



# Project Report

*CSC320 Summer 2018*

*Project - SAT Based Sudoku Solving*

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## Introduction

In this project, we will write two programs with a given SAT solver to generate a solution for a unsolved Sudoku puzzles.

## Method

We decided to implement both programs using Java language. The first program, sud2sat read in a Sudoku puzzle and translate it to a CNF formula which allow us to input it into a miniSAT SAT solver. The miniSAT solver will output a solution file which the the second program, sat2sud takes in, decode and output the solved puzzle.

## Experiment

In this experiment we test our program using 50 unsolved Sudoku puzzle, check the correctness and record time required to solve for each puzzle.

## Result

50 unsolved Sudoku puzzle (input). The input is formatted as the following, each line contains 81 integers indicating one Sudoku puzzle.

```
003020600900305001001806400008102900700000008006708200002609500800203009005010300
200080300060070084030500209000105408000000000402706000301007040720040060004010003
000000907000420180000705026100904000050000040000507009920108000034059000507000000
030050040008010500460000012070502080000603000040109030250000098001020600080060020
020810740700003100090002805009040087400208003160030200302700060005600008076051090
100920000524010000000000070050008102000000000402700090060000000000030945000071006
0430802506000000000000001094900004070000608000010200003820500000000000005034090710
480006902002008001900370060840010200003704100001060049020085007700900600609200018
000900002050123400030000160908000000070000090000000205091000050007439020400007000
001900003900700160030005007050000009004302600200000070600100030042007006500006800
000125400008400000420800000030000095060902010510000060000003049000007200001298000
```

062340750100005600570000040000094800400000006005830000030000091006400007059083260  
300000000005009000200504000020000700160000058704310600000890100000067080000005437  
630000000000500008005674000000020000003401020000000345000007004080300902947100080  
000020040008035000000070602031046970200000000000501203049000730000000010800004000  
361025900080960010400000057008000471000603000259000800740000005020018060005470329  
050807020600010090702540006070020301504000908103080070900076205060090003080103040  
0800050000000003457000070809060400903007010500408007020901020000842300000000100080  
003502900000040000106000305900251008070408030800763001308000104000020000005104800  
000000000009805100051907420290401065000000000140508093026709580005103600000000000  
020030090000907000900208005004806500607000208003102900800605007000309000030020050  
005000006070009020000500107804150000000803000000092805907006000030400010200000600  
040000050001943600009000300600050002103000506800020007005000200002436700030000040  
004000000000030002390700080400009001209801307600200008010008053900040000000000800  
360020089000361000000000000803000602400603007607000108000000000000418000970030014  
500400060009000800640020000000001008208000501700500000000090084003000600060003002  
007256400400000005010030060000508000008060200000107000030070090200000004006312700  
000000000079050180800000007007306800450708096003502700700000005016030420000000000  
030000080009000500007509200700105008020090030900402001004207100002000800070000090  
200170603050000100000006079000040700000801000009050000310400000005000060906037002  
000000080800701040040020030374000900000030000005000321010060050050802006080000000  
000000085000210009960080100500800016000000000890006007009070052300054000480000000  
608070502050608070002000300500090006040302050800050003005000200010704090409060701  
050010040107000602000905000208030501040070020901080406000401000304000709020060010  
053000790009753400100000002090080010000907000080030070500000003007641200061000940  
006080300049070250000405000600317004007000800100826009000702000075040190003090600  
005080700700204005320000084060105040008000500070803010450000091600508007003010600  
000900800128006400070800060800430007500000009600079008090004010003600284001007000  
000080000270000054095000810009806400020403060006905100017000620460000038000090000  
000602000400050001085010620038206710000000000019407350026040530900020007000809000

000900002050123400030000160908000000070000090000000205091000050007439020400007000  
 380000000000400785009020300060090000800302009000040070001070500495006000000000092  
 000158000002060800030000040027030510000000000046080790050000080004070100000325000  
 010500200900001000002008030500030007008000500600080004040100700000700006003004050  
 080000040000469000400000007005904600070608030008502100900000005000781000060000010  
 904200007010000000000706500000800090020904060040002000001607000000000030300005702  
 000700800006000031040002000024070000010030080000060290000800070860000500002006000  
 001007090590080001030000080000005800050060020004100000080000030100020079020700400  
 000003017015009008060000000100007000009000200000500004000000020500600340340200000  
 300200000000107000706030500070009080900020004010800050009040301000702000000008006

