Models of Creative Destruction
Firm Dynamics

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Motivation

We extend the Schumpeterian model to have innovation by incumbents.

This produces a model of firm size dynamics.

Environment

Demographics, preferences, commodities: unchanged.

Resource constraint:

$$Y = C + X + Z \tag{1}$$

where

$$X(t) = \int_0^1 \psi x(v,t)\,dv \tag{2}$$

$$Z(t) = \int_0^1 [z(v,t) + \hat{z}(v,t)]\,q(v,t)\,dv \tag{3}$$

z and $\hat{z}$ are innovation inputs by incumbents and their rivals.

Final goods technology

$$Y(t) = \frac{1}{1-\beta} L(t)^{\beta} \int_0^1 q(v,t)^{\beta} x(v,t|q)^{1-\beta} dv \tag{4}$$

- the only change: quality is taken to power β
- implies: sales vary with quality (so the model has firm size implications)

Intermediate goods technology

- constant marginal cost ψ

Innovation technology for incumbents

- let $q(v,s)$ be the quality at the time the incumbent invented it
- investing zq implies a flow probability of innovation of ϕz
- the quality step is λ

Innovation technology for entrants

- investing $\hat{z}q$ implies a flow probability of innovation of $\eta(\hat{z})\hat{z}$
- η is decreasing
- marginal cost of innovation is rising in $\hat{z}$
- the quality step is $\kappa > \lambda$ (leapfrogging)
- innovators take η as given (an externality)

Solving each agent's problem

Solving each agents' problem

Household:

$$g(C) = \frac{r - \rho}{\theta} \tag{5}$$

Final goods producer:

$$x(v,t|q) = p^x(v,t|q)^{-1/\beta} q(v,t) L \tag{6}$$
$$w(t) = \beta Y(t) / L(t) \tag{7}$$

Intermediate goods producer

Assume drastic innovation

$$p^{x}(v,t|q) = \frac{\psi}{1-\beta} = 1 \tag{8}$$

Innovation by entrants

Free entry:

Investing $q\hat{z}$ gives a flow of $\eta\hat{z}$ new patents "per period"

$$\underbrace{\eta(\hat{z})\hat{z}}_{\text{probability}}\ \underbrace{V(v,t|\kappa q)}_{\text{payoff}} = \underbrace{q(v,t)\hat{z}}_{\text{cost}} \tag{9}$$

or

$$\boxed{V(v,t|\kappa q) = \frac{q}{\eta(\hat{z})}} \tag{10}$$

Note the κq.

This assumes an equilibrium with entry.

The flow probability that any competitor replaces the incumbent is $\hat{z}\eta(\hat{z})$.

Innovation by incumbents

Again assuming positive innovation.

Increase z until the marginal value equals marginal cost:

$$\underbrace{\phi z(v,t|q)}_{\text{probability}}\underbrace{[V(v,t|\lambda q)-V(v,t|q)]}_{\text{payoff}}=\underbrace{q(v,t)z(v,t|q)}_{\text{cost}} \tag{11}$$

We show later that V is proportional to quality q. Then

$$\phi V(v,t|q)[\lambda-1]=q(v,t) \tag{12}$$

or

$$\boxed{V(v,t|q)=\frac{q}{\phi(\lambda-1)}} \tag{13}$$

Value of the firm

Expected discounted value or profits

$$rV(v,t|q) = \underbrace{\pi(v,t|q)}_{\text{flow profit}} + \underbrace{\dot{V}(v,t|q)}_{0} - \underbrace{z(v,t|q)\,q(v,t)}_{\text{R\&D cost}} \tag{14}$$

$$+ \underbrace{\phi z(v,t|q)}_{\text{prob success}} \underbrace{[V(v,t|\lambda q) - V(v,t|q)]}_{\text{payoff}} \tag{15}$$

$$- \underbrace{\hat{z}(v,t|q)\,\eta(\hat{z}(v,t|q))}_{\text{prob lost patent}} \underbrace{V(v,t|q)}_{\text{loss}} \tag{16}$$

Note: Terms 3 and 4 cancel by the incumbent's FOC.

Generic derivation I

Take the generic discounted present value

$$V = \mathbb{E}\int_0^{\infty} e^{-rt}\pi(t)\,dt \tag{17}$$

where profits change stochastically according to a Poission process.

With flow probability ρ, profits change so that the continuation value becomes $\hat{V}$.

Evaluate the flow payoffs over a short period Δt:

$$V = \int_0^{\Delta t} e^{-(r+\rho)t}\pi_t dt \tag{18}$$

$$+ e^{-r\Delta t}\left[e^{-\rho\Delta t}V_{\Delta t} + \left[1 - e^{-\rho\Delta t}\right]\hat{V}\right] \tag{19}$$

At the end of the interval, discounted by $e^{-r\Delta t}$, the payoffs are

Generic derivation II

- $V_{\Delta t}$: the value of continuing at the end of Δt; with probability $e^{-\rho \Delta t}$
- $\hat{V}$: the value of continuing with a shock; with complementarity probability.

