Inverse Problem Theory

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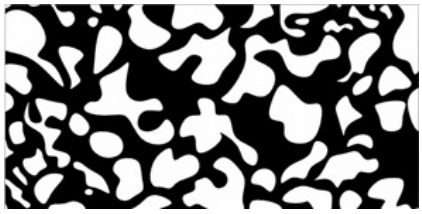
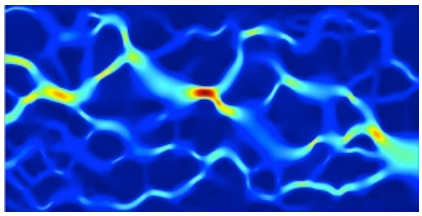
Complex geological priors from geostatistics

WHY IS COMPLEXITY IMPORTANT?

Connectivity on large scales depends on connectivity on small scales

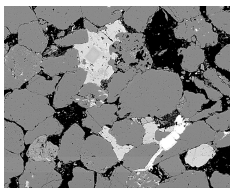


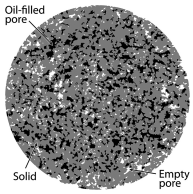
Earth structure

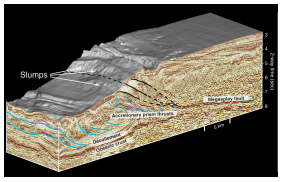


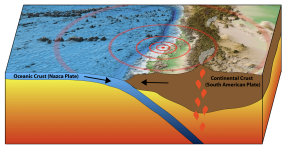
Flow pattern

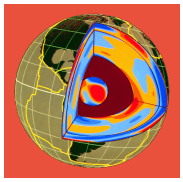
Geological Complexity on Multiple Scales





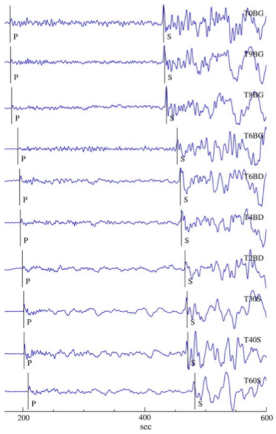


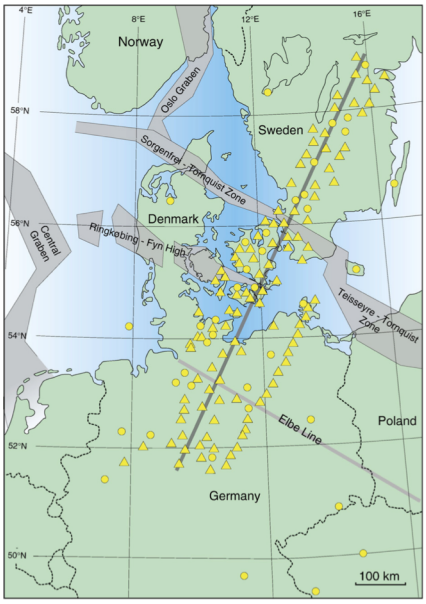




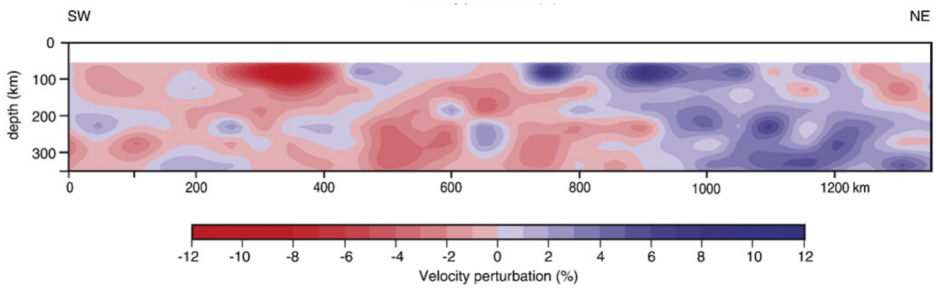
A classical example: Seismic tomography

- Seismic signals from distant earthquakes, recorded along a profile
- Time delays compared to a reference Earth model





Seismic tomography: A least-squares result



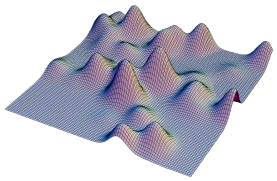
- Deviation from a reference Earth model
- Smooth models (least-squares/Gaussian)

How can we quantify geological information and use it to obtain high-resolution models from geophysical data inversion?

