

# Money in the Utility Function

Prof. Lutz Hendricks

Econ720

October 14, 2020

## Money in the utility function

- ▶ A shortcut for getting money valued in equilibrium: assume that households gain utility from holding money.
- ▶ "Sidrauski" model.
- ▶ Benefits: Tractability.
- ▶ Drawbacks: Arbitrary specification of utility affects results.

# The Economic Environment

- ▶ Much of the model is a standard growth model.
- ▶ The government prints paper (costlessly).
- ▶ Households gain utility from holding paper.

# Environment

- ▶ Demographics: 1 representative household
- ▶ Endowments:
  - ▶ 1 unit of work time at each instant
  - ▶  $k_0$  units of the good
  - ▶  $M_0$  bits of paper
- ▶ Technology:
  - ▶  $F(K, L) - \delta K = c + \dot{K}$

# Environment

- ▶ Preferences:

$$\int_{t=0}^{\infty} e^{-\rho t} u(c_t, m_t) dt \quad (1)$$

$$m_t = M_t / p_t$$

- ▶ Government:

- ▶ prints  $\dot{M}_t$  and hands it to households

- ▶ Markets:

- ▶ goods, labor, capital rental, money

# Household

Households solve:

$$\max \int_{t=0}^{\infty} e^{-\rho t} u(c_t, m_t) dt \quad (2)$$

subject to  $k_0, m_0$  given and

$$p(c + \dot{k}) + \dot{M} = p(w + rk + x) \quad (3)$$

$x$  are lump-sum transfers (of money).

## Budget constraint in real terms

$$\dot{k} + \dot{M}/p = w + rk + x - c \quad (4)$$

Note that

$$\begin{aligned} \dot{m} &= \dot{M}/p - (M/p^2)\dot{p} \\ &= \dot{M}/p - m\pi \end{aligned}$$

where  $\pi$  is the inflation rate ( $\pi = \dot{p}/p$ ).

Therefore

$$\dot{k} + \dot{m} = w + rk + x - c - \pi m \quad (5)$$

## Budget constraint

- ▶ We seem to have 2 state variables  $(k, m)$  but only one law of motion.
- ▶ The reason: the correct state variable is wealth:  $A = k + m$ .
- ▶ To transform the budget constraint into a law of motion for  $A$ , write it as

$$\dot{A} = w + rA + x - c - (r + \pi)m \quad (6)$$

- ▶ Every unit of wealth held in money reduces income by the nominal interest rate  $(r + \pi)$ .



# An Equivalence

- ▶ The household problem is exactly the same as in a real two-good economy.
- ▶ Money is like a consumption good with price  $r + \pi$ .

## Solving the household problem

$$H = u(c, m) + \lambda[w + rA + x - c - (r + \pi)m] \quad (7)$$

FOC:

$$\begin{aligned} u_c &= \lambda \\ u_m &= \lambda(r + \pi) \\ \dot{\lambda} &= (\rho - r)\lambda \end{aligned}$$

TVC:

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t A_t = 0$$

## Household optimality

Static condition:

$$u_c = u_m / (r + \pi) \quad (8)$$

Intuition?

Intertemporal condition:

$$\dot{\lambda} / \lambda = g(u_c) = -(r - \rho) \quad (9)$$

where  $g(z) \equiv \dot{z}/z$  denotes a growth rate.

## Household: Separable utility

If the utility function is **separable**,

$$u(c, m) = v(c) + \bar{v}(m) \quad (10)$$

then

$$u_c = v'(c) \quad (11)$$

and

$$g(u_c) = v''(c) \dot{c} / v'(c) = -\sigma g_c \quad (12)$$

Then a very common expression emerges:

$$g(c) = (r - \rho) / \sigma \quad (13)$$

# Equilibrium

**Firms** solve the standard static profit maximization problem:

$$r = f'(k) - \delta \quad (14)$$

$$w = f(k) - f'(k)k \quad (15)$$

# Government

- ▶ The government grows the money supply at the constant rate  $\mu = g(M)$ .
- ▶ Implied lump-sum transfers are

$$x = \dot{M}/p = \mu m \quad (16)$$

# Market clearing

- ▶ Money and factor market clearing are implicit in the notation.
- ▶ Goods market clearing is feasibility:

$$\dot{k} + c = f(k) - \delta k \quad (17)$$

# Equilibrium

An equilibrium is a set of functions of time that satisfy

These are 9 variables and 10 equations.

