Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **4** questions. Answer all questions.
- The total time is 3 hours. The total number of points is 180.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 Endogenous Growth with Two Capital Goods (35 points)

Demographics: The economy is populated by a representative consumer who lives forever.

Preferences:

$$\int_0^\infty e^{-\rho t} \frac{c(t)^{1-\sigma}}{1-\sigma} dt$$

Endowments:

- At $t = 0$, households are endowed with two types of capital, $K_1(0)$ and $K_2(0)$.

Technology:

- Output is used for consumption and two types of investment:

$$Y(t) = c(t) + x_1(t) + x_2(t)$$

- The production function is

$$Y(t) = \mathcal{F}(K_1(t), K_2(t)) = K_1(t)^\alpha K_2(t)^{1-\alpha}$$

- Capital is accumulated according to

$$\dot{K}_i(t) = x_i(t) - \delta K_i(t)$$

  where $i \in \{1, 2\}$. Investment may be negative.

Government:

- The government taxes income from capital good 2 at rate τ. Revenues are rebated to the household as lump-sum transfers $T(t)$. The government budget constraint is $\tau r_2 K_2 = T$.

Markets:

- There are competitive markets for final goods (numeraire) and rental markets for capital (prices r_i).
- The household owns the capital goods and rents them to the firm.

Questions:

1. [15 points] Write down the household's Hamiltonian and solve for the optimal consumption growth rate as a function of prices and tax rates.

   Answer ___

   The household's budget constraint is given by

$$c(t) + x_1(t) + x_2(t) = r_1(t)K_1(t) + r_2(t)[1-\tau]K_2(t) + T(t)$$

where $\dot{k}_j = x_j - \delta K_j$. The household's Hamiltonian is given by

$$\begin{aligned} H = & \frac{c(t)^{1-\sigma}}{1-\sigma} \\ & + \lambda(t) \left\{ [r_1(t) - \delta] K_1(t) + r_2(t) \left[1 - \tau\right] K_2(t) + T(t) - x_2(t) - c(t) \right\} \\ & + \mu(t) \left\{ x_2(t) - \delta K_2(t) \right\} \end{aligned}$$

with controls c and x_2 (it could be written with other controls).

The first-order conditions imply two versions of the standard Euler equation:

$$\begin{aligned} g(c(t)) &= \left\{ r_1(t) - \delta - \rho \right\} / \sigma \\ &= \left\{ [r_2(t)(1-\tau) - \delta] - \rho \right\} / \sigma \end{aligned}$$

and therefore $r_1/r_2 = 1 - \tau$. If the tax rises, r_2 must rise to compensate.

It's also OK to set up the problem with one state $(K_1 + K_2)$.[1]

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2. [10 points] Show that in equilibrium

$$r_1(t) = \alpha \left[\frac{(1-\tau)(1-\alpha)}{\alpha} \right]^{1-\alpha}$$

Explain in words why this is true. Would your answer change if it were costly to move capital between sectors?

Answer

The firm's first-order conditions are

$$\begin{aligned} r_1(t) &= \alpha \left[K_1(t)/K_2(t) \right]^{\alpha - 1} \\ r_2(t) &= (1-\alpha) \left[K_1(t)/K_2(t) \right]^{\alpha} \end{aligned}$$

so that $r_1/r_2 = \frac{\alpha}{1-\alpha} \frac{K_2}{K_1}$. From the household solution, we know that $r_1/r_2 = 1 - \tau$. Therefore,

$$\frac{K_2}{K_1} = \frac{(1-\tau)(1-\alpha)}{\alpha} \tag{1}$$

and thus the target expression for r_1.

Intuition: The relative rental of capital goods is fixed at $1 - \tau$ to equalize rates of return. Given that the final goods are produced with constant returns to scale, this fixes K_2/K_1.

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[1] The question is based on a problem due to Dirk Krueger.

3. [5 points] Derive the growth rate of consumption in competitive equilibrium for a given τ. Why does it not depend on the capital endowments?

