Applying the Solow Model
Part 2

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Non-renewable Resources

Non-renewable Resources

What happens when production uses essential resources that are in **fixed supply**?

- oil, coal, rare metals, ...

Does the economy eventually run out of resources?

Does growth come to a halt?

Model with Non-renewables

Modify the Solow model as follows:

1. The economy is endowed with a resource stock R_0.
2. Over time, oil is extracted at rate E_t (barrels/year).
3. Each barrel extracted reduces the remaining stock:

$$\dot{R}(t) = -E(t) \tag{1}$$

4. The fraction of the stock R extracted each period is a constant s_E:

$$E(t) = s_E R(t) \tag{2}$$

5. E is used in production:

$$Y(t) = B(t) K(t)^{\alpha} E(t)^{\gamma} L(t)^{1-\alpha-\gamma} \tag{3}$$

Everything else is unchanged.

The Solow Law of Motion

$\dot{R} = -E = -s_E R$ implies constant decline of the stock:

$$R(t) = R_0 e^{-s_E t} \tag{4}$$

- To prove this: differentiate to that $\dot{R} = s_E R$.

Therefore, resource input is declining exponentially:

$$E(t) = s_E R(t) = s_E R_0 e^{-s_E t} \tag{5}$$

In the limit, $E(t) \to 0$, which does not look promising.

Balanced Growth Path

Output is given by

$$Y = BK^{\alpha}E^{\gamma}L^{1-\alpha-\gamma} \tag{6}$$

Take growth rates:

$$g(Y) = g(B) + \alpha g(K) + \gamma g(E) + (1-\alpha-\gamma)g(L) \tag{7}$$

From $\dot{K} = sY - \delta K$, it follows that K/Y converges to a constant.

- Therefore, $g(K) = g(Y)$.

We showed that $g(E) = -s_E$ and $g(L) = n$.

Therefore:

$$(1-\alpha)g(Y) = g(B) - \gamma s_E + (1-\alpha-\gamma)n \tag{8}$$

Balanced Growth Path

Or in per capita terms:

$$g(y) = g(Y) - n \tag{9}$$
$$= \frac{g(B)}{1-\alpha} - \frac{\gamma}{1-\alpha} + \frac{1-\alpha-\gamma}{1-\alpha}n - n \tag{10}$$
$$= \frac{g(B)}{1-\alpha} - \frac{\gamma}{1-\alpha}(s_E + n) \tag{11}$$

In words:

- Output grows due to productivity growth $g(B)$.
- There is a drag due to population growth and resource depletion.

Note: Even if $s_E > 0$ (resource extraction grows over time), we still have a drag term just due to population growth.

Intuition

Consider $s_E = 0$ – resource extraction is constant over time.

Output per worker:

$$
\begin{aligned}
y &= (BE^{\gamma}) K^{\alpha} L^{1-\alpha-\gamma}/L && (12) \\
&= (BE^{\gamma}) k^{\alpha} L^{-\gamma} && (13)
\end{aligned}
$$

This looks like a Solow model with

- productivity BE^{γ} – this still grows at rate $g(B)$
- diminishing returns to scale: capital share + labor share < 1
    - $\alpha + (1-\alpha-\gamma) = 1-\gamma < 1$
- even with constant E this causes a drag on growth

Intuition

We can make this look more like a Solow model.

$$y = \left(B(E/L)^{\gamma}\right) k^{\alpha} \tag{14}$$

Now we exactly have a Solow model, but with productivity $B(E/L)^{\gamma}$.

Productivity grows at rate

$$g\left(B(E/L)^{\gamma}\right) = g(B) - \gamma(s_E + n) \tag{15}$$

Therefore: **non-renewable resources have the same effect as slower productivity growth**.

How Big Is the Drag on Growth?

We need parameter values for α, γ, s_E.

Empirical estimates (Nordhaus et al., 1992):

- $\alpha = 1/3$ is the capital share (as before);
- $\gamma = 0.1$ is the share of renewables.
- $n = 0.01$ is negligible
- $s_E = 1/200$: 0.5% of the stock is extracted each year.

The growth drag is then

$$\frac{(\gamma + n) s_E}{1 - \alpha} = \frac{0.11/200}{2/3} = 0.3\% \tag{16}$$

Why is this so small?

Resource Prices

If this model is correct, the relative price of resources should rise over time.

