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Jupiter Made Abstract, and then Refined

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Abstract Collaborative text editing systems allow multiple users to concurrently edit the same document, which can be modelled by a replicated list object. In the literature, there is a family of operational transformation (OT)-based Jupiter protocols for replicated lists, including AJupiter, XJupiter, and CJupiter. They are hard to understand due to the subtle OT technique, and little work has been done on formal verification of complete Jupiter protocols. Worse still, they use quite different data structures. It is unclear how they are related to each other, and it would be laborious to verify each Jupiter protocol separately. In this work, we make contributions towards a better understanding of Jupiter protocols and the relation among them. We first identify the key OT issue in Jupiter and present a generic solution. We summarize several techniques for carrying out the solution, including the data structures to maintain OT results and to guide OTs. Then, we propose an implementation-independent AbsJupiter protocol. Finally, we establish the (data) refinement relation among these Jupiter protocols (AbsJupiter included). We also formally specify and verify the family of Jupiter protocols and the refinement relation among them using TLA$^+$ (TLA stands for "Temporal Logic of Actions") and the TLC model checker. To our knowledge, this is the first work to formally specify and verify a family of OT-based Jupiter protocols and the refinement relation among them. It would be helpful to promote a rigorous study of OT-based protocols.

Keywords Jupiter protocols, operational transformation, refinement, replicated list, TLA$^+$

1 Introduction

Collaborative text editing systems, such as Google Docs [1], Firepad [2], Overleaf [3], and SubEthaEdit [4], allow multiple users to concurrently edit the same document. For availability, such systems often replicate the document at several replicas. For low latency, replicas are required to respond to user operations immediately and updates are propagated asynchronously [1, 2].

The replicated list object is frequently used to model the core functionality (e.g., insertion and deletion) of replicated collaborative text editing systems [1–4]. A common specification for it is strong eventual consistency (*SEC*) [5]. It requires that whenever two replicas have processed the same set of updates, they have the same list. A family of Jupiter protocols [3] for implementing such a replicated list have been proposed, including XJupiter [4] (a multi-client version of [3] given by Xu et al.), AJupiter (another multi-client version of [3] given by Attiya et al.), and CJupiter [6] (short for Compact Jupiter). They adopt the client/server ar-

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[1]GoogleDocs. `https://docs.google.com`, accessed in 09/2020.

[2]Firepad. `https://firepad.io/`, accessed in 09/2020.

[3]Overleaf. `https://www.overleaf.com/`, accessed in 09/2020.

[4]SubEthaEdit. `https://subethaedit.net/`, accessed in 09/2020.

chitecture, where the server serializes operations and propagates them from one client to others (Fig. 1). Note that since replicas are required to respond to user operations immediately, the C/S architecture does not imply that clients process operations in the same order. To achieve convergence, Jupiter adopts the operational transformation (OT) technique [1,7] to resolve the conflicts caused by concurrent operations. The idea of OT is, for each replica, to process local operations immediately and to transform received operations according to the effects of previously processed concurrent operations. The transformation rules are called OT functions [1,3].

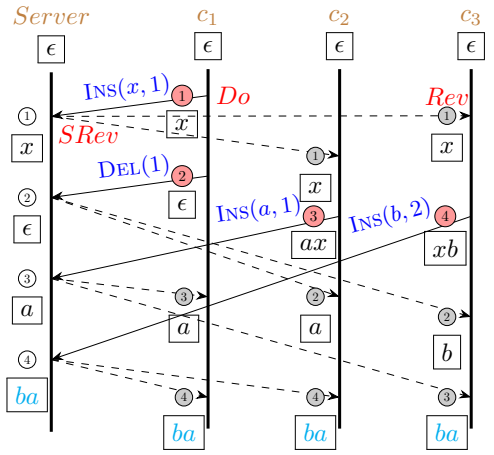


Fig.1. System model. The circled numbers indicate the serialization order (SO) in which the operations are received at the server (Section 3). The list produced by Jupiter protocols are shown in boxes. (Adapted from [6].)

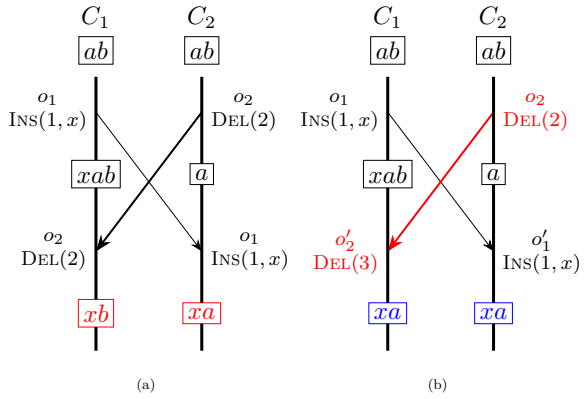


Fig.2. Example for OT. The positions are indexed from 1. (The server is not shown.) (a) Without OT, the states of C_1 and C_2 diverge. (b) With OT, C_1 and C_2 converge to the same state.

***Example* 1** *(Illustration of OT).* Fig. 2 shows a replicated list system with two client replicas C_1 and C_2 which initially hold the same list "ab". Suppose that user 1 issues $o_1 = \text{INS}(1, x)$ at C_1 and concurrently user 2 issues $o_2 = \text{DEL}(2)$ at C_2. After being executed locally, each operation is sent to the other replica. Without OT, C_1 and C_2 wind up with different lists (i.e., "xb" and "xa", respectively). With OT, o_2 is transformed to $o_2' = \text{DEL}(3)$ at C_1, taking into account the fact that o_1 has inserted an element at position 1. Meanwhile, o_1 remains unchanged after OT at C_2. As a result, two replicas converge to the same list "xa".

When several replicas diverge by multiple operations, OT becomes much more subtle and error-prone. Some published OT-based protocols [1, 8] were even later shown incorrect [9–11]. The intrinsic complexity in concurrency control makes the OT-based Jupiter protocols hard to understand. Moreover, little has been done on formal verification of complete OT-based protocols (not only of OT functions). Worse still, Jupiter protocols use quite different data structures, rendering the relation among them unclear. It would be also laborious and wasteful to prove or verify that the Jupiter protocols satisfy a certain property one by one. In this work, we make contributions towards a better understanding of Jupiter protocols and the relation among them. Specifically (Fig. 3),

- (Section 3) We first identify the key issue involving OT that Jupiter needs to address as follows: When a replica r receives an operation op, which operations should op be transformed against and in what order before it is applied? We also present a generic solution to this issue: Transform op against the set of concurrent operations previously executed at r in the serialization order established at the server. Then, we summarize sev-

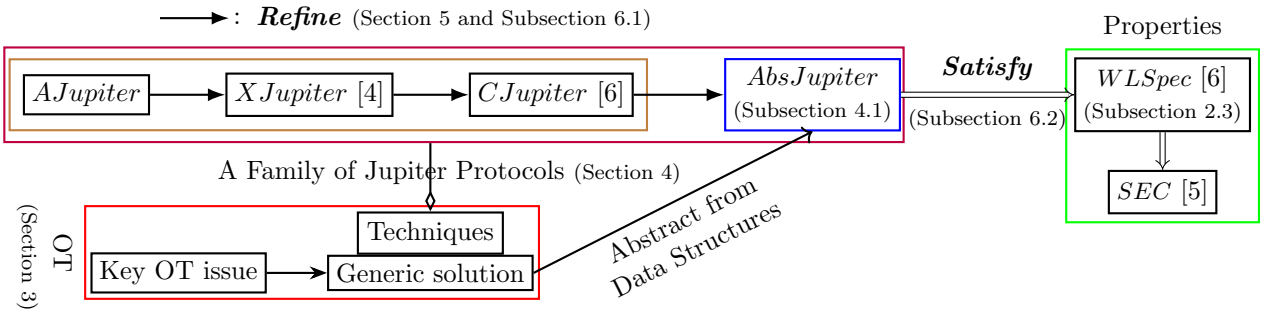


Fig.3. Overview of contributions.

eral techniques that the Jupiter protocols adopt to carry out the solution, including those for deciding whether two operations are concurrent, for determining the serialization order, and the data structures to maintain (intermediate) OT results and to guide OTs.

