

# Growth Through Product Creation

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# Issues

We study models where **intentional innovation** drives productivity growth.

Background:

- ▶ Historians often view innovation as the result of research that is not profit driven.
- ▶ Economists treat innovation as producing goods that are sold in markets ("blueprints").
- ▶ There are historical examples of both types of innovation.
- ▶ How important are the 2 cases? – An open question.

# How to model innovation

- ▶ Current models are somewhat reduced form.
- ▶ The issue how existing knowledge feeds into future innovation is treated as a **knowledge spillover**.
- ▶ Knowledge is treated as a scalar - like capital.
- ▶ In fact, the only difference between blueprints and machines is **non-rivalry**:
  - ▶ blueprints can be used simultaneously in the production of several goods.

# How to model innovation

There are  $N$  consumption goods (or intermediate inputs).

The goods are imperfect substitutes in preferences (or final output production).

- ▶ Therefore downward sloping demand curves

## Approach 1: **Quality ladders**

- ▶ Each good can be made by many firms.
- ▶ Firms can invest to improve quality (equivalently: lower the cost) of 1 good.

## Approach 2: **Increasing variety**

- ▶ Each firm can invest to create a new variety ( $N \rightarrow N + 1$ )
- ▶ Then it becomes the monopolist for that variety

## 2. A Model of Increasing Varieties

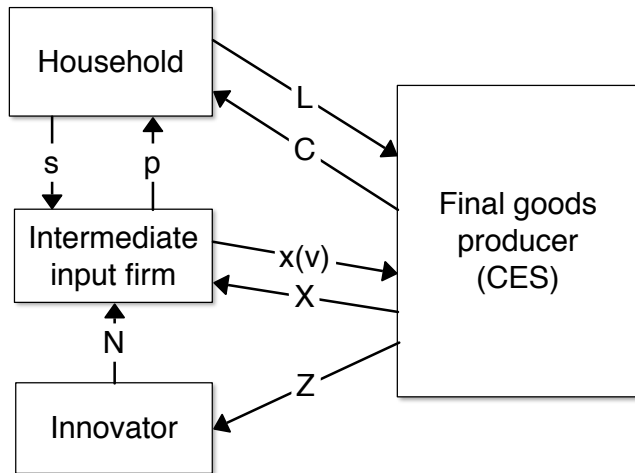
# Setup

Agents:

1. A representative **household** supplies labor to firms
2. **Final** goods firms use labor and intermediate inputs
3. **Intermediate** inputs are produced from final goods
4. **Innovators**  
create new intermediates from final goods  
receive permanent monopolies

Note: Now that models get more complicated, it really pays off to be pedantic about details.

## Model structure



# The Story Line

## Innovators

- ▶ buy goods from the final goods firm ( $Z$ ).
- ▶ invent a new variety  $j$
- ▶ receive a permanent patent for good  $j$

## Intermediate goods producers

- ▶ buy the patent from the innovator
- ▶ forever make  $x_j$  using the final good ( $X$ )
- ▶ sell it as monopolist to the final goods firm
- ▶ profits go to households



# Demographics and Preferences

Demographics:

- ▶ A representative household.

Preferences:

$$\int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\theta} - 1}{1-\theta} dt \quad (1)$$

- ▶  $C$ : the final good

## Technology: Final Goods

Resource constraint:

$$C_t + X_t + Z_t = Y_t$$

Final goods  $Y$  are used for

- ▶  $Z$ : R&D investment.
- ▶  $X$ : Inputs into the production of intermediates  $x$ .
- ▶  $C$ : consumption

## Technology: Final Goods

Production of **final goods** from intermediates and labor:

$$Y_t = \underbrace{(1 - \beta)^{-1}}_{\text{normalization}} \mathcal{Q}^{1-\beta} L^\beta \quad (2)$$

where

$$\mathcal{Q} \equiv \left[ \int_0^N x(v)^{1-\beta} dv \right]^{\frac{1}{1-\beta}} \quad (3)$$

is a CES aggregator of intermediate inputs  $x$ .

Simplify:

$$Y_t = (1 - \beta)^{-1} \left[ \int_0^{N_t} x(v, t)^{1-\beta} dv \right] L^\beta \quad (4)$$

## Technology: Final Goods

This is the key trick of the model:

- ▶ the CES aggregator  $\mathcal{Q}$  implies a **constant price elasticity** of demand for  $x$
- ▶ assume that the marginal cost of  $x$  is constant
- ▶ if the suppliers of  $x$  are monopolists, their prices are a fixed markup over marginal costs

Then the price of  $x$  becomes a known function of parameters.

