# Practice Problems: Solow Model

Econ520. Fall 2022. Prof. Lutz Hendricks. November 1, 2022

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Jones, Economic Growth, exercises 2.1 to 2.6.

Blanchard/Johnson, Macroeconomics, ch. 11, questions 1-4, 6-9.

Jones, Macroeconomics, exercises 5.1-5.1, 5.7, 5.10.

## 1 Comparative Dynamics

1. Examine the effect of a one-time **increase in the labor force** ($L$) in the Solow model without technical change. Plot the time paths of $Y/L$, wages, and interest rates. [Jones 2.2] [Answer: the same as a decrease in $K$ or an increase in $A$. The reason: only $\bar{k}$ appears in the law of motion.]

2. **Income tax:** Examine the short-run and long-run effects of an income tax. Instead of receiving $wL+rK$, households receive $(1-\tau)(wL+rK)$. The government consumes the tax revenues. [Jones 2.3] [Answer: the same as a decline in $s$ from its original value to $s(1-\tau)$].

   (a) How does your answer change if the government instead invests the tax revenues? [Answer: the same as a rise in $s$.]

## 2 Varying the capital share

Compare two Solow economies. Economy A has $\alpha = 1/3$. Economy B has $\alpha = 2/3$. Otherwise, the economies are identical.

1. Draw the Solow diagrams ($sy_t$ and $(n+\delta)k_t$ against $k_t$) for both cases. Explain how they differ.

2. For both economies, assume that $k_0$ equals 10% of the steady state value of $k$. In which economy do you expect growth to be faster at date

0? Which economy do you expect to approach its steady state faster? Explain the intuition. You need not derive your answer. *Hint:* Find $\dot{k}$ in the Solow diagram for some low value of $k$. For which economy is it larger?

3. Derive an equation for the growth rate of $k$ as a function of the distance from the steady state $(k/k^*)$. How does the growth rate of $k$ depend on $\alpha$? Verify your intuition of part 2.

## 2.1 Answer

1. Low $\alpha$: the production function has lots of curvature.

2. We know that $g(k) = A^{1-\alpha} s k^{\alpha-1} - (n+\delta)$. For small $\alpha$, $k^{\alpha-1}$ gets very large when $k$ is small. Or in the graph: with more curvature, the gap between $sy$ and $(n+\delta)k$ is larger at low $k$. High MPK means that the economy can grow fast.

3. Just multiply and divide by $(k^*)^{1-\alpha}$:

$$g(k) = A^{1-\alpha} s \left(k^*\right)^{\alpha-1} \left(k/k^*\right)^{\alpha-1} - (n+\delta) \tag{1}$$

Now think about what happens to $(k/k^*)^{\alpha-1}$ for small and large $\alpha$. If $\alpha$ is very close to 1, $(k/k^*)^{\alpha-1} \approx (k/k^*)^0 = 1$ and the growth rate does not respond much when $k$ changes. But if $\alpha$ is very close to 0, $(k/k^*)^{\alpha-1} \approx (k/k^*)^{-1}$ and the growth rate responds strongly when $k$ changes.

# 3 Solow Model With Subsistence

Consider a version of our Solow model where households require a subsistence level of consumption. Here are the details. The production function is

$$Y = K^{\alpha} \ (AL)^{1-\alpha} \tag{2}$$

and capital is accumulated according to

$$\dot{K}_t = sY_t - \delta K_t \tag{3}$$

Labor input grows at a constant rate $n$. Productivity is constant at $A$. The new part is the saving rate. While income is lower than a threshold $\bar{y}$ the household does not save. Of the income above $\bar{y}$ the household saves a constant fraction $\bar{s}$. Per capita saving is therefore given by $sy = 0$ if $y < \bar{y}$ and

$$sy = \bar{s}\ (y - \bar{y}) \tag{4}$$

if $s \geq \bar{y}$.

1. Graph the saving rate against $y$.

2. Plot the Solow diagram for this model. By this I mean a diagram depicting $sy$ and $(n + \delta)\, k$. For comparison also plot the $sy$ line without subsistence ($\bar{y} = 0$).

3. How many steady states does the model have? Assume that $\bar{y}$ is not too large – otherwise the economy has no steady states with $k > 0$.

4. For various values of initial capital, characterize which steady states the economy may converge to.

## 3.1 Answer: Solow Model with Subsistence

1. $s = \max\{0, \bar{s}(1 - \bar{y}/y)\}$. This starts at 0 until $y$ reaches $\bar{y}$. Then it rises towards $\bar{s}$.

2. Solow diagram: Note that $sy = \bar{s}A^{1-\alpha}k^{\alpha} - \bar{s}\bar{y}$. The $sy$ line is simply shifted down by a constant. It crosses the x-axis where $y = \bar{y}$. See figure 1.

3. The model has 3 steady states (including $k = 0$).

4. The highest steady state is similar to the regular Solow model and locally stable. The steady state at $k = 0$ is also stable. The middle steady state is unstable. The model has a poverty trap.

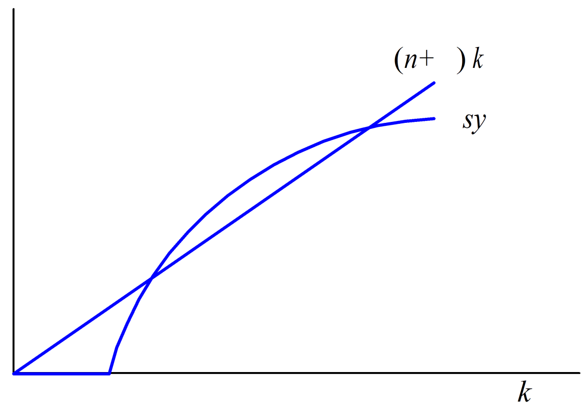


Figure 1: Solow Model with Subsistence Consumption

# 4 Convergence

## 4.1 Galton's Fallacy

[Based on Jones 3.4]

1. Imagine you have data on the height of mothers ($h_m$) and daughters ($h_d$). You plot the change in daughter's height ($h_d - h_m$) against mother's height and find a negative relationship. Tall mothers tend to have daughters that are less tall than their mothers. Does this imply that all persons converge over time to the same height? Why or why not?

2. Now imagine that mother's height and daughter's height are drawn independently from the same distributions. What will you find if you plot the change in height against mother's height?

3. What does all this imply for the interpretation of cross-country growth regressions that find a negative relationship between initial income and subsequent income growth?

### 4.1.1 Answer: Galton's Fallacy

**1.** Heights need not converge. Counter-example: each person draws her height from a fixed distribution, independently of mother's height. The reason why the change in height is small when the mother is tall is simply that everyone has (on average) average height daughters. Tall mothers therefore have daughters that are shorter than themselves.

**2.** See #1.

**3.** In some samples of countries we find that $g(y)$ is negatively related to $y$ at the beginning of the sample. This does not imply $\sigma$ convergence.