- (Subsection 4.1) We propose AbsJupiter, an abstract Jupiter protocol which captures the OT essence of existing Jupiter protocols. Specifically, it addresses the key OT issue in a way abstract from concrete data structures by using mathematical sets.

- (Section 5) For different purposes such as performance or ease of correctness proof, existing Jupiter protocols use quite different data structures. The implementation details in data structures have obscured the similarities among them. We show that the existing Jupiter protocols are actually (data) refinements [12–14] of AbsJupiter in data structures. Specifically, we show that AJupiter is a refinement (a.k.a. implementation) of XJupiter, which is a refinement of CJupiter, which is a refinement of AbsJupiter. As a consequence, the properties like *SEC* and *WLSpec* (weak list specification defined in Subsection 2.3) that hold for AbsJupiter also automatically hold for other Jupiter protocols.

- (Sections 4 and 6) In Section 4, we formally specify the family of Jupiter protocols in TLA$^+$ [15]. The refinement mappings among Jupiter protocols are also expressed in TLA$^+$ in Section 5. [5] Section 6 presents the model checking results conducted by TLC [16] (the model checker [17] for TLA$^+$) of verifying both the properties for Jupiter protocols and refinement relations among them.

Section 2 provides a brief introduction to TLA$^+$ and covers preliminaries on system model, OT, and list specifications. Section 7 discusses related work. Section 8 concludes.

2 Preliminaries

2.1 TLA$^+$

The specification language TLA$^+$ was designed by Lamport for modelling and reasoning about concurrent and distributed programs [15]. In TLA$^+$, systems are modelled as state machines. A state machine is described by its initial states and actions. A state is an assignment of values to variables. An action is a relation between old states and new states, and is represented by a formula over unprimed variables referring to the old state and primed variables referring to the

[5] The whole project can be found at `https://github.com/hengxin/jupiter-refinement-project`.

Table 1. A Summary of TLA$^+$ Operators Used in this Paper

Category	Operator	Meaning
Logic	CHOOSE $x \in S : P(x)$	An x in S satisfying $P(x)$ [6]
Set	SUBSET S	Powerset (i.e., set of subsets) of S
	$\{e : x \in S\}$	Set of elements e such that x is in S
	$\{x \in S : p\}$	Set of elements x in S satisfying p
Function	$f[e]$	Function application
	$[x \in S \mapsto e]$	Function f such that $f[x] = e$ for $x \in S$
	$[S \to T]$	Set of functions mapping from S to T
	$[f$ EXCEPT $![e_1] = e_2]$, where e_2 may contain @	Function $\hat{f}$ equal to f except that $\hat{f}[e_1] = e_2$, where any occurrence of @ in e_2 stands for $f[e_1]$
Record	$e.h$	The h-field of record e
	$[h_1 \mapsto e_1, \cdots, h_n \mapsto e_n]$	The record whose h_i field is e_i
	$[h_1 : S_1, \cdots, h_n : S_n]$	Set of all records with h_i field in S_i
	$[r$ EXCEPT $!.h = e]$, where e may contain @	Record $\hat{r}$ equal to r except that $\hat{r}.h = e$, where any occurrence of @ in e stands for $r.h$
Tuple	$e[i]$	The i-th component of tuple e
	$\langle e_1, \cdots, e_n \rangle$	The n-tuple whose i-th component is e_i
	$S_1 \times \cdots \times S_n$	The set of all n-tuples with i-th component in S_i
Sequence	$Seq(S)$	The set of all sequences of elements of the set S
	$Head(s)$	The first element of sequence s
	$Last(s)$	The last element of sequence s
	$Tail(s)$	The tail of sequence s, which consists of s with its head removed
	$Range(s)$	The set of elements of sequence s
Action Operator	e'	The value of e in the new state of an action
	$[A]_e$	$A \vee (e' = e)$
Temporal Operator	$\Box F$	F is always true

new state. For example, $x' = y + 42$ is the relation asserting that the value of x in the new state is 42 greater than that of y in the old state.

TLA$^+$ is based on TLA, the Temporal Logic of Actions [18]. A program is specified in TLA$^+$ as a temporal formula of TLA of the form $Spec \triangleq Init \wedge \Box[Next]_{vars} \wedge L$, where *Init* is a predicate specifying all possible initial states of the program, *Next* specifies the next-state relation of the program, $\Box$ is the temporal operator read "Always", *vars* is the tuple of all variables used in the program, and L is a fairness property (not used in this paper). The next-state relation *Next* is typically a disjunction of all the actions of the program. The expression $[Next]_{vars}$ is true if either *Next* is true, meaning that some action is true and thus taken, or *vars* stutters, meaning that their values are unchanged. A behavior of the program specified by *Spec* (ignoring L) of the above form is a sequence of states that satisfies *Spec*; namely, the *Init* predicate holds in the first state of this sequence, and the next-state relation $[Next]_{var}$ holds for any two consecutive states of this sequence.

TLA$^+$ combines TLA with first-order logic and Zermelo-Fraenkel set theory. Table 1 summarizes the operators in the logic and set theory we use in this

paper. It is an excerpt from the complete summary of TLA$^+$ [7] and shows only the operators that have special notations in TLA$^+$.

Specifications of programs are grouped into modules. In a module, we can declare constants (CONSTANTS) and variables (VARIABLES), define operators ($F(x_1, \cdots, x_n) \triangleq \cdots$), and claim theorems (THEOREM). A module M can import the declarations, definitions, and theorems from other modules $M_1, \cdots, M_n$ by extending them, namely writing EXTENDS $M_1, \cdots, M_n$ in M. Modules can also be instantiated. Consider the following INSTANCE statement in module M:

$$IM_1 \triangleq \text{INSTANCE } M_1 \text{ WITH } p_1 \leftarrow e_1, \cdots, p_n \leftarrow e_n$$

where p_i consist of all declared constants and variables of M_1 and e_i are valid expressions in M. [8] For each operator F and its definition d of module M_1, this defines F to be the operator, denoted $IM_1!F$, whose definition is obtained from d by replacing each p_i of M_1 with e_i.

TLC is an explicit-state model checker for TLA$^+$ [16]. It can compute and explore the state space of finite-state instances of TLA$^+$ specifications. These finite-state instances are called TLC models of TLA$^+$ specifications. For example, a TLC model of a specification describing a distributed system consisting of a set of processors declared as CONSTANTS *Proc* should instantiate *Proc* with a set consisting of a fixed number of processors, like $Proc \triangleq \{1, 2, 3\}$. We can also represent a process by a TLC model value, which is considered to be unequal to any other values in TLA$^+$. So, we can instantiate *Proc* with a set of model values $Proc \triangleq \{p1, p2, p3\}$. Moreover, if permuting the elements in a set of model values does not change whether a behavior satisfies a desired specification, we can further use the symmetry set technique to reduce the state space that TLC has to check [15].

