Money in OLG Models

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Topics

As an application, we study several models of money.

Philiosophy question:

What is money?

Money: A Fiction

Macro models usually contain two types of riskless assets:

- **money** and **bonds**.

Bonds pay interest; money usually not.

- "rate of return dominance"

The central question of monetary economics:

Why and when is money valued in equilibrium?

Why does money have value?

"**Liquidity**" - what is that?

In reality, "liquidity" is not discrete

- e.g., short term t-bills versus long-term bonds
- credit is money.

But macro models have not caught up with this.

Money in the OLG model

In the OLG model, money means:

- Pieces of paper with pictures of dead presidents.

Key features of OLG money:

- It is useless.
- It can be produced (by the government only) at no cost.

How then do we get agents to pay for money?

Money in the OLG model

Money is a **bubble**.

- Its value derives solely from the expectation that money will be valued tomorrow.

Money is valued like any other asset.

- It is only held if no other asset offers a higher rate of return.

Other Theories of Money

1. Cash-in-advance constraints:
   - Money is required for transactions
   - A technological requirement
2. Money in the utility function:
   - People just like the stuff
   - A reduced form
3. Search
   - Transactions benefit from having a means of exchange that everyone accepts.
   - What object is used as money is a social convention.
   - Not clear why “money” is (was?) used as money (as opposed to credit).

Dynamic efficiency

Can money alleviate dynamic inefficiency?

- Previous models lacked a long-lived asset that would facilitate intergenerational trade.
- Money could solve this problem.

An OLG Model of Money

Start with the standard two period OLG model without production or bonds.

Demographics: $N_t = (1+n)^t$

Preferences: $u\left(c_t^y, c_{t+1}^o\right)$

Endowments: e_1 when young; e_2 when old

Technologies: Goods can only be eaten:

$$N_t c_t^y + N_{t-1} c_t^o = N_c e_1 + N_{t-1} e_2 \tag{1}$$

Markets:

- goods (P_t), money (numeraire)

Introducing Money

In period 1, the initial old are given M_0 bits of green paper

In every subsequent period, the government prints additional paper and hands it to the current old *in proportion* to current paper holdings.

Effectively, money pays (nominal) interest if held from young to old age.

The money growth rate is constant:

$$M_{t+1}/M_t = 1 + \theta$$

Therefore: θ is the nominal interest rate earned by holding money.

Timing

Beginning of t:

- N_t young are born and receive e_1 goods.
- N_{t-1} old
  - receive e_2 goods
  - carry over M_{t-1}/N_{t-1} units of money
  - receive money transfer $\theta M_{t-1}/N_{t-1}$.
  - they now hold M_t/N_{t-1} each

Timing

During t

- the young sell goods to the old
- the old sell money to the young

At the end of t:

- the young hold M_t/N_t each

Note: This is the intergenerational transfer that is missing in the non-monetary model.

Young budget constraint

$$P_t e_1 = P_t \left(c_t^y + x_t \right) \tag{2}$$

or

$$e_1 = c_t^y + x_t \tag{3}$$

$x_t P_t$: nominal purchases of money (household saving).

Old budget constraint

$$P_{t+1}c^o_{t+1} = P_{t+1}e_2 + x_t(1+\theta)P_t \tag{4}$$

or

$$c^o_{t+1} = e_2 + x_t R_{t+1} \tag{5}$$

where

$$R_{t+1} = (1+\theta)P_t/P_{t+1}$$

is the gross real interest rate earned by holding money.

Lifetime budget constraint

$$e_1 - c_t^y = \frac{c_{t+1}^o - e_2}{R_{t+1}}$$

The household looks exactly like a model with a one-period bond.

The household does not care where R comes from.

Euler equation:

$$u_1(t) = R_{t+1} u_2(t)$$

Household: Solution

A solution to the household problem is a triple (c_t^y, c_{t+1}^o, x_t) which satisfies

- the Euler equation and
- the two budget constraints.

Optimal behavior can be characterized by a savings function

$$x_t = s(R_{t+1}, e_1, e_2) \tag{6}$$

The only new item: x is real money purchases (not bond or capital purchases).

General point

The household problem looks the same, regardless of the assets available.

An asset is simply a rate of return.

Without uncertainty, all assets must pay the same rate of return (otherwise: arbitrage).

Equilibrium

The government is simply described by a money growth rule:

$$M_{t+1}/M_t = 1+\theta$$

Market clearing:

- Money market:

$$M_t = N_t P_t x_t$$

  or

$$m_t = M_t/(N_t P_t) = s(R_{t+1})$$

- Goods market: RC

Careful about notation: the young in t take M_t into $t+1$.

