Problem Set 2: OLG Models with Money

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1 Money and Storage

Consider a two-period OLG model with fiat money and a storage technology.

Demographics: In each period $N_t = (1+n)^t$ persons are born. Each lives for 2 periods.

Endowments: The initial old hold capital K_0 and money M_0. Each young person is endowed with a e units of the good.

Preferences: $u(c_t^y) + \beta u(c_{t+1}^o)$.

Technology: Storing k_t units of the good in t yields $f(k_t)$ units in $t+1$. f obeys Inada conditions. The resource constraint is $N_t k_{t+1} = N_t e + N_{t-1} f(k_t) - C_t$ where $C_t = N_t\, c_t^y + N_{t-1}\, c_t^o$.

Government: The government pays a lump-sum transfer of $x_t p_t$ units of money to each old person: $M_{t+1} = M_t + N_{t-1} x_t p_t$. The aggregate money supply grows at the constant rate μ: $M_{t+1} = (1+\mu)\, M_t$.

Markets: In each period, agents buy/sell goods and money in spot markets.

The timing in period t is a follows:

- The old enter period t holding aggregate capital $K_t = N_{t-1} k_t$ and nominal money balances of $M_t = m_t\ N_{t-1}$.
- Each old person produces $f(k_t)$.
- The young buy money (m_{t+1}/p_t) from the old, consume c_t^y and save k_{t+1}.
- The old consume their income.

Questions:

1. State the household's budget constraints when young and old.
2. Derive the household's optimality conditions. Define a solution to the household problem.
3. Define a competitive equilibrium.
4. Does an equilibrium with positive inflation exist? Intuition?
5. Define a steady state as a system of 6 equations in 6 unknowns.
6. Find the money growth rate (μ) that maximizes steady state consumption per young person, $(N_t\, c_t^y + N_{t-1}\, c_t^o)/N_t$.

2 Money in the Utility Function in an OLG Model

Demographics: In each period a cohort of constant size N is born. Each person lives for 2 periods.

Endowments: The initial old hold capital K_0 and money M. No new money is ever issued. The young are endowed with one unit of work time.

Preferences: $u(c_t^y) + \beta\, u(c_{t+1}^o) + v(m_t^d/p_t)$. Assume $v' > 0$. Agents derive utility from real money balances as defined below.

Technology: Output is produced with a constant returns to scale production function $F(K_t, L_t)$. The resource constraint is standard. Capital depreciates at rate δ.

Markets: There are spot markets for goods (price p_t), money, labor (wage w_t), and capital rental (price q_t).

Timing:

- The old enter period t holding money M and capital K_t.
- Production takes place.
- The old sell money to the young. m_t^d is the nominal per capita money holding of a young person.
- Consumption takes place.

Questions:

1. Derive a set of 4 equations that characterize optimal household behavior. Show that the household's first-order conditions imply rate of return dominance, i.e., the real return on money is less than the real return on capital (assuming both capital and money are held in equilibrium).
2. Solve the firm's problem.
3. Define a competitive equilibrium.
4. Assume that the utility functions u and v are logarithmic. Solve *in closed form* for the household's money demand function, $m_t^d/p_t = \varphi(w_t, r_{t+1}, \pi_{t+1})$, and for its saving function, $s_{t+1} = \phi(w_t, r_{t+1}, \pi_{t+1})$. $\pi_{t+1} \equiv p_{t+1}/p_t$.