In TLA$^+$, refinement is logical implication. Suppose we have two specifications: *AbsSpec* defined in module *AbsModule* with variables $x_1, \cdots, x_m, y_1, \cdots, y_n$, and *ImplSpec* defined in module *ImplModule* with variables $x_1, \cdots, x_m, z_1, \cdots, z_p$. Let X, Y, and Z denote $x_1, \cdots, x_m$, $y_1, \cdots, y_n$, and $z_1, \cdots, z_p$, respectively. To verify that *ImplSpec* refines *AbsSpec*, formally $ImplSpec \implies AbsSpec$, we need to show that for each behavior satisfying *ImplSpec*, there is some way to assign values of the variables Y in each state so that the resulting behavior satisfies *AbsSpec* [13]. This can be done by explicitly specifying those values of Y in terms of X and Z. Specifically, for each y_i, we define an expression $\overline{y_i}$ in terms of X and Z, substitute $y_i \leftarrow \overline{y_i}$ in *AbsSpec* to get $\overline{AbsSpec}$, and show that *ImplSpec* refines $\overline{AbsSpec}$. The substitution $y_i \leftarrow \overline{y_i}$ is called a refinement mapping. To verify the assertion that *ImplSpec* refines *AbsSpec* under such a refinement mapping in TLA$^+$, we can add the following definition to module *ImplModule* (*AbsSub* is a fresh identifier),

$$AbsSub \triangleq \text{INSTANCE } AbsModule \text{ WITH } y_1 \leftarrow \overline{y_1}, \cdots, y_n \leftarrow \overline{y_n}$$

and let TLC check the theorem

$$\text{THEOREM } ImplSpec \implies AbsSub!AbsSpec$$

which is added to module *ImplModule*.

There are two kinds of refinement [14], namely data refinement [12] and step refinement. In data refinement,

[7] Leslie Lamport. Summary of TLA$^+$. `http://lamport.azurewebsites.net/tla/summary-standalone.pdf`, accessed in 09/2020.

[7] The most common use of the CHOOSE operator is to select a unique value satisfying $P(x)$ [15]. If there is no element $x \in S$ satisfying $P(x)$, then TLC will report an error. On the other hand, if there are several such x's, then an arbitrary one is chosen.

[8] Note that constant parameters p_i must be instantiated by constant-level expressions built up from constants and constant operators and variable parameters by state-level expressions which may contain variables and the ENABLED operator (not used in this paper). For simplicity, we omit the formal definitions of levels [15].

MODULE *Op*

$Del \triangleq [type : \{\text{“Del”}\},\ pos : Nat]$ The positions (pos) are indexed from 1.
$Ins \triangleq [type : \{\text{“Ins”}\},\ pos : Nat,\ ch : Char,\ pr : 1\ ..\ Cardinality(Client)]$
$Op \triangleq Ins \cup Del$ The set of all possible update operations.
$Nop \triangleq \text{CHOOSE } o : o \notin Op$

“abstract” data of a high-level protocol is refined by a “concrete” representation of a lower-level protocol [12]. In step refinement, a single step (i.e., actions in terms of TLA$^+$) of a high-level protocol is refined by multiple steps of a lower-level protocol [14].

Constructing a refinement mapping may require adding auxiliary variables to the (lower-level) protocols [13,19]. One kind of auxiliary variables that we will use in data refinement among Jupiter protocols is called history variables [13, 19]. Intuitively, history variables record information about past behaviors of a protocol, and are typically not used by the actual variables of the protocol. Therefore, it is safe to add history variables to protocols, without altering their behaviors [13].

2.2 System Model

We let *Client* denote the set of client replicas, *Server* the unique server replica, and $Replica \triangleq Client \cup \{Server\}$ the set of all replicas. Client replicas are connected to the server replica via FIFO channels. The set of messages is denoted by M. A replica is modelled as a state machine. Each replica r maintains its current list $list[r]$ (initially empty; denoted ϵ) and interacts with three kinds of actions from users and other replicas:

- $Do(c \in Client, op \in Op)$: Client c receives an operation $op \in Op$ (defined below) from an unspecified user (we also sometimes say that client c generates the operation op) and responds to the user immediately. It then sends the update in a message $m \in M$ to the server asynchronously.
- $Rev(c \in Client, m \in M)$: Client c receives and processes a message m from the server.
- $SRev(m \in M)$: The server receives a message m from a client. It will produce and broadcast a new message to other clients.

***Example* 2** *(Behaviors of Replicas).* Consider client c_3 in Fig. 1. First, in $Rev(c_3, \_)$, client c_3 receives a message containing information about o_1 (maybe transformed) of client c_1 from the server. Next, in $Do(c_3, o_4)$, it generates operation o_4 ($\text{INS}(b, 2)$), applies o_4 locally, and sends o_4 to the server. Then, in $Rev(c_3, \_)$, it receives messages containing information about o_2 and o_3 of client c_1 and c_2 respectively, from the server. The list $list[c_3]$ at c_3 is updated accordingly.

2.3 List, OT, and Weak List Specification

A replicated list object supports two types of update operations: *Del* and *Ins*, defined as records in module *Op*. Following [2], we assume that all inserted elements are unique, which can be achieved by attaching replica identifiers and local sequence numbers. The priority field “*pr*” of *Ins* helps to resolve the conflicts caused by two concurrent *Ins* operations that are intended to insert different elements at the same position.

Module *OT* shows a complete definition of OT functions for lists [1,3]. $OT(lop, rop)$ transforms *lop* against *rop* by calling the appropriate OT function according to the types of *lop* and *rop*. For example, *OTID* defines how an *Ins* operation *ins* is transformed against a *Del* operation *del*. It adjusts the insertion position of *ins* according to the deletion position of *del*.

We consider the weak list specification *WLSpec* [2],

MODULE OT

$OTII(lins,\ rins) \triangleq$ *lins* is transformed against *rins*; *II* is for *Ins vs. Ins.*
IF $lins.pos < rins.pos$ THEN $lins$
ELSE IF $lins.pos > rins.pos$
THEN $[lins$ EXCEPT $!.pos = @ + 1]$
ELSE IF $lins.ch = rins.ch$ THEN Nop
ELSE IF $lins.pr > rins.pr$ THEN $lins$ using "priority"
ELSE $[lins$ EXCEPT $!.pos = @ + 1]$

$OTID(ins,\ del) \triangleq$ *ins* is transformed against *del*
IF $ins.pos \leq del.pos$ THEN ins
ELSE $[ins$ EXCEPT $!.pos = @ - 1]$

$OTDI(del,\ ins) \triangleq$ *del* is transformed against *ins*
IF $del.pos < ins.pos$ THEN del
ELSE $[del$ EXCEPT $!.pos = @ + 1]$

$OTDD(ldel,\ rdel) \triangleq$ *ldel* is transformed against *rdel*; *DD* is for *Del vs. Del.*
IF $ldel.pos < rdel.pos$ THEN $ldel$
ELSE IF $ldel.pos = rdel.pos$ THEN Nop
ELSE $[ldel$ EXCEPT $!.pos = @ - 1]$