50 solved Sudoku puzzle with their solving time:

Solution	CPU Time
4 8 3 9 2 1 6 5 7 9 6 7 3 4 5 8 2 1 2 5 1 8 7 6 4 9 3  5 4 8 1 3 2 9 7 6 7 2 9 5 6 4 1 3 8 1 3 6 7 9 8 2 4 5  3 7 2 6 8 9 5 1 4 8 1 4 2 5 3 7 6 9 6 9 5 4 1 7 3 8 2	0s
2 4 5 9 8 1 3 7 6 1 6 9 2 7 3 5 8 4 8 3 7 5 6 4 2 1 9  9 7 6 1 2 5 4 3 8 5 1 3 4 9 8 6 2 7 4 8 2 7 3 6 9 5 1  3 9 1 6 5 7 8 4 2 7 2 8 3 4 9 1 6 5 6 5 4 8 1 2 7 9 3	0.004114 s
4 6 2 8 3 1 9 5 7 7 9 5 4 2 6 1 8 3 3 8 1 7 9 5 4 2 6  1 7 3 9 8 4 2 6 5 6 5 9 3 1 2 7 4 8 2 4 8 5 6 7 3 1 9  9 2 6 1 7 8 5 3 4 8 3 4 2 5 9 6 7 1 5 1 7 6 4 3 8 9 2	0.001328 s
1 3 7 2 5 6 8 4 9	0.005948 s

928 314 567 465 897 312  673 542 981 819 673 254 542 189 736  256 731 498 391 428 675 784 965 123	
523 816 749 784 593 126 691 472 835  239 145 687 457 268 913 168 937 254  342 789 561 915 624 378 876 351 492	0.003188 s
176 923 584 524 817 639 893 654 271  957 348 162 638 192 457 412 765 398  265 489 713 781 236 945 349 571 826	0.00557 s
143 986 257 679 425 381 285 731 694  962 354 178 357 618 942 418 279 563  821 567 439 796 143 825 534 892 716	0.005825 s
487 156 932 362 498 751 915 372 864  846 519 273 593 724 186 271 863 549  124 685 397 738 941 625 659 237 418	0.001164 s
814 976 532 659 123 478 732 854 169  948 265 317 275 341 896 163 798 245	0.005632 s

391 682 754 587 439 621 426 517 983	
761 928 453 925 743 168 438 615 927  357 461 289 894 372 615 216 589 374  689 154 732 142 837 596 573 296 841	0.002033 s
976 125 438 158 436 927 423 879 156  234 761 895 867 952 314 519 384 762  782 513 649 395 647 281 641 298 573	0.005895 s
962 341 758 148 975 623 573 268 149  321 694 875 487 512 936 695 837 412  834 726 591 216 459 387 759 183 264	0.00412 s
397 681 524 645 279 813 218 534 976  823 956 741 169 742 358 754 318 692  472 893 165 531 467 289 986 125 437	0.001802 s
639 218 457 471 539 268 825 674 139  564 823 791 793 451 826 218 796 345  352 987 614 186 345 972 947 162 583	0.004754 s
697 128 345	0.005914 s

428 635 197 315 479 682  531 246 978 286 397 451 974 581 263  149 852 736 752 963 814 863 714 529	
361 725 948 587 964 213 492 831 657  638 259 471 174 683 592 259 147 836  746 392 185 923 518 764 815 476 329	0.004056 s
359 867 124 648 312 597 712 549 836  876 924 351 524 731 968 193 685 472  931 476 285 465 298 713 287 153 649	0.001307 s
786 945 312 219 863 457 534 271 869  165 482 973 327 619 548 498 537 126  951 728 634 842 356 791 673 194 285	0.004051 s
743 512 986 589 346 217 126 987 345  934 251 768 671 498 532 852 763 491  398 675 124 417 829 653 265 134 879	0.004005 s
782 614 359 439 825 176 651 937 428  293 471 865 568 392 714 147 568 293	0.004246 s