Classical Least-Squares Inversion

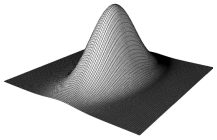
Likelihood Function:

$$L(\mathbf{m}) = C \cdot \exp\left(-\frac{\left\|\mathbf{d}_{obs} - f(\mathbf{m})\right\|^2}{2\sigma_d^2}\right)$$



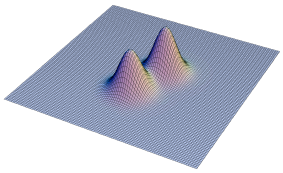
A priori distribution:

Gaussian distribution

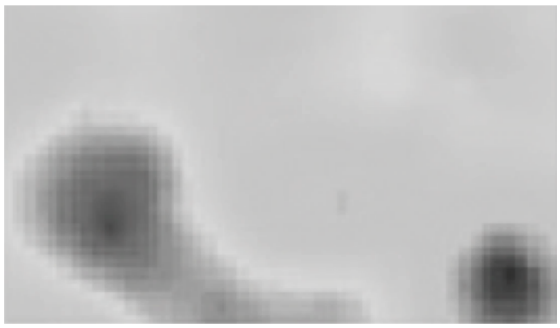


A posteriori distribution

$$\sigma(\mathbf{m}) = C \cdot L(\mathbf{m})\rho(\mathbf{m})$$



A Classical Least-Squares Solution

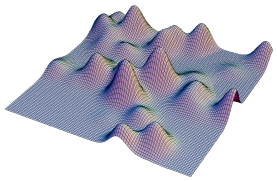


Least-squares solutions are usually too smooth to be geologically acceptable.

Probabilistic Inversion

Likelihood Function:

$$L(\mathbf{m}) = C \cdot \exp\left(-\frac{\left\|\mathbf{d}_{obs} - f(\mathbf{m})\right\|^2}{2\sigma_d^2}\right)$$

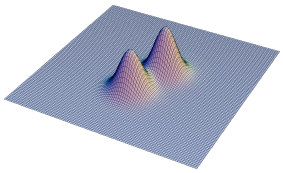


A priori distribution:

Prior information defined by geological prototype examples

A posteriori distribution

$$\sigma(\mathbf{m}) = C \cdot L(\mathbf{m})\rho(\mathbf{m})$$



Prior information defined by a geological prototype example

"The subsurface model **m** is statistically similar to a training image"

The Geostatistical Prior

Training Image

A statistically similar image

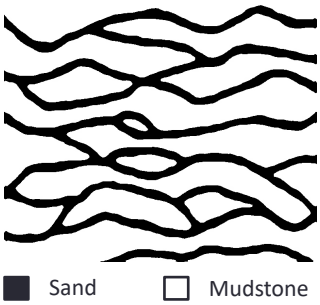
"The subsurface model **m** is statistically similar to a training image"

Examples of geo-information: Braided rivers

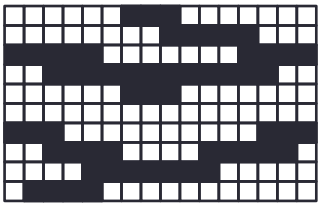
Rakaia River, New Zealand. (Google Earth)

Congo River at the border of Congo and Democratic Republic of Congo. (Google Earth)