$OT(lop,\ rop) \triangleq$ *lop* is transformed against *rop*
CASE $lop = Nop \lor rop = Nop \to lop$
$\square\ lop.type =$ "Ins" $\land\ rop.type =$ "Ins" $\to OTII(lop,\ rop)$
$\square\ lop.type =$ "Ins" $\land\ rop.type =$ "Del" $\to OTID(lop,\ rop)$
$\square\ lop.type =$ "Del" $\land\ rop.type =$ "Ins" $\to OTDI(lop,\ rop)$
$\square\ lop.type =$ "Del" $\land\ rop.type =$ "Del" $\to OTDD(lop,\ rop)$

MODULE *WLSpec*

$Compatible(l1,\ l2) \triangleq$ Are $l1$ and $l2$ compatible?
$\lor\ l1 = l2$ Obviously true
$\lor$ LET $commonElements \triangleq Range(l1) \cap Range(l2)$
IN $\forall\, e1,\ e2 \in commonElements :$
$\lor\ e1 = e2$
$\lor\ FirstIndexOfElement(l1,\ e1) < FirstIndexOfElement(l1,\ e2)$
$\equiv\ FirstIndexOfElement(l2,\ e1) < FirstIndexOfElement(l2,\ e2)$

which is stronger than strong eventual consistency (*SEC*) [5]. *WLSpec* is equivalent to the "pairwise state compatibility property" [6]. It requires any pair of lists across the system to be compatible. Two lists l_1 and l_2 are compatible if for any two common elements e_1 and e_2 of l_1 and l_2, the relative ordering of e_1 and e_2 are the same in l_1 and l_2; see module *WLSpec* for the formal specification of *Compatible*. Let *hlist* be a set of lists. *WLSpec* is defined as $WLSpec \triangleq \forall l_1, l_2 \in hlist : Compatible(l_1, l_2)$; see also module *AbsJupiterH* in Subsection 6.2.

***Example* 3** *(Weak List Specification; Adapted from [6]).* Consider the execution in Fig. 1. There exist three replica states with list $l_1 = ba$, $l_2 = ax$, and $l_3 = xb$, respectively. This is allowed by *WLSpec*, since the lists are pairwise compatible. However, an execution is not allowed by *WLSpec*, if it contained two states with, say, $l = ab$ and $l' = ba$.

3 Jupiter Family

The key issue for Jupiter protocols to address is as follows: When a replica r receives an operation op,

Table 2. Techniques Adopted by Jupiter Protocols to Address the Key OT Issue

Protocol	Concurrent Operation	SO Order	Data Structure
AbsJupiter	COT	SV	Set
CJupiter [6]	COT	SV	n-ary digraph
XJupiter [4]	COT	COT	$2D$ digraph
AJupiter	ACK	Buffer	$1D$ buffer

MODULE COT

$Oid \triangleq [c : Client, seq : Nat]$

$Cop \triangleq [op : Op, oid : Oid, ctx : \text{SUBSET } Oid]$

the client that generates cop

$ClientOf(cop) \triangleq cop.oid.c$

$COT(lcop, rcop) \triangleq$

$[lcop \text{ EXCEPT } !.op = OT(lcop.op, rcop.op), !.ctx = @ \cup \{rcop.oid\}]$

which operations should op be transformed against and in what order before it is applied? The solution is to transform op against the set of operations that are concurrent with it and have been previously executed at r in their serialization order, denoted SO, i.e., the order in which they are received by the server. The four Jupiter protocols we study differ in the way they carry out the solution. Table 2 summarizes several key techniques that they adopt to carry out the solution, including those for deciding whether two operations are concurrent, for determining the serialization order, and the data structures to maintain (intermediate) OT results and to guide OTs. We explain them in the following.

3.1 Context-based OT (COT)

According to whether they use context-based operations (Cop) and context-based OT (COT) [20], Jupiter protocols fall into two categories: AbsJupiter, CJupiter, and XJupiter are all context-based, while AJupiter is not. In this section, we define Cop and COT. How they are used to decide whether two operations are concurrent or not is explained in Subsection 3.3, along with the concrete data structures.

Each operation $op \in Op$ is associated with a unique operation identifier (oid, for short) in Oid, which is a record of client c that generates op and a local sequence number $cseq[c]$ of c. Each replica r maintains its document state $ds[r]$ as the set of operation identifiers it has processed. The document state $ds[r]$ is updated to include oid whenever the replica r receives and processes an operation with oid.

Operations in $ds[r]$ of each replica r are related to each other via contexts. Intuitively, the context of an operation is a set of operations that it is aware of. Formally, in module COT, a context-based operation $cop \in Cop$ is a record of operation $op \in Op$, its oid $oid \in Oid$, and its context $ctx \subseteq Oid$ representing a document state. When an operation is generated by client c, its context is set to be the current document state $ds[c]$ of c. When a context-based operation $lcop$ is transformed against another one $rcop$, $lcop.ctx$ will be updated to include $rcop.oid$; see module COT. Note that according to the context-based condition (CC) [20], two context-based operations can be transformed against each other, only if they have the same context. This will be guaranteed by context-based Jupiter protocols.

MODULE SV

$so(oid1,\ oid2,\ sv) \triangleq$ Is $oid1$ totally ordered before $oid2$ according to sv?
  LET $pos1 \triangleq FirstIndexOfElementSafe(sv,\ oid1)$ 0 if $oid1$ is not in sv
      $pos2 \triangleq FirstIndexOfElementSafe(sv,\ oid2)$ 0 if $oid2$ is not in sv
  IN IF $pos1 \neq 0 \land pos2 \neq 0$ Case (1): both have been at the server
     THEN $pos1 < pos2$ using the order in which they arrived at the server
     ELSE IF $pos1 = 0 \land pos2 = 0$ Case (2): none has been at the server
          THEN $oid1.seq < oid2.seq$ using the order they were generated
          ELSE $pos1 \neq 0$ Case (3): the one that has been at the server is ahead

MODULE Set

$xForm(r,\ cop) \triangleq$ Transform cop at replica r
  LET $ctxDiff \triangleq ds[r] \setminus cop.ctx$ calculate concurrent operations
      $xFormHelper(coph,\ ctxDiffh,\ copssh) \triangleq$
        IF $ctxDiffh = \{\}$ THEN $[xcop \mapsto coph,\ xcopss \mapsto copssh]$
        ELSE LET $foidh \triangleq$ CHOOSE $oid \in ctxDiffh :$
                  $\forall\, id \in ctxDiffh \setminus \{oid\} : so(oid,\ id,\ serial[r])$
             $fcoph \triangleq$ CHOOSE $fcop \in copss[r] :$
                  $fcop.oid = foidh \land fcop.ctx = coph.ctx$
             $xcoph \triangleq COT(coph,\ fcoph)$
             $xfcoph \triangleq COT(fcoph,\ coph)$
          IN $xFormHelper(xcoph,\ ctxDiffh \setminus \{fcoph.oid\},$
                $copssh \cup \{xcoph,\ xfcoph\})$
  IN $xFormHelper(cop,\ ctxDiff,\ copss[r] \cup \{cop\})$

3.2 Serial Views (SV)

In AbsJupiter and CJupiter, replicas need to decide the SO order among operations (i.e., the order in which they are received by the server) with local knowledge. To do this, each replica r maintains a serial view $serial[r]$ which is a sequence of oids, representing its own knowledge about SO. The server always has the latest serial view $serial[Server]$ and updates it in $SRev$ by each time appending to it the recently received oid. In addition, $serial[Server]$ will be broadcast to clients along with actual messages. Each client c synchronizes its serial view with the server by updating $serial[c]$ to the latest $serial[Server]$ that it receives in $Rev(c, \_)$.