3 2 6 7 4 9 5 8 1 9 7 5 1 8 3 6 4 2 8 1 4 2 5 6 9 3 7	
4 2 8 5 3 1 7 9 6 3 6 5 9 4 7 1 8 2 9 7 1 2 6 8 4 3 5  2 1 4 8 9 6 5 7 3 6 9 7 4 5 3 2 1 8 5 8 3 1 7 2 9 6 4  8 4 9 6 1 5 3 2 7 7 5 2 3 8 9 6 4 1 1 3 6 7 2 4 8 5 9	0.002075 s
4 2 5 7 8 1 9 3 6 1 7 8 3 6 9 5 2 4 3 6 9 5 2 4 1 8 7  8 9 4 1 5 7 3 6 2 6 5 2 8 4 3 7 9 1 7 1 3 6 9 2 8 4 5  9 8 7 2 1 6 4 5 3 5 3 6 4 7 8 2 1 9 2 4 1 9 3 5 6 7 8	0.004597 s
3 4 8 2 6 7 9 5 1 5 7 1 9 4 3 6 2 8 2 6 9 1 8 5 3 7 4  6 9 7 3 5 1 4 8 2 1 2 3 8 7 4 5 9 6 8 5 4 6 2 9 1 3 7  4 1 5 7 9 8 2 6 3 9 8 2 4 3 6 7 1 5 7 3 6 5 1 2 8 4 9	0.00408 s
1 2 4 9 8 6 7 3 5 8 6 7 4 3 5 9 1 2 3 9 5 7 1 2 6 8 4  4 7 8 3 5 9 2 6 1 2 5 9 8 6 1 3 4 7 6 3 1 2 7 4 5 9 8  7 1 2 6 9 8 4 5 3 9 8 3 5 4 7 1 2 6 5 4 6 1 2 3 8 7 9	0.005004 s
3 6 1 5 2 4 7 8 9 7 8 9 3 6 1 4 2 5 5 2 4 8 7 9 3 6 1  8 9 3 1 5 7 6 4 2 4 1 2 6 8 3 5 9 7 6 5 7 9 4 2 1 3 8  1 4 8 7 9 6 2 5 3 2 3 5 4 1 8 9 7 6 9 7 6 2 3 5 8 1 4	0.004901 s
5 8 1 4 7 9 2 6 3	0.005705 s



329 156 847 647 328 159  956 731 428 238 964 571 714 582 936  172 695 384 893 247 615 465 813 792	
387 256 419 469 781 325 512 439 867  123 548 976 758 963 241 694 127 583  835 674 192 271 895 634 946 312 758	0 s
345 871 269 279 653 184 861 429 537  197 346 852 452 718 396 683 592 741  738 264 915 516 937 428 924 185 673	0.006232 s
235 761 489 419 328 576 867 549 213  746 135 928 521 896 734 983 472 651  394 287 165 652 913 847 178 654 392	0.007798 s
298 175 643 657 394 128 134 286 579  821 649 735 573 821 496 469 753 281  312 468 957 785 912 364 946 537 812	0.005037 s
761 543 289 832 791 645 549 628 137  374 215 968 128 936 574 695 487 321	0.006078 s

417 369 852 953 872 416 286 154 793	
132 649 785 758 213 649 964 785 123  543 897 216 276 531 894 891 426 537  619 378 452 327 154 968 485 962 371	0.004562 s
698 173 542 354 628 179 172 549 368  531 897 426 946 312 857 827 456 913  765 931 284 213 784 695 489 265 731	0.005901 s
852 716 943 197 843 652 463 925 187  278 634 591 645 179 328 931 582 476  786 491 235 314 258 769 529 367 814	0.004444 s
453 218 796 629 753 481 178 496 532  796 582 314 314 967 825 285 134 679  542 879 163 937 641 258 861 325 947	0.006334 s
516 289 347 849 173 256 732 465 918  698 317 524 327 954 861 154 826 739  961 732 485 275 648 193 483 591 672	0.003352 s
945 681 723	0 s

