu(\mathbf{m})}$$

Voices of the West Coast

George Backus

Freeman Gilbert

Voices of the West Coast

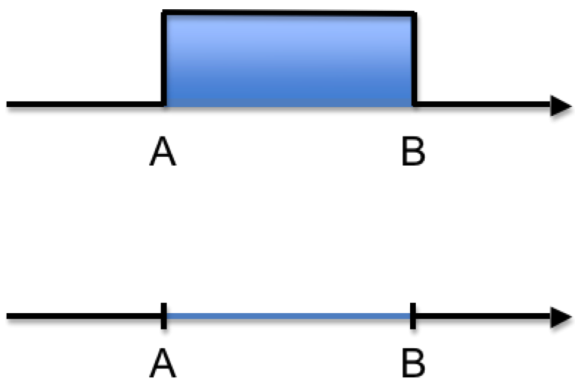
George Backus

Freeman Gilbert

Albert:

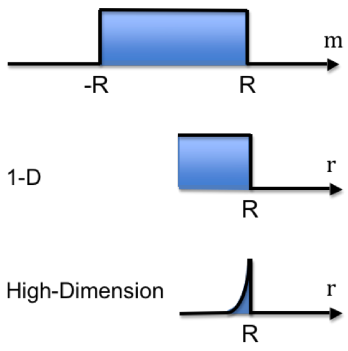
> In his paper "Inference from inadequate and inaccurate data," Backus (1970a,b,c) made the first effort to formalize the probabilistic approach to inverse problems. Although indigestible, the paper is historically important. The essentials of the theory are as follows.

Which ignorance is most profound?



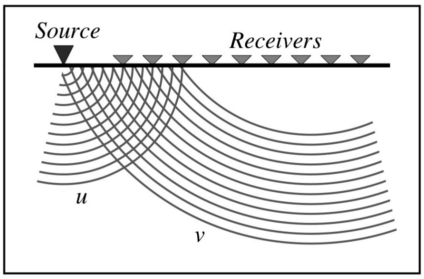
The ”Backus Effect”

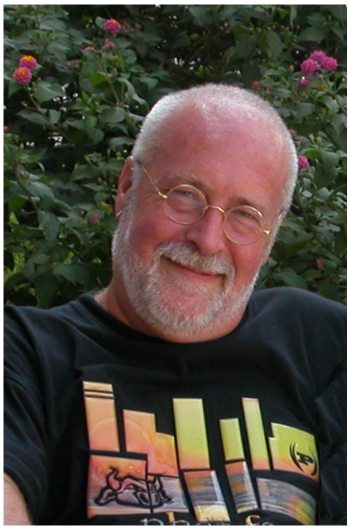
- In 1988 **George Backus** published a paper where he showed that turning a hard constraint into a soft (probability) constraint could give “unexpected” results.
- A homogeneous prior in a ball of radius R will, in high dimensions, assign almost all its probability close to the shell of radius R.



The seminal papers on waveform inversion (Tarantola, 1984, 1986 and 1988)

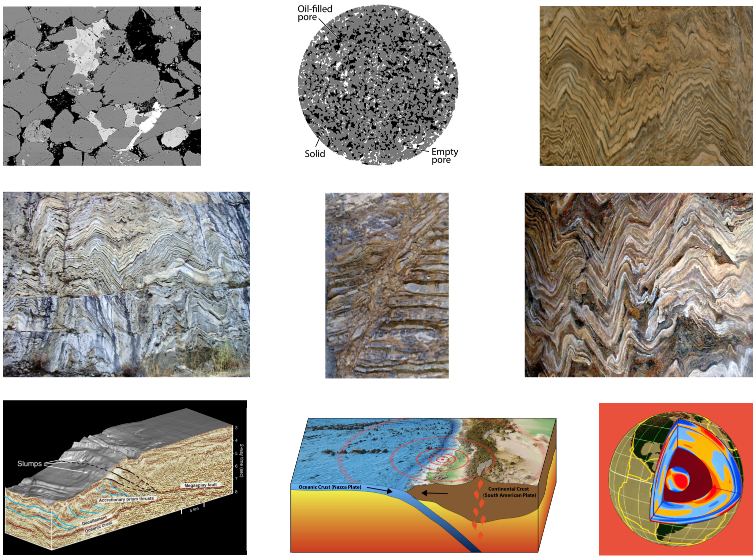
- One of the most difficult inverse problems in geoscience
- The adjoint method





WHY IS COMPLEXITY IMPORTANT?

