The Romer Model

Prof. Lutz Hendricks

Econ520

November 2, 2022

Issues

- We study models where **intentional innovation** drives productivity growth.
- **Romer model**:
  - The standard model of R&D goes back to **Romer** (1990).
  - Innovations are produced like any other good using R&D labor as input.
- **Policy effects**
  - Policies, such as R&D subsidies, can change the rate at which innovations are produced.
  - Surprisingly, it turns out that **policies** have **no effect on long-run growth**.

Learning Objectives

In this section you will learn:

1. how to analyze the Romer model
2. why R&D policies do not change the long-run growth rate of the economy

The Romer model
Solow block

- Production of goods works exactly like in the Solow Model
- Aggregate production function:

$$Y_t = K_t^{\alpha} \; (A_t L_{Yt})^{1-\alpha} \tag{1}$$

- **Capital accumulation** as in the Solow model

$$\dot{K}_t = s_K Y_t - \delta K_t \tag{2}$$

- **Labor input** grows at a constant rate

$$g(L) = n \tag{3}$$

Solow Block

What has changed?

Final goods production function has:

- constant returns to rival inputs: K and L_Y.
- has **increasing returns** to all inputs (including A)

Labor is divided into production (L_Y) and R&D (L_A).

R&D Block

- Ideas are produced just like other goods.
- The input is labor (L_{At})
    - not much changes if capital is an input, too.
- The output is a number of new ideas.
    - A_t is the number of ideas that have been invented up to t.
    - $\dot{A}_t$ is the number of ideas discovered today (or the rate at which they are discovered).

R&D Block

- The **ideas production function**:

$$\dot{A}_t = \bar{B} L_{At}^{\lambda} \tag{4}$$

- λ determines returns to scale.
- $\bar{B}$ is a productivity parameter.

Ideas are inputs to innovation

How easy it is to produce a new idea depends on how much has already been discovered.

$$\bar{B} = B\ A^{\phi} \tag{5}$$

If ideas help produce new ideas: $\phi > 0$: $A \uparrow \Longrightarrow \bar{B} \uparrow$.

If there is "fishing out:" $\phi < 0$.

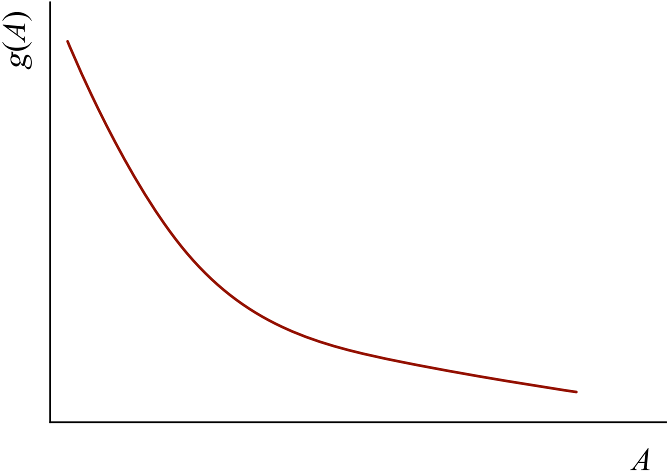
Assume $\phi \leq 1$. (If $\phi > 1$ odd things happen...).

The ideas production function is then

$$\dot{A} = B\ L_A^{\lambda}\ A^{\phi} \tag{6}$$

$$g(A) \equiv \dot{A}/A = B\ L_A^{\lambda} A^{\phi-1} \tag{7}$$

Ideas production function



Even though ideas foster innovation $(\phi > 0)$, more ideas imply **slower** $g(A)$.

Ideas production function

Note how similar this is to the law of motion for capital in the Solow model

Model			Productivity	"Capital"	Labor	Depreciation
Solow	$\dot{K}_t$	$=$	$sA^{1-\alpha}$	K_t^{α}	$L_t^{1-\alpha}$	$-\delta K_t$
Romer	$\dot{A}_t$	$=$	B	A_t^{ϕ}	L_{At}^{λ}	-0

It follows that there cannot be long-run growth in A/L when $\lambda + \phi < 1$ (details follow).

But we still can get long-run growth in Y/L.

The Romer model
Behavior

So far we have described technologies.

To describe behavior, we make a **Solow assumption**:

- A constant saving rate

$$S/Y = I/Y = s_K$$

- A constant labor allocation:

$$L_A = s_A L \tag{8}$$
$$L_Y = (1 - s_A)\ L \tag{9}$$

Model summary

The Solow block:

$$Y = K^{\alpha} \ (A \ L_Y)^{1-\alpha} \tag{10}$$

$$\dot{K} = s_K \ Y - \delta \ K \tag{11}$$

$$L_t = L_0 \ e^{nt} \tag{12}$$

Production of ideas:

$$\dot{A} = B \ L_A^{\lambda} \ A^{\phi} \tag{13}$$

Constant behavior:

$$L_Y = s_Y \ L; \quad L_A = s_A \ L \tag{14}$$

The growth rate of ideas:

$$g(A) = B \ (s_A \ L)^{\lambda} \ A^{\phi - 1} \tag{15}$$

Model summary

- This looks complicated, but isn't.
- We have tricked the model such that Y and K don't matter for how A evolves.

$$\dot{A} = B \ L_A^{\lambda} \ A^{\phi} \tag{16}$$

- This would change, if we let $\dot{A}$ depend on K
  - but that would not affect the results
  - only the algebra would be more complicated (see Romer 2011)

Does the Model Make Sense?

