Problem Set 6: Innovation and Growth

Econ720. Fall 2022. Prof. Lutz Hendricks

1 Stochastic patent duration[1]

Consider a version of the "Expanding Variety of Goods" model in which innovators' monopoly power diminishes over time. Otherwise the model is standard.

Demographics: There is a single representative household.

Endowments: The household is endowed with L units of labor, which can only be used for work.

Preferences:

$$U = \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\theta} - 1}{1-\theta} dt. \tag{1}$$

Technology:

- Final goods are produced from labor and intermediate inputs according to

$$Y = (1-\beta)^{-1} L^\beta \int_0^N x(v)^{1-\beta} dv \tag{2}$$

where $0 < \beta < 1$, Y is output, L is labor input, $x(v)$ is the input of the vth type of the intermediate good, and N is the number of varieties.

- Intermediates: It takes ψ units of the final good to make an intermediate good.
- Innovation: It costs $1/\eta$ units of the final good to create a new type of intermediate good.

Market arrangements:

- The final goods sector is perfectly competitive.
- Intermediate goods producers hold monopolies.
- There is free entry for innovators.
- Households own all firms in the economy.

Patents: Upon innovation, the innovator receives a patent. If intermediate good v is currently monopolized, it becomes competitive in the next instant dT with probability $\delta \cdot dT$, where $\delta \geq 0$. Thus, if good j is invented at time t, the probability of it still being monopolized at some future date $v \geq t$ is $e^{-\delta \cdot (v-t)}$.

Notation: Denote by N_1, the number of intermediate goods produced by monopolists and by N_2 the number that is produced competitively. $N = N_1 + N_2$.

Note that the household problem is the same as in the model discussed in class. It is characterized by the Euler equation

$$g(C) = \frac{r-\rho}{\theta} \tag{3}$$

[1] Due to Matt Doyle.

Questions:

1. Solve the problem of the final goods producer. Note that it faces different prices for competitive versus monopolized goods.

2. Solve the problem of a monopolist intermediate input producer.

3. The value of a new blueprint is now given by

$$rV(v) = \pi(v) + \dot{V}(v) - \delta V(v) \tag{4}$$

   The new term here is $\delta V(v)$ which reflects the fact that the monopolist loses the patent (and its value) with flow probability δ. Assuming (and verifying later) that $\dot{V}(v) = 0$ while the patent lives, show that free entry implies

$$r = \left(\frac{\psi}{1-\beta}\right)^{-(1-\beta)/\beta} \eta\beta L - \delta \tag{5}$$

4. Derive the equilibrium growth rate. Which patent duration (or δ) maximizes growth? Does this also maximize welfare (you need to guess here, unless you want to follow the hint at the end of the question and solve for consumption).

5. Consider the balanced growth path. Show that

$$\frac{N_1}{N_2} = \frac{g}{\delta} \tag{6}$$

   Hint: Write out differential equation for N_1 and N_2.

6. Define a competitive equilibrium.

With symmetry, it is easy, but tedious, to show that

$$Y = AL(N_1\omega + N_2) \tag{7}$$

and

$$X = A(1-\beta)(N_1\omega + N_2) \tag{8}$$

where $A \equiv (1-\beta) L\psi^{-(1-\beta)/\beta}$ and $\omega \equiv (1-\beta)^{1/\beta}$. From, this one can derive balanced growth consumption, but this is also quite tedious, so I am not asking you to do this.