Problem Set 4: Cash-in-Advance Model

Econ720. Fall 2020. Prof. Lutz Hendricks

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1 CIA Model

Demographics: A single, infinitely lived household.

Preferences: $\sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$. c is consumption; h is land.

Endowments: At $t=0$: M_0 units of money, k_0 units of goods, H units of land.

Technology: $c_t + k_{t+1} = Rk_t + F(h_t)$. F satisfies Inada conditions. $R > 1/\beta$ is exogenous.

Government: prints money and hands it out as lump sum transfer: $p_t \tau_t = M_{t+1} - M_t$.

Markets: competitive markets for goods (p_t), land purchase (q_t), land rental (r_t), money (numeraire).

CIA constraint: $c_t \leq m_t = M_t/P_t$.

Questions:

1. Write down the household budget constraint.
2. Write down the household's Bellman equation.
3. Write down the first-order conditions and envelope conditions.
4. Eliminate the value functions from those conditions.
5. Interpret the resulting first-order conditions.
6. What can be said about the relative returns of the three assets when the CIA constraint does not bind?
7. Derive and interpret the condition
$$U_c = \beta R U_c(.') \frac{\pi}{\pi'} \tag{1}$$
What happens to the intertemporal allocation of consumption when the inflation rate is rising over time? Why?

1.1 Answer: CIA model

1. Budget constraint:
$$M_t + p_t\tau_t + Rp_t k_t + p_t(q_t + r_t)h_t = p_t c_t + p_t k_{t+1} + M_{t+1} + p_t q_t h_{t+1} \tag{2}$$
or in real terms
$$m_t + \tau_t + Rk_t + (q_t + r_t)h_t = c_t + k_{t+1} + m_{t+1}\pi_{t+1} + q_t h_{t+1} \tag{3}$$
2. Bellman
$$\begin{aligned} V(k,h,m) &= \max U(c,h) \quad (4)\\ &+ \beta V(k',h',m') + \mu\{m - c\} \quad (5)\\ &+ \lambda\{m + \tau + Rk + (q+r)h - c - k' - m'\pi' - qh'\} \quad (6) \end{aligned}$$

3. First-order conditions

$$U_c = \lambda + \mu \tag{7}$$
$$\beta V_k \left(.'\right) = \lambda \tag{8}$$
$$\beta V_h \left(.'\right) = \lambda q \tag{9}$$
$$\beta V_m \left(.'\right) = \lambda \pi' \tag{10}$$

Envelope:

$$V_k = \lambda R \tag{11}$$
$$V_h = \lambda \left(q + r\right) + U_h \tag{12}$$
$$V_m = \lambda + \mu \tag{13}$$

4. Simplify:

$$U_c = \lambda + \mu \tag{14}$$
$$\lambda = \beta R \lambda' \tag{15}$$
$$\lambda q = \beta \left[U_h \left(.'\right) + \lambda' \left(q' + r'\right)\right] \tag{16}$$
$$\lambda \pi' = \beta U_c \left(.'\right) \tag{17}$$

Therefore

$$U_c = \beta R U_c \left(.'\right) \frac{\pi}{\pi'} \tag{18}$$

5. In words:
   (a) a unit of consumption requires a unit of money;
   (b) the household can transfer income across periods at rate R
   (c) give up q units of capital and buy 1 unit of land; eat and sell that tomorrow;
   (d) the household can buy π' units of money and eat 1 unit of consumption tomorrow.
6. If CIA does not bind: money and capital have the same return. Land returns are lower because of U_h.
7. Derivation above. If inflation is constant, it does not distort the intertemporal allocation. There is no way to escape the inflation tax. Buying consumption requires holding money for one period at opportunity cost $R\pi$. If inflation is rising, it pays to eat early.