Answer

Substitute $r_1(t)$ into the household Euler equation to obtain the equilibrium growth rate

$$g(c(t)) = \{r_1 - \delta - \rho\} / \sigma$$

The growth rate does not depend on capital endowments because capital can be costlessly moved between sectors. For a fixed ratio K_2/K_1, the model becomes AK. Hence, the marginal product of capital does not depend on the level of K. The interest rate and growth rate are therefore constant.

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4. [5 points] Subsidizing investment by setting $\tau < 0$ increases growth and therefore output for all t. Should the government do that?

Answer

No. From the Welfare Theorems, the undistorted equilibrium is welfare-maximizing.

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2 Heterogeneous Firms (45 points)

Demographics: A unit mass of identical, infinitely lived households. A unit mass of firms.

Preferences: Households value consumption according to $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}(C_t)$.

Firms:

- Each firm enters the period with capital stock k, which differs across firms.
- Next, it draws an idiosyncratic productivity shock z from distribution Γ.
- It then produces output $z\mathcal{F}(k)$. $\mathcal{F}$ is strictly concave.
- Finally, the firm decides k'. A firm that wants to change its capital stock must buy (or sell) $k'-k$ from (or to) other firms at price p. It also incurs an adjustment cost $\mathcal{G}(k'-k)$ that is positive and strictly convex with $\mathcal{G}(0)=0$.
- Dividends are paid out to households who own the firm:

$$\pi = z\mathcal{F}(k) - \mathcal{G}(k'-k) - p \times (k'-k) \tag{2}$$

- Firms maximize the expected discounted present value of dividends.

The resource constraint for goods is $Y = C + G$ where Y is total output (the sum of all $z\mathcal{F}$ across firms) and G is the total adjustment cost (the sum of $\mathcal{G}$ across firms).

The aggregate capital stock is fixed at K. Firms can trade capital, but not augment it.

Markets: There are competitive markets for goods (numeraire), capital (price p), bonds (gross interest rate R), and shares of all firms (prices q vary with firm states).

We consider a recursive competitive equilibrium. Be sure to write your answers recursively.

Questions:

1. [5 points] What is the aggregate state of the economy?

   Answer

   The aggregate state is the joint distribution of (k,z). But since the distribution of z is fixed, the aggregate state S is really the conditional distribution of $k|z$. There is no aggregate uncertainty. So we can write the law of motion $S' = \mathcal{H}(S)$.

2. [10 points] Write out the firm's dynamic program. Explain why discounting future dividends at the risk-free bond rate is correct.

   Answer

$$V(k,z,S) = \max_{k'} \pi(k',k,z,S) + R'(S)^{-1} \mathbb{E}V'(k',z',\mathcal{H}(S)) \tag{3}$$

where profits are given by (2). $R'(S)$ is the gross interest rate on bonds issued today. This is known today. It is correct to discount future dividends using the risk-free rate because firm shares are viewed as riskless by the household. Holding a diversified portfolio of shares yields a certain return.

Solution: Value function V and decision rule $k' = \kappa(k, z, S)$. Implied is a profit function $\pi(k, z, S)$.

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3. [10 points] Derive the firm's first-order conditions and state what they say in words.

Answer

FOC:

- $\partial\pi/\partial k' + R'(S)^{-1}\,\mathbb{E}\partial V/\partial k(.') = 0$, where the expectation is taken over z'.
- $\partial\pi/\partial k' = -p - \mathcal{G}'(k' - k)$
- $\partial V/\partial k = z\mathcal{F}'(k) + p(S)\,\mathcal{G}'(k' - k)$
- Combine: $p(S) + \mathcal{G}'(k' - k) = R'(S)^{-1}\,\mathbb{E}\{z'\mathcal{F}'(k') + p(S') + \mathcal{G}'(k'' - k')\}$

In words: Buying more capital today reduces profits by the purchase price p and by the marginal adjustment cost $\mathcal{G}'$. The payoff next period is the marginal product of capital $z'\mathcal{F}'(k')$ plus the marginal cost of buying the same amount of capital tomorrow.

