

# Overlapping Generations Model

## Bequests and Altruism

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# Topics

We introduce intergenerational links into the OLG model:

- ▶ parents leave bequests to their children

The main **goal** is to learn the model setup.

We study whether bequests solve the **dynamic inefficiency** problem

- ▶ The answer is no
- ▶ Bequests can only increase the capital stock

# A key result

A key result:

- ▶ when parents leave bequests, they behave as if they lived forever
- ▶ some view this as micro-foundation for models where households live forever (though that seems misguided to me)

# Bequest Motives

Why do parents leave bequests to their children?

Theoretically, there are various ways of modeling bequests:

1. **Altruism**: parents value their children's utility.
  - 1.1 **Warm glow**: parents value the bequest itself (a reduced form).
  - 1.2 **Strategic**: parents promise bequests so kids behave well.

Empirically, we don't know (a possible research question).

## OLG Model With Altruism

# Model Elements

- ▶ We study the standard endowment economy, just with different preferences.
- ▶ Demographics: Each household has  $(1+n)$  children when old.
- ▶ Endowments:  $e_1$  when young,  $e_2$  when old.
- ▶ Technology: none.
- ▶ Markets: goods, bonds

# Preferences

The household values own consumption according to

$$u(c_t^y, c_{t+1}^o)$$

The household also values the utility of the child.

Preferences are defined recursively:

$$V(t) = u(c_t^y, c_{t+1}^o) + \omega V(t+1)$$

$\omega > 0$  governs the strength of altruism.

# Household

Expanding this we find that the parent values utility of all future generations:

$$\begin{aligned} V(t) &= u(c_t^y, c_{t+1}^o) + \omega[u(c_{t+1}^y, c_{t+2}^o) + \omega V(t+2)] \\ &= u(c_t^y, c_{t+1}^o) + \omega u(c_{t+1}^y, c_{t+2}^o) \\ &\quad + \omega^2[u(c_{t+2}^y, c_{t+3}^o) + \omega V(t+3)] \end{aligned}$$

and therefore

$$V(t) = \sum_{j=0}^{\infty} \omega^j u(c_{t+j}^y, c_{t+j+1}^o) \quad (1)$$



# Household

This looks like

- ▶ the **planner's** welfare function,
- ▶ the utility function of a household who **lives forever**.

Next, we write the sequence of budget constraints to look like a single budget constraint.

## Household problem

Period budget constraints are

$$c_t^y + s_t = e_1 + b_t \quad (2)$$

$$c_{t+1}^o + (1+n)b_{t+1} = e_2 + R_{t+1}s_t \quad (3)$$

$b_{t+1}$  is the bequest left to each child by cohort  $t$ .

Present value budget constraint (set  $n = 0$  for simplicity):

$$b_t = \underbrace{c_t^y - e_1 + (c_{t+1}^o - e_2)/R_{t+1}}_{z_t} + b_{t+1}/R_{t+1} \quad (4)$$

$$= z_t + b_{t+1}/R_{t+1} \quad (5)$$

$$= z_t + (z_{t+1} + b_{t+2}/R_{t+2})/R_{t+1} \quad (6)$$

$$= z_t + \frac{z_{t+1}}{R_{t+1}} + \frac{b_{t+2}}{R_{t+1}R_{t+2}} \quad (7)$$

## Budget constraint

Successively replace the  $b_{t+j}$  with  $z_{t+j} + b_{t+j+1}/R_{t+j+1}$  to obtain

$$b_t = \sum_{j=0}^J \frac{z_{t+j}}{D_{t,j}} + \frac{b_{t+J+1}}{D_{t,t+J+1}}$$

where

$$D_{t,j} = \prod_{i=1}^j R_{t+i}$$

is a discount factor.

## Budget constraint

Take  $J \rightarrow \infty$  and assume that

$$\lim_{J \rightarrow \infty} \frac{b_{t+J}}{D_{t,t+J}} = 0$$

We discuss (much) later why we might want to assume this.