The boundary conditions are initial values for  $M$  and  $k$  and the TVC.



# Characterization

- ▶ We reduce the CE to 4 equations in  $(c, p, k, m)$ .
- ▶ Household first-order conditions:

$$\begin{aligned}g(u_c[c, m]) &= -(f'(k) - \delta - \rho) \\ u_c(c, m) &= u_m(c, m)/(f'(k) - \delta + \pi)\end{aligned}$$

- ▶ Goods market clearing:

$$\dot{k} + c = f(k) - \delta k$$

- ▶ Money growth rule:

$$\dot{m} = (\mu - \pi)m$$

# Monetary Neutrality

- ▶ Assume: the utility function is additively separable

$$u(c, m) = \bar{u}(c) + v(m) \quad (18)$$

- ▶ Then money has absolutely no effect on the real sector.
- ▶ The evolution of  $c$  and  $k$  is determined by the Euler equation and the goods market clearing condition alone.
- ▶ Intuition?

## Steady state

In steady state  $c, k, m$  are constant.

The Euler equation then determines the steady state capital stock:

$$r = f'(k) - \delta = \rho \quad (19)$$

Goods market clearing then yields consumption:

$$c = f(k) - \delta k$$

Constant real balances require  $\pi = \mu$ .

The static optimality condition yields an implicit equation for  $m$ :

$$u_m(c_{SS}, m_{SS}) = (\rho + \mu) u_c(c_{SS}, m_{SS}) \quad (20)$$

$\Rightarrow$

$$m_{SS} = m^d(c_{SS}, \rho + \mu) \quad (21)$$

## Super-neutral money

- ▶ Changes in money growth ( $\mu$ ) only affect the inflation rate, but not real variables ( $k_{ss}, c_{ss}$ ).
- ▶ Intuition: inflation does not alter the intertemporal tradeoff between consumption today and tomorrow.
- ▶ Inflation only affects the relative levels of goods and money consumed

## Inflation and welfare

- ▶ What is the effect of inflation on real money balances?  
Differentiate (20) to obtain

$$u_{mm}dm = (\rho + \mu)u_{cm}dm + u_c d\mu \quad (22)$$

$\Rightarrow$

$$dm/d\mu = u_c/[u_{mm} - (\rho + \mu)u_{cm}] \quad (23)$$

- ▶ Unless money and consumption are too strong complements ( $u_{cm}$  large and positive), higher inflation is associated with lower real money balances and thus lower steady state utility.

# The Friedman Rule

- ▶ Which money growth rate maximizes steady state utility?
- ▶ Since  $\mu$  does not affect  $c_{ss}$ , we only need to know how to maximize  $m_{ss}$ .
- ▶ If we set  $\rho + \mu = 0$ , then  $u_m = 0$ , which is the best we can do: satiate the household with money.
- ▶ If  $u_m > 0$  even asymptotically, the problem does not have a solution.
- ▶ The intuition is quite general:
  - ▶ If money provides some kind of benefit, the best we can do is to make it costless to hold money.
  - ▶ That will be the case when money pays the same rate of return as capital (the Friedman rule).

# Is This a Good Theory of Money?

Pros:

- ▶ tractable

Cons:

- ▶ the value of money is assumed  
therefore: no non-monetary equilibrium / hyperinflation
- ▶ money is not used in transactions  
it's really a consumption good

## Where Is this Used?

Models of the financial sector, where the details why households hold money play a minor role

- ▶ Van den Heuvel, Skander J. "The welfare cost of bank capital requirements." *Journal of Monetary Economics* 55.2 (2008): 298-320.



# Reading

- ▶ Blanchard and Fischer (1989), ch. 4.5

## References I

Blanchard, O. J. and S. Fischer (1989): *Lectures on macroeconomics*, MIT press.