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4. [10 points] Write out the household's dynamic program.

Answer

Let $s \equiv (k, z)$. The household takes as given a price function $q(s, S)$ which prices all feasible (k, z) combinations. Together with the firm's decision rules for k' this implies a rate of return function

$$R'(s', s, S) = \frac{q(s', S') + \pi(s', S')}{q(s, S)} \tag{4}$$

where $S' = \mathcal{H}(S)$.

The household's individual state is the mass of shares held by each firm $(x(s))$ and the amount of bonds held (b).

Budget constraint:

$$b' + \int q(s, S)(v(s) - x(s))\,ds + C = R(S)\,b + \int \pi(s, S)\,x(s)\,ds \tag{5}$$

Here, $v(s)$ denotes share purchases. One wrinkle: The household cannot (directly) choose $x'(s')$ because, for each firm, the transition $s \to s'$ is stochastic.

Let $\mathcal{G}$ denote the law of motion that links z and x'. That is, $x'(s') = \mathcal{G}(s'|v,S)$. If the household purchases $v(s,S)$ shares (for all s), then they end up with $x'(s')$ shares of s' tomorrow. Since the firm's shocks average out and since S' is a known function of S, $\mathcal{G}$ is **not stochastic**. That is, the household knows exactly how many shares in state s' they will own tomorrow, given v and S.

The Bellman equation is now:

$$W(b, x(s), S) = \max_{b', v(s)} \mathcal{U}(C) + \beta W(b', \mathcal{G}(s'|v,S), \mathcal{H}(S)) \tag{6}$$

where x and v are functions of s.

What does $\mathcal{G}$ look like? It solves

$$x'(s') = \mathcal{G}(s'|v,S) = \int v(s,S)\,\phi(s'|s,S)\,ds \tag{7}$$

where the transition probability satisfies $\phi(s'|s,S) = \Pr(z'(s')|z(s)) \times \mathbb{I}\{\kappa(s,S) = k'(s')\}$. In words: The mass of firms that moves from that $s \to s'$ is equal to the fraction that move between the matching zs and choose the right k'.

Note: It is tempting to simply write $\int q_j\left(x_j' - x_j\right) dj$. But that does not work well in RCE. The RCE needs price functions, which are not a function of j but of (z,k).

Note: Full credit for answers that simply let the household choose $x'(s)$. Because all shares have the same expected returns, that also yields the right answer.

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5. [10 points] Derive and interpret the household's first-order conditions, which should not contain derivatives of the value function.

Answer

Euler equation (one for each share):

$$\mathcal{U}'(C(b,x,S)) = \beta' \mathcal{U}'(C(b',x',S'))\,\mathbb{E}R'(s',s,S) \tag{8}$$

and the same for bonds. Note that C' is not stochastic. These are just Lucas asset pricing equations.

Household solution: Value function and policy functions $b' = \mathcal{B}(b,x,S)$ and $x' = \mathcal{X}(b,x,S)$. They solve the Bellman equation in the usual sense.

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3 Government Spending and the Business Cycle (55 points)

Consider a real business cycle model with government spending. The economy consists of a large number of identical infinitely-lived households with utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln\left(c_t - \frac{1}{2}n_t^2\right), \quad 0 < \beta < 1,$$

where c_t is consumption and n_t is labor. The representative firm produces output using capital K_t and labor N_t according to a technology $Y_t = F(K_t, N_t)$, where F is strictly increasing, strictly concave, and has constant returns to scale. Capital accumulates through investment and depreciates at rate δ.

Every period, a given **fraction** of aggregate output must be allocated to government spending. Specifically, if Y_t is the output in period t, the required government expenditures in period t are $g_t Y_t$, where g_t follows an exogenous Markov process and satisfies $0 < g_t < 1$ at all times. The government as well as private agents take g_t as given. Government spending is completely wasteful and is just thrown away.

