

# The Romer Model

Prof. Lutz Hendricks

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# Issues

- ▶ We study models where **intentional innovation** drives productivity growth.
- ▶ **Romer model:**
  - ▶ The standard model of R&D goes back to **Romer** (1990).
  - ▶ Innovations are produced like any other good using R&D labor as input.
- ▶ **Policy effects**
  - ▶ Policies, such as R&D subsidies, can change the rate at which innovations are produced.
  - ▶ Surprisingly, it turns out that **policies have no effect on long-run growth.**

# Learning Objectives

In this section you will learn:

1. how to analyze the Romer model
2. why R&D policies do not change the long-run growth rate of the economy

# The Romer model

## Solow block

- ▶ Production of goods works exactly like in the Solow Model
- ▶ Aggregate production function:

$$Y_t = K_t^\alpha (A_t L_{Yt})^{1-\alpha} \quad (1)$$

- ▶ **Capital accumulation** as in the Solow model

$$\dot{K}_t = s_K Y_t - \delta K_t \quad (2)$$

- ▶ **Labor input** grows at a constant rate

$$g(L) = n \quad (3)$$

# Solow Block

What has changed?

Final goods production function has:

- ▶ constant returns to rival inputs:  $K$  and  $L_Y$ .
- ▶ has **increasing returns** to all inputs (including  $A$ )

Labor is divided into production ( $L_Y$ ) and R&D ( $L_A$ ).

# R&D Block

- ▶ Ideas are produced just like other goods.
- ▶ The input is labor ( $L_{At}$ )
  - ▶ not much changes if capital is an input, too.
- ▶ The output is a number of new ideas.
  - ▶  $A_t$  is the number of ideas that have been invented up to  $t$ .
  - ▶  $\dot{A}_t$  is the number of ideas discovered today (or the rate at which they are discovered).

- ▶ The **ideas production function**:

$$\dot{A}_t = \bar{B} L_{At}^{\lambda} \quad (4)$$

- ▶  $\lambda$  determines returns to scale.
- ▶  $\bar{B}$  is a productivity parameter.

## Ideas are inputs to innovation

How easy it is to produce a new idea depends on how much has already been discovered.

$$\bar{B} = B A^{\phi} \quad (5)$$

If ideas help produce new ideas:  $\phi > 0$ :  $A \uparrow \implies \bar{B} \uparrow$ .

If there is "fishing out":  $\phi < 0$ .

Assume  $\phi \leq 1$ . (If  $\phi > 1$  odd things happen...).

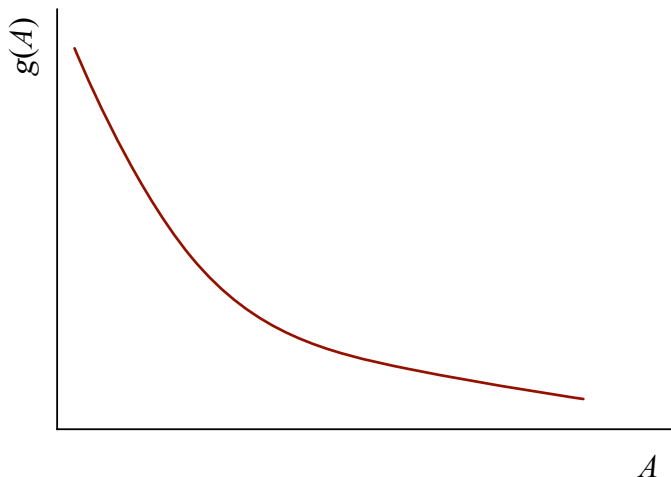
The ideas production function is then

$$\dot{A} = B L_A^{\lambda} A^{\phi} \quad (6)$$

$$g(A) \equiv \dot{A}/A = B L_A^{\lambda} A^{\phi-1} \quad (7)$$



## Ideas production function



Even though ideas foster innovation ( $\phi > 0$ ), more ideas imply slower  $g(A)$ .

