

# Knowledge Spillovers and Scale Effects

Prof. Lutz Hendricks

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# Issues

- ▶ What happens when innovation takes labor (a non-reproducible factor)?
- ▶ Then we need a knowledge spillover to sustain growth.
- ▶ It takes some tricks to prevent the model from exhibiting explosive growth.

## 2. Knowledge Spillovers

## Ideas Produced From Labor

The previous model had endogenous growth because ideas were produced with constant return from a **reproducible factor**: ideas (embodied in goods).

If ideas are produced from (non-reproducible) labor: there is no sustained growth.

### Example

Assume  $\dot{N}_t = \eta Z_t^\alpha L_{Rt}^{1-\alpha}$ . Show that the balanced growth rate is 0 unless  $\alpha = 1$ .

# Knowledge Spillovers

We need a mechanism that offsets diminishing returns to ideas in the production of ideas.

Knowledge spillover:  $N$  appears in the innovation production function for  $N$ .

This is an **externality**: firms do not pay for the  $N$  input.

This is possible because  $N$  is non-rival.

The idea: "standing on the shoulders of giants"

Problem: A **knife-edge** parameter assumption is needed for endogenous growth.

- ▶ Some parameters must sum to 1.
- ▶ This is always true because we need **constant returns to reproducible factors**.

## Knowledge spillover model

Keep everything the same, except the production of ideas:

$$\dot{N}_t = \eta N_t L_{Rt} \quad (1)$$

We show later: linearity in  $N$  is required for endogenous growth.

Labor now has 2 uses:

- ▶ produce goods:  $L_E$
- ▶ produce ideas:  $L_R$

Resource constraint:

$$L = L_{Rt} + L_{Et} \quad (2)$$

Note: this does not change the problems of household, final goods firms, or intermediate input firms.

## Balanced growth rate

Euler equation is still:  $g(C) = (r - \rho)/\theta$ .

Interest rate is determined by free entry:  $V = \pi/r$ .

But now the cost of creating a new patent is different:

$$\eta N_t V_t = w_t \quad (3)$$

- hire a unit of labor and produce a flow of  $\eta N_t$  patents per “period”

## Balanced growth rate

Wage rate (unchanged):

$$w_t = \frac{\beta}{1 - \beta} N_t \quad (4)$$

Profits earned by monopolists (unchanged):

$$\pi = \beta L_E \quad (5)$$

Sub wage rate into free entry:

$$\eta N_t \frac{\beta L_E}{r} = w = \frac{\beta}{1 - \beta} N_t \quad (6)$$

$\Rightarrow$

$$r^* = (1 - \beta) \eta L_E^* \quad (7)$$

Intuition ...



## Balanced growth rate

Euler equation (unchanged):

$$g^* = g(C) = \frac{(1 - \beta) \eta L_E^* - \rho}{\theta} \quad (8)$$

Almost done - just need to find  $L_E$ .

Balanced growth requires

$$g(C) = g(Y) = g(N) \quad (9)$$

Ideas production function:

$$g(N) = \eta L_R^* = \eta (L - L_E^*) \quad (10)$$

## Balanced growth

Solve for the growth rate.

$$\begin{aligned}g(C) &= \frac{(1-\beta)L_E^* - \rho}{\theta} \\ &= \eta(1-L_E^*)\end{aligned}$$

Intuition ...



$$L_E^* = \frac{\theta\eta L + \rho}{(1-\beta)\eta + \theta\eta} \quad (11)$$

Scale effects: larger economies grow faster.

With population growth, output growth explodes.

### 3. Growth without scale effects

- ▶ The previous models do not have balanced growth paths when there is population growth.
- ▶ The reason is the scale effect:
  - ▶ Larger population  $\rightarrow$  more R&D  $\rightarrow$  faster growth.
- ▶ Diminishing returns to reproducible factors avoid the scale effect, but also kill endogenous growth.

## Growth without scale effects

To avoid scale effects, modify the model as follows.

Innovation:

$$\dot{N}_t = \eta N_t^\phi L_{Rt} \quad (12)$$

$$0 < \phi \leq 1 \quad (13)$$

Demographics:

$$L_t = e^{nt} \quad (14)$$

$$= L_{Rt} + L_{Et} \quad (15)$$

## Balanced growth

From the innovation technology:

$$g(N) = \eta N_t^{\phi-1} L_{Rt} \quad (16)$$

Constant growth requires constant  $N^{\phi-1} L_R$  and

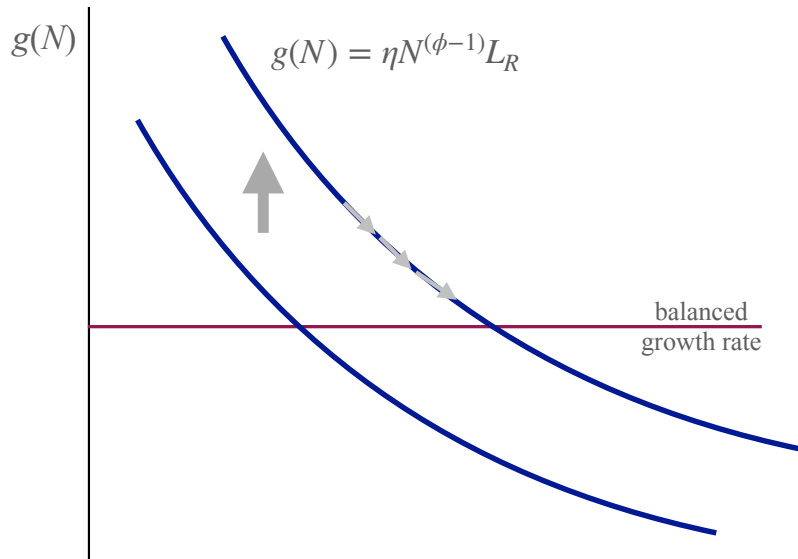
$$g(N) = \frac{n}{1-\phi} \quad (17)$$

The growth rate is "**semi-endogenous**:" endogenous, but not responding to changes in agents' choice variables.

There are still scale effects:

- Larger economies tend towards higher levels of output per person.

## Balanced growth



## Avoiding scale effects

It is possible to write down models that have endogenous growth, but no scale effects (growth does not increase with  $L$ ).

The idea: Prevent innovator profits from increasing with  $L$ .

One approach: the number of products increases with  $L$  exactly so that the market size for each variety remains the same (Young, 1998).

Avoiding scale effects requires knife-edge assumptions like this.

## Reading

- ▶ Acemoglu (2009), ch. 13.
- ▶ Romer (2011), ch. 3.1-3.4.
- ▶ Jones (2005)



## References I

- Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.
- Jones, C. I. (2005): "Growth and ideas," *Handbook of economic growth*, 1, 1063–1111.
- Romer, D. (2011): *Advanced macroeconomics*, McGraw-Hill/Irwin.
- Young, A. (1998): "Growth without scale effects," *The Journal of Political Economy*, 106, 41.