Consider two operation identifiers $oid1$ and $oid2$ that are generated or received by some replica r. The operator $so(oid1, oid2, sv)$ in module SV decides whether $oid1$ precedes (or will precede) $oid2$ in SO order given the local serial view sv of r. There are three cases: (1) If both have been at the server, we use the order in which they arrived at the server, which is captured by the positions they are in sv; (2) If none has been at the server, they must be generated by the same client, and we use the order they were generated; (3) Otherwise, the one that has been at the server precedes the other that has not.

3.3 Data Structures

3.3.1 Set

In AbsJupiter, each replica r maintains a set $copss[r]$ of context-based operations. When a replica r receives a context-based operation cop, it calls $xForm(r, cop)$ of module Set to transform cop against a subset of context-based operations in $copss[r]$ that are concurrent with cop in their SO order.

Due to the FIFO communication, we have that

$cop.ctx \subseteq ds[r]$. Thus, *xForm* first calculates the set of (oids of) concurrent operations with *cop* as the set difference *ctxDiff* between $ds[r]$ and $cop.ctx$. Then it recursively transforms *cop* against the context-based operations in $copss[r]$ whose oids are in *ctxDiff* in their SO order according to the serial view $serial[r]$. This is done in $xFormHelper(coph, ctxDiffh, copssh)$:

1) If *ctxDiffh* is empty, the most recently transformed *coph* and the latest data structure *copssh* are returned.

2) Otherwise, *xFormHelper* chooses the next operation *fcoph* against which *coph* is to be transformed, such that $fcoph.oid$ is the first one in the current *ctxDiffh* and that $fcoph.ctx = coph.ctx$. Because the communication in the client/server model is FIFO, when an operation *cop* is received by some replica, the operations in its context have already been in this replica. Thus, such *fcoph* satisfying $fcoph.ctx = cop.ctx$ exists. The existence of *fcoph* in recursion can be further justified by induction.

3) *coph* and *fcoph* are transformed against each other. The intermediate transformed operation *xcoph* is recursively transformed against the remaining concurrent operations (with *oid*) in $ctxDiffh \setminus \{foph.oid\}$.

3.3.2 Digraph

In CJupiter and XJupiter, the set of context-based operations are organized into edge-labeled digraphs. A digraph is represented by a record with *node* and *edge* fields; see *IsDigraph* of module *Digraph*. Each node in $G.node$ of a digraph G represents a document state. Each directed edge e in $G.edge$ is labeled with a context-based operation *cop* satisfying $cop.ctx = e.from$, meaning that when applied, *cop* changes the document state from $e.from$ to $e.to = e.from \cup \{cop.oid\}$. The operator $\oplus$ takes the union of two records with *node* and *edge* fields.

In CJupiter and XJupiter, when a replica r (either client or server) receives a context-based operation *cop*, it calls $xForm(NextEdge, r, cop, g)$ of module *Digraph* to iteratively transform *cop* against a sequence of context-based operations along a path in some digraph g maintained by r. This path starts with the node u equal to $cop.ctx$ and ends with the one equal to $ds[r]$. Each such path contains the operations whose oids are in $ds[r] \setminus cop.ctx$, which are concurrent with *cop* due to the FIFO communication. The next edge is chosen by *NextEdge* specific to CJupiter and XJupiter to ensure the SO order. $xFormHelper(uh, vh, coph, gh)$ starts the transformation with $uh \leftarrow u$ (Fig. 4 and module *Digraph*):

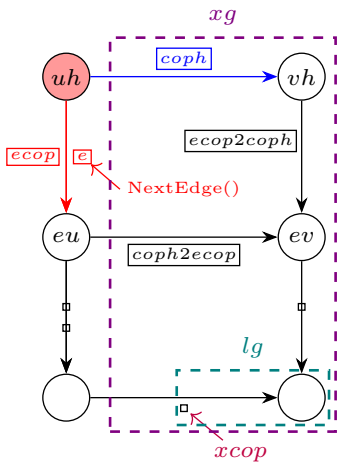


Fig.4. Illustration of *xForm* of module *Digraph*.

1) If $uh = ds[r]$, the most recently transformed operation *coph*, the record *gh* consisting of nodes and edges produced in *xForm* so far, and the node and edge (collected in *lg*) produced in the last iteration of transformation are returned.

2) Otherwise, the next edge e (and its associated operation $ecop \triangleq e.cop$) outgoing from *uh* is chosen using $NextEdge(r, uh, g)$ specific to CJupiter and XJupiter.

MODULE *Digraph*

$IsDigraph(G) \triangleq$ G is a record with *node* and *edge* fields
$\quad \land\ G.node \subseteq (\text{SUBSET } Oid)$ each node represents a document state
$\quad \land\ G.edge \subseteq [from : G.node,\ to : G.node,\ cop : Cop]$
$EmptyGraph \triangleq [node \mapsto \{\{\}\},\ edge \mapsto \{\}]$

$g \oplus h \triangleq [node \mapsto g.node \cup h.node,\ edge \mapsto g.edge \cup h.edge]$

$xForm(NextEdge(\_, \_, \_),\ r,\ cop,\ g) \triangleq$ Transform *cop* in g at replica r
$\quad$ LET $u \triangleq$ CHOOSE $n \in g.node : n = cop.ctx \quad v \triangleq u \cup \{cop.oid\}$
$\quad\quad xFormHelper(uh,\ vh,\ coph,\ gh) \triangleq$
$\quad\quad\quad$ IF $uh = ds[r]$ THEN $[xcop \mapsto coph,\ xg \mapsto gh,$
$\quad\quad\quad\quad lg \mapsto [node \mapsto \{vh\},$
$\quad\quad\quad\quad edge \mapsto \{[from \mapsto uh,\ to \mapsto vh,\ cop \mapsto coph]\}]]$
$\quad\quad\quad$ ELSE LET $e \triangleq NextEdge(r,\ uh,\ g)$ specific to *CJupiter* and *XJupiter*
$\quad\quad\quad\quad ecop \triangleq e.cop \quad eu \triangleq e.to \quad ev \triangleq vh \cup \{ecop.oid\}$
$\quad\quad\quad\quad coph2ecop \triangleq COT(coph,\ ecop)$
$\quad\quad\quad\quad ecop2coph \triangleq COT(ecop,\ coph)$
$\quad\quad\quad\quad$ IN $xFormHelper(eu,\ ev,\ coph2ecop,$
$\quad\quad\quad\quad gh \oplus [node \mapsto \{ev\},$
$\quad\quad\quad\quad\quad edge \mapsto \{[from \mapsto vh,\ to \mapsto ev,\ cop \mapsto ecop2coph],$
$\quad\quad\quad\quad\quad\quad [from \mapsto eu,\ to \mapsto ev,\ cop \mapsto coph2ecop]\}])$
$\quad$ IN $xFormHelper(u,\ v,\ cop,\ [node \mapsto \{v\},$
$\quad\quad\quad\quad edge \mapsto \{[from \mapsto u,\ to \mapsto v,\ cop \mapsto cop]\}])$

3) *coph* and *ecop* are transformed against each other. The intermediate transformed operation *coph*2*ecop* is then recursively transformed against the sequence of operations starting with the node $eu \triangleq e.to$, the successor of *uh* along the edge e.

