Problem Set 2: OLG Models with Money

Econ720. Fall 2022. Prof. Lutz Hendricks. August 30, 2022

1 Money and Heterogeneity

Consider a two-period OLG model with fiat money.

Demographics: In each period $N_t = (1+n)^t$ persons are born. Each lives for 2 periods. Half of the agents are of type I, the other half of type II.

Endowments: The initial old hold M_0 units of money, evenly distributed across agents. Each person is endowed with (e_i^y, e_i^o) units of consumption when (young, old).

Preferences: $\ln(c_t^y) + \beta \ln(c_{t+1}^o)$.

Technology: Goods can only be eaten the day they drop from the sky.

Government: The government pays a lump-sum transfer of $x_t p_t$ units of money to each old person: $M_t = M_{t-1} + N_{t-1} x_t p_t$. The aggregate money supply grows at the constant rate μ: $M_{t+1} = (1+\mu) M_t$.

Markets: In each period, agents buy/sell goods and money in spot markets.

Questions:

1. Define a competitive equilibrium.
2. Derive the household consumption function.
3. Derive a difference equation for the equilibrium interest rate when $\mu = 0$.
4. Is the monetary steady state dynamically efficient?

2 Money and Storage

Consider a two-period OLG model with fiat money and a storage technology.

Demographics: In each period $N_t = (1+n)^t$ persons are born. Each lives for 2 periods.

Endowments: The initial old hold capital K_0 and money M_0. Each young person is endowed with a e units of the good.

Preferences: $u(c_t^y) + \beta u(c_{t+1}^o)$.

Technology: Storing k_t units of the good in t yields $f(k_t)$ units in $t+1$. f obeys Inada conditions. The resource constraint is $N_t k_{t+1} = N_t e + N_{t-1} f(k_t) - C_t$ where $C_t = N_t\, c_t^y + N_{t-1}\, c_t^o$.

Government: The government pays a lump-sum transfer of $x_t p_t$ units of money to each old person: $M_{t+1} = M_t + N_{t-1} x_t p_t$. The aggregate money supply grows at the constant rate μ: $M_{t+1} = (1+\mu) M_t$.

Markets: In each period, agents buy/sell goods and money in spot markets.

The timing in period t is a follows:

- The old enter period t holding aggregate capital $K_t = N_{t-1} k_t$ and nominal money balances of $M_t = m_t \ N_{t-1}$.
- The old receive the money transfer from the government and now hold M_{t+1}.
- Each old person produces $f(k_t)$.
- The young buy money (m_{t+1}/p_t) from the old, consume c_t^y and save k_{t+1}.
- The old consume their income.

Questions:

1. State the household's budget constraints when young and old.
2. Derive the household's optimality conditions. Define a solution to the household problem.
3. Define a competitive equilibrium.
4. Does an equilibrium with positive inflation exist? Intuition?
5. Define a steady state as a system of 6 equations in 6 unknowns.
6. Find the money growth rate (μ) that maximizes steady state consumption per young person, $(N_t \, c_t^y + N_{t-1} \, c_t^o)/N_t$.