Questions:

1. (5 points) Write down the social planner's problem in recursive form. Assume that the social planner, just like the private agents, takes the process for g_t as given.

   Answer ______

   $$\mathcal{V}(g, K) = \max_{C,K'N} \ln\left(C - \frac{1}{2}N^2\right) + \beta\mathbb{E}\mathcal{V}(g', K')$$

   subject to

   $$C + K' = (1-g)F(K, N) + (1-\delta)K$$

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2. (15 points) How does a shock to g_t affect the period-t labor hours chosen by the social planner? Prove your answer. Describe the income and substitution effects. Briefly explain how and why the results are different from the model of government spending shocks covered in ECON 721.

   Answer ______

   The first-order condition is

   $$(1-g)F_N(K, N) = N$$

   so that a shock to g acts as a decrease in productivity, decreasing N. Due to GHH preferences, there is *no* income effect.

In the usual model of government spending shocks, shocks are to the level of G; in that case, the shock would be an additive deduction from output and therefore would generate no substitution effect. With preferences that rule out income effects (as they do in this case), there would be *no* effect on N.

Note that, with more general preferences, level shocks to G would have an effect on N (a pure income effect). This effect would usually be to *increase* N by making agents in the economy poorer. So, GHH preferences are key to the result that (i) proportional positive shocks to g (as in this problem) necessarily decrease N, since the only effect is a substitution effect; and (ii) level shocks to G (as in the model from ECON 721) have no effect on N. However, GHH preferences are not key to the result that proportional and level shocks have *different* effects on N.

Key mistakes and omissions for this question were:

- Not understanding the definitions of income and substitution effect.
  - Not understanding which effect is operative in this problem.
  - Not understanding which effect is operative in the model from class.
  - Not understanding the role of the form of the utility function for these effects.

Now, consider a recursive competitive equilibrium. Households own the capital and rent it out to firms in a competitive market, as well as supplying labor in a competitive market. The government uses a **lump-sum tax** to finance government expenditures.

3. (15 points) Write down (i) the household's problem, and (ii) the government budget constraint, in recursive language. What are the individual and aggregate state variables? If any objects are functions of the aggregate state, you must make this explicit.

Answer

The household's recursive problem is

$$V(g, K, k) = \max \ln \left(c - \frac{1}{2} n^2 \right) + \beta \mathbb{E} V(g', K', k')$$

subject to

$$c + k' = (1 - \delta)k + r(g, K)k + w(g, K)n - T(g, K)$$

and the law of motion $K' = \mathcal{H}(g, K)$. The individual state is the household's own capital, k. The aggregate states are the shock to government spending g and the endogenous aggregate capital K.

The government budget constraint is:

$$gF(K, N(g, K)) = T(g, K)$$

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4. (10 points) Does the equilibrium coincide with the solution to the planner's problem? How (if at all) does the equilibrium response of period-t labor hours to a g_t shock differ from the social planner's? Justify your answers. Provide some intuition.

Answer

The household optimality condition (after substituting in for w) is

$$F_N(K_t, N_t) = N_t$$

This means: (i) the equilibrium is inefficient. The social planner internalizes the fact that higher output leads to higher wasteful government expenditures, while private agents do not. (ii) shocks to g_t now have no effect on labor hours. This is precisely because, from the individual household's point of view, the tax used to finance government expenditures is independent of labor hours and the household therefore perceives it as non-distortionary.

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5. (10 points) Propose an alternative taxation scheme such that the competitive equilibrium allocation is efficient.

