## Modern Macro

Prof. Lutz Hendricks

Econ720

August 6, 2020

## What Econ720 is about

Macro is built around a small number of workhorse models:

- 1. Overlapping generations
- 2. "Standard growth" in continuous and discrete time
- 3. Stochastic growth
- 4. Endogenous growth
- 5. Search and matching models

We study basic versions of the **models** and the **tools** needed to analyze them.

## What is not covered

Computational issues

see Econ821 (sometimes)

Empirical issues

How to take models to data?

We will talk about empirical applications later in the semester (and in Econ721).

## Modern Macro

(Special Advertisement Section)

Or:

Why Most of Your Undergraduate Macro Courses Were Useless

## Modern macro

Let's start by talking about how macroeconomists approach questions.

The main point is: *Macro is micro.* 

## Macro Before Lucas

## Goods market primitives:

- ► Consumption function:  $C = C_0 + cY$ .
- ▶ Investment function:  $I = I_0 bi$ .
- ldentity: Y = C + I + G.

IS curve:

$$(1-c)Y = C_0 + I_0 + G - bi$$

## Money market primitives:

- Money demand:  $L = L_0 + kY di$ .
- ▶ Money supply: M/P.

#### LM curve:

$$M/P = L_0 + kY - di$$

# Key Features

## Aggregate relationships are taken as primitives

such as the demand for goods or money

The **parameters** inside the aggregate relationships are taken as primitives

- ightharpoonup such as  $C_0$  or the marginal propensity to consume c
- these parameters are thought of as (more or less) constant

## Expectations are taken as given

▶ in dynamic versions of the model

# What is Missing?

- 1. Constraints: Where are the budget constraints of consumers and the government?
- Responses to changing government policies:
   E.g., consumption should depend on government debt.
- Expectations
   What happens in the future matters for today's decisions.
   E.g., saving depends on the solvency of social security.
- 4. Expectations have to be consistent with actions

  This cannot be right!

## Modern Macro

Modern macro builds models bottom-up (micro-foundations).

A model is an artificial economy.

- Agents interact in markets.
- Aggregate outcomes result from individual decisions.

The primitives are the "physical" environment and agents' preferences.

## Model Primitives

An economy is described by

- the list of agents,
- their demographics,
- their preferences,
- their endowments,
- the technologies they have access to
- the markets in which they can trade.

Important note: every model description should start with these elements.

You are not allowed to analyze anything until you have described these model elements.

# How agents behave

Individual behavior is the result of an optimization problem.

• e.g., maximize utility subject to budget constraints

Agents have rational expectations.

- ▶ They understand how the economy works.
- ► Their expectations are the best possible forecasts.

# Digression

Are people really this rational?

# Digression

What is economics?

## Competitive Equilibrium

Once we put all the pieces together and let agents interact in markets, we get a Competitive Equilibrium.

A key skill we will learn:

How to translate the description of an economy into a set of equations that characterize the **competitive equilibrium**.

## Definition

A competitive equilibrium is an **allocation** (a list of quantities) and a **price system** (a list of prices) such that

- the quantities solve all agents' problems, given the prices;
- all markets clear.

# How to Set Up a Competitive Equilibrium

## Steps:

- 1. Describe the economy
- 2. Solve each agent's problem
- 3. State the market clearing conditions
- 4. Define an equilibrium

All of this is really mechanical.

The hard part is to say something about what the equilibrium looks like.

(Ask your computer.)

- 1. List the agents (households, firms).
- 2. For each agent define:
  - **Demographics**: e.g., population grows at rate n.
  - **Preferences**: e.g., households maximize utility u(c).
  - ► **Endowments**: e.g., each household has one unit of time each period.
  - **Technologies**: e.g., output is produced using f(k).
- 3. Define the **markets** in which agents interact.
  - E.g., households work for firms; households purchase goods from firms.

# Step 2: Solve Each Agent's Problem

Write down the maximization problem each agent solves.

Derive a set of equations that determine the agent's choice variables.

- E.g.: A consumption function, saving function.
- ► We call these **policy functions** or **decision rules**.

## Example

A household chooses *c* and *s* to maximize utility, subject to a budget constraint.

Policy rules: c = f(y, p) and s = g(y, p)

# Step 3: Market Clearing

For each market, calculate supply and demand by each agent.