## Technology: Intermediate Inputs

Each unit of  $x$  requires  $\psi$  units of  $Y$ .

- This is the constant marginal cost of  $x$  assumption.

The total amount of goods used to make intermediates is

$$X = \psi \int_0^{N_t} x(v, t) dv \quad (5)$$

Intermediate inputs fully depreciate in use.

## Technology: Innovation

Investing the final good yields a flow of new patents:

$$\dot{N} = \eta Z_t \quad (6)$$

Think of this as the aggregate (deterministic) outcome of the (stochastic) innovation efforts of many firms.

# Market arrangements

- ▶ Final goods and labor markets are competitive.
- ▶ Intermediates are sold by **monopolists** (the innovators).
  - ▶ Monopolies are permanent.
  - ▶ What the monopolists do with their profits is not clear.
- ▶ Free entry into innovation
  - ▶ ensures zero present value of profits
- ▶ The household owns the innovating firms.
- ▶ Asset markets are complicated
  - ▶ there is often no need to spell out the details

# Notes

Production is cyclical:

- ▶ today's  $Y$  is used to make  $X$  which makes  $Y$
- ▶ the alternative: durable  $X$  (more complicated)
- ▶ implication: the efficient allocation maximizes  $Y - X = C + Z$

The only long-lived object is a patent

- ▶ this keeps the model simple

Assuming that intermediates are made from final goods fixes marginal costs (and prices)



### 3. Solving Each Agent's Problem

## Final goods producers

- ▶ Maximize period profits by choosing  $L$  and  $x(v, t)$ .
- ▶ Take prices  $p(v, t)$  as given.
- ▶ Normalize the price  $Y$  to 1.
- ▶ Profits

$$Y_t - w_t L_t - \int_0^{N_t} p^x(v, t) x(v, t) dv \quad (7)$$

where

$$Y_t = (1 - \beta)^{-1} \left[ \int_0^{N_t} x(v, t)^{1-\beta} dv \right] L^\beta \quad (8)$$

## Final goods producers

FOCs:

$$\blacktriangleright \partial Y / \partial x(v) = L^\beta x(v)^{-\beta} = p^x(v)$$

$$\blacktriangleright \partial Y / \partial L = \beta Y / L = w$$

Demand function:

$$x(v, t) = L p^x(v, t)^{-1/\beta} \quad (9)$$

Note the constant price elasticity  $1/\beta$ .

**Solution** to the firm's problem:  $L_t, x(v, t)$  that satisfy the "2" first-order conditions.

## Intermediate input producers

Problem after inventing a variety.

$x$  is produced at constant marginal cost  $\psi$ .

Maximize present value of profits

$$V(v, t) = \int_t^{\infty} e^{-rs} \pi(v, s) ds \quad (10)$$

Instantaneous profits are

$$\pi(v, t) = (p^x(v, t) - \psi) x(v, t) \quad (11)$$

where  $x(v, t) = Lp^x(v, t)^{-1/\beta}$

This is a sequence of static problems

## Intermediate input producers

- ▶ First order condition (standard monopoly pricing formula):

$$p^x = \psi / (1 - \beta) \quad (12)$$

- ▶ Profits are

$$\pi(v, t) = \psi \frac{\beta}{1 - \beta} x(v, t) \quad (13)$$

- ▶ Solution: A constant  $p^x$ .

# Innovators

Technology:

$$\dot{N} = \eta Z_t \quad (14)$$

Spend 1 unit of the good for one unit of time at total cost 1.

Obtain  $\eta$  new patents, each valued (initially) at  $V(v, t)$ .

Innovate as long as

$$\eta V(v, t) \geq 1 \quad (15)$$

In equilibrium, free entry makes (15) hold with equality.

# Household

- ▶ The household holds shares of all intermediate input firms.
- ▶ Each firm produces a stream of profits.
- ▶ New firms issue new shares.
- ▶ But: the details don't matter to the household.
- ▶ There simply is an asset with rate of return  $r$ .
- ▶ Euler equation is standard:

$$g(C) = \frac{r - \rho}{\theta} \quad (16)$$

- ▶ Invoke Walras' law - so you never have to write down the budget constraint!

## 4. Equilibrium

Objects:

- ▶ allocation:  $C_t, X_t, Z_t, x(v, t), V(v, t), N_t$
- ▶ prices:  $p^x(v, t), r(t), w(t)$ .