Answer

Efficiency can be achieved by replacing the lump-sum tax with a proportional output tax, or taxing labor and capital income at the same rate. A tax on only labor or only capital would not work. This is because both the intra-temporal optimality condition and the inter-temporal optimality condition need to be coincide between the equilibrium and the efficient allocation. The inter-temporal condition in equilibrium would read

$$\frac{1}{C_t - \frac{1}{2}N_t^2} = \beta \mathbb{E}_t[1 - \delta + (1 - \tau_{t+1})F_K(K_{t+1}, N_{t+1})]\frac{1}{C_{t+1} - \frac{1}{2}N_{t+1}^2}$$

whereas for the social planner it is

$$\frac{1}{C_t - \frac{1}{2}N_t^2} = \beta \mathbb{E}_t[1 - \delta + (1 - g_{t+1})F_K(K_{t+1}, N_{t+1})]\frac{1}{C_{t+1} - \frac{1}{2}N_{t+1}^2}$$

The intra-temporal condition in equilibrium would read

$$(1 - \tau_t)F_N(K_t, N_t) = N_t$$

whereas for the social planner it is

$$(1 - g_t)F_N(K_t, N_t) = N_t$$

Therefore a proportional tax on both labor and capital income equal to $\tau_t = g_t$ would achieve the efficient allocation in equilibrium.

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4 Credit Markets and Self-insurance (45 points)

An economy lasts for three periods: $t = 0, 1, 2$. There is a continuum of ex ante identical households, who value consumption in each period according to the utility function

$$\mathbb{E}_0 \left[\ln(c_0) + \beta \ln(c_1) + \beta^2 \ln(c_2) \right], \quad 0 < \beta < 1$$

In period 0, each household receives a deterministic endowment $y_0 > 0$. In period 1, each household's endowment $\tilde{y}_1$ is stochastic: each household receives

$$\tilde{y}_1 = \begin{cases} y^h, & \text{with probability } \pi, \\ y^\ell, & \text{with probability } 1 - \pi \end{cases}$$

where $y^h > y^\ell > 0$ and $\pi \in (0, 1)$. There is no aggregate uncertainty: the endowment shock $\tilde{y}_1$ is an idiosyncratic shock independent across households. In period 2, each household receives a deterministic endowment $y_2 > 0$. Output cannot be stored across periods.

Financial markets work as follows. Households cannot trade contingent claims, and can only borrow and lend in an uncontingent bond in zero net supply.

- In period 0, households can lend and borrow among themselves. Denote by r_0 the equilibrium interest rate for lending/borrowing between periods 0 and 1.
- In period 1, households can also lend and borrow, but only up to a borrowing limit $\bar{A}$. Denote by r_1 the equilibrium interest rate for lending/borrowing between periods 1 and 2.

Everything except each household's $\tilde{y}_1$ realization is known in period 0. In other words, the only uncertainty each household faces is about period-1 income.

Questions:

1. (5 points) Write down the household's problem.

Answer

The household chooses

$$\{c_0, c_1^h, c_1^\ell, c_2^h, c_2^\ell, a_1, a_2^h, a_2^\ell\}$$

to maximize

$$\ln(c_0) + \beta\{\pi[\ln(c_1^h) + \beta \ln(c_2^h)] + (1 - \pi)[\ln(c_1^\ell) + \beta \ln(c_2^\ell)]\}$$

subject to

$$c_0 + a_1 = y_0$$

$$c_1^h + a_2^h = y^h + (1 + r_0)a_1$$

$$c_1^\ell + a_2^\ell = y^\ell + (1 + r_0)a_1$$

$$c_2^h = y_2 + (1 + r_1)a_2^h$$

$$c_2^\ell = y_2 + (1+r_1)a_2^\ell$$
$$a_2^h \geq -\bar{A}$$
$$a_2^\ell \geq -\bar{A}$$

The most common error here was not recognizing that the household chooses *state-contingent* consumption in period 1 and 2.

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2. (10 points) What is the highest possible equilibrium value of r_1?