Assume that π is constant over the interval Δt. Then the first integral is

$$\frac{1-e^{-(r+\rho)\Delta t}}{r+\rho}\pi \tag{20}$$

Add and subtract V in the second term and it becomes

$$e^{-\rho \Delta t}(V_{\Delta t}-V)+\left[1-e^{-\rho \Delta t}\right]\hat{V}+e^{-\rho \Delta t}V \tag{21}$$

Substituting back into the definition of V gives

Generic derivation III

$$V\left[1-e^{-(r+\rho)\Delta t}\right]=\frac{1-e^{-(r+\rho)\Delta t}}{r+\rho}\pi \tag{22}$$

$$+e^{-r\Delta t}\left[e^{-\rho\Delta t}\left[V_{\Delta t}-V\right]+\left[1-e^{-\rho\Delta t}\right]\hat{V}\right] \tag{23}$$

Divide by $\left[1-e^{-(r+\rho)\Delta t}\right]$ and take $\Delta t\to 0$.

The first term becomes $\frac{\pi}{r+\rho}$.

Set $[V_{\Delta t}-V]=\dot{V}\Delta t$. Then the second term becomes

$$\frac{e^{-(r+\rho)\Delta t}}{1-e^{-(r+\rho)\Delta t}}\dot{V}\Delta t \tag{24}$$

Using L'Hopital's rule this becomes:

$$\frac{-(r+\rho)e^{-(r+\rho)\Delta t}\Delta t+e^{-(r+\rho)\Delta t}}{(r+\rho)e^{-(r+\rho)\Delta t}}=\frac{1}{r+\rho} \tag{25}$$

Generic derivation IV

Similarly, using L'Hopital's rule the third term becomes

$$\frac{\rho}{r+\rho}\hat{V} \tag{26}$$

Putting it all together gives

$$(r+\rho)V = \pi + \dot{V} + \rho\hat{V} \tag{27}$$

or

$$rV = \pi + \dot{V} + \rho\left[\hat{V} - V\right] \tag{28}$$

Value of the firm

Profit

$$\pi(v,t|q) = [p^x(v,t|q) - \psi]\, x(v,t|q) \quad (29)$$
$$= \beta q L \quad (30)$$

because $p^x = 1$ and $x = qL$.

Therefore

$$rV = \beta q L - \hat{z}\eta(\hat{z})V \quad (31)$$

or

$$\boxed{V = \frac{\beta q L}{r + \hat{z}\eta(\hat{z})}} \quad (32)$$

The usual story: losing the patent just increases the effective interest rate.

Equilibrium

Allocation

$\{C(t), X(t), Z(t), Y(t), L(t), z(v,t), \hat{z}(v,t), x(v,t), \pi(v,t), V(v,t)\}$

Prices $\{p^x(v,t), w(t), r(t)\}$

that satisfy:

- household: Euler (and TVC)
- final goods firm: 3
- intermediate goods firm: 1
- free entry of incumbents and entrants: 2
- market clearing: goods, labor (2)
- definitions of X, Z, π (3)
- definition of V (differential equation) (1)

Balanced Growth Path

Euler equation

$$g(C) = \frac{r-\rho}{\theta} \tag{33}$$

We now have 3 expressions for the value of the firm:

1. Free entry by incumbents (13)
2. Free entry by entrants (10)
3. The present value of profits (32)

$$V(q) = \underbrace{\frac{\beta q L}{r + \hat{z}\eta(\hat{z})}}_{\text{incumbents}} = \underbrace{\frac{q/\kappa}{\eta(\hat{z})}}_{\text{entrants}} = \underbrace{\frac{q}{\phi(\lambda - 1)}}_{\text{present value}} \tag{34}$$

These jointly solve for $r, \hat{z}$.

The Euler equation (33) then gives the growth rate.

Implications for firm dynamics

We now begin to have a model of firm dynamics.

- We have firm entry and exit (innovation by entrants)
- We have firm sales growth (stochastic) with firm age

Firm sales are given by$x(v,t|q) = qL$.

For a given firm: x

- increases by factor λ with probability $\phi z \Delta t$
- stays the same with probability $\hat{z}\eta(\hat{z})\Delta t$
- drops to 0 with complementary probability

Applications

Garcia-Macia et al. (2016)

- how much of output growth is due to innovation by incumbents vs competitors?

Acemoglu et al. (2013)

- tax policy in a model with R&D and firm quality heterogeneity

Hottman et al. (2016)

- measures sources of firm heterogeneity

Reading

- Acemoglu (2009), ch. 14.3.
- Aghion et al. (2014), survey of Schumpeterian growth models

References I

Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.

Acemoglu, D., U. Akcigit, N. Bloom, and W. R. Kerr (2013): "Innovation, reallocation and growth," Tech. rep., National Bureau of Economic Research.

Aghion, P., U. Akcigit, and P. Howitt (2014): "What Do We Learn From Schumpeterian Growth Theory?" in *Handbook of Economic Growth*, ed. by P. Aghion and S. N. Durlauf, Elsevier, vol. 2 of *Handbook of Economic Growth*, 515–563, dOI: 10.1016/B978-0-444-53540-5.00001-X.

Garcia-Macia, D., C.-T. Hsieh, and P. J. Klenow (2016): "How Destructive is Innovation?" Working Paper 22953, National Bureau of Economic Research.

Hottman, C. J., S. J. Redding, and D. E. Weinstein (2016): "Quantifying the sources of firm heterogeneity," *The Quarterly Journal of Economics*, 131, 1291–1364.