

# Arrow-Debreu and Sequential Trading

Prof. Lutz Hendricks

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# Introduction

Macro models are dynamic (have many periods).

Then we have a choice of how to represent equilibrium:

- ▶ Arrow-Debreu: all trading takes place at date 0
- ▶ Sequential trading: markets open in each period

This is where the details matter (units of account, Walras' law, ...)

# Two Period Example

Demographics:

- ▶  $N$  identical households live for 2 periods,  $t = 1, 2$ .

Commodities:

- ▶ there is one good in each period

Preferences:  $u(c_1, c_2)$

Endowments:  $e_t$

“Technology”:  $c_t = e_t$

# Markets

Now we have a choice between 2 equivalent arrangements

- ▶ Arrow-Debreu: all trades take place at  $t = 1$
- ▶ Sequential trading: markets open in each period

# Arrow-Debreu Trading

The arrangement:

- ▶ All trades take place at  $t = 1$
- ▶ Agents can buy and sell goods for delivery at any date  $t$
- ▶ Prices are  $p_t$

Can we normalize prices to 1?

Surprise:

If we write out this model, it **looks exactly like the static 2 good model** (see above).

# Arrow-Debreu Equilibrium

Household budget constraint:

$$\sum_t p_t e_t = \sum_t p_t c_t \quad (1)$$

Interpretation:

The household sells  $e_t$  to and buys  $c_t$  from the Walrasian auctioneer at a single trading date.

Market clearing:

$$e_t = c_t \quad (2)$$

- ▶ Again the same as resource constraints.

# Equilibrium

Objects:  $c_t, p_t, t = 1, 2$

Equations:

- ▶ Household policy rules:  $c_t(p_1, p_2)$   
implicitly defined by first-order condition and budget constraint
- ▶ Market clearing:  $e_t = c_t$

Notes:

- ▶ only  $p_2/p_1$  is determined in equilibrium (choice of unit of account)
- ▶ only **one** equation is redundant by Walras' law (why?)

# Equivalence of Dates and Goods

## Fact

*A model with  $T$  goods is equivalent to a model with  $T$  periods.*

Why is this true? What is a “good?”

This is only true under “**complete markets**”

- ▶ roughly: there are markets that allow agents to trade goods across all periods and states of the world
- ▶ we will talk about details later



# Sequential Trading

An alternative trading arrangement.

Markets open at each date.

Only the date  $t$  good can be purchased in the period  $t$  market.

Now we have **one numeraire for each trading period**:  $p_t = 1$ .

We need assets to transfer resources between periods.

# Markets

At each date we have

1. a market for goods ( $p_t = 1$ );
2. a market for 1 period discount bonds (price  $q_t$ )

A discount bond pays 1 unit of  $t + 1$  consumption.

# Digression: Modeling bonds

## Definition

A one period bond promises to pay one unit of consumption in  $t+1$ .

Call its price  $q_t$ .

Then the real interest rate is:  $R_{t+1} = 1/q_t$ .

## Econ101 question

What is a real interest rate?

Alternative normalization:

- ▶ set  $q_t = 1$  and let each bond pay  $R_{t+1}$  units of consumption
- ▶ why can I do this?

## Household problem

Now we have one budget constraint per period:

$$e_t + b_{t-1} = c_t + b_t q_t \quad (3)$$

with  $b_0 = 0$ .

### Fact

*Agents have one budget constraint for each trading period*

Household solves:

$$\max_{b_1} u(e_1 - b_1 q_1, e_2 + b_1) \quad (4)$$

## Household solution

FOC:

$$u_1 q_1 = u_2 \quad (5)$$

$q_1$  is the relative price of period 2 consumption.

Give up 1 unit of  $c_1$  and get  $1/q_1$  units of  $c_2$ .

Solution:  $c_1, c_2, b_1$  that solve FOC and 2 budget constraints.

# Market Clearing

- ▶ Goods:  $e_t = c_t$
- ▶ Bonds:  $b_t = 0$

Why does bond market clearing look so odd?

# Equivalence

Note that the relative price is the same under both trading arrangements:

$$p = q = u_2/u_1 \quad (6)$$

## Fact

*When markets are complete, Arrow-Debreu and sequential trading equilibria are identical.*

# Why two trading arrangements?

When building a model, choose the most convenient trading arrangement.

In many cases, it does not matter

- ▶ equilibrium allocations are the same
- ▶ both are similarly convenient

It matters when there is uncertainty.

- ▶ with complete markets, Arrow-Debreu is much simpler
- ▶ with incomplete markets, sequential trading makes it easy to specify which assets exist



# Summary

*Macro is micro*

*or*

*IS-LM is dead. Long-live general equilibrium*

- ▶ The method outlined here is central to all of (macro) economics.
- ▶ Being able to translate a description of an economy into the definition of a competitive equilibrium is an important skill.

## Final example

Demographics: There are  $N$  households. Each lives for  $T > 1$  periods.

Preferences:  $\sum_{t=1}^T u(c_{1,t}, \dots, c_{J,t})$  where  $J$  is the number of goods available in each period.

Endowments: Household  $i$  receives  $e_{i,j,t}$ .

Technologies: Endowments can only be eaten in the period they are received.

- ▶ Resource constraint:

Markets:

- ▶ Sequential trading: there are competitive markets for the  $J$  goods; there are one period discount bonds in each period.
- ▶ Arrow-Debreu: the  $J \times T$  goods are traded in  $t = 1$ .

## Final example: Arrow-Debreu Equilibrium

## Reading

Krusell (2014), ch. 5 talks about Arrow-Debreu versus sequential trading.

## References

Krusell, P. (2014): “Real Macroeconomic Theory,” Unpublished.