Answer

The period-1 Euler equations are

$$\frac{1}{c_1^h} \geq \beta(1+r_1)\frac{1}{c_2^h} \tag{9}$$

$$\frac{1}{c_1^\ell} \geq \beta(1+r_1)\frac{1}{c_2^\ell} \tag{10}$$

Rearranging and adding up, we get

$$\pi c_2^h + (1-\pi)c_2^\ell \geq \beta(1+r_1)[\pi c_1^h + (1-\pi)c_1^\ell] \tag{11}$$

But then market clearing implies

$$y_2 \geq \beta(1+r_1)[\pi y^h + (1-\pi)y^\ell], \tag{12}$$

Rearranging, we get

$$r_1 \leq \frac{y_2}{\beta[\pi y^h + (1-\pi)y^\ell]} - 1, \tag{13}$$

giving an upper bound on r_1 (which is achieved with equality if the borrowing constraint does not bind).

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3. (15 points) Determine how an increase in $\bar{A}$ affects r_1. Provide some intuition.

Answer

If the borrowing constraint does not bind for anyone in period 1, then an increase in $\bar{A}$ clearly has no effect. If the borrowing constraint binds, it binds for the low-income households and does not bind for the high-income households (this can be shown by contradiction). This means that for high-income households, the Euler equation holds with equality, and moreover,

$$c_1^h = y^h - \frac{1-\pi}{\pi}\bar{A}$$

and

$$c_2^h = y_2 + (1+r_1)\frac{1-\pi}{\pi}\bar{A}$$

An increase in $\bar{A}$, all else equal, lowers c_1^h and raises c_2^h. So, r_1 has to rise to satisfy the **lender's** Euler equation

$$\frac{1}{c_1^h} = \beta(1+r_1)\frac{1}{c_2^h}, \tag{14}$$

which holds with equality. Intuitively, the borrowing constraint puts downward pressure on the interest rate, by lowering the demand for borrowing. Relaxing the borrowing limit relaxes this downward pressure. Alternatively, relaxing the borrowing limit increases borrowing, so the interest rate has to rise to incentivize the lenders to save a larger amount.

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4. (10 points) Determine how an increase in $\bar{A}$ affects r_0. Provide some intuition.

Answer

If the borrowing limit binds in period 1, the period-0 Euler equation reads

$$\frac{1}{y_0} = \beta(1+r_0)\left[\pi\frac{1}{y^h - \frac{1-\pi}{\pi}\bar{A}} + (1-\pi)\frac{1}{y^\ell + \bar{A}}\right] \tag{15}$$

The constraint binding implies $y^h - \frac{1-\pi}{\pi}\bar{A} > y^\ell + \bar{A}$. Differentiating the expression in square brackets with respect to $\bar{A}$, we get

$$(1-\pi)\left[\frac{1}{\left(y^h - \frac{1-\pi}{\pi}\bar{A}\right)^2} - \frac{1}{\left(y^\ell + \bar{A}\right)^2}\right] < 0$$

So an increase in $\bar{A}$ lowers the expression in brackets and therefore must raise r_0 to still satisfy the Euler equation. Intuitively, an increase in $\bar{A}$ narrows the difference in marginal utilities in good and bad states, which lowers the incentive for precautionary saving. In equilibrium, there is no precautionary saving since this is an endowment economy, so instead the interest rate rises.

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5. (5 points) Suppose that, between periods 0 and 1, a storage technology is available that converts one unit of stored goods in period 0 to R units of goods in period 1. Explain how the equilibrium would change, and determine how $\bar{A}$ affects r_0 and the amount of storage.

Answer

Now, in period 0 there will be some storage, and the interest rate must equal the return on storage to preclude arbitrage; so the Euler equation becomes

$$\frac{1}{y_0 - s} = \beta R\left[\pi\frac{1}{y^h + Rs - \frac{1-\pi}{\pi}\bar{A}} + (1-\pi)\frac{1}{y^\ell + Rs + \bar{A}}\right] \tag{16}$$

In this case, an increase in $\bar{A}$ does not affect r_0 (which is pinned down by R) and instead decreases s.