Examples of geo-information: Braided rivers

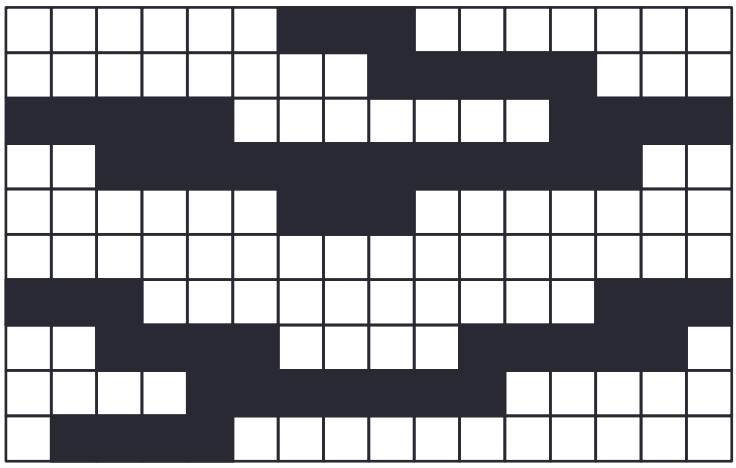


A simple model of a braided river (Strebelle, 2002)



A close-up of part of the pixeled model

Pattern statistics from a geological model



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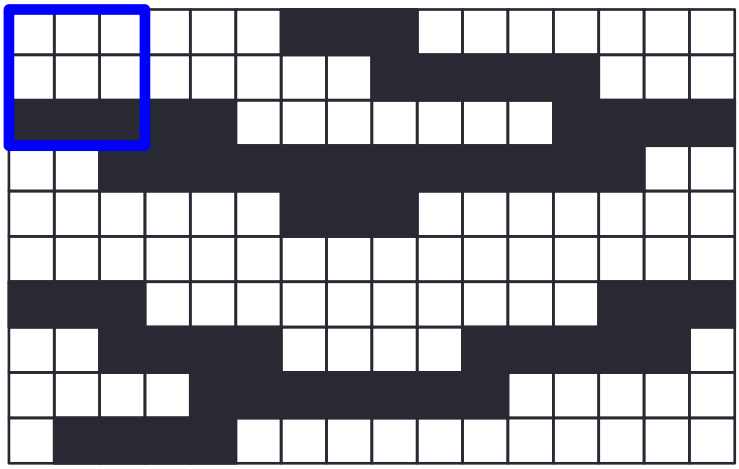
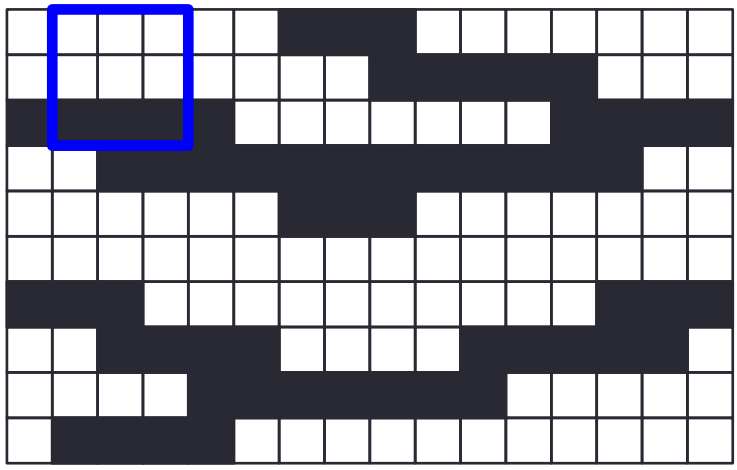
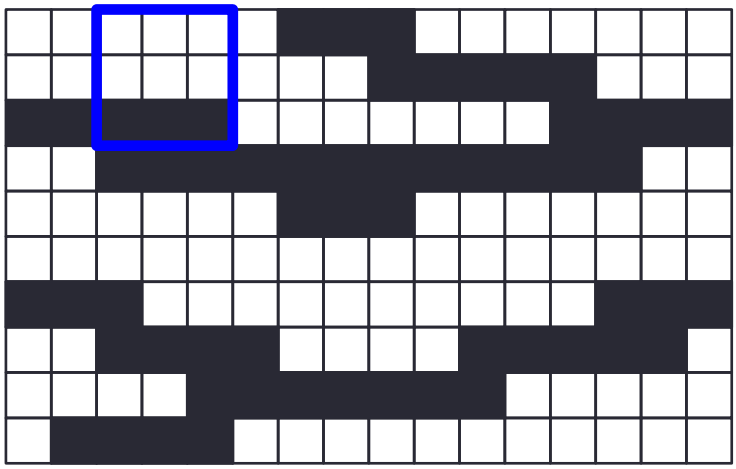


