

Models of Creative Destruction

Firm Dynamics

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Motivation

We extend the Schumpeterian model to have innovation by incumbents.

This produces a model of firm size dynamics.

Environment

Demographics, preferences, commodities: unchanged.

Resource constraint:

$$Y = C + X + Z \quad (1)$$

where

$$X(t) = \int_0^1 \psi x(v, t) dv \quad (2)$$

$$Z(t) = \int_0^1 [z(v, t) + \hat{z}(v, t)] q(v, t) dv \quad (3)$$

z and \hat{z} are innovation inputs by incumbents and their rivals.

Final goods technology

$$Y(t) = \frac{1}{1-\beta} L(t)^\beta \int_0^1 q(v,t)^\beta x(v,t|q)^{1-\beta} dv \quad (4)$$

- ▶ the only change: quality is taken to power β
- ▶ implies: sales vary with quality (so the model has firm size implications)

Intermediate goods technology

- ▶ constant marginal cost ψ
- ▶ unchanged

Innovation technology for incumbents

- ▶ let $q(v, s)$ be the quality at the time the incumbent invented it
- ▶ investing zq implies a flow probability of innovation of ϕz
- ▶ the quality step is λ

Innovation technology for entrants

- ▶ investing $\hat{z}q$ implies a flow probability of innovation of $\eta(\hat{z})\hat{z}$
- ▶ η is **decreasing**
- ▶ marginal cost of innovation is rising in \hat{z}
- ▶ the quality step is $\kappa > \lambda$ (leapfrogging)
- ▶ innovators take η as given (an externality)

Solving each agent's problem

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Household (unchanged):

$$g(C) = \frac{r - \rho}{\theta} \quad (5)$$

Final goods producer (almost unchanged):

$$x(v, t|q) = p^x(v, t|q)^{-1/\beta} q(v, t) L \quad (6)$$

$$w(t) = \beta Y(t) / L(t) \quad (7)$$

Intermediate goods producer

Assume drastic innovation.

Then price follows the usual monopoly formula (unchanged):

$$p^x(v, t|q) = \frac{\psi}{1 - \beta} = 1 \quad (8)$$

Innovation by entrants

Free entry:

Investing $q\hat{z}$ gives a flow of $\eta\hat{z}$ new patents “per period”

$$\underbrace{\eta(\hat{z})\hat{z}}_{\text{probability}} \underbrace{V(v,t|\kappa q)}_{\text{payoff}} = \underbrace{q(v,t)\hat{z}}_{\text{cost}} \quad (9)$$

or

$$\boxed{V(v,t|\kappa q) = \frac{q}{\eta(\hat{z})}} \quad (10)$$

Note the κq .

This assumes an equilibrium with entry.

The flow probability that any competitor replaces the incumbent is $\hat{z}\eta(\hat{z})$.

Innovation by incumbents

Again assuming positive innovation.

Increase z until the marginal value equals marginal cost:

$$\underbrace{\phi z(v, t|q)}_{\text{probability}} \underbrace{[V(v, t|\lambda q) - V(v, t|q)]}_{\text{payoff}} = \underbrace{q(v, t) z(v, t|q)}_{\text{cost}} \quad (11)$$

We show later that V is proportional to quality q . Then

$$\phi V(v, t|q) [\lambda - 1] = q(v, t) \quad (12)$$

or

$$\boxed{V(v, t|q) = \frac{q}{\phi(\lambda - 1)}} \quad (13)$$

Value of the firm

Expected discounted value or profits

$$rV(v, t|q) = \underbrace{\pi(v, t|q)}_{\text{flow profit}} + \underbrace{\dot{V}(v, t|q)}_0 - \underbrace{z(v, t|q)q(v, t)}_{\text{R\&D cost}} \quad (14)$$

$$+ \underbrace{\phi z(v, t|q)}_{\text{prob success}} \underbrace{[V(v, t|\lambda q) - V(v, t|q)]}_{\text{payoff}} \quad (15)$$

$$- \underbrace{\hat{z}(v, t|q)\eta(\hat{z}(v, t|q))}_{\text{prob lost patent}} \underbrace{V(v, t|q)}_{\text{loss}} \quad (16)$$

Note: Terms 3 and 4 cancel by the incumbent's FOC.

Generic derivation

Take the generic discounted present value

$$V = \mathbb{E} \int_0^{\infty} e^{-rt} \pi(t) dt \quad (17)$$

where profits change stochastically according to a Poisson process.

With flow probability ρ , profits change so that the continuation value becomes \hat{V} .

We show that

$$rV = \pi + \dot{V} + \rho (\hat{V} - V) \quad (18)$$

Generic derivation I

Evaluate the flow payoffs over a short period Δt :

$$V = \int_0^{\Delta t} e^{-(r+\rho)t} \pi_t dt \quad (19)$$

$$+ e^{-r\Delta t} \left[e^{-\rho\Delta t} V_{\Delta t} + \left[1 - e^{-\rho\Delta t} \right] \hat{V} \right] \quad (20)$$

Note the discounting at $r + \rho$.