# Ideas production function

Note how similar this is to the law of motion for capital in the Solow model

Model			Productivity	"Capital"	Labor	Depreciation
Solow	$\dot{K}_t$	=	$sA^{1-\alpha}$	$K_t^\alpha$	$L_t^{1-\alpha}$	$-\delta K_t$
Romer	$\dot{A}_t$	=	$B$	$A_t^\phi$	$L_{At}^\lambda$	$-0$

It follows that there cannot be long-run growth in  $A/L$  when  $\lambda + \phi < 1$  (details follow).

But we still can get long-run growth in  $Y/L$ .

# The Romer model

## Behavior

So far we have described technologies.

To describe behavior, we make a **Solow assumption**:

- ▶ A constant saving rate

$$S/Y = I/Y = s_K$$

- ▶ A constant labor allocation:

$$L_A = s_A L \tag{8}$$

$$L_Y = (1 - s_A) L \tag{9}$$

## Model summary

The Solow block:

$$Y = K^\alpha (A L_Y)^{1-\alpha} \quad (10)$$

$$\dot{K} = s_K Y - \delta K \quad (11)$$

$$L_t = L_0 e^{nt} \quad (12)$$

Production of ideas:

$$\dot{A} = B L_A^\lambda A^\phi \quad (13)$$

Constant behavior:

$$L_Y = s_Y L; \quad L_A = s_A L \quad (14)$$

The growth rate of ideas:

$$g(A) = B (s_A L)^\lambda A^{\phi-1} \quad (15)$$

## Model summary

- ▶ This looks complicated, but isn't.
- ▶ We have tricked the model such that  $Y$  and  $K$  don't matter for how  $A$  evolves.

$$\dot{A} = B L_A^\lambda A^\phi \quad (16)$$

- ▶ This would change, if we let  $\dot{A}$  depend on  $K$ 
  - ▶ but that would not affect the results
  - ▶ only the algebra would be more complicated (see Romer 2011)

# Does the Model Make Sense?

- ▶ The production functions are arbitrary.
  - ▶ But what matters are certain qualitative features, not the exact functional form.
  - ▶ We will get back to this.
- ▶ There is only one input. Only one good.
  - ▶ All of this can be relaxed without changing anything too important.
- ▶ Where are the households, consumption, population growth ...
  - ▶ We can add those - it does not make any difference.
- ▶ The labor allocation is fixed.
  - ▶ This is important.
  - ▶ The literature does not make this assumption. It can talk about patents, policy, ...
- ▶ Ideas are produced like goods.

# Balanced growth path

## Definition

A BGP is a path along which all variables grow at **constant rates**.

Why might this be interesting?

## Balanced growth path

At what rates do the endogenous objects grow on the BGP?

Result 1:  $g(k) = g(y)$

Proof:

- ▶ Law of motion:  $g(k) = s y/k - \delta$ .
- ▶ Constant  $g(k)$  requires constant  $k/y$ .



## Balanced growth path

Result 2:  $g(y) = g(A)$

Proof:

- ▶ Production function:  $y = k^\alpha A^{1-\alpha}$ .
- ▶ Take growth rates:  $g(y) = \alpha g(y) + (1 - \alpha) g(A)$

### Result

All long-run growth is due to R&D.

## Growth rate of ideas

$$g(A) = \frac{\lambda n}{1 - \phi} \quad (17)$$

**Proof:**

Ideas production:

$$g(A) = B \frac{L_A^\lambda}{A^{1-\phi}} \quad (18)$$

BGP:  $g(A)$  is constant  $\implies L_A^\lambda A^{\phi-1}$  is constant

Take growth rates of that

$$g(g(A)) = \lambda g(L_A) - (1 - \phi) g(A) = 0 \quad (19)$$

With constant time allocation,  $s_A$ :  $g(L_A) = n$ .

Solve for  $g(A)$ . Done.

## Summary: Balanced growth

Balanced growth in the Romer model is characterized by:

$$g(y) = g(k) = g(A) \quad (20)$$

$$g(A) = \frac{\lambda n}{1 - \phi} \quad (21)$$

**All growth is due to innovation.**

Why is this true?

