Knowledge Spillovers and Scale Effects

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Issues

- What happens when innovation takes labor (a non-reproducible factor)?
- Then we need a knowledge spillover to sustain growth.
- It takes some tricks to prevent the model from exhibiting explosive growth.

Knowledge Spillovers

Ideas Produced From Labor

The previous model had endogenous growth because ideas were produced with constant return from a **reproducible factor**: ideas (embodied in goods).

If ideas are produced from (non-reproducible) labor: there is no sustained growth.

Example

Assume $\dot{N}_t = \eta \; Z_t^{\alpha} \; L_{Rt}^{1-\alpha}$. Show that the balanced growth rate is 0 unless $\alpha = 1$.

Knowledge Spillovers

We need a mechanism that offsets diminishing returns to ideas in the production of ideas.

Knowledge spillover: N appears in the innovation production function for N.

This is an **externality**: firms do not pay for the N input.

This is possible because N is non-rival.

The idea: "standing on the shoulders of giants"

Problem: A **knife-edge** parameter assumption is needed for endogenous growth.

- Some parameters must sum to 1.
- This is always true because we need **constant returns to reproducible factors**.

Knowledge spillover model

Keep everything the same, except the production of ideas:

$$\dot{N}_t = \eta N_t L_{Rt} \tag{1}$$

We show later: linearity in N is required for endogenous growth.

Labor now has 2 uses:

- produce goods: L_E
- produce ideas: L_R

Resource constraint:

$$L = L_{Rt} + L_{Et} \tag{2}$$

Note: this does not change the problems of household, final goods firms, or intermediate input firms.

Balanced growth rate

Euler equation is still: $g(C) = (r - \rho)/\theta$.

Interest rate is determined by free entry: $V = \pi/r$.

But now the cost of creating a new patent is different:

$$\eta N_t V_t = w_t \tag{3}$$

- hire a unit of labor and produce a flow of ηN_t patents per "period"

Balanced growth rate

Wage rate (unchanged):

$$w_t = \frac{\beta}{1-\beta} N_t \tag{4}$$

Profits earned by monopolists (unchanged):

$$\pi = \beta L_E \tag{5}$$

Sub wage rate into free entry:

$$\eta N_t \frac{\beta L_E}{r} = w = \frac{\beta}{1-\beta} N_t \tag{6}$$

$\Longrightarrow$

$$r^* = (1-\beta)\,\eta L_E^* \tag{7}$$

Intuition ...

Balanced growth rate

Euler equation (unchanged):

$$g^* = g(C) = \frac{(1-\beta) L_E^* - \rho}{\theta} \tag{8}$$

Almost done - just need to find L_E.

Balanced growth requires

$$g(C) = g(Y) = g(N) \tag{9}$$

Ideas production function:

$$g(N) = \eta L_R^* = \eta (L - L_E^*) \tag{10}$$

Balanced growth

Solve for the growth rate.

$$\begin{aligned} g(C) &= \frac{(1-\beta)L_E^* - \rho}{\theta} \\ &= \eta(1-L_E^*) \end{aligned}$$

Intuition ...

$\Longrightarrow$

$$L_E^* = \frac{\theta\eta L + \rho}{(1-\beta)\eta + \theta\eta} \tag{11}$$

Scale effects: larger economies grow faster.

With population growth, output growth explodes.

Growth without scale effects

- The previous models do not have balanced growth paths when there is population growth.
- The reason is the scale effect:
  - Larger population $\rightarrow$ more R&D $\rightarrow$ faster growth.
- Diminishing returns to reproducible factors avoid the scale effect, but also kill endogenous growth.

Growth without scale effects

To avoid scale effects, modify the model as follows.

Innovation:

$$\begin{aligned} \dot{N}_t &= \eta N_t^{\phi} L_{Rt} && (12) \\ 0 &< \phi \leq 1 && (13) \end{aligned}$$

Demographics:

$$\begin{aligned} L_t &= e^{nt} && (14) \\ &= L_{Rt} + L_{Et} && (15) \end{aligned}$$

Balanced growth

From the innovation technology:

$$g(N) = \eta \ N_t^{\phi-1} \ L_{Rt} \tag{16}$$

Constant growth requires constant $N^{\phi-1}L_R$ and

$$g(N) = \frac{n}{1-\phi} \tag{17}$$

The growth rate is "**semi-endogenous**:" endogenous, but not responding to changes in agents' choice variables.