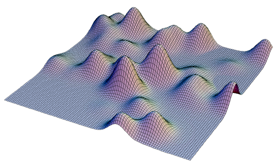
Geological Complexity on Multiple Scales



Classical Probabilistic Inversion

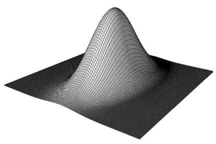
Data information:

Likelihood function $L(\mathbf{m})$



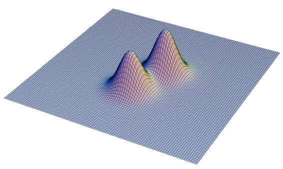
A priori information

Prior distribution $\rho(\mathbf{m})$



A posteriori probability

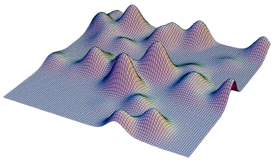
$$\sigma(\mathbf{m}) = c\, L(\mathbf{m})\rho(\mathbf{m})$$



Probabilistic Inversion with Complex Priors

Data information:

Likelihood function $L(\mathbf{m})$

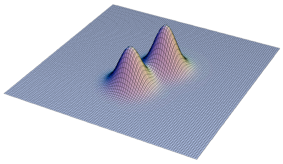


A priori information

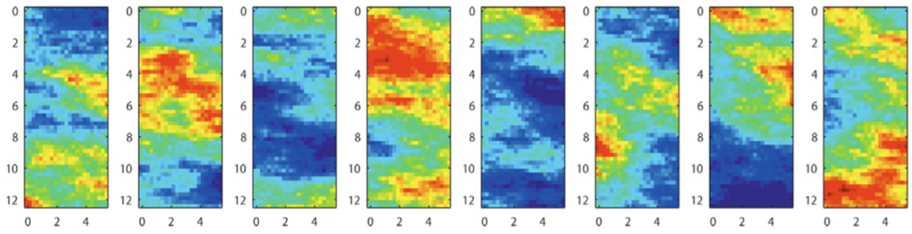
Prior distribution $\rho(\mathbf{m})$

A posteriori probability

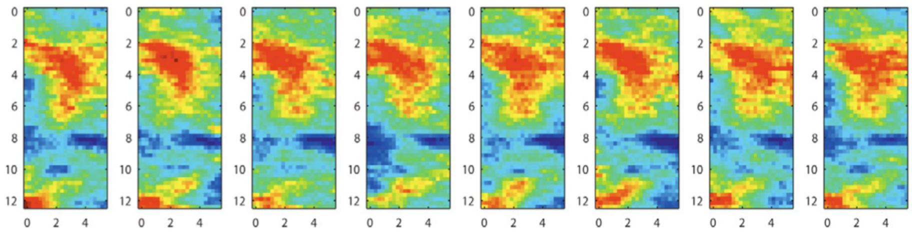
$$\sigma(\mathbf{m}) = c\, L(\mathbf{m})\rho(\mathbf{m})$$



A priori acceptable geological structure



Geological structure, also **consistent with** data:



Hansen et al. 2008

The future of the grand scheme

- Is there a pragmatic way of integrating geostatistical and geophysical data?
- The vision of a *Bayesian data base*

A Change of Mode..

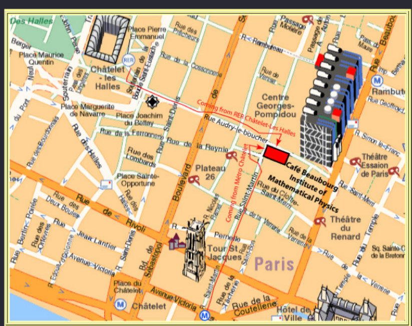
Responsibilities:

Jan. 1987 - Dec. 1992:	Chairman, Committee on Mathematical Geophysics, Internationa
Jan. 1990 - Dec. 1992:	Editor, Journal of Geophysical Research, American Geophysical U
Jun. 1991 - Jun. 1996:	Director of Graduate Studies at the Institut de Physique du Globe
Jan. 1993 - Dec. 1993:	Member of the Editorial Board, Inverse Problems, Institute of Phy
Jan. 1994 - Jun. 1996:	Chairman of the Department of Geophysical Observatories (inclu
Jan. 1995 - Jun. 2000:	Member of the Editorial Board, Journal of Volcanology.
Jun. 2000 - Nov. 2008:	Time has come to be selfish, and take some time for myself.

