

# Example: Money in OLG Models

Econ720. Prof. Lutz Hendricks. July 21, 2020

## 1 Government Purchases

Consider a two-period OLG model with money.

Demographics: At each date  $t$  a cohort of size  $N_t = (1+n)^t$  is born. Each household lives for two periods.

Preferences: Individuals derive utility from consumption at both ages according to

$$u(c_t^y, c_{t+1}^o)$$

Endowments: Households receive endowments of goods when young ( $e_t$ ) that cannot be stored.

Technologies: None.

Government: The government buys  $G_t = g_t N_t$  goods in each period and throws them into the ocean. It prints money to finance the purchases.

Markets: There are markets for goods and for money. Money is the only asset.

(a) Write down the government budget constraint. is. Show that it can be written as

$$P_t/P_{t+1} = (1+n)(\bar{m}_{t+1} - g_{t+1})/\bar{m}_t$$

where  $\bar{m} = M/(PN)$  denotes real, per capita money balances. We are doing this because we need to know the rate of return of holding money. It will be the argument of the saving function.

(b) Write down the household problem and derive an expression that implicitly defines a saving function.

(c) Define a competitive equilibrium.

(d) The money market clearing condition is

$$\bar{m}_t = s(P_t/P_{t+1})$$

where  $s$  is a saving function. For what follows assume that  $g$  is constant over time and that  $s(1+n) > 0$  (Samuelson case). Also assume that the offer curve is convex, but not backward bending. Using

$$\bar{m}_t = s((1+n)(\bar{m}_{t+1} - g_{t+1})/\bar{m}_t), \quad (1)$$

which can be thought of as a difference equation for  $m$ , show that there is a limit to steady state seignorage. I.e.,  $g$  is bounded above by  $g_{max}$ .

(e) Show that, if  $0 < g < g_{max}$ , there are two stationary monetary equilibria. The one with higher money stock (call it  $m^*$ ) has a lower rate of inflation.

(f) There is a continuum of equilibria indexed by

$$\bar{m}_1 \in (0, m^*)$$

(g) Why is there no equilibrium where money is not valued? Isn't that weird?

## 1.1 Answer: Government Purchases

(a) The government budget constraint is

$$M_{t+1} - M_t = P_{t+1}G_{t+1}$$

Divide both sides by  $P_{t+1}N_{t+1}$ :

$$\bar{m}_{t+1} = \bar{m}_t / [(1+n)(1+\pi_{t+1})] + g_{t+1}$$

Rearrange to obtain the answer for (a).

(b) There is actually nothing new in this part of the question. The household problem is exactly the same as in the OLG model with one period bonds. The young budget constraint is  $c_t^y = e_1 - m_t/P_t$ . The old budget constraint is

$$c_{t+1}^o = e_2 + m_t/P_{t+1}$$

The lifetime budget constraint is

$$c_t^y + c_{t+1}^o / R_{t+1} = e_1 + e_2 / R_{t+1}$$

where the real interest rate is

$$R_{t+1} = P_t / P_{t+1}.$$

We get the saving function  $s(R_{t+1}; e_1, e_2)$  as usual from the Euler equation and the budget constraints.

(c) A CE consists of sequences  $\{c_t^y, c_t^o, m_t, \bar{m}_t, P_t\}$  that satisfy

- 3 household conditions (2 b.c. and saving function);
- government budget constraint;
- Goods market clearing:

$$c_t^y + c_t^o / (1+n) = e_1 + e_2 / (1+n) + g_t$$

- Money market clearing:

$$m_t / P_t = \bar{m}_t = s(P_t / P_{t+1})$$

[Strictly speaking:  $\bar{m}_t = s(R_{t+1})$  is the household decision rule and  $m_t / P_t = \bar{m}_t$  is the market clearing condition.]

(d) The offer curve now relates  $\bar{m}_{t+1} - g_{t+1}$  to  $\bar{m}_t$ . Consider first the case where  $g = 0$ . Then we are back to the case studied in class. Now raise  $g$ . The offer curve shifts up (for a given  $\bar{m}_t$  we have a higher  $\bar{m}_{t+1}$ ). Hence there is a family of offer curves. All are parallel and indexed by  $g$ . Increasing  $g$  shifts the offer curve up.

Given the assumption that the offer curve is not backward bending, the two steady states move closer together and finally coincide when the offer curve is tangent to the 45-degree line (see Figure 1).

(e) Shifting the offer curve shows that there are two steady states. Since in steady state

$$1 = 1 / [(1+n)(1+\pi)] + g / \bar{m}$$

a higher money stock (given  $g$ ) requires a lower rate of inflation.

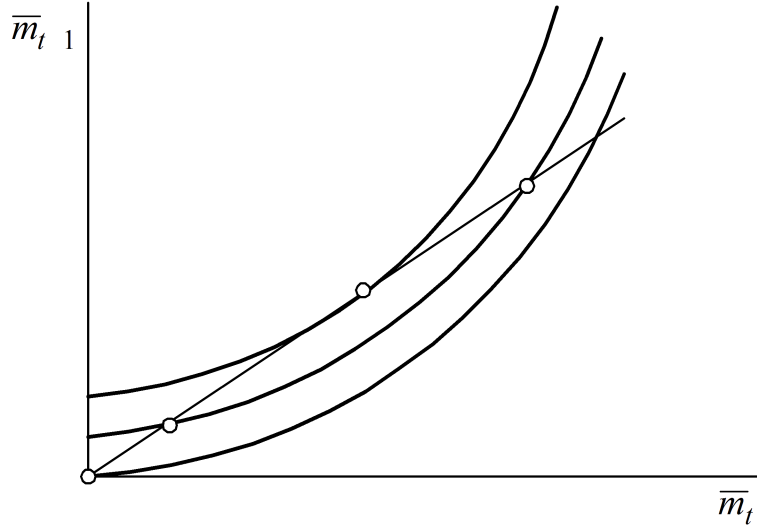


Figure 1: Offer curve

(f) The equilibrium conditions leave the initial money stock (i.e. price level) indeterminate. The lower steady state is stable, while the higher one is not.

(g) Technically, the answer is that such an equilibrium violates the government budget constraint. This is an example of the “Fiscal Theory of the Price Level.” Government spending, via the budget constraint, determines the value of money in equilibrium. There is something questionable about the concept. Essentially, the government promises to violate its budget constraint in states of the world that are not on the equilibrium path. Doing so, it rules out such states as equilibria. A lot of people think this type of equilibrium selection does not make sense.