- ▶ see transversality conditions

Then the present value budget constraint becomes

$$\underbrace{\sum_{j=0}^{\infty} \frac{c_{t+j}^y + c_{t+j+1}^o / R_{t+j+1}}{D_{t,j}}}_{\text{pv of consumption}} = \underbrace{\sum_{j=0}^{\infty} \frac{e_1 + e_2 / R_{t+j+1}}{D_{t,j}}}_{\text{pv of "earnings"}} + \underbrace{b_t}_{\text{initial assets}}$$

## Budget constraint

This is a common result:

$$\textit{Present value of spending} = [\textit{Present value of income}] \\ + [\textit{Initial assets}]$$

This looks like the budget constraint of an infinitely lived household.

# Infinitely lived dynasty

The parent therefore behaves exactly like an infinitely lived individual

- ▶ maximizing a single utility function over an infinite horizon
- ▶ subject to a single present value budget constraint.

This only works if

- ▶ households can borrow and lend at the same interest rate;
- ▶ bequests can be negative or are always intended to be positive
- ▶ parents are altruistic (not warm glow etc)

## Exercise

Show that the equilibrium allocation is the same as the planner's allocation.

# Implications

Why is this important?

- ▶ If we think bequests are positive, we can ignore finite lifetimes and write down models with a single, infinitely lived household.

One potential problem:

- ▶ We set up the parent's problem as if he could choose the child's actions.
- ▶ Later, we talk about why this is correct (see Dynamic Programming)



# When Are Bequests Positive?

And do they help with dynamic inefficiency?

## When are bequests positive? I

Bequests are positive, if a small bequests raises parental utility.

Consider the following perturbation of the optimal plan with  $b = 0$ :

1. Reduce old age consumption by  $\varepsilon$ . The utility loss is  $-u_2(t)\varepsilon$ .
2. Give  $\varepsilon/(1+n)$  to each child as a bequest.
3. Assume the child eats the bequest when young [what if not?] and gains

$$\omega u_1(t+1) \cdot \varepsilon / (1+n) \quad (8)$$

4. The household wants to leave a bequest if

$$\omega u_1(t+1) / (1+n) > u_2(t) \quad (9)$$

Does this expression look familiar?

## When are bequests positive? II

5. Apply the parent's FOC to express both gain and loss in terms of  $u_1$ . The FOC is

$$u_1(t) = (1 + r_{t+1})u_2(t)$$

Thus the parent increases his bequest if

$$\omega u_1(t+1)/(1+n) > u_1(t)/(1+r_{t+1})$$

or

$$u_1(t) < \frac{1+r_{t+1}}{1+n} \omega u_1(t+1) \quad (10)$$

6. In steady state this reduces to  $\omega(1+r) > (1+n)$ .

$\omega(1+r) = (1+n)$  is the **modified golden rule** (the planner's FOC).

# Dynamic inefficiency

This means:

- ▶ A situation where  $\omega R = \omega(1+r) > (1+n)$  can never be a steady state.
  - ▶ Every parent would want to increase his bequest until the MGR holds with equality
  - ▶ Then the economy is dynamically *efficient*.
- ▶ If without bequests  $\omega R < (1+n)$ , households don't want to leave bequests and the bequest motive is irrelevant.
  - ▶ Dynamic inefficiency remains.

The same holds in a production economy (the household does problem is the same).

# Summary

If the bequest motive is operative ( $b > 0$ ), then:

- ▶ The economy attains the modified golden rule.
- ▶ Therefore it is dynamically efficient.
- ▶ The market equilibrium coincides with the planner's solution (show this!).
- ▶ Ricardian equivalence holds even across generations. (We haven't shown that, but it follows directly from the fact that there is a present value budget constraint that holds across generations.)

If the bequest motive is not operative, it does not matter.

- ▶ This happens when the economy is initially dynamically inefficient.

# Reading

- ▶ Acemoglu (2009), ch. 5.3, 9.

## References I

Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.