Headquaters at Parisian Cafés

Café Costes

Cefé Beaubourg



Café Beaubourg Institute of Mathematical Physics

Café Beaubourg Institute of Mathematical Physics

The *Café Beaubourg Institute of Mathematical Physics* promotes the advancement of our understanding of the Physical World.

Intellectual rigor, freedom of thought, and humanity are among the most respected values of the Institute.

By "Physical World" we understand the current subjects of Physics, but also human history, psychology, economy, sociology and geopolitics. We like Quantum Mechanics, but adopt a "wait and see" attitude. We like Big-Bang theory of the universe, but believe that there are infinitely many more layers in the physical Universe that our current models integrate. We perceive Mathematics as an (unavoidable) extension of the human natural language.

Better than in the "office" ambiance of present-day university laboratories, we prefer to engage scientific discussions in the warm atmosphere of a Parisian Café, our main headquarters being at Café Beaubourg.

The members of the Institute are co-opted.

Members:

Bartolomé Coll (founding member), also working at *Observatoire de Paris*.
Albert Tarantola (founding member), also working at *Institut de Physique du Globe de Paris*.
Klaus Mosegaard, also working at the *Niels Bohr Institute*, Copenhagen.
Juan-Antonio Morales, also working at the *Universitat de València*.
Argyris Nicolaidis, also working at the *Aristotle University* of Thessaloniki.
Loring Tu, also working at Tufts University, Boston.
Joan Ferrando, also working at the *Universitat de València*.
Chus Martín, also working at the *Universidad de Salamanca*.
Fernando San José, also working at the *Universidad Politécnica de Madrid*.
José Maria Pozo, also working at the *Observatoire de Paris* and at the *Institut de Physique du Globe de Paris*

Honorary Members:

Nuria Morales Ruiz, also working at ...

The electronic adress of all members is formed as **firstname.secondname@cb-institute.org**

Click here for a map indicating the location of the Institute.

Café Beaubourg Institute of Mathematical Physics

The *Café Beaubourg Institute of Mathematical Physics* promotes the advancement of our understanding of the Physical World.

Intellectual rigor, freedom of thought, and humanity are among the most respected values of the Institute.

By "Physical World" we understand the current subjects of Physics, but also human history, psychology, economy, sociology and geopolitics. We like Quantum Mechanics, but adopt a "wait and see" attitude. We like Big-Bang theory of the universe, but believe that there are infinitely many more layers in the physical Universe that our current models integrate. We perceive Mathematics as an (unavoidable) extension of the human natural language.

Better than in the "office" ambiance of present-day university laboratories, we prefer to engage scientific discussions in the warm atmosphere of a Parisian Café, our main headquarters being at Café Beaubourg.

The members of the Institute are co-opted.

Members:

Bartolomé Coll (founding member), also working at *Observatoire de Paris*.
Albert Tarantola (founding member), also working at *Institut de Physique du Globe de Paris*.
Klaus Mosegaard, also working at the *Niels Bohr Institute*, Copenhagen.
Juan-Antonio Morales, also working at the *Universitat de València*.
Argyris Nicolaidis, also working at the *Aristotle University* of Thessaloniki.
Loring Tu, also working at Tufts University, Boston.
Joan Ferrando, also working at the *Universitat de València*.
Chus Martín, also working at the *Universidad de Salamanca*.
Fernando San José, also working at the *Universidad Politécnica de Madrid*.
José Maria Pozo, also working at the *Observatoire de Paris* and at the *Institut de Physique du Globe de Paris*

Honorary Members:

Nuria Morales Ruiz, also working at ...


Video Chat with Albert Tarantola
Albert Tarantola
has left the Video Chat.