- Aggregate supply =  $\sum$  individual supplies.
- ► Aggregate demand =  $\sum$  individual demands.

Market clearing is simply:

Aggregate supply = aggregate demand.

## Tip

The market clearing condition for apples contains only quantities of apples.

▶ If there are prices or bananas, it's wrong.

# Step 4: Define the Equilibrium

From steps 2-3:

Collect all endogenous objects

e.g., consumption, output, wage rate, ...

Collect all equations

- first order conditions or policy functions
- market clearing conditions

You should have N equations that could (in principle) be solved for N endogenous objects

- prices
- quantities (the allocation)

# What do we gain from this approach?

## Consistency:

- Aggregate relationships by construction satisfy individual constraints.
- Example: the aggregate consumption function cannot violate any person's budget constraint.

## Transparency:

► The assumptions about the fundamentals are clearly stated.

# What do we gain from this approach?

## Non-arbitrary behavior:

- ▶ In old macro, results depend on the assumed behavior.
- ▶ In modern macro, behavior is derived.

## **Expectations**:

- Expectations are endogenous.
- They are automatically consistent with the way the economy behaves.

# What do we gain from this approach?

#### Welfare:

▶ It is possible to figure out how a policy change affects the welfare (utility) of each agent.

## Testing:

▶ Models can be tested against micro data.

Micro and macro become the same thing.

# Static example

# Static Example

- We study a very simple one period economy.
- There are many identical households.
- ► They receive **endowments** which they eat in each period.
- Nothing interesting happens in this economy it merely illustrates the method.

## Demographics:

- ► There are N identical households.
- ► They live for one period.
- For now, there are no other agents (firms, government, ...).

#### Preferences:

▶ Households value consumption of two goods according to a utility function  $u(c_1,c_2)$ 

#### **Endowments:**

▶ Each agent receives endowments of the two goods  $(e_1, e_2)$ .

## Technology:

- There is no production. Endowments cannot be stored.
- Resource constraint:  $Ne_1 = Nc_1$  and  $Ne_2 = Nc_2$ .

## Tip

A technology is always described by a **resource constraint** that equates how much stuff is made with how much stuff is used. Resource constraints usually become market clearing conditions in equilibrium.

#### Markets:

- ▶ There are competitive markets for the two goods
- ▶ The prices of the two goods are  $p_1$  and  $p_2$ .

What are prices denoted in?

# Step 2: Solve each Agent's Problem

There is only one agent: the household.

Households maximize  $u(c_1,c_2)$  subject to a budget constraint.

State variables the household takes as given:

- $\blacktriangleright$  market prices for the two goods,  $p_1$  and  $p_2$ .
- ightharpoonup endowments  $e_1$  and  $e_2$ .

The choice variables are  $c_1$  and  $c_2$ .

- We can normalize the price of one good to one (numeraire):  $p_1 = 1$ .
- ► Call the relative price  $p = p_2/p_1$ .

## Household problem

Budget constraint: Value of endowments = value of consumption.

The household solves the **problem**:

$$\max u(c_1, c_2)$$
  
s.t.  $c_1 + p \ c_2 = e_1 + p \ e_2$ 

# Solving the household problem

- $\blacktriangleright$  A solution to the household problem is a pair  $(c_1,c_2)$ .
- ► To find the optimal choices set up a Lagrangean:

$$\Gamma = u(c_1, c_2) + \lambda [e_1 + p e_2 - c_1 - p c_2]$$

▶ It would actually be easier to substitute the constraint into the objective function and solve the unconstrained problem

$$\max u(e_1 + p e_2 - p c_2, c_2)$$

but the Lagrangean is instructive.

## Household first-order conditions

The first order conditions are

$$\partial \Gamma / \partial c_i = u_i(c_1, c_2) - \lambda p_i = 0 \tag{1}$$

- The multiplier  $\lambda$  has a useful interpretation: It is the marginal utility of relaxing the constraint a bit, i.e. the marginal utility of wealth.
- The solution to the household problem is then a vector  $(c_1, c_2, \lambda)$  that solves
  - 2 FOCs
  - the budget constraint.