Equilibrium Definition

A sequence of prices and quantities
such that

Characterizing Equilibrium

- We look for a difference equation in terms of the economy's state variables.
- State variables are M and P.
- But in this model (and typically) only the ratio $m = M/PN$ matters.

Characterizing Equilibrium

Start from the money market clearing condition

$$m_t = s(R_{t+1}) \tag{7}$$

Subsitute out R using

$$R_{t+1} = (1+\theta)P_t/P_{t+1} \tag{8}$$

We need an expression for inflation. From

$$1+\theta = \frac{M_{t+1}}{M_t} = \frac{m_{t+1}}{m_t}\frac{P_{t+1}}{P_t}\frac{N_{t+1}}{N_t}$$

we have

$$R_{t+1} = (1+\theta)P_t/P_{t+1} = (1+n)m_{t+1}/m_t$$

The law of motion is

$$m_t = s((1+n)m_{t+1}/m_t) \tag{9}$$

Characterizing Equilibrium

A more explicit way of deriving this.

For ease of notation assume

$$u\left(c_t^y, c_{t+1}^o\right) = v\left(c_t^y\right) + \beta v\left(c_{t+1}^o\right) \tag{10}$$

Sub budget constraint into Euler equation:

$$v'\left(e_1 - x_t\right) = R_{t+1}\beta v'\left(e_2 + R_{t+1}x_t\right) \tag{11}$$

Sub in $m_t = x_t$ and $R_{t+1} = (1+n)m_{t+1}/m_t$:

$$v'\left(e_1 - m_t\right) = (1+n)\frac{m_{t+1}}{m_t}\beta v'\left(e_2 + (1+n)m_{t+1}\right) \tag{12}$$

Intuition

$$m_t = s((1+n)m_{t+1}/m_t) \quad (13)$$

Why is this true?

- we really have $m_t = s(R_{t+1})$
- fixed: nominal money growth and n
- higher growth in $m = M/PN \implies$ lower inflation $\implies$ higher return

The Offer Curve

The Offer Curve

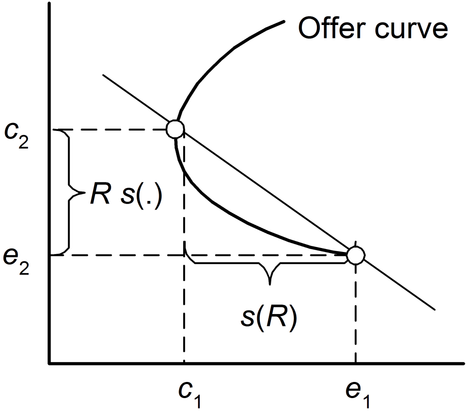
We want to determine the shape of the law of motion.

The key idea is to use the household's intertemporal consumption allocation to figure out how money evolves over time.

Household consumption choice

The lifetime budget constraint has slope $-R_{t+1}$.

Plot the tangencies between budget constraint and indifference curves for all interest rates $\rightarrow$ offer curve.



Offer curve

What do we know about the offer curve?

1. It goes through the endowment point.
2. At low levels of R the household would like to borrow (but cannot).
3. For interest rates where the household saves very little, income effects are small
   $\Longrightarrow$ savings rise with R_{t+1}
   $\Longrightarrow$ the offer curve is upward sloping
4. The offer curve intersects each budget line only once.

Law of motion for m

The offer curve maps $s(R_{t+1})$ into $R_{t+1}s(R_{t+1})$.

But we want to map m_t into m_{t+1}.

The horizontal axis actually shows m_t.

Because money demand equals saving of the young:

$$m_t = s(R_{t+1}) = e_1 - c_t^y \tag{14}$$

Law of motion for m

The vertical axis actually shows $(1+n)m_{t+1}$.

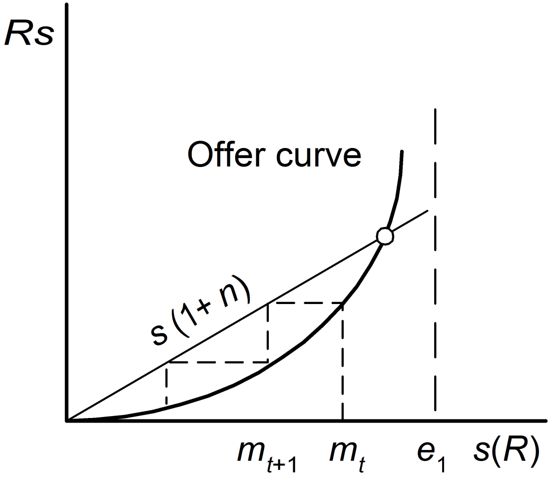
Because for the old:

$$
\begin{aligned}
c_{t+1}^{o} - e_2 &= R_{t+1} s(R_{t+1}) && (15)\\
&= (1+n)\frac{m_{t+1}}{m_t} m_t && (16)\\
&= (1+n) m_{t+1} && (17)
\end{aligned}
$$

Implicitly, the offer curve gives a law of motion for m:

$$(1+n)m_{t+1} = F(m_t)$$

Law of motion



Using a line of slope $(1+n)$ we can find the path of m_t for any start value m_0.