There are still scale effects:

- Larger economies tend towards higher levels of output per person.

Avoiding scale effects

It is possible to write down models that have endogenous growth, but no scale effects (growth does not increase with L).

The idea: Prevent innovator profits from increasing with L.

One approach: the number of products increases with L exactly so that the market size for each variety remains the same (Young, 1998).

Avoiding scale effects requires knife-edge assumptions like this.

Final Example:
Durable Intermediate Inputs

Environment

We study a final example where intermediates are durable (the model has capital).

Demographics: There is a representative household who lives forever.

Preferences:

$$\int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1 - \sigma} dt \tag{18}$$

Endowments: The household works one unit of time at each instant.

Technologies: Final goods

$$Y_t = AL_t^{\beta} \int_0^{N_t} x(v,t)^{1-\beta}\, dv = C_t + X_t + Z_t$$

where

- $X_t = \int_0^{N_t} I(v,t)\, dv$
- $I(v,t)$ is investment in intermediates of type v
- Z_t is investment in R&D

Technologies: Intermediates

- Upon invention, the inventor is endowed with x_0 units of $x(v)$.
- Additional units are accumulated according to

$$\dot{x}(v,t) = \eta I(v,t)^{\varphi} - \delta x(v,t) \tag{19}$$

- $0 < \varphi < 1$
- Diminishing returns imply smooth adjustment of x over time.
- Intermediates are *rented* to final goods firms at price $q(v,t)$.

Technologies: R&D

New varieties are invented according to:

$$\dot{N}_t = Z_t/B \tag{20}$$

where Z denotes goods devoted to R&D.

Market arrangements

Markets:

- Final goods: price 1
- Labor: w_t
- Intermediate input rental: $R_{j,t}$

Each intermediate input producer has a permanent monopoly for his variety.

Free entry into the market for innovation

Household

- Standard, with complicated budget constraint.
- Euler:

$$\dot{c}_t/c_t = \frac{r_t - \rho}{\sigma} \tag{21}$$

- TVC $\lim_{t\to\infty} e^{-\rho t} u'(c_t)\, a_t = 0$.

Final goods firm

$$\max AL_t^{\beta}\int_0^{N_t} x(v,t)^{1-\beta}\,dv - w_t L_t - \int_0^{N_t} R(v,t)\,x(v,t)_{,}\,dv \tag{22}$$

$\{y_t, L_t, x(v,t)\}$ solve the production function and the FOCs

$$\begin{aligned} w_t &= \beta y_t / L_t \qquad (23)\\ q(v,t) &= (1-\beta) A L^{\beta} x(v,t)^{-\beta} \qquad (24)\end{aligned}$$

Constant elasticity demand function:

$$x(v,t) = L_t\left[(1-\beta)A/q(v,t)\right]^{1/\beta} \tag{25}$$

Price elasticity: $-d\ln x / d\ln q = 1/\beta$

Revenue for intermediates

$$R(x) = q(x)x \tag{26}$$
$$= A(1-\beta)L^{\beta}x^{1-\beta} \tag{27}$$

Marginal revenue:

$$R'(x) = (1-\beta)A(1-\beta)L^{\beta}x^{-\beta} \tag{28}$$
$$= (1-\beta)q(x) \tag{29}$$

Intermediate input producer

Now a truly dynamic problem (v index suppressed)

$$V_t = \max \int_t^\infty e^{-r\tau}[R(x(\tau)) - I(\tau)]d\tau$$

subject to

$$\dot{x} = \eta I^\varphi - \delta x \tag{30}$$

Hamiltonian:

$$H = R(x) - I + \mu[\eta I^\varphi - \delta x] \tag{31}$$

Intermediate input producer

FOCs:

$$\begin{aligned} \partial H/\partial I &= -1+\mu\eta\varphi I^{\varphi-1}=0 \\ \dot{\mu} &= (r+\delta)\mu - R'(x) \end{aligned}$$

Intuition...