# Some tips

- Always explicitly state what variables constitute a solution and which equations do they have to satisfy.
  - ► This makes it easy to keep track of the pieces that go into the competitive equilibrium.
- You should have a FOC for each choice variable and all the constraints.
- Make sure you have the same number of variables and equations.
- When you write down an equation, pause and think.
  - Make sure you understand what the equation says in words.
  - If you cannot make sense of it, it's probably wrong!

# Simplify the optimality conditions

- lt is useful to substitute out the Lagrange multiplier  $\lambda$ .
- ► The ratio of the FOCs implies

$$u_2/u_1 = p \tag{2}$$

- ► This is the familiar tangency condition: marginal rate of substitution equals relative price. [Graph]
- Now the solution is a pair  $(c_1, c_2)$  that satisfies (2) and the budget constraint.
- Note: I can keep the Lagrange multiplier or drop it. If I keep it, I also need to keep another equation (e.g., the FOC for  $c_1$ ).

## Log utility example

Assume log utility:

$$u(c_1,c_2) = \ln(c_1) + \beta \ln(c_2)$$

▶ Then the problem can be solved in closed form:

$$\frac{u_2}{u_1} = \beta \frac{c_1}{c_2} = p$$

Substitute this back into the budget constraint:

$$c_1 + \beta c_1 = W = e_1 + p e_2$$

$$c_1 = \frac{W}{1 + \beta}$$

$$c_2 = \frac{\beta W}{1 + \beta}$$

## Log utility example

- ➤ Tip: This is a peculiar (and often very useful) feature of log utility: the expenditure shares are independent of p. The reason is exactly the same as that of constant expenditure shares resulting from a Cobb-Douglas production function: unit elasticity of substitution.
- ► Tip: Recall that taking a monotone transformation of u doesn't change the optimal policy functions. In particular, we can replace u by

$$u(c_1,c_2)=c_1 c_2^{\beta}$$

Convince yourself that this yields exactly the same consumption functions.

# Step 3: Market Clearing

There are two markets (for goods 1 and 2).

▶ Why isn't there just 1 market where good 1 is traded for good 2?

## Each agent

- ightharpoonup supplies the endowments  $e_i$  and
- demands consumption c<sub>i</sub> in those markets.

Goods are traded for units of account.

I don't use the word **money** because there is no such thing in this economy.

## Market Clearing

The market clearing condition is

"aggregate supply = aggregate demand."

Aggregate supply is simply the sum of individual supplies:

$$S_i = \sum_{h=1}^{N} e_i = N \ e_i \tag{3}$$

Aggregate demand:

$$D_i(p, e_1, e_2) = \sum_{h=1}^{N} c_i = N \ c_i(p, e_1, e_2)$$
 (4)

Market clearing:

$$c_i = e_i \tag{5}$$

Everybody eats their own endowments.

# Definition of Equilibrium

A competitive equilibrium is an allocation  $(c_1, c_2)$  and a price p that satisfy:

- 2 household optimality conditions (FOC and budget constraint).
- 2 goods markets clearing conditions.

Now we count equations and variables.

- ▶ We have 2N + 1 objects: 2N consumption levels and one price.
- We have 2N household optimality conditions and 2 market clearing conditions.

Why do we have one equation too many?

# Recap of key points

- 1. A macro model consists of exactly these parts:
  - 1.1 Demographics
  - 1.2 Preferences
  - 1.3 Endowments
  - 1.4 Technologies
  - 1.5 Markets
- 2. A competitive equilibrium is an allocation (think quantities) and a price system such that
  - 2.1 all agents solve their optimization problems, given prices;
  - 2.2 markets clear.
- Market clearing conditions only contain quantities of one good (no prices!).
- 4. Prices are in units of account that can be chosen arbitrarily at each trading date.
- 5. Walras' law allows you to drop one market clearing condition **or** one budget constraint.

# Where Is the Money?

We just wrote down a model without money.

The vast majority of macro models have no money.

But money is important ... are macroeconomists crazy?<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>They may well be, but not because they write down models without money.

# Reading

There is a remarkable shortage of discussions on how to properly set up a model in textbooks. But see

- ► Krusell (2014), ch. 2 describes the ingredients of modern macro models.
- Ch. 5 talks about Arrow-Debreu versus sequential trading.

## References

Krusell, P. (2014): "Real Macroeconomic Theory," Unpublished.