Arrow-Debreu and Sequential Trading

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Introduction

Macro models are dynamic (have many periods).

Then we have a choice of how to represent equilibrium:

- Arrow-Debreu: all trading takes place at date 0
- Sequential trading: markets open in each period

This is where the details matter (units of account, Walras' law, ...)

Two Period Example

Demographics:

- N identical households live for 2 periods, $t = 1, 2$.

Commodities:

- there is one good in each period

Preferences: $u(c_1, c_2)$

Endowments: e_t

"Technology": $c_t = e_t$

Markets

Now we have a choice between 2 equivalent arrangements

- Arrow-Debreu: all trades take place at $t = 1$
- Sequential trading: markets open in each period

Arrow-Debreu Trading

The arrangement:

- All trades take place at $t = 1$
- Agents can buy and sell goods for delivery at any date t
- Prices are p_t

Can we normalize prices to 1?

Surprise:
If we write out this model, it **looks exactly like the static 2 good model** (see above).

Arrow-Debreu Equilibrium

Household budget constraint:

$$\sum_t p_t e_t = \sum_t p_t c_t \tag{1}$$

Interpretation:
The household sells e_t to and buys c_t from the Walrasian auctioneer at a single trading date.

Market clearing:

$$e_t = c_t \tag{2}$$

- Again the same as resource constraints.

Equilibrium

Objects: $c_{t,}p_t$, $t = 1,2$

Equations:

- Household policy rules: $c_t(p_1,p_2)$
  implicitly defined by first-order condition and budget constraint
- Market clearing: $e_t = c_t$

Notes:

- only p_2/p_1 is determined in equilibrium (choice of unit of account)
- only **one** equation is redundant by Walras' law (why?)

Equivalence of Dates and Goods

Fact

A model with T goods is equivalent to a model with T periods.

This is only true under "**complete markets**"

- roughly: there are markets that allow agents to trade goods across all periods and states of the world
- we will talk about details later

Sequential Trading

An alternative trading arrangement.

Markets open at each date.

Only the date t good can be purchased in the period t market.

Now we have **one numeraire for each trading period**: $p_t = 1$.

We need assets to transfer resources between periods.

Markets

At each date we have

1. a market for goods $(p_t = 1)$;
2. a market for 1 period discount bonds (price q_t)

A discount bond pays 1 unit of $t+1$ consumption.

Digression: Modeling bonds

Definition

A one period bond promises to pay one unit of consumption in $t+1$.

Call its price q_t.

Then the real interest rate is: $R_{t+1} = 1/q_t$.

What is a real interest rate?

Alternative normalization:

- set $q_t = 1$ and let each bond pay R_{t+1} units of consumption
- why can I do this?

Household problem

Now we have one budget constraint per period:

$$e_t + b_{t-1} = c_t + b_t q_t \tag{3}$$

With $b_0 = 0$.

Household solves:

$$\max_{b_1} u(e_1 - b_1 q_1, e_2 + b_1) \tag{4}$$

Household solution

FOC:

$$u_1 q_1 = u_2 \tag{5}$$

q_1 is the relative price of period 2 consumption.

Give up 1 unit of c_1 and get $1/q_1$ units of c_2.

Solution: c_1, c_2, b_1 that solve FOC and 2 budget constraints.

Market Clearing

- Goods: $e_t = c_t$
- Bonds: $b_t = 0$

Why does bond market clearing look so odd?

Equivalence

Note that the relative price is the same under both trading arrangements:

$$p = q = u_2/u_1 \tag{6}$$

Fact

When markets are complete, Arrow-Debreu and sequential trading equilibria are identical.

Summary

Macro is micro
or
IS-LM is dead. Long-live general equilibrium

- The method outlined here is central to all of (macro) economics.
- Being able to translate a description of an economy into the definition of a competitive equilibrium is an important skill.

Final example

Demographics: There are N households. Each lives for $T > 1$ periods.

Preferences: $\sum_{t=1}^{T} u(c_{1,t}, \ldots, c_{J,t})$ where J is the number of goods available in each period.

Endowments: Household i receives $e_{i,j,t}$.

Technologies: Endowments can only be eaten in the period they are received.

- Resource constraint:

Markets:

- Sequential trading: there are competitive markets for the J goods; there are one period discount bonds in each period.
- Arrow-Debreu: the $J \times T$ goods are traded in $t = 1$.

Final example: Equilibrium

Reading

Krusell (2014), ch. 5 talks about Arrow-Debreu versus sequential trading.

References

Krusell, P. (2014): "Real Macroeconomic Theory," Unpublished.