Pattern statistics from a geological model



Pattern statistics from a geological model



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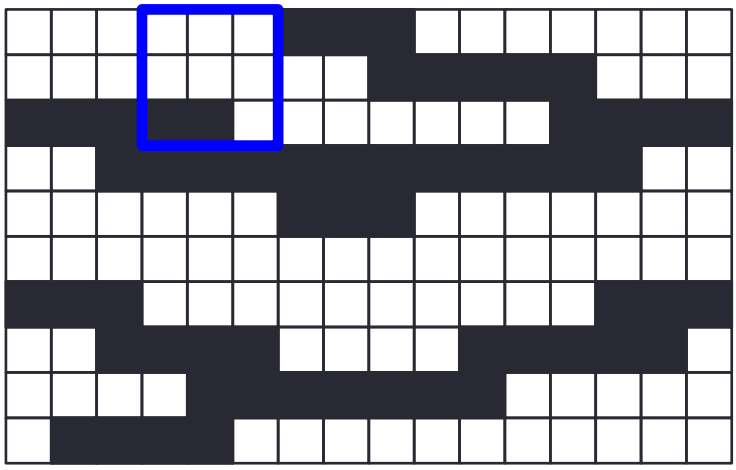
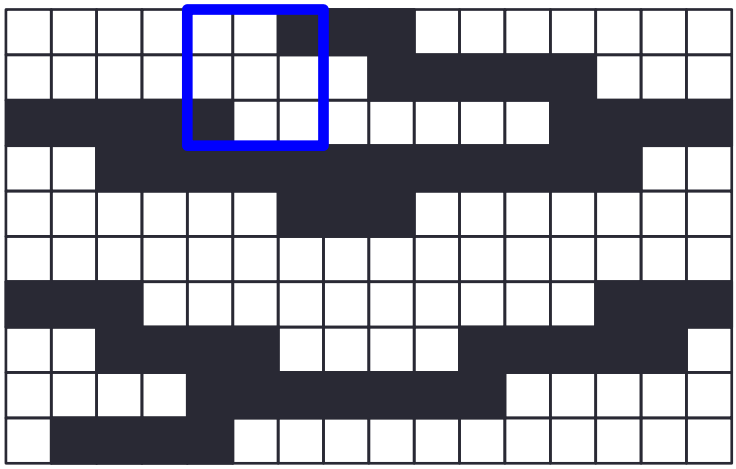
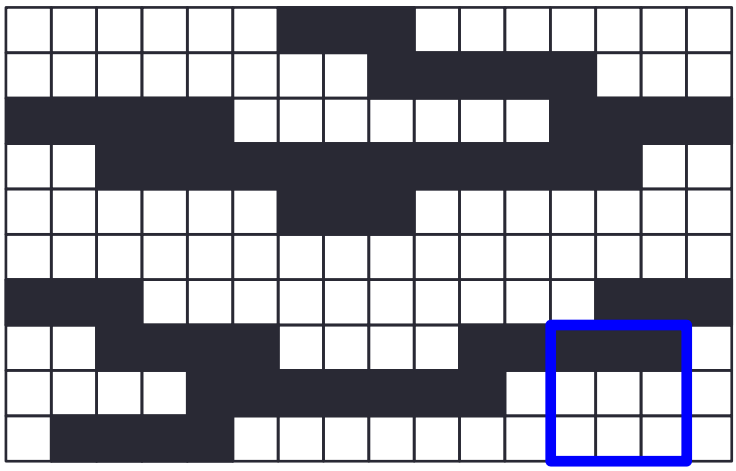