Solution: $\{I_t, x_t, \mu_t\}$ that solve 2 FOCs and law of motion for x.

Boundary conditions:

- $x(0)=0$ given,
- $\lim_{t\to\infty} e^{-rt}\mu_t x_t = 0$.

Free entry of innovators

Technology:

$$\dot{N} = B^{-1}Z \tag{32}$$

Free entry:

- Spend $B\ dt$ to obtain $dN = B/B\ dt$ new patents worth $V\ dt$.
- Equate cost and profits:

$$B = V \tag{33}$$

Equilibrium

Objects: $\{q(v,t), x(v,t), N_t, I(v,t), \mu(v,t), y_t, L_t, r_t, c_t, w_t\}$

Equilibrium conditions:

- Household: Euler (1)
- Final goods firm: 3
- Intermediate goods firm: 3
- Free entry: $B = V = \int e^{-rt}[R(x_t) - I_t]\,dt$
- Market clearing

Market clearing

1. Final goods: Resource constraint or $Y = C + NI + \dot{N}B$.
2. Intermediates: implicit in notation.
3. Labor: $L = 1$.
4. Asset markets: suppressed (details not specified)

Case $\varphi = 1$

Assume that the same equilibrium conditions hold for $\varphi = 1$ (not obvious).

Then FOC for investment in x becomes

$$1 = \mu \eta \varphi I^{\varphi - 1} = \mu \eta \tag{34}$$

μ must be constant over time (assuming investment takes place at all times; not obvious).

Constant μ implies:

$$\dot{\mu} = (r + \delta)\mu - R'(x) = 0 \tag{35}$$

x must be constant over time.

Case $\varphi = 1$

Demand function implies (cf. (29)):

$$R'(x) = (1-\beta)\,q(x) \tag{36}$$

Therefore:

$$R'(x) = (1-\beta)\,q(x) = (r+\delta)\,\mu \tag{37}$$

where $\mu = 1/\eta$ so that

$$q = \frac{r+\delta}{(1-\beta)\,\eta} \tag{38}$$

Then we know x from the demand function

$$x = L\left[(1-\beta)A/q\right]^{1/\beta} \tag{39}$$

With a linear technology, the best approach is to build all x in one shot, then keep x constant.

Symmetric equilibrium I

With $\varphi = 1$ there is a symmetric equilibrium because it does not take time to build up the stock of x.

Start from the Euler equation: $g(c) = (r - \rho)/\sigma$.

Free entry pins down r:

$$B = V = \int_0^\infty e^{-rt} [R(x_t) - I_t] \, dt - \underbrace{\frac{x - x_0}{\eta}}_{I_0} \tag{40}$$

Assume $x_0 = 0$.

Stationary x:

$$I_t = x\delta/\eta \tag{41}$$

Symmetric equilibrium II

From marginal revenue (37) we have:

$$R(x) = \frac{r+\delta}{(1-\beta)\,\eta}x \tag{42}$$

Therefore the integrand becomes:

$$R(x) - I = x\left[\frac{r+\delta}{\eta\,(1-\beta)} - \frac{\delta}{\eta}\right] \tag{43}$$

and free entry implies

$$B = V = \frac{R(x) - \frac{\delta}{\eta}x}{r} - \frac{x}{\eta} \tag{44}$$

or

$$B = \frac{1}{r}x\left[\frac{r+\delta}{\eta\,(1-\beta)} - \frac{\delta}{\eta}\right] - \frac{x}{\eta} \tag{45}$$

Symmetric equilibrium III

Demand for intermediates (39) gives x.

Now we have 3 equations in (q, r, x):

1. Demand for intermediates (39)
2. Marginal revenue: (37)
3. Free entry (45)

These could, in principle, be solved for the equilibrium values.

Reading

- Acemoglu (2009), ch. 13.
- Romer (2011), ch. 3.1-3.4.
- Jones (2005)

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