

Arrow-Debreu and Sequential Trading

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Introduction

Macro models are dynamic (have many periods).

Then we have a choice of how to represent equilibrium:

- ▶ Arrow-Debreu: all trading takes place at date 0
- ▶ Sequential trading: markets open in each period

This is where the details matter (units of account, Walras' law, ...)

Two Period Example

Demographics:

- ▶ N identical households live for 2 periods, $t = 1, 2$.

Commodities:

- ▶ there is one good in each period

Preferences: $u(c_1, c_2)$

Endowments: e_t

“Technology”: $c_t = e_t$

Markets

Now we have a choice between 2 equivalent arrangements

- ▶ Arrow-Debreu: all trades take place at $t = 1$
- ▶ Sequential trading: markets open in each period

Arrow-Debreu Trading

The arrangement:

- ▶ All trades take place at $t = 1$
- ▶ Agents can buy and sell goods for delivery at any date t
- ▶ Prices are p_t

Can we normalize prices to 1?

Surprise:

If we write out this model, it **looks exactly like the static 2 good model** (see above).

Arrow-Debreu Equilibrium

Household budget constraint:

$$\sum_t p_t e_t = \sum_t p_t c_t \quad (1)$$

Interpretation:

The household sells e_t to and buys c_t from the Walrasian auctioneer at a single trading date.

Market clearing:

$$e_t = c_t \quad (2)$$

- ▶ Again the same as resource constraints.

Equilibrium

Objects: $c_t, p_t, t = 1, 2$

Equations:

- ▶ Household policy rules: $c_t(p_1, p_2)$
implicitly defined by first-order condition and budget constraint
- ▶ Market clearing: $e_t = c_t$

Notes:

- ▶ only p_2/p_1 is determined in equilibrium (choice of unit of account)
- ▶ only **one** equation is redundant by Walras' law (why?)

Equivalence of Dates and Goods

Fact

A model with T goods is equivalent to a model with T periods.

Why is this true? What is a “good?”

This is only true under “**complete markets**”

- ▶ roughly: there are markets that allow agents to trade goods across all periods and states of the world
- ▶ we will talk about details later

Sequential Trading

An alternative trading arrangement.

Markets open at each date.

Only the date t good can be purchased in the period t market.

Now we have **one numeraire for each trading period**: $p_t = 1$.

We need assets to transfer resources between periods.

Markets

At each date we have

1. a market for goods ($p_t = 1$);
2. a market for 1 period discount bonds (price q_t)

A discount bond pays 1 unit of $t + 1$ consumption.

Digression: Modeling bonds

Definition

A one period bond promises to pay one unit of consumption in $t+1$.

Call its price q_t .

Then the real interest rate is: $R_{t+1} = 1/q_t$.

Econ101 question

What is a real interest rate?

Alternative normalization:

- ▶ set $q_t = 1$ and let each bond pay R_{t+1} units of consumption
- ▶ why can I do this?

Household problem

Now we have one budget constraint per period:

$$e_t + b_{t-1} = c_t + b_t q_t \quad (3)$$

with $b_0 = 0$.

Fact

Agents have one budget constraint for each trading period

Household solves:

$$\max_{b_1} u(e_1 - b_1 q_1, e_2 + b_1) \quad (4)$$

Household solution

FOC:

$$u_1 q_1 = u_2 \quad (5)$$

q_1 is the relative price of period 2 consumption.

Give up 1 unit of c_1 and get $1/q_1$ units of c_2 .

Solution: c_1, c_2, b_1 that solve FOC and 2 budget constraints.

Market Clearing

- ▶ Goods: $e_t = c_t$
- ▶ Bonds: $b_t = 0$

Why does bond market clearing look so odd?

Equivalence

Note that the relative price is the same under both trading arrangements:

$$p = q = u_2/u_1 \quad (6)$$

Fact

When markets are complete, Arrow-Debreu and sequential trading equilibria are identical.

Why two trading arrangements?

When building a model, choose the most convenient trading arrangement.

In many cases, it does not matter

- ▶ equilibrium allocations are the same
- ▶ both are similarly convenient

It matters when there is uncertainty.

- ▶ with complete markets, Arrow-Debreu is much simpler
- ▶ with incomplete markets, sequential trading makes it easy to specify which assets exist

Summary

Macro is micro

- ▶ The method outlined here is central to all of (macro) economics.
- ▶ Being able to translate a description of an economy into the definition of a competitive equilibrium is an important skill.

Final example

Demographics: There are N households. Each lives for $T > 1$ periods.

Preferences: $\sum_{t=1}^T u(c_{1,t}, \dots, c_{J,t})$ where J is the number of goods available in each period.

Endowments: Household i receives $e_{i,j,t}$.

Technologies: Endowments can only be eaten in the period they are received.

- ▶ Resource constraint:

Markets:

- ▶ Sequential trading: there are competitive markets for the J goods; there are one period discount bonds in each period.
- ▶ Arrow-Debreu: the $J \times T$ goods are traded in $t = 1$.

Final example: Arrow-Debreu Equilibrium

Household problem:

FOCs:

Household solution:

Equilibrium:

Reading

Krusell (2014), ch. 5 talks about Arrow-Debreu versus sequential trading.

References

Krusell, P. (2014): “Real Macroeconomic Theory,” Unpublished.