Conditions:

- ▶ "Everybody maximizes." (see above)
- ▶ Markets clear.
  1. Goods: resource constraint.
  2. Shares: omitted b/c I did not write out the household budget constraint.
  3. Intermediates: implicit in notation.
- ▶ Innovation effort satisfies a **free entry** condition: present value of profits equals 0.



# Symmetric Equilibrium

We assume (and then show) that all varieties  $v$  share the same  $x, V$  and  $p^x$ .

## Intuition:

- ▶  $p^x$ : monopoly pricing with a constant elasticity
- ▶  $x$ : varieties enter final goods production symmetrically
- ▶  $V$ : the age of a variety does not matter  
(no stock of  $x$  to build; permanent patents)

# Simplifications

Normalize marginal cost  $\psi = 1 - \beta$

- ▶ so that  $p^x = 1$ .
- ▶ Why can I do that?

Focus on balanced growth paths.

## 5. Equilibrium: Characterization

There is an algorithm ...

- ▶ The growth rate follows from the Euler equation:  
 $g(C) = (r - \rho)/\theta$ .
- ▶ We get  $r$  from free entry by innovators: present value of profits = cost of creating a variety.

## Equilibrium: Characterization

Free entry will determine the interest rate

Innovators enter until marginal cost equals value of patents created:

$$\eta V(v, t) = 1 \quad (17)$$

Then  $V$  is constant over time.

- This assumes that innovation takes place.

With balanced growth and constant profits (to be shown):

$$V = \int_0^{\infty} e^{-rt} \pi dt = \pi/r \quad (18)$$

# Profits

With a fixed markup, profits are a multiple of revenues.

We derived (cf (13)):

$$\pi(t) = \psi \frac{\beta}{1-\beta} x(t) \quad (19)$$

$$= \beta x(t) \quad (20)$$

Demand for intermediates (cf (9)):

$$\begin{aligned} x(t) &= L p^x(t)^{-1/\beta} \\ &= L \end{aligned}$$

Profits:  $\pi = \beta L$ .

## Free Entry

Free entry:

$$\eta V = \eta \pi / r = \eta \beta L / r = 1 \quad (21)$$

This is the closed form solution for

$$r = \eta \beta L \quad (22)$$

Balanced **growth** rate then follows from the Euler equation.

$$g(C) = \frac{\eta \beta L - \rho}{\theta} \quad (23)$$

## Equilibrium: Characterization

Production function for final goods with  $x = L$ :

$$Y = \frac{1}{1 - \beta} N_t L \quad (24)$$

Wage (from firm's FOC):

$$w_t = \beta \frac{Y_t}{L_t} = \frac{\beta}{1 - \beta} N_t \quad (25)$$

Total expenditure on intermediates:

$$X_t = \psi N_t x_t = (1 - \beta) N_t L \quad (26)$$

# Summary of Equilibrium

Prices and quantities of intermediate inputs are constant.

- ▶ the model is rigged to deliver this
- ▶ for tractability

Growth comes from rising  $N$



# No Transition Dynamics

The equilibrium looks like an AK model with production function

$$\begin{aligned} Y_t - X_t &= \left[ \frac{L}{1-\beta} - (1-\beta) \right] N_t \\ \dot{N}_t &= \eta s_z (Y_t - X_t) \end{aligned}$$

Intuition:

- ▶ Period profits  $\pi$  are constant at  $\beta L$ .
- ▶ At any moment we need  $\eta V = 1$ .
- ▶  $V$  is the present value of (constant) profits.
- ▶ Constant  $V$  is only possible with constant  $r$ .
- ▶ Then at least consumption growth must be constant.

# Scale Effects

$$g(C) = \frac{\eta\beta L - \rho}{\theta}$$

Larger economies ( $L$ ) grow faster.

Population growth implies exploding income growth (!)

Mechanical reason:

- ▶ Innovation technology is linear in goods.
- ▶ Larger economy  $\rightarrow$  higher  $Y \rightarrow$  higher  $Z \rightarrow$  faster growth.

We will return to this later.

# Reading

- ▶ Acemoglu (2009), ch. 13.
- ▶ Krusell (2014), ch. 9
- ▶ Romer (2011), ch. 3.1-3.4.
- ▶ Jones (2005)

## References I

- Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.
- Jones, C. I. (2005): "Growth and ideas," *Handbook of economic growth*, 1, 1063–1111.
- Krusell, P. (2014): "Real Macroeconomic Theory," Unpublished.
- Romer, D. (2011): *Advanced macroeconomics*, McGraw-Hill/Irwin.