- The production functions are arbitrary.
  - But what matters are certain qualitative features, not the exact functional form.
  - We will get back to this.
- There is only one input. Only one good.
  - All of this can be relaxed without changing anything too important.
- Where are the households, consumption, population growth ...
  - We can add those - it does not make any difference.
- The labor allocation is fixed.
  - This is important.
  - The literature does not make this assumption. It can talk about patents, policy, ...
- Ideas are produced like goods.

Balanced growth path

Definition

A BGP is a path along which all variables grow at **constant rates**.

Why might this be interesting?

Balanced growth path

At what rates do the endogenous objects grow on the BGP?

Result 1: $g(k) = g(y)$

- as in the Solow model (same technology)

Proof:

- Law of motion: $g(k) = s\ y/k - \delta$.
- Constant $g(k)$ requires constant k/y.

Balanced growth path

Result 2: $g(y) = g(A)$

- as in the Solow model (same technology)

Proof:

- Production function: $y = k^{\alpha} A^{1-\alpha}$.
- Take growth rates: $g(y) = \alpha\ g(y) + (1-\alpha)\ g(A)$

Result

All long-run growth is due to R&D.

Growth rate of ideas

This is the key result of the model:

On the **balanced growth path**

$$g(A) = \frac{\lambda \ n}{1 - \phi} \tag{17}$$

Important:

This only holds on the balanced growth path.

Derivation

Ideas production:

$$g(A) = B\frac{L_A^{\lambda}}{A^{1-\phi}} \tag{18}$$

BGP: $g(A)$ is constant $\implies L_A^{\lambda}/A^{1-\phi}$ is constant.

Or:

$$g\left(L_A^{\lambda}\right) = g\left(A^{1-\phi}\right) \tag{19}$$

$$\lambda g(L_A) = (1-\phi)\, g(A) \tag{20}$$

With constant time allocation, s_A: $g(L_A) = n$. Therefore

$$\lambda n = (1-\phi)\, g(A) \tag{21}$$

Rearrange. Done.

Summary: Balanced growth

Balanced growth in the Romer model is characterized by:

$$g(y) = g(k) = g(A) \tag{22}$$

$$g(A) = \frac{\lambda \ n}{1 - \phi} \tag{23}$$

All growth is due to innovation.

Why is this true?

Why is all growth due to innovation?

Solow model:

- K is **rival**
- What matters for per capita output is K/L
- K does growth in the Solow model (due to population growth), but not K/L (diminishing returns)

Romer model:

- A is **non-rival**
- What matters for per capita output is A, NOT A/L
- A grows (due to population growth), even if A/L falls over time

Balanced growth: Intuition

$$g(A) = \frac{\lambda \ n}{1-\phi} \tag{24}$$

Growth is simply a multiple of population growth

Behavior does not matter: s_K and s_A do not appear in (24).

Intuition

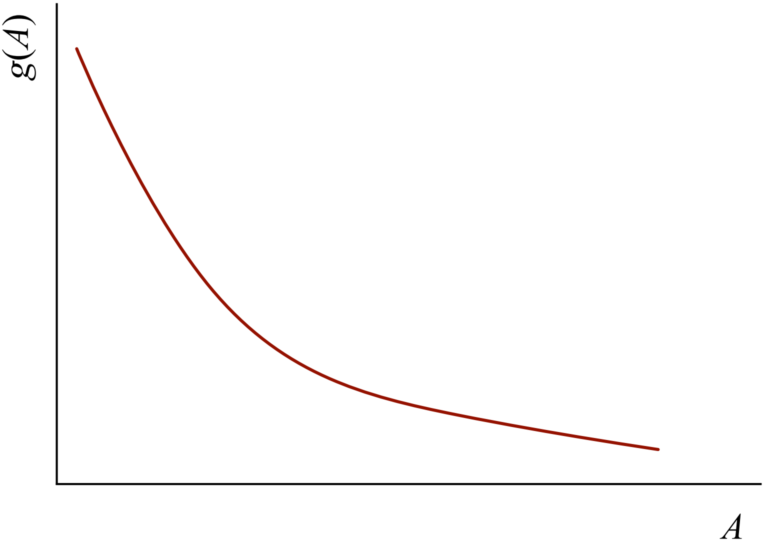
- Consider the case $\phi = 0$.
- Ideas production is then

$$\dot{A} = B\ L_A^{\lambda} \tag{25}$$

- If the population is **constant**, L_A is constant.
- In each period, the economy produces a constant number of ideas.
- The growth rate of ideas, $g(A) = B\ L_A / A$, falls to zero over time.
- A fixed number of people cannot produce a growing stream of ideas.

Population growth is necessary for sustained innovation (at a constant rate).

How growth is sustained



$g(A) = BA^{\phi-1}L_A^{\lambda}$

Special Case: Phi = 1

With $\phi = 1$, idea production becomes

$$g(A) = B\ L_A^{\lambda} \tag{26}$$

This is the case studied by Romer (1990).

The model has exploding growth, unless the population is constant.

This is clearly contradicted by post-war data: L_A rose dramatically, while $g(y)$ was at best constant.

Reality check

1. The model says: constant population - no growth.
   - But we are still producing new ideas all the time.
   - How can we reconcile this?
2. What if the population shrinks over time?
   - Is the long-run growth rate negative?

Reading

- Jones (2013b), ch. 5.

Optional:

- Romer (2011), ch. 3.1-3.4
- Jones (2013a), ch. 6

Advanced Reading

- Jones (2005) talks in some detail about the economics of ideas.
- Lucas (2009) and McGrattan and Prescott (2009) on openness and growth

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