Problem Set 3: Infinite Horizon Model in Discrete Time

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1 Wealth in the utility function

Consider the following modification of the standard growth model where the households derives utility from holding wealth.

Demographics: There is a representative household of unit mass who lives forever.

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t, k_{t-1})$ where c_t is consumption and k_{t-1} is last period's capital (wealth). The utility function is strictly concave and increasing in both arguments.

Endowments: At $t = 0$ the household is endowed with capital K_0. In each period the household works 1 unit of time.

Technologies:

$$K_{t+1} = A\,F(K_t, L_t) + (1 - \delta)\,K_t - c_t \tag{1}$$

The production function has constant returns to scale.

Markets: Production takes place in a representative firm which rents capital and labor from households. There are competitive markets for goods (price 1), capital rental (r_t), and labor rental (w_t).

1. State the household's dynamic program.
2. Derive and explain the conditions that characterize a solution to the household problem (in sequence language).
3. Define a competitive equilibrium.
4. Derive a single equation that determines the steady state capital stock.
5. Is the steady state unique? Explain the intuition why the steady state is or is not unique.

2 Ben-Porath Model

We study the decision problem of an infinitely lived agent in discrete time. At $t = 0$, the agent is endowed with h_0 units of human capital. In each period, he can invest l_t units of time, so that human capital evolves according to

$$h_{t+1} = (1 - \delta)\,h_t + F\,(h_t l_t) \tag{2}$$

$$F\,(hl) = (hl)^{\alpha} \tag{3}$$

with $0 < \alpha, \delta < 1$. The objective is to maximize the present value of lifetime earnings, given by

$$Y = \sum_{t=0}^{\infty} R^{-t} w h_t\,(1 - l_t) \tag{4}$$

where $R > 0$ and $w > 0$ are taken as given.

Questions:

1. Write down the agent's Dynamic Program.
2. Derive and interpret the first-order condition for l.
3. Derive $V'(h) = w + (1-\delta) R^{-1} V'(h')$.
4. Derive and interpret $V'(h) = w \frac{R}{r+\delta}$ where $R = 1 + r$.
5. How do the wage and the interest rate affect steady state h and l?