Problem Set 7: Asset Pricing

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1 Lucas Fruit Trees With Crashes

Demographics: There is a single, representative household who lives forever.

Preferences: $U = \mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t)$ where $u(c) = c^{1-\sigma}/(1-\sigma)$.

Endowments: The agent is endowed at $t = 0$ with 1 tree. In each period, the tree yields stochastic consumption d_t, which cannot be stored. d_t evolves as follows:

- If $d_t = d_{t-1}$, then $d_{t+1} = d_t$ forever after.
- If $d_t \neq d_{t-1}$, then $d_{t+1} = \gamma d_t$ with probability π and $d_{t+1} = d_t$ with probability $1-\pi$. $\gamma > 1$.

In words: d grows at rate $\gamma - 1$ until some random event occurs (with probability $1-\pi$), at which point growth stops forever.

Markets: There are competitive markets for consumption (numeraire) and trees (price p_t). Assume that p_t is *cum dividend*, meaning that d_t accrues to the household who buys the tree in t and holds it into $t+1$.

Questions:

1. State the household's dynamic program.
2. Derive the Euler equation.
3. Define a recursive competitive equilibrium. Key: what is the state vector?
4. Characterize the stochastic process of p_t. Is p_t a Markov process? Hint: there are 2 phases: before and after dividends have stopped growing. Assume that p/d is constant during the phase with growth.
5. What happens to the stock market when the economy stops growing? Does it crash (does the price decline)? Under what condition?

2 Two stocks

Demographics: There is a single representative household who lives forever.

Preferences:

$$E\sum_{t=0}^{\infty}\beta^t u(c_t) \tag{1}$$

where u is strictly increasing, strictly concave, and satisfies $u'(0) = \infty$.

Technologies: None (endowment economy).

Endowments: There are two fruit trees. In each period, tree j yields $\theta_{j,t}y$ units of the consumption good. $y > 0$ is a constant. The $\theta_{j,t}$ are i.i.d. random variables.

Markets: There are competitive markets for goods (numeraire) and trees with prices $p_{j,t}$. Shares are traded after the $\theta_{j,t}$ are realized and dividends are paid. Households also trade a riskless bond in zero net supply with endogenous price q_t.

Questions:

1. State the household problem in recursive form.
2. Derive the Euler equations. Assume an interior solution where the household holds both stocks.
3. From now on assume that $\sum_j \theta_j = 1$. Assume that there are no bubbles. Solve for the equilibrium asset prices.
4. Derive the equity premium. Explain your finding.
5. Now assume that y_t is random and drawn from a finite Markov chain. Utility is $u(c) = \ln(c)$. Solve for the equilibrium stock prices and expected returns. What can you say about the connection between expected stock returns and growth, i.e., are stocks expected to do well when there are strong growth prospects? What is the intuition for this?