Intuition:

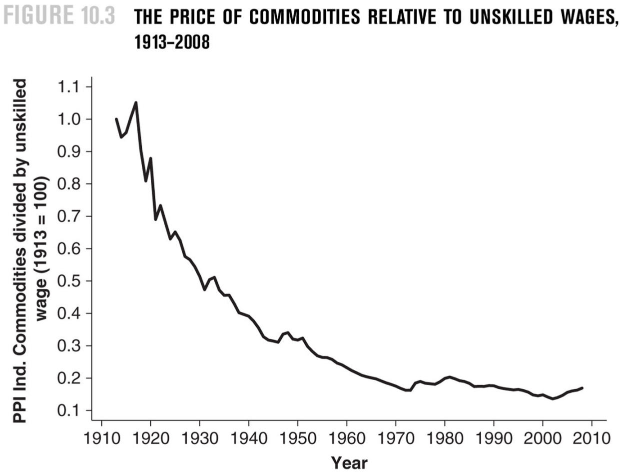
- resource input (demand) is falling as the economy is growing
- this can only happen as resource prices are rising

Derivation:

- the income share of resources is constant: $\gamma = P_E E/Y$
- labor share: $1-\alpha = wL/Y$
- ratio: $\gamma/(1-\alpha) = (P_E E)/(wL)$ should be constant
- E/L is falling, so P_E/w should be rising

Evidence: resource prices are **falling** instead.

Resource Prices



Source: Jones (2013)

Implication: the share of non-renewables γ must be falling over time.

Why Is the Renewables Share Declining?

One possibility: renewables and other inputs are **highly substitutable**.

- using less E then reduces its income share (its price does not rise much)
- that would imply a smaller drag on growth (E is not "essential" in production).
- but then its price has been falling, not rising

Resource conserving technical change

- even though E declines over time, its efficiency rises
- directed technical change

Conclusion:

- the direct growth drag from non-renewables is not likely large

Discussion

What is missing in this discussion?

The End of Economic Growth?

The Issues

We discuss the claims made in Frey (2015): "How to Prevent the End of Economic Growth"

What does the article claim?

Proposed policy solutions

1. Support investment in labor intensive industries (!)
2. Redistribute income to raise aggregate demand
3. Encourage more entrepreneurial risk taking (how does that fit in?)

A Solow Interpretation

Innovations raise productivity (presumably, which is why they are worth a lot).

- A rises.

But the additional income accrues to neither capital nor labor.

- it goes to innovators
- their saving rate is high

Defer concerns about aggregate demand (this is a long-run model)

A modified Solow model

There is a new input X that represents innovation

$$Y = AX^{1-\beta}K^{\beta\alpha}L^{\beta(1-\alpha)} \tag{17}$$

or

$$y = Y/L = Ax^{1-\beta}k^{\beta\alpha} \tag{18}$$

Early innovation: A rises; $1-\beta = 0$.

"New economy:" A rises; $1-\beta$ rises.

- x gets a larger income share.

Law of motion

Capital accumulation is unchanged $\dot{k} = sy - (n+\delta)k$

- This fixes steady state $k/y = s/(n+\delta)$.

Production function:

$$y/k = Ax^{1-\beta}k^{\beta\alpha-1} \tag{19}$$

Steady state capital stock:

$$k^{1-\alpha\beta} = Ax^{1-\beta}\frac{s}{n+\delta} \tag{20}$$

Factors are paid marginal products

As always with Cobb-Douglas: factor income shares are constant

- capital gets $\beta\alpha$
- labor gets $1-\beta\alpha$
- x gets the rest: $1-\beta$

Details: factor shares

- Labor:

$$
\begin{aligned}
w &= \beta(1-\alpha)y && (21)\\
&= \beta(1-\alpha)(y/k)k && (22)
\end{aligned}
$$

- Capital gets share $\beta\alpha$:

$$q = \beta\alpha y/k \tag{23}$$

- x gets share $1-\beta$:

$$p = (1-\beta)(y/k)k/x \tag{24}$$

"Old fashioned" innovation

A rises by factor $\lambda > 1$

β unchanged.

Implications:

- k/y unchanged
- k rises by $\lambda^{1/(1-\alpha\beta)}$
  - from the steady state k solution
- w and p and y do the same
  - from the factor price equations
- q unchanged

“New economy:” Lower β

To focus on redistributional effect: adjust A so that y unchanged

k/y unchanged

Then k unchanged

w, q fall;

p rises

Redistribution of income from factors to x

Combined Effect

"New economy:" A rises while income is redistributed from factors to x (β falls).

- or: A is constant, but X rises due to innovation (at the same time β falls)
- we probably don't have the right production function for that!

x owners (innovators) get richer.

Wages: may stagnate, even though output rises

- labor share declines (true in the data!)

Investment

- marginal product of capital q falls
- I/Y may fall (but then c/y would have to rise!)

Policy implications

What has changed relative to old-fashioned A growth?

Should we subsidize labor intensive industries?

Policy implications

A key idea of economic policy

Separate redistribution from efficiency
If you want to redistribute income, use transfers, not subsidies.