Steady state

- There is a unique monetary **steady state** (intersection of offer curve and ray through origin).
- It is ***unstable***.

Properties of the steady state

m is constant over time.

The gross rate of return on money is

$$\begin{aligned} R_{t+1} &= (1+\theta)P_t/P_{t+1} \\ &= (1+n)m_{t+1}/m_t \end{aligned}$$

Therefore, in steady state, the Golden Rule holds:

$$R = 1+n$$

Steady state inflation is

$$P_{t+1}/P_t = \frac{1+\theta}{1+n}$$

Dynamics

Assumption: the offer curve is not backward bending.

Take m_0 as given for now.

What if $m_0 > m_{ss}$?

- This cannot happen because m_t would blow up towards ∞.
- But then consumption will exceed total output at some point.

Dynamics

If $m_0 < m_{ss}$: m_t collapses towards 0.

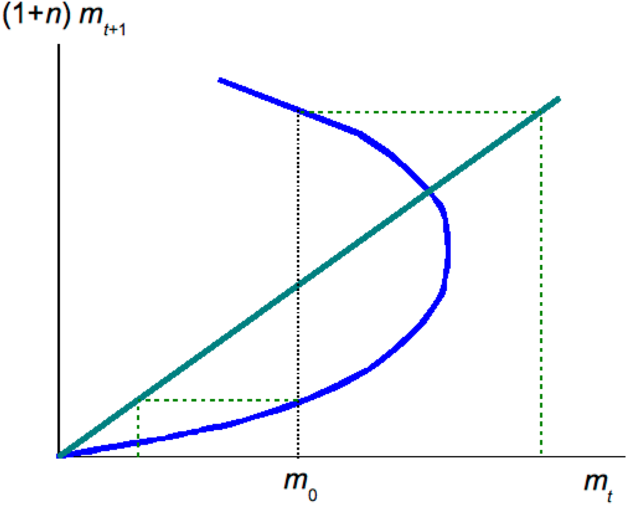
- Because M grows at a constant rate, this must happen through inflation.
- Along this path R falls over time $\Rightarrow$ inflation accelerates.

Intuition:

- If $m_0 = m_{ss}$ people save just enough to keep m constant.
- If m_0 is a bit lower, then R is a bit lower. People save less.
- That requires a lower m_1, hence more inflation.
- That leads people to save less again, etc.

Dynamics: Backward bending Offer Curve

We have **multiple equilibria** and **complex dynamics**.



Initial money stock

- Nothing in the model pins down m_0. Any value below m_{ss} is acceptable.
- There is a continuum of equilibrium paths.
- The reason: money is a bubble.
- As long as expectations are such that people are willing to hold m_0, we have an equilibrium.
- $m_0 = 0$ is also an equilibrium.

Dynamic Efficiency

Does money solve the dynamic inefficiency problem?

- It might because it permits intergenerational trade.

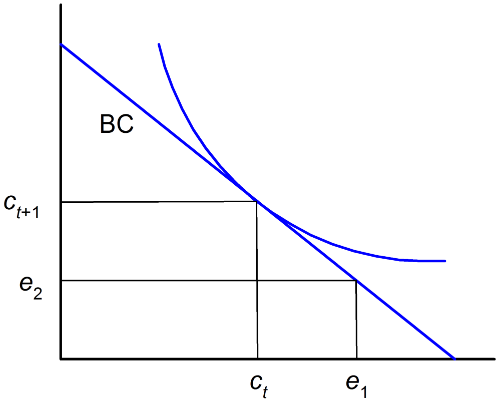
Two cases:

1. Samuelson case: the offer curve at the origin is flatter than $1+n$;
2. Classical case: it is steeper than $1+n$.

Dynamic Efficiency

- Why is the slope of the offer curve at the origin interesting?
- Because it is the interest rate in the non-monetary economy.
- Therefore:
  - Samuelson case: non-monetary economy is dyn. **inefficient**
  - Classical case: it is **efficient**

Samuelson case



Non-monetary interest rate is $< 1+n$

At $R = 1+n$ households want to save

Money can be valued in equilibrium

Classical Case

- The non-monetary economy is dynamically **efficient**.
- The offer curve is too steep to intersect the $1+n$ line.
- A monetary equilibrium does not exist.