3.3.3 *Buffer*

AJupiter maintains buffers (i.e., sequences) of operations of type *Op*. $xForm(op, ops)$ of module *Buffer* transforms an operation *op* against a buffer *ops* of operations; see Fig. 5. It utilizes $xFormOpOps\ (op, ops)$ and $xFormOpsOp(ops, op)$ to obtain the last transformed operation *xop* and the transformed buffer *xops*, respectively. Specifically, *xFormOpOps* returns the sequence of intermediate transformed operations, the last one of which is the desired *xop*:

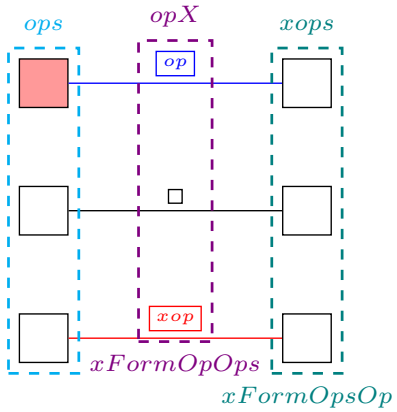


Fig.5. Illustration of *xForm* of module *Buffer*.

1) If *ops* is empty, $\langle op \rangle$ is returned;

2) Otherwise, it prepends *op* to the resulting sequence obtained by recursively transforming $OT(op, Head(ops))$ against the tail $Tail(ops)$ of *ops*.

It also facilitates *xFormOpsOp* to generate *xops* by transforming each operation in *ops* against the corresponding one in $opX \triangleq xFormOpOps(op, ops)$. Fi-

MODULE *Buffer*

$xFormOpOps(op,\ ops) \triangleq$ Transform op against ops
  IF $ops = \langle\rangle$ THEN $\langle op\rangle$ and return intermediate transformed operations.
  ELSE $\langle op\rangle \circ xFormOpOps(OT(op,\ Head(ops)),\ Tail(ops))$
$xFormOpsOp(ops,\ op) \triangleq$ Transform ops against op and return the transformed ops.
  LET $opX \triangleq xFormOpOps(op,\ ops)$
  IN $[i \in 1\,..\,Len(ops) \mapsto OT(ops[i],\ opX[i])]$
$xForm(op,\ ops) \triangleq$
  $[xop \mapsto Last(xFormOpOps(op,\ ops)),\ xops \mapsto xFormOpsOp(ops,\ op)]$
$xFormShift(op,\ ops,\ shift) \triangleq xForm(op,\ SubSeq(ops,\ shift + 1,\ Len(ops)))$

MODULE *AbsJupiter*

VARIABLES $copss$ $copss[r]$: the set of context-based operations maintained at replica r

$Perform(r,\ cop) \triangleq$ LET $xform \triangleq xForm(r,\ cop)$ $xform : [xcop, xcopss]$
  IN $\wedge\ copss' = [copss$ EXCEPT $![r] = xform.xcopss]$
  $\wedge$ apply $xform.xcop.op$ to $list[r]$
$Do(c,\ op) \triangleq$ LET $cop \triangleq [op \mapsto op,\ oid \mapsto [c \mapsto c,\ seq \mapsto cseq[c]],\ ctx \mapsto ds[c]]$
  IN $\wedge\ copss = [copss$ EXCEPT $![c] = @ \cup \{cop\}]$
  $\wedge$ apply op to $list[c]$; send cop to the $Server$
$Rev(c,\ cop) \triangleq Perform(c,\ cop)$
$SRev(cop) \triangleq \wedge\ Perform(Server,\ cop)$
  $\wedge$ broadcast cop to clients other than $ClientOf(cop)$

nally, $xFormShift(op, ops, shift)$ transforms op against the subsequence of ops obtained by shifting the first $shift$ operations out of ops.

4 Jupiter Protocols

In this section, we formally specify Jupiter protocols in TLA$^+$, including AbsJupiter that we propose as an abstract solution. We focus on when and how OTs are performed and on the data structures supporting OTs. As running examples, we will illustrate the behaviors of client c_3 in different Jupiter protocols under the schedule of Fig. 1.

4.1 AbsJupiter

In AbsJupiter, each replica r maintains a set $copss[r]$ of context-based operations. The operator $Perform(r, cop)$ calls $xForm(r, cop)$ of module *Set* to transform cop in $copss[r]$. The transformed operation $xform.xcop.op$ is applied to $list[r]$ and $copss[r]$ is updated to $xform.xcopss$.

In $Do(c, op)$, the client c first wraps op into a context-based operation cop by attaching oid and $ctx = ds[c]$ to it. Then it updates $copss[c]$ to include cop, applies op to $list[c]$, and sends cop to the server. When the server receives a context-based operation cop from client c, it calls $Perform(Server, cop)$ and then broadcasts cop to other clients than c; see $SRev(cop)$. In $Rev(c, cop)$, the client c just calls $Perform(c, cop)$.

Thanks to the mathematical set it uses, AbsJupiter is abstract from implementations with concrete data structures. As shown in Section 5, it embraces the other three Jupiter protocols as refinements.

***Example* 4** *(Illustration of AbsJupiter).* We illustrate client c_3 in *AbsJupiter* under the schedule of Fig. 1; see also Fig. 6 (a). For convenience, we denote, for instance, an operation o_3 with context $\{o_1, o_2, o_4\}$ by $o_3\{o_1, o_2, o_4\}$.

MODULE *CJupiter*

VARIABLES *css* *css*[*r*]: the *n*-ary digraph maintained at replica *r*

$NextEdge(r,\ u,\ g) \triangleq$ CHOOSE $e \in g.edge : \land\ e.from = u$
$\land\ \forall\, ue \in g.edge \setminus \{e\} :$
$(ue.from = u) \Rightarrow so(e.cop.oid,\ ue.cop.oid,\ serial[r])$

$Perform(r,\ cop) \triangleq$ LET $xform \triangleq xForm(NextEdge,\ r,\ cop,\ css[r])$
IN $\land\ css' = [css$ EXCEPT $![r] = @ \oplus xform.xg]$
$\land$ apply $xform.xcop.op$ to $list[r]$

$Do(c,\ op) \triangleq$ LET $cop \triangleq [op \mapsto op,\ oid \mapsto [c \mapsto c,\ seq \mapsto cseq[c]],\ ctx \mapsto ds[c]]$
$u \triangleq ds[c] \quad v \triangleq u \cup \{cop.oid\}$
IN $\land\ css' = [css$ EXCEPT $![c] =$
$@ \oplus [node \mapsto \{v\},$
$edge \mapsto \{[from \mapsto u,\ to \mapsto v,\ cop \mapsto cop]\}]]$
$\land$ apply op to $list[c]$; send cop to the *Server*

$Rev(c,\ cop) \triangleq Perform(c,\ cop)$

$SRev(cop) \triangleq \land\ Perform(Server,\ cop)$
$\land$ broadcast cop to clients other than $ClientOf(cop)$

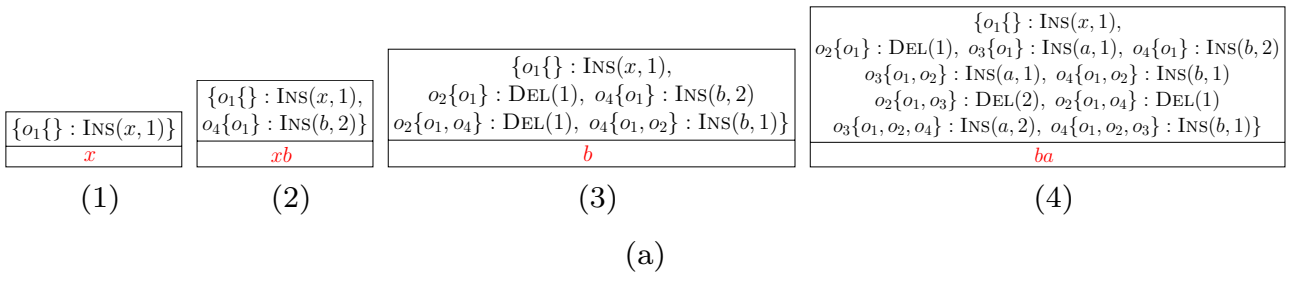
After receiving and applying $o_1\{\}$ (INS$(x, 1)$) of client c_1 from the server, client c_3 generates o_4 (INS$(b, 2)$). It wraps o_4 into a context-based operation $o_4\{o_1\}$ with context o_1, adds $o_4\{o_1\}$ to $copss[c_3] = \{o_1\{\}\}$, applies o_4 locally, and then sends $o_4\{o_1\}$ to the server.