781 234 965 326 759 184  269 175 348 138 942 576 574 863 219  457 326 891 612 598 437 893 417 652	
365 942 871 128 756 493 974 813 562  819 435 627 537 268 149 642 179 358  296 384 715 753 691 284 481 527 936	0.003976 s
134 587 296 278 169 354 695 234 817  359 816 472 821 473 569 746 925 183  917 348 625 462 751 938 583 692 741	0.001424 s
193 672 485 462 358 971 785 914 623  538 296 714 674 135 298 219 487 356  826 741 539 941 523 867 357 869 142	0.001135 s
814 976 532 659 123 478 732 854 169  948 265 317 275 341 896 163 798 245  391 682 754 587 439 621 426 517 983	0.005472 s
384 567 921 126 439 785 759 821 346  563 798 214 847 312 659 912 645 873	0.006087 s

231 974 568 495 286 137 678 153 492	
469 158 372 712 463 859 538 297 641  927 634 518 385 719 426 146 582 793  653 941 287 294 876 135 871 325 964	0.001992 s
316 549 278 987 321 645 452 678 931  594 236 817 238 417 569 671 985 324  845 162 793 129 753 486 763 894 152	0.006149 s
586 127 943 723 469 851 491 853 267  135 974 628 279 618 534 648 532 179  917 246 385 352 781 496 864 395 712	0.005894 s
954 213 687 617 548 923 832 796 541  763 851 294 128 974 365 549 362 178  281 637 459 475 129 836 396 485 712	0.001786 s
159 743 862 276 589 431 348 612 759  624 978 315 917 235 684 583 164 297  435 821 976 861 497 523 792 356 148	0.0057 s
861 357 294	0.004436 s

597 482 361 432 619 785  916 275 843 358 964 127 274 138 956  789 541 632 143 826 579 625 793 418	
294 863 517 715 429 638 863 751 492  152 947 863 479 386 251 638 512 974  986 134 725 521 678 349 347 295 186	0.008426 s
351 286 497 492 157 638 786 934 512  275 469 183 938 521 764 614 873 259  829 645 371 163 792 845 547 318 926	0.006742 s

## Analysis

The total time for solving the easy 50 Sudoku puzzle is 0.210271 and average time for each puzzle is 0.00420542s.

## Extended Tasks

To further study our project the team completed three extended tasks.

- Task 1: Solved the “hard” input provided at [magictour.free.fr/top95](http://magictour.free.fr/top95)

Solution and time required of 95 hard Sudoku puzzles:

See *top95.txt.sol* in the *output* folder for each Sudoku puzzles solution and required time.

The total time is 0.478208s, and the average time is 0.00503377s.

- Task 2: Make at least one alternation to the minimal encoding.

Originally in the minimal encoding, we have clauses ensure each number appears at most once in every row and each number appears at most once in every

column, with formula:  $\bigwedge_{i=1}^9 \bigwedge_{k=1}^9 \bigwedge_{j=1}^8 \bigwedge_{\ell=j+1}^9 (\neg x_{ijk} \vee \neg x_{i\ell k})$  and  $\bigwedge_{j=1}^9 \bigwedge_{k=1}^9 \bigwedge_{i=1}^8 \bigwedge_{\ell=i+1}^9 (\neg x_{ijk} \vee \neg x_{\ell j k})$ .

We modify these two parts into clauses with each number appears at least once

in each row, with formula  $\bigwedge_{y=1}^9 \bigwedge_{z=1}^9 \bigvee_{x=1}^9 s_{xyz}$ , and each number appears at

least once in each column, with formula  $\bigwedge_{x=1}^9 \bigwedge_{z=1}^9 \bigvee_{y=1}^9 s_{xyz}$ . See

sud2sat\_v2.java for detailed implementation. Both implementations provide correct solutions. The implementation with alternation produces cnf with less clauses count but increases running time. We take the first Sudoku puzzle in the easy50 file, clauses count before the modification is 2968, and after modification is 2217. The required time before is 0.00306s, and after is 0.00533s.

- Task 3: Use of SAT-solvers other than miniSAT.

We used Satz231 (new version) contributed by Chu-Min Li found at <http://www.cs.ubc.ca/~hoos/SATLIB/solvers.html> to solve the easy 50 Sudoku puzzles. Some modification was made to the original code by the team. All 'CLK\_TCK' were changed to CLOCKS\_PER\_SECOND due to compiled error in the original code. See `satz.c` for detailed implementation.