Result:
Money is valued in equilibrium only in an economy that would be dynamically inefficient without money.

Is this a good theory of money?

Good features of the OLG model of money are:

1. The outcome that money is valued in equilibrium is not assumed (e.g. because money yields utility or is simply required for transactions).
2. The value of money depends on expectations and is fragile.

The problem:

1. The model does not generate rate of return dominance.
2. A key feature of money seems to be missing: liquidity.

How to construct a theory of money that resolves the problems without introducing new ones is an open question.

Fiscal Theory of the Price Level

Model With Government Spending

We add government spending to the model and get an odd result.

Preferences, endowments, demographics are unchanged

Government

- buys $G_t = g_t N_t$ goods
- prints money to finance the purchases

Markets: There are markets for goods and for money.

Government Budget Constraint

The government budget constraint is

$$M_{t+1} - M_t = P_{t+1} G_{t+1}$$

Divide both sides by $P_{t+1} N_{t+1}$:

$$m_{t+1} = m_t / [(1+n)(1+\pi_{t+1})] + g_{t+1} \tag{18}$$

where $m = M/(PN)$

Household

The household problem is exactly the same as in the OLG model with one period bonds.

Young budget constraint: $c_t^y = e_1 - x_t$.

Old budget constraint: $c_{t+1}^o = e_2 + x_t P_t / P_{t+1}$

Lifetime budget constraint:

$$c_t^y + c_{t+1}^o / R_{t+1} = e_1 + e_2 / R_{t+1}$$

where the real interest rate is now

$$R_{t+1} = P_t / P_{t+1}.$$

We get the saving function $s(R_{t+1}; e_1, e_2)$ as usual from the Euler equation and the budget constraints.

Equilibrium

A CE consists of sequences $\{c_t^y, c_t^o, x_t, m_t, P_t\}$ that satisfy

- 3 household conditions (2 b.c. and saving function);
- government budget constraint (18);
- Goods market clearing:

$$c_t^y + c_t^o/(1+n) + \underbrace{g_t}_{\text{new}} = e_1 + e_2/(1+n)$$

- Money market clearing: $M_t = N_t P_t x_t$ or

$$x_t \quad = m_t = \quad s(P_t/P_{t+1})$$

Offer Curve

Start from $m_t = s(P_t/P_{t+1})$.

Rewrite the government budget constraint as

$$P_t/P_{t+1} = (1+n)(m_{t+1} - g_{t+1})/m_t$$

Then:

$$m_t = s((1+n)(m_{t+1} - g_{t+1})/m_t), \tag{19}$$

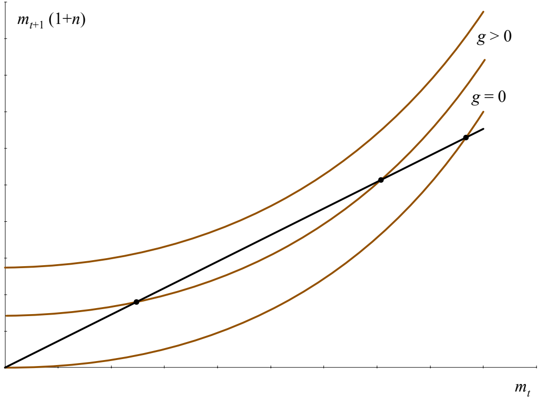
The offer curve now relates $m_{t+1} - g_{t+1}$ to m_t.

With $g = 0$ this is the model we studied earlier.

Assume

- g is constant over time
- $s(1+n) > 0$ (Samuelson case).
- the offer curve is convex, but not backward bending.

Offer Curve: Varying g



$g \uparrow$ shifts the offer curve up
There is a continuum of equilibria indexed by $m_1 \in (0, m^*)$

Multiple Steady States

- There are two steady states.
- How do the 2 steady states differ?
- The lower steady state is stable, while the higher one is not.
- From the government budget constraint

$$1 = 1/[(1+n)(1+\pi)] + g/m$$

- A higher m implies a lower π (given g).

No Non-monetary Equilibrium

- The odd finding:
  With $g > 0$ the non-monetary equilibria have disappeared!
- The reason: the government promises to violate its budget constraint in equilibria it does not like
- This is the essence of the "Fiscal Theory of the Price Level."
- Government spending, via the budget constraint, determines the value of money in equilibrium.

Reading

- Blanchard and Fischer (1989), ch. 4.1 [A clear exposition.]
- Krueger, "Macroeconomic Theory," ch. 8 discusses offer curves (can be found online).
- Ljungqvist and Sargent (2004), ch. 9 [Detailed.]
- McCandless and Wallace (1991)

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