# Models of Creative Destruction
# Firm Dynamics

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# Motivation

We extend the Schumpeterian model to have innovation by incumbents.

This produces a model of firm size dynamics.

# Environment

Demographics, preferences, commodities: unchanged.

Resource constraint:

$$Y = C + X + Z \tag{1}$$

where

$$X(t) = \int_0^1 \psi x(v,t)\,dv \tag{2}$$

$$Z(t) = \int_0^1 [z(v,t) + \hat{z}(v,t)]\,q(v,t)\,dv \tag{3}$$

$z$ and $\hat{z}$ are innovation inputs by incumbents and their rivals.

# Final goods technology

$$Y(t) = \frac{1}{1-\beta} L(t)^{\beta} \int_0^1 q(v,t)^{\beta} x(v,t|q)^{1-\beta} dv \tag{4}$$

- the only change: quality is taken to power $\beta$
- implies: sales vary with quality (so the model has firm size implications)

# Intermediate goods technology

- constant marginal cost $\psi$

# Innovation technology for incumbents

- let $q(v,s)$ be the quality at the time the incumbent invented it
- investing $zq$ implies a flow probability of innovation of $\phi z$
- the quality step is $\lambda$

# Innovation technology for entrants

- investing $\hat{z}q$ implies a flow probability of innovation of $\eta(\hat{z})\hat{z}$
- $\eta$ is decreasing
- marginal cost of innovation is rising in $\hat{z}$
- the quality step is $\kappa > \lambda$ (leapfrogging)
- innovators take $\eta$ as given (an externality)

# Solving each agent's problem

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Household:

$$g(C) = \frac{r - \rho}{\theta} \tag{5}$$

Final goods producer:

$$x(v, t|q) = p^x(v, t|q)^{-1/\beta} q(v, t) L \tag{6}$$
$$w(t) = \beta Y(t) / L(t) \tag{7}$$

# Intermediate goods producer

Assume drastic innovation

$$p^x(v,t|q) = \frac{\psi}{1-\beta} = 1 \tag{8}$$

# Innovation by entrants

Free entry:

Investing $q\hat{z}$ gives a flow of $\eta\hat{z}$ new patents "per period"

$$\underbrace{\eta(\hat{z})\hat{z}}_{\text{probability}}\ \underbrace{V(v,t|\kappa q)}_{\text{payoff}} = \underbrace{q(v,t)\hat{z}}_{\text{cost}} \tag{9}$$

or

$$\boxed{V(v,t|\kappa q) = \frac{q}{\eta(\hat{z})}} \tag{10}$$

Note the $\kappa q$.

This assumes an equilibrium with entry.

The flow probability that any competitor replaces the incumbent is $\hat{z}\eta(\hat{z})$.

# Innovation by incumbents

Again assuming positive innovation.

Increase $z$ until the marginal value equals marginal cost:

$$\underbrace{\phi z(v,t|q)}_{\text{probability}}\underbrace{[V(v,t|\lambda q)-V(v,t|q)]}_{\text{payoff}}=\underbrace{q(v,t)z(v,t|q)}_{\text{cost}} \tag{11}$$

We show later that $V$ is proportional to quality $q$. Then

$$\phi V(v,t|q)[\lambda-1]=q(v,t) \tag{12}$$

or

$$\boxed{V(v,t|q)=\frac{q}{\phi(\lambda-1)}} \tag{13}$$

# Value of the firm

Expected discounted value or profits

$$rV(v,t|q) = \underbrace{\pi(v,t|q)}_{\text{flow profit}} + \underbrace{\dot{V}(v,t|q)}_{0} - \underbrace{z(v,t|q)\,q(v,t)}_{\text{R\&D cost}} \tag{14}$$

$$+ \underbrace{\phi z(v,t|q)}_{\text{prob success}} \underbrace{[V(v,t|\lambda q) - V(v,t|q)]}_{\text{payoff}} \tag{15}$$

$$- \underbrace{\hat{z}(v,t|q)\,\eta(\hat{z}(v,t|q))}_{\text{prob lost patent}} \underbrace{V(v,t|q)}_{\text{loss}} \tag{16}$$

Note: Terms 3 and 4 cancel by the incumbent's FOC.

# Value of the firm

Profit

$$\pi(v,t|q) = [p^x(v,t|q) - \psi]\, x(v,t|q) \tag{17}$$
$$= \beta q L \tag{18}$$

because $p^x = 1$ and $x = qL$.

Therefore

$$rV = \beta q L - \hat{z}\eta(\hat{z})V \tag{19}$$

or

$$\boxed{V = \frac{\beta q L}{r + \hat{z}\eta(\hat{z})}} \tag{20}$$

The usual story: losing the patent just increases the effective interest rate.

# Equilibrium

Allocation

$\{C(t), X(t), Z(t), Y(t), L(t), z(v,t), \hat{z}(v,t), x(v,t), \pi(v,t), V(v,t)\}$

Prices $\{p^x(v,t), w(t), r(t)\}$

that satisfy:

- household: Euler (and TVC)
- final goods firm: 3
- intermediate goods firm: 1
- free entry of incumbents and entrants: 2
- market clearing: goods, labor (2)
- definitions of $X, Z, \pi$ (3)
- definition of $V$ (differential equation) (1)

# Balanced Growth Path

Euler equation

$$g(C) = \frac{r - \rho}{\theta} \tag{21}$$

We now have 3 expressions for the value of the firm:

1. Free entry by incumbents (13)
2. Free entry by entrants (10)
3. The present value of profits (20)

$$V(q) = \underbrace{\frac{\beta q L}{r + \hat{z}\eta(\hat{z})}}_{\text{incumbents}} = \underbrace{\frac{q/\kappa}{\eta(\hat{z})}}_{\text{entrants}} = \underbrace{\frac{q}{\phi(\lambda - 1)}}_{\text{present value}} \tag{22}$$

These jointly solve for $r, \hat{z}$.

The Euler equation (21) then gives the growth rate.

# Implications for firm dynamics

We now begin to have a model of firm dynamics.

- We have firm entry and exit (innovation by entrants)
- We have firm sales growth (stochastic) with firm age

Firm sales are given by$x(v,t|q) = qL$.

For a given firm: $x$

- increases by factor $\lambda$ with probability $\phi z \Delta t$
- stays the same with probability $\hat{z}\eta(\hat{z})\Delta t$
- drops to 0 with complementary probability

# Applications

Garcia-Macia et al. (2016)

- how much of output growth is due to innovation by incumbents vs competitors?

Acemoglu et al. (2013)

- tax policy in a model with R&D and firm quality heterogeneity

Hottman et al. (2016)

- measures sources of firm heterogeneity

# Reading

- Acemoglu (2009), ch. 14.3.
- Aghion et al. (2014), survey of Schumpeterian growth models

# References I


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