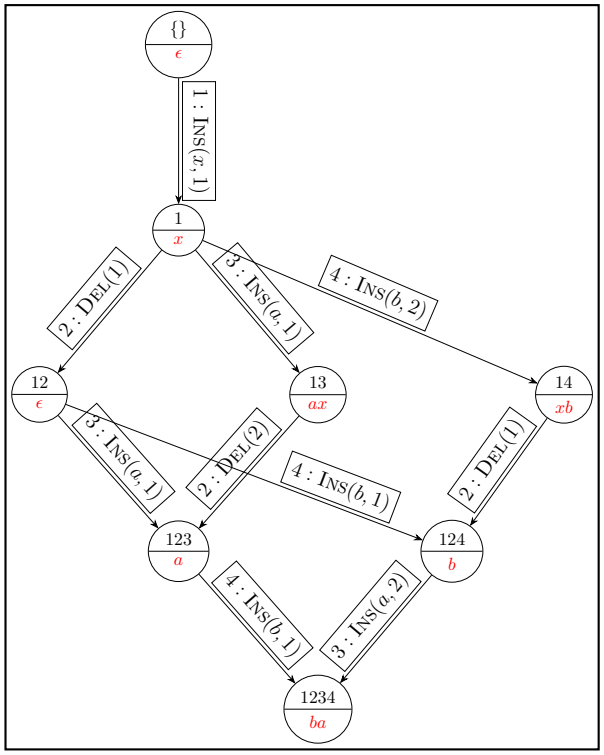
Next, client c_3 receives $o_2\{o_1\}$ (DEL(1)) of client c_1 from the server. By $xForm(c_3, o_2\{o_1\})$, it transforms $o_2\{o_1\}$ against the set of context-based operations in $copss[c_3] = \{o_1\{\}, o_4\{o_1\}\}$. Since o_4 is the only concurrent operation with o_2 in $copss[c_3]$, $o_2\{o_1\}$ and $o_4\{o_1\}$ are transformed against each other. As a result, the new context-based operations $o_2\{o_1, o_4\}$ (DEL(1)) and $o_4\{o_1, o_2\}$ (INS$(b, 1)$) are added into $copss[c_3]$. The transformed operation DEL(1) is applied locally.

Finally, client c_3 receives $o_3\{o_1\}$ (INS$(a, 1)$) of client c_2 from the server. By $xForm(c_3, o_3\{o_1\})$, it transforms $o_3\{o_1\}$ against the set of context-based operations in $copss[c_3] = \{o_1\{\}, o_4\{o_1\}, o_2\{o_1\}, o_4\{o_1, o_2\}, o_2\{o_1, o_4\}\}$. The set of oids of concurrent operations with o_3 in $copss[c_3]$ is calculated as $\{o_1, o_2, o_4\} \setminus \{o_1\} = \{o_2, o_4\}$. Since o_2 precedes o_4 in the SO order according to $serial[c_3] = \langle o_1, o_2\rangle$, $o_3\{o_1\}$ is first transformed with $o_2\{o_1\}$, yielding $o_3\{o_1, o_2\}$ (INS$(a, 1)$) and $o_2\{o_1, o_3\}$ (DEL(2)). Then, $o_3\{o_1, o_2\}$ is transformed with $o_4\{o_1, o_2\}$ (INS$(b, 1)$), yielding $o_3\{o_1, o_2, o_4\}$ (INS$(a, 2)$) and $o_4\{o_1, o_2, o_3\}$ (INS$(b, 1)$). At last, c_3 applies the transformed operation INS$(a, 2)$ locally, obtaining the list content ba.

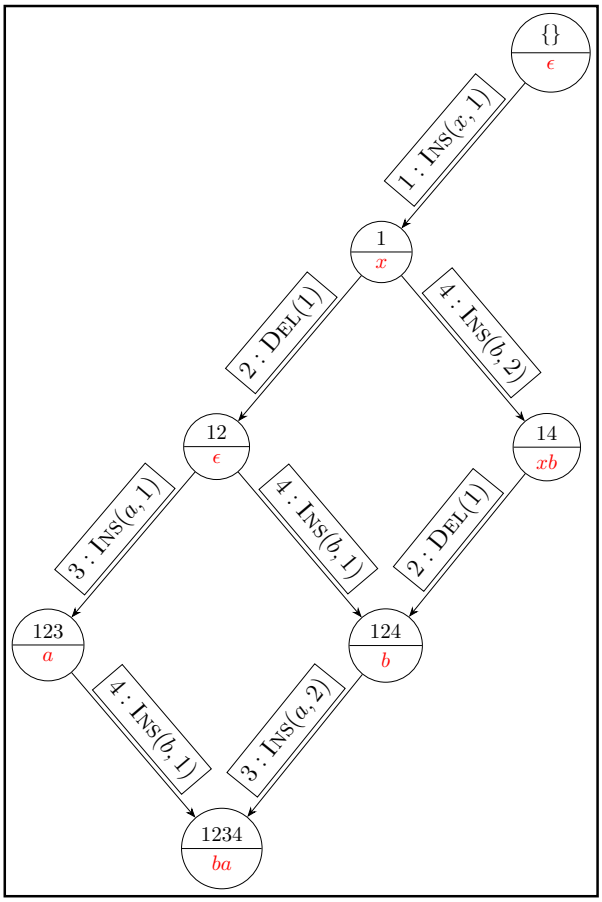
4.2 CJupiter

In CJupiter, each replica r maintains an n-ary digraph $css[r]$ (initially *EmptyGraph*), a digraph where the outdegree of each node can be at most n; see module *CJupiter*. In $Do(c, op)$, the client c first wraps op into a context-based operation cop. Then it applies op to $list[c]$, inserts an edge labeled by cop from the node $ds[c]$ in $css[c]$, and sends cop to the server. The definitions of *Rev* and *SRev* of *CJupiter* are the same as those in *AbsJupiter*, except that $xForm(NextEdge, r, cop, css[r])$ of module *Digraph* is called by replica r to transform cop against a sequence of context-based operations with cop along a path in digraph $css[r]$. The next edge from a given node chosen in *NextEdge* is the first one in terms of SO according to the serial view $serial[r]$ of r. The intermediate

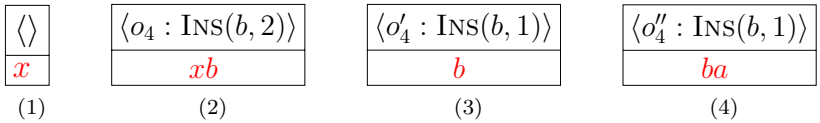




(b)



(c)



(d)

Fig.6. Illustration of client c_3 in Jupiter protocols under the schedule of Fig. 1. (a) AbsJupiter. (b) CJupiter. (c) XJupiter. (d) AJupiter.

xform.xg produced in *xForm* is integrated into $css[r]$ and the transformed operation *xform.xcop.op* is applied to $list[r]$.

