

Growth Through Product Creation

Part 2

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Econ720

December 5, 2025

1. Pareto Efficient Allocation

Efficiency

Two distortions prevent efficiency of equilibrium:

1. Monopoly pricing \Rightarrow high profits \Rightarrow too much innovation.
2. Knowledge spillover \Rightarrow too little innovation

Planner's Problem

Resource constraint:

$$Y = C + X + Z \quad (1)$$

$$\underbrace{Y - X}_{\text{net income}} = \underbrace{C + \dot{N}/\eta}_{\text{useful spending}} \quad (2)$$

Solve in two stages:

1. Given N , find optimal static allocation $x(v, t)$.
 - ▶ That is: maximize $Y - X$ which is available for consumption and investment.
 - ▶ An odd feature of the model: goods are produced from goods without delay.
2. Given the reduced form production function from #1, find optimal Z .

Static Allocation

Given N , choose $x(v, t)$ to maximize $Y - X$:

$$\max (1 - \beta)^{-1} L^\beta \int_0^{N_t} x(v, t)^{1-\beta} dv - \int_0^{N_t} \psi x(v, t) dv \quad (3)$$

L is fixed.

First-order condition

$$L^\beta x^{-\beta} = \psi \quad (4)$$

with $\psi = 1 - \beta$:

$$x = (1 - \beta)^{-1/\beta} L \quad (5)$$

The planner's x is larger than the equilibrium x (Intuition?)

Static Allocation

Next: find $Y - X$.

$$X = \underbrace{\psi N x}_{\text{symmetry}} = \underbrace{(1 - \beta) N}_{\psi} \underbrace{(1 - \beta)^{-1/\beta} L}_x \quad (6)$$

Reduced form production function:

$$Y_t = (1 - \beta)^{-1} L^\beta N [(1 - \beta)^{1-1/\beta} L]^{1-\beta} \quad (7)$$

$$= (1 - \beta)^{-1/\beta} L N_t \quad (8)$$

Net output

$$\begin{aligned} Y - X &= (1 - \beta)^{-1/\beta} L N - (1 - \beta)^{1-1/\beta} L N \\ &= (1 - \beta)^{-1/\beta} \beta L N \end{aligned} \quad (9)$$

Planner: Dynamic Optimization

$$\max \int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\theta} - 1}{1-\theta} dt$$

subject to

$$\begin{aligned}\dot{N} &= \eta Z \\ Y - X &= (1-\beta)^{-1/\beta} \beta L N = C + Z\end{aligned}$$

Or

$$\dot{N} = A N - \eta C \quad (10)$$

$$A = \eta (1-\beta)^{-1/\beta} \beta L \quad (11)$$

Hamiltonian

$$H = \frac{C^{1-\theta} - 1}{1-\theta} + \mu [AN - \eta C] \quad (12)$$

FOC

$$\partial H / \partial C = C^{-\theta} - \mu \eta = 0 \quad (13)$$

$$\partial H / \partial N = \rho \mu - \dot{\mu} = \mu A \quad (14)$$

Optimal growth

The same as in an AK model with

$$A = \eta (1 - \beta)^{-1/\beta} \beta L \quad (15)$$

we have

$$\dot{C}/C = \frac{A - \rho}{\theta} \quad (16)$$

Comparison with CE

- ▶ CE interest rate: $\eta\beta L$.
- ▶ Planner's "interest rate:" $(1 - \beta)^{-1/\beta} \eta\beta L$.
- ▶ The planner chooses faster growth.
- ▶ Intuition:
 - ▶ CE under-utilizes the fruits of innovation: x is too low.
 - ▶ This reduces the value of innovation.

Policy Implications

One might be tempted to reduce monopoly power.

- ▶ A policy that encourages competition (e.g. less patent protection, forcing lower p^x) reduces the static price distortion.
- ▶ But it also reduces growth: innovation is less valuable.

Similar result for shorter patents.

Policy trades off static efficiency and incentives for innovation.

2. Example: Durable Intermediate Inputs

Environment

We study an example where intermediates are durable (the model has capital).