- ▶ Because the probability of still receiving profits is $e^{-\rho t}$

At the end of the interval, discounted by $e^{-r\Delta t}$, the payoffs are

- ▶ $V_{\Delta t}$: the value of continuing at the end of Δt ; with probability $e^{-\rho\Delta t}$
- ▶ \hat{V} : the value of continuing with a shock; with complementarity probability.

Generic derivation II

Assume that π is constant over the interval Δt . Then the first integral is

$$\frac{1 - e^{-(r+\rho)\Delta t}}{r + \rho} \pi \quad (21)$$

Add and subtract V in the second term and it becomes

$$e^{-\rho\Delta t} (V_{\Delta t} - V) + \left[1 - e^{-\rho\Delta t}\right] \hat{V} + e^{-\rho\Delta t} V \quad (22)$$

Substituting back into the definition of V gives

$$V \left[1 - e^{-(r+\rho)\Delta t}\right] = \frac{1 - e^{-(r+\rho)\Delta t}}{r + \rho} \pi \quad (23)$$

$$+ e^{-r\Delta t} \left[e^{-\rho\Delta t} [V_{\Delta t} - V] + \left[1 - e^{-\rho\Delta t}\right] \hat{V} \right] \quad (24)$$

Generic derivation III

Divide by $[1 - e^{-(r+\rho)\Delta t}]$ and take $\Delta t \rightarrow 0$.

The first term becomes $\frac{\pi}{r+\rho}$.

Set $[V_{\Delta t} - V] = \dot{V}\Delta t$. Then the second term becomes

$$\frac{e^{-(r+\rho)\Delta t}}{1 - e^{-(r+\rho)\Delta t}} \dot{V}\Delta t \quad (25)$$

Using L'Hopital's rule this becomes:

$$\frac{-(r+\rho)e^{-(r+\rho)\Delta t}\Delta t + e^{-(r+\rho)\Delta t}}{(r+\rho)e^{-(r+\rho)\Delta t}} = \frac{1}{r+\rho} \quad (26)$$

Similarly, using L'Hopital's rule the third term becomes

$$\frac{\rho}{r+\rho} \hat{V} \quad (27)$$

Generic derivation IV

Putting it all together gives

$$(r + \rho)V = \pi + \dot{V} + \rho \hat{V} \quad (28)$$

or

$$rV = \pi + \dot{V} + \rho [\hat{V} - V] \quad (29)$$

Value of the firm

Using the generic formula:

$$rV = \pi + \underbrace{\dot{V}}_{=0} - \hat{z}\eta(\hat{z}) \times V \quad (30)$$

Profit (unchanged):

$$\pi(v, t|q) = [p^x(v, t|q) - \psi]x(v, t|q) \quad (31)$$

$$= \beta qL \quad (32)$$

because $p^x = 1$ and $x = qL$. Therefore

$$rV = \beta qL - \hat{z}\eta(\hat{z}) V \quad (33)$$

or

$$V = \frac{\beta qL}{r + \hat{z}\eta(\hat{z})} \quad (34)$$

The usual story: losing the patent just increases the effective interest rate.

Equilibrium

Allocation

$\{C(t), X(t), Z(t), Y(t), L(t), z(v, t), \hat{z}(v, t), x(v, t), \pi(v, t), V(v, t)\}$

Prices $\{p^x(v, t), w(t), r(t)\}$

that satisfy:

- ▶ household: Euler (and TVC)
- ▶ final goods firm: 3
- ▶ intermediate goods firm: 1
- ▶ free entry of incumbents and entrants: 2
- ▶ market clearing: goods, labor (2)
- ▶ definitions of X, Z, π (3)
- ▶ definition of V (differential equation) (1)

Balanced Growth Path

Euler equation

$$g(C) = \frac{r - \rho}{\theta} \quad (35)$$

We now have 3 expressions for the value of the firm:

1. Free entry by incumbents (13)
2. Free entry by entrants (10)
3. The present value of profits (34)

$$V(q) = \underbrace{\frac{\beta q L}{r + \hat{z} \eta(\hat{z})}}_{\text{incumbents}} = \underbrace{\frac{q/\kappa}{\eta(\hat{z})}}_{\text{entrants}} = \underbrace{\frac{q}{\phi(\lambda - 1)}}_{\text{present value}} \quad (36)$$

These jointly solve for r, \hat{z} .

The Euler equation (35) then gives the growth rate.

Implications for firm dynamics

We now begin to have a model of firm dynamics.

- ▶ We have firm entry and exit (innovation by entrants)
- ▶ We have firm sales growth (stochastic) with firm age

Firm sales are given by $x(v, t|q) = qL$.

For a given firm: x

- ▶ increases by factor λ with probability $\phi z \Delta t$
- ▶ stays the same with probability $\hat{z} \eta(\hat{z}) \Delta t$
- ▶ drops to 0 with complementary probability

Applications

Garcia-Macia et al. (2016)

- ▶ how much of output growth is due to innovation by incumbents vs competitors?

Acemoglu et al. (2013)

- ▶ tax policy in a model with R&D and firm quality heterogeneity

Hottman et al. (2016)

- ▶ measures sources of firm heterogeneity

Reading

- ▶ Acemoglu (2009), ch. 14.3.
- ▶ Aghion et al. (2014), survey of Schumpeterian growth models

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