Applying the Solow Model
Part 2

Prof. Lutz Hendricks

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Non-renewable Resources

Non-renewable Resources

What happens when production uses essential resources that are in **fixed supply**?

- oil, coal, rare metals, ...

Does the economy eventually run out of resources?

Does growth come to a halt?

Model with Non-renewables

Modify the Solow model as follows:

1. The economy is endowed with a resource stock R_0.
2. It digs up R at a rate of E:

$$\dot{R} = -E \tag{1}$$

3. The rate of extraction is constant:

$$E = s_E R \tag{2}$$

4. E is used in production:

$$Y = BK^{\alpha}E^{\gamma}L^{1-\alpha-\gamma} \tag{3}$$

Everything else is unchanged

The Solow Law of Motion

$\dot{R} = -E = -s_E R$ implies

$$R(t) = R_0 e^{-s_E t} \tag{4}$$

The stock is depleted at a constant exponential rate.

- to prove this: differentiate to find $\dot{R}$

Therefore, resource input is declining exponentially:

$$E(t) = s_E R(t) = s_E R_0 e^{-s_E t} \tag{5}$$

In the limit, $E(t) \to 0$, which does not look promising

Balanced Growth Path

From $\dot{K} = sY - \delta K$, it follows that K/Y converges to a constant. Output is given by

$$Y^{1-\alpha} = B(K/Y)^{\alpha} \underbrace{\left(s_E R_0 e^{-s_E t}\right)^{\gamma}}_{E} L^{1-\alpha-\gamma} \tag{6}$$

Take growth rates:

$$(1-\alpha)\,g(Y) = g(B) - \gamma s_E + (1-\alpha-\gamma)\,n \tag{7}$$

Or in per capita terms:

$$g(y) = \frac{g(B)}{1-\alpha} - \frac{\gamma}{1-\alpha}(s_E + n) \tag{8}$$

Interpretation: faster resource extraction permantly slows down growth.

Intuition

Output per worker is

$$y = Bk^{\alpha} (E/L)^{\gamma} \tag{9}$$

Population growth has the same effect as in the Solow model: capital dilution.

E shows up as negative productivity growth

$$y = \left(B(E/L)^{\gamma}\right) k^{\alpha} \tag{10}$$

with growth rate of the productivity term given by

$$g\left(B(E/L)^{\gamma}\right) = g(B) - \gamma(s_E + n) \tag{11}$$

Therefore: non-renewable resources have the same effect as slower productivity growth.

How Big Is the Drag on Growth?

We need parameter values for α, γ, s_E.

Key assumption: factors (including E) are paid their marginal products.

Then: α is the capital share (as before); γ is the share of renewables.

Empirical estimates (Nordhaus et al., 1992):

- $\alpha = 0.2$
- $\gamma = 0.1$
- there is also a fixed factor (land) with a share of 0.1.
- $s_E = 1/200$
- $n = 0.01$

The growth drag is then

$$\frac{0.1n + (\gamma + n)\, s_E}{1 - \alpha} = 0.3\% \tag{12}$$

Resource Prices

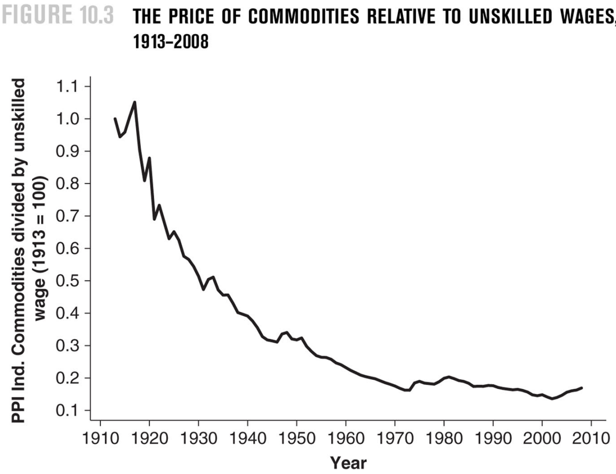
If this model is correct, the relative price of resources should rise over time.

Intuition:

- the income share of resources is constant: $\gamma = P_E E / Y$
- labor share: $1 - \alpha = wL/Y$
- ratio: $\gamma / (1 - \alpha) = (P_E E) / (wL)$ should be constant
- E/L is falling, so P_E/w should be rising

Evidence: resource prices are **falling** instead.

Resource Prices



Source: Jones (2013)

Implication: the share of renewables γ must be falling over time.

Why Is the Renewables Share Declining?

One possibility: renewables and other inputs are **highly substitutable**.

- using less E then reduces its income share (its price does not rise much)
- buth then its price has been falling, not rising

Resource conserving technical change

- even though E declines over time, its efficiency rises
- directed technical change

Conclusion:

- the direct growth drag from non-renewables is not likely large

Discussion

What is missing in this discussion?