Pattern statistics from a geological model



Pattern statistics from a geological model



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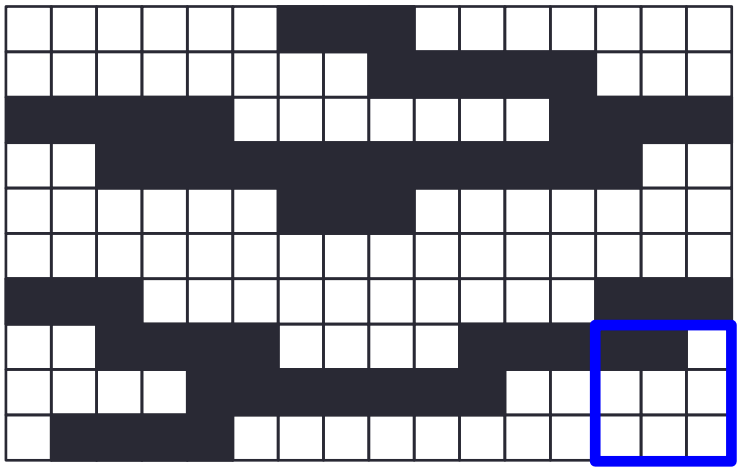
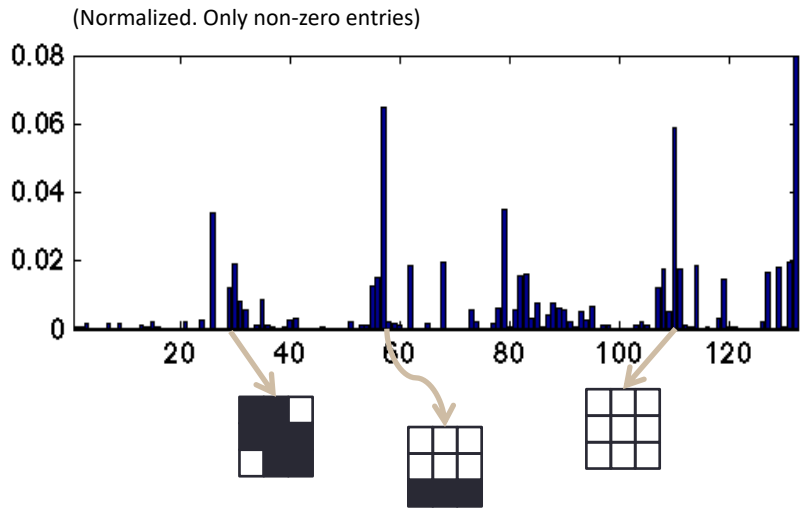
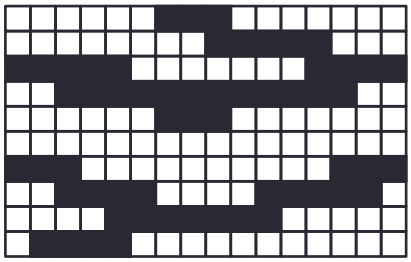
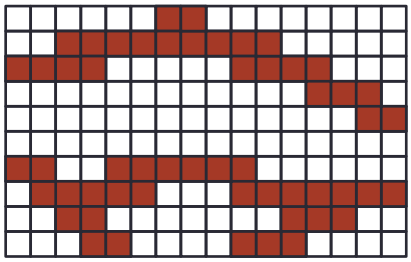


The frequency distribution



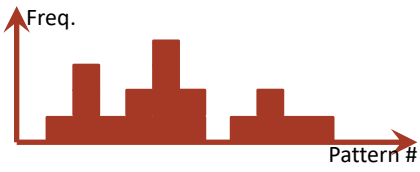
Computing the probability of an image





Reference Model

What is the probability that this model is a realization of the same random process?



Combining flow data and the braided river model into a solution to the inverse problem

$$\sigma(\mathbf{m}) = \rho_R(\mathbf{m})\, \mathrm{L}(\mathbf{m})$$

where $\mathbf{m}$ is the Earth model, L is a likelihood function, and $\rho_R(\mathbf{m})$ is the prior probability of $\mathbf{m}$, given the reference model.