One additional concern:

What if the marginal product of some workers falls so much to make them unemployable?

Automation

Automation has replaced "routine" jobs.

Figure 6. Employment Growth Has Polarized Between High- and Low-Paid Occupations

CHANGES IN OCCUPATIONAL EMPLOYMENT SHARES AMONG WORKING-AGE ADULTS, 1980–2015

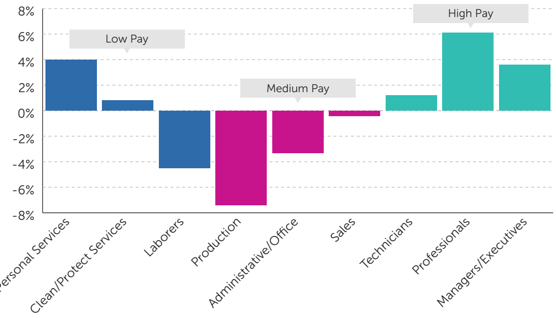
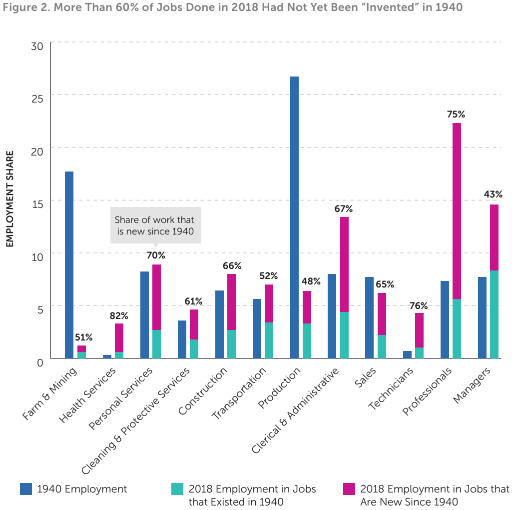


Figure is constructed using U.S. Census of Population data for 1980, 1990, and 2000, and pooled American Community Survey (ACS) data for years 2014 through 2016, sourced from IPUMS (Ruggles et al., 2018). Sample includes working-age adults ages 16 – 64 excluding those in the military. Occupational classifications are harmonized across decades using the classification scheme developed by Dorn (2009).

Source: Autor (2020)

Automation also creates new jobs



Source: Autor (2020)

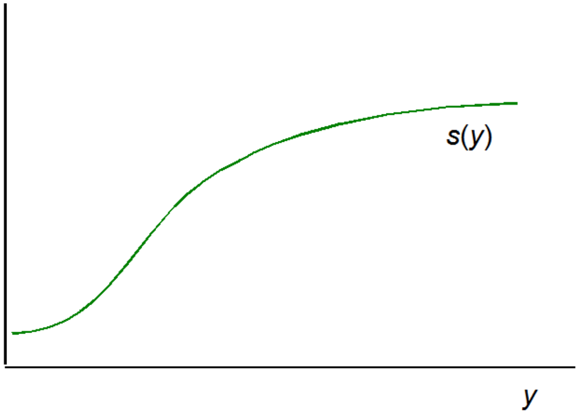
What does the future hold?

We don't know.

> *"No economic law dictates that the creation of new work must equal or exceed the elimination of old work. Still, history shows that they tend to evolve together." – Autor (2020), p. 12*

Exercise: The Saving Rate Depends on Income

- Consider an alternative version of the Solow model.
- The saving rate depends on income.
- What happens?



Conclusion: Is the Solow Model Useful?

- As a model of growth or large cross-country income differences, the model is a failure.
- But its failure contains important insights:
  1. Capital does not drive growth.
  2. Capital does not drive large fractions of cross-country income gaps.
- Both findings are surprising - and often not understood in the policy debate.

Conclusion: Is the Solow Model Useful?

- But the main significance of the Solow model itself is as a **building block** for macro models.
- We always have to keep track of how capital is accumulated.
- A Solow block is therefore part of virtually every model.
- The same logic extends to other accumulated factors: human capital, knowledge capital, organization capital.
- The Solow transition dynamics is an important piece for understanding business cycle dynamics.

Reading

- Non-renewable resources: Jones (2013), ch. 10.
- Frey (2015)

References I

Autor, D. (2020): "The Work of the Future," Tech. rep., MIT Work of the Future Task Force.

Frey, C. B. (2015): "How to Prevent the End of Economic Growth," *Scientific American*.

Jones, Charles; Vollrath, D. (2013): *Introduction To Economic Growth*, W W Norton, 3rd ed.

Nordhaus, W. D., R. N. Stavins, and M. L. Weitzman (1992): "Lethal model 2: the limits to growth revisited," *Brookings Papers on Economic Activity*, 1–59.