The End of Economic Growth?

The Issues

We discuss the claims made in Frey (2015): "How to Prevent the End of Economic Growth"

What does the article claim?

Proposed policy solutions

1. Support investment in labor intensive industries (!)
2. Redistribute income to raise aggregate demand
3. Encourage more entrepreneurial risk taking (how does that fit in?)

A Solow Interpretation

Innovations raise productivity (presumably, which is why they are worth a lot).

- A rises.

But the additional income accrues to neither capital nor labor.

- it goes to innovators
- their saving rate is high

Defer concerns about aggregate demand (this is a long-run model)

A modified Solow model

There is a new input X that represents innovation

- $Y = AX^{1-\beta}K^{\beta\alpha}L^{\beta(1-\alpha)}$

Capital accumulation is unchanged $\dot{k} = sy - (n+\delta)k$

- This fixes steady state $k/y = s/(n+\delta)$.
- Steady state $k^{1-\alpha\beta} = \left(sAx^{1-\beta}\right)/(n+\delta)$
  - from $k = \frac{s}{n+\delta}Ax^{1-\beta}k^{\alpha\beta}$

Factors are paid marginal products

$$w = \beta(1-\alpha)y \tag{13}$$
$$= \beta(1-\alpha)(y/k)k \tag{14}$$

Capital gets share $\beta\alpha$:

$$q = \beta\alpha y/k \tag{15}$$

x gets share $1-\beta$:

$$p = (1-\beta)(y/k)k/x \tag{16}$$

Innovation

A rises by factor $\lambda > 1$

k/y unchanged

k rises by $\lambda^{1/(1-\alpha\beta)}$

- from the steady state k solution

w and p and y do the same

- from the factor price equations

q unchanged

Lower β

To focus on redistributional effect: adjust A so that y unchanged

k/y unchanged

Then k unchanged

w, q fall;

p rises

Redistribution of income from factors to x

Combined Effect

"New economy:" A rises while income is redistributed from factors to x (β falls).

- or: A is constant, but X rises due to innovation (at the same time β falls)
- we probably don't have the right production function for that!

Can get stagnant wages, even though output rises

At the same time, the x owners (innovators) get richer.

If investment reponds to falling q, I/Y may falll (but then c/y would have to rise!)

Policy implications

What has changed relative to old-fashioned A growth?

Should we subsidize labor intensive industries?

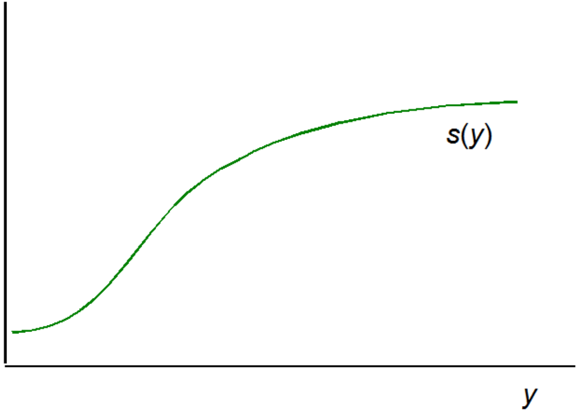
Policy implications

A key idea of economic policy

Separate redistribution from efficiency
If you want to redistribute income, use transfers, not subsidies.

Exercise: The Saving Rate Depends on Income

- Consider an alternative version of the Solow model.
- The saving rate depends on income.
- What happens?



Conclusion: Is the Solow Model Useful?

- As a model of growth or large cross-country income differences, the model is a failure.
- But its failure contains important insights:
    1. Capital does not drive growth.
    2. Capital does not drive large fractions of cross-country income gaps.
- Both findings are surprising - and often not understood in the policy debate.

Conclusion: Is the Solow Model Useful?

- But the main significance of the Solow model itself is as a **building block** for macro models.
- We always have to keep track of how capital is accumulated.
- A Solow block is therefore part of virtually every model.
- The same logic extends to other accumulated factors: human capital, knowledge capital, organization capital.
- The Solow transition dynamics is an important piece for understanding business cycle dynamics.

Reading

- Non-renewable resources: Jones (2013), ch. 10.
- Frey (2015)

References I

Frey, C. B. (2015): "How to Prevent the End of Economic Growth," *Scientific American*.

Jones, Charles; Vollrath, D. (2013): *Introduction To Economic Growth*, W W Norton, 3rd ed.

Nordhaus, W. D., R. N. Stavins, and M. L. Weitzman (1992): "Lethal model 2: the limits to growth revisited," *Brookings Papers on Economic Activity*, 1–59.