# Why is all growth due to innovation?

Solow model:

- ▶  $K$  is **rival**
- ▶ What matters for per capita output is  $K/L$
- ▶  $K$  does growth in the Solow model (due to population growth), but not  $K/L$  (diminishing returns)

Romer model:

- ▶  $A$  is **non-rival**
- ▶ What matters for per capita output is  $A$ , NOT  $A/L$
- ▶  $A$  grows (due to population growth), even if  $A/L$  falls over time

## Balanced growth: Intuition

$$g(A) = \frac{\lambda n}{1 - \phi} \quad (22)$$

Growth is simply a multiple of population growth

Behavior does not matter:  $s_K$  and  $s_A$  do not appear in (22).

# Intuition

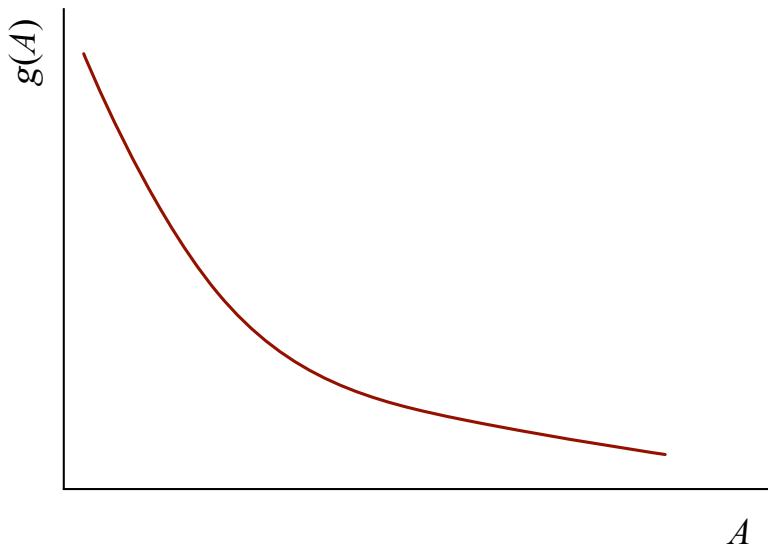
- ▶ Consider the case  $\phi = 0$ .
- ▶ Ideas production is then

$$\dot{A} = B L_A^\lambda \quad (23)$$

- ▶ If the population is **constant**,  $L_A$  is constant.
- ▶ In each period, the economy produces a constant number of ideas.
- ▶ The growth rate of ideas,  $g(A) = B L_A / A$ , falls to zero over time.
- ▶ A fixed number of people cannot produce a growing stream of ideas.

**Population growth** is necessary for sustained innovation (at a constant rate).

## How growth is sustained



$$g(A) = BA^{\phi-1}L_A^\lambda$$

## Special Case: $\Phi = 1$

With  $\phi = 1$ , idea production becomes

$$g(A) = B L_A^\lambda \quad (24)$$

This is the case studied by Romer (1990).

The model has exploding growth, unless the population is constant.

This is clearly contradicted by post-war data:  $L_A$  rose dramatically, while  $g(y)$  was at best constant.



# Reality check

1. The model says: constant population - no growth.
  - ▶ But we are still producing new ideas all the time.
  - ▶ How can we reconcile this?
2. What if the population shrinks over time?
  - ▶ Is the long-run growth rate negative?

# Reading

- ▶ Jones (2013b), ch. 5.

Optional:

- ▶ Romer (2011), ch. 3.1-3.4
- ▶ Jones (2013a), ch. 6

## Advanced Reading

- ▶ Jones (2005) talks in some detail about the economics of ideas.
- ▶ Lucas (2009) and McGrattan and Prescott (2009) on openness and growth

## References I

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- McGrattan, E. R. and E. C. Prescott (2009): “Openness, technology capital, and development,” *Journal of Economic Theory*, 144, 2454–2476.
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