Unchanged relative to previous model:

- ▶ demographics
- ▶ preferences
- ▶ endowments
- ▶ final goods technology
- ▶ innovation technology

Technologies: Intermediates

- ▶ Upon invention, the inventor is endowed with $x_0 = 0$ units of $x(v)$.
- ▶ Additional units are accumulated according to

$$\dot{x}(v, t) = \omega I(v, t)^\varphi - \delta x(v, t) \quad (17)$$

- ▶ $0 < \varphi < 1$
- ▶ Diminishing returns imply smooth adjustment of x over time.
- ▶ Intermediates are *rented* to final goods firms at price $q(v, t)$.
- ▶ Total input of final goods: $X_t = \int_0^{N_t} I(v, t) dv$

Market arrangements

Markets:

- ▶ Final goods: price 1
- ▶ Labor: w_t
- ▶ Intermediate input rental: $q(v, t)$

Each intermediate input producer has a permanent monopoly for his variety.

Free entry into the market for innovation

2.1. Agents' Problems

Unchanged:

- ▶ Household
- ▶ Final goods firm
- ▶ Free entry of innovator

Changed:

- ▶ Intermediate goods firm

Intermediate input producer

Now a truly dynamic problem (v index suppressed)

$$V_t = \max \int_t^\infty e^{-r\tau} [R(x(\tau)) - I(\tau)] d\tau$$

subject to

$$\dot{x} = \omega I^\varphi - \delta x \quad (18)$$

where $R(x)$ is the revenue from renting out x .

Revenue

Final goods firm's demand (unchanged):

$$q(x) = L^{\beta} x^{-\beta} \quad (19)$$

Revenue:

$$R(x) = q(x)x \quad (20)$$

$$= L^{\beta} x^{1-\beta} \quad (21)$$

Marginal revenue:

$$R'(x) = (1 - \beta) L^{\beta} x^{-\beta} \quad (22)$$

$$= (1 - \beta) q(x) \quad (23)$$

Intermediate input producer

Hamiltonian:

$$H = R(x) - I + \mu [\omega I^\varphi - \delta x] \quad (24)$$

FOCs:

$$\begin{aligned} \partial H / \partial I &= -1 + \mu \omega \varphi I^{\varphi-1} = 0 \\ \dot{\mu} &= (r + \delta) \mu - R'(x) \end{aligned}$$

Intuition...

Solution: $\{I_t, x_t, \mu_t\}$ that solve 2 FOCs and law of motion for x .

Boundary conditions:

- ▶ $x(0)$ given,
- ▶ $\lim_{t \rightarrow \infty} e^{-rt} \mu_t x_t = 0$.

Free entry of innovators

Technology (unchanged):

$$\dot{N} = \eta Z \quad (25)$$

Free entry:

- ▶ Spend $1/\eta$ for period dt to obtain $dN = \eta/\eta dt$ new patents worth $V dt$.
- ▶ Equate cost and profits:

$$1/\eta = V \quad (26)$$

2.2. Equilibrium

Objects: $\{q(v, t), x(v, t), N_t, I(v, t), \mu(v, t), y_t, L_t, r_t, c_t, w_t\}$

Equilibrium conditions:

- ▶ Household: Euler (1)
- ▶ Final goods firm: 3
- ▶ Intermediate goods firm: 3
- ▶ Free entry:

$$1/\eta = V = \int e^{-rt} [R(x_t) - I_t] dt \quad (27)$$

where R defined above

- ▶ Market clearing

Market clearing

1. Final goods: Resource constraint or $Y = C + NI + \dot{N}/\eta$.
2. Intermediates: implicit in notation.
3. Labor: $L = 1$.
4. Asset markets: suppressed (details not specified)

Applications

Designing optimal patents

- ▶ to trade off static costs vs innovation benefits of patents

Designing incentives for innovation

- ▶ Akcigit et al. (2021)

Implications of tax policies

- ▶ Akcigit et al. (2022), Jones (2019)

Implications for the future / sustainability of growth

- ▶ Jones (2022)

Reading

- ▶ **Macroeconomics**, ch. 13.5
- ▶ Acemoglu (2009), ch. 13.
- ▶ Krusell (2014), ch. 9
- ▶ Romer (2011), ch. 3.1-3.4.
- ▶ Jones (2005)

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