It is remarkable that although $(n+1)$ n-ary digraphs are maintained by CJupiter, they are (eventually) all the same. In other words, at a high level, CJupiter maintains only a single n-ary digraph, which contains exactly all replica states across the system [6]. This makes it feasible to reason about global properties like weak list specification [2, 6].

***Example* 5** *(Illustration of CJupiter; Adapted from [6]).* We illustrate client c_3 in CJupiter under the schedule of Fig. 1; see also Fig. 6 (b). For convenience, we denote, for instance, a node v with document state $\{o_1, o_4\}$ by v_{14}.

After receiving and applying $o_1\{\}$ of client c_1 redirected by the server, client c_3 generates o_4 ($\textsc{Ins}(b, 2)$). It wraps o_4 into a context-based operation $o_4\{o_1\}$, links a new node v_{14} to v_1 via an edge labeled by $o_4\{o_1\}$, and then sends $o_4\{o_1\}$ to the server.

Next, client c_3 receives $o_2\{o_1\}$ ($\textsc{Del}(1)$) of client c_1 from the server. The context of $o_2\{o_1\}$ matches node v_1. By *xForm*, $o_2\{o_1\}$ and $o_4\{o_1\}$ are transformed against each other. As a result, v_{124} is created and is linked to v_{12} and v_{14} via the edges labeled with $o_4\{o_1, o_2\}$ ($\textsc{Ins}(b, 1)$) and $o_2\{o_1, o_4\}$ ($\textsc{Del}(1)$), respectively.

Finally, client c_3 receives $o_3\{o_1\}$ ($\textsc{Ins}(a, 1)$) of client c_2 from the server. The context of $o_3\{o_1\}$ matches node v_1. By *xForm*, $o_3\{o_1\}$ will be transformed with the operation sequence consisting of operations along the "first" (in terms of SO with $serial[c_3] = \langle o_1, o_2\rangle$) edges from v_1 to v_{124}. Specifically, $o_3\{o_1\}$ is first transformed with $o_2\{o_1\}$. Then, $o_3\{o_1, o_2\}$ ($\textsc{Ins}(a, 1)$) is transformed with $o_4\{o_1, o_2\}$ ($\textsc{Ins}(b, 1)$), yielding v_{1234}, $o_3\{o_1, o_2, o_4\}$ ($\textsc{Ins}(a, 2)$), and $o_4\{o_1, o_2, o_3\}$ ($\textsc{Ins}(b, 1)$). Client c_3 applies $\textsc{Ins}(a, 2)$, obtaining the list content ba.

4.3 XJupiter

XJupiter uses $2D$ digraphs where the outdegree of each node is at most 2. Each client c maintains a single $2D$ digraph $c2ss[c]$, and the server maintains n $2D$ digraphs, one digraph $s2ss[c]$ per client c. Conceptually, a $2D$ digraph, either $c2ss[c]$ or $s2ss[c]$, has two dimensions: a local dimension for storing operations generated by c and a remote dimension for storing operations generated by other clients.

In $Do(c, op)$, the client c first wraps op into a context-based operation *cop* by attaching *oid* and $ctx = ds[c]$ to it. Then it applies op to $list[c]$, inserts an edge labeled by *cop* from the node $ds[c]$ in $c2ss[c]$ along the local dimension, and sends *cop* to the server.

When the server receives a context-based operation *cop* from client c, it transforms *cop* against the context-based operations along the remote dimension from node $u \triangleq cop.ctx$ to $ds[Server]$ in $s2ss[c]$. In $SRev(cop)$, this is done in $xForm(NextEdge, Server, cop, s2ss[c])$ of module *Digraph*, where *NextEdge* returns the unique outgoing edge of a given node. Then, the transformed operation *xform.xcop.op* is applied to $list[Server]$, $s2ss[c]$ is updated to integrate *xform.xg*, and *xform.lg* is inserted to the remote dimension of each digraph $s2ss[cl \neq c]$. Finally, the server broadcasts the transformed context-based operation *xform.xcop* to other clients than c.

When client c receives a context-based operation *cop* from the server, it calls $xForm(NextEdge, c, cop, c2ss[c])$ of module *Digraph* to transform *cop* against the operations along the local dimension from node $u \triangleq cop.ctx$ to $ds[c]$ in $c2ss[c]$. The intermediate *xform.xg* is integrated into $c2ss[c]$ and the transformed operation *xform.xcop.op* is ap-

MODULE *XJupiter*

VARIABLES $c2ss$, $c2ss[c]$: the $2D$ digraph maintained at client c
$s2ss$ $s2ss[c]$: the $2D$ digraph maintained by the *Server* for client c

$NextEdge(\_, u, g) \triangleq$ CHOOSE $e \in g.edge : e.from = u$
$Do(c, op) \triangleq$ LET $cop \triangleq [op \mapsto op, oid \mapsto [c \mapsto c, seq \mapsto cseq[c]], ctx \mapsto ds[c]]$
$u \triangleq ds[c] \quad v \triangleq u \cup \{cop.oid\}$
IN $\wedge\ c2ss' = [c2ss$ EXCEPT $![c] =$
$@ \oplus [node \mapsto \{v\},$
$edge \mapsto \{[from \mapsto u, to \mapsto v, cop \mapsto cop]\}]]$
$\wedge$ apply op to $list[c]$; send cop to the *Server*
$Rev(c, cop) \triangleq$ LET $xform \triangleq xForm(NextEdge, c, cop, c2ss[c])$
$xform$: $[xcop, xg, lg]$
IN $\wedge\ c2ss' = [c2ss$ EXCEPT $![c] = @ \oplus xform.xg]$
$\wedge$ apply $xform.xcop.op$ to $list[c]$
$SRev(cop) \triangleq$
LET $c \triangleq ClientOf(cop)$
$xform \triangleq xForm(NextEdge, Server, cop, s2ss[c])$ $xform$: $[xcop, xg, lg]$
IN $\wedge\ s2ss' = [cl \in Client \mapsto$ IF $cl = c$ THEN $s2ss[cl] \oplus xform.xg$
ELSE $s2ss[cl] \oplus xform.lg]$
$\wedge$ apply $xform.xcop.op$ to $list[Server]$
$\wedge$ broadcast the *transformed* operation $xform.xcop$ to clients other than c

plied to $list[c]$.

Since the transformed context-based operations are broadcast by the server in XJupiter, XJupiter is slightly optimized in implementation at clients with respect to CJupiter, by eliminating redundant OTs that have already been performed at the server [6]. More importantly, this improvement makes it possible to reduce n-ary digraphs to $2D$-digraphs.

***Example* 6** *(Illustration of XJupiter; Adapted from [6]).* We illustrate client c_3, as well as the *Server*, in XJupiter under the schedule of Fig. 1; see Fig. 7 and Fig. 6 (c). Client c_3 in XJupiter behaves similarly as it does in CJupiter, when it receives o_1 of client c_1, o_4 generated by itself, and o_2 of client c_1.

We now explain what c_3 does when it receives o_3 of client c_2 redirected by the server. Client c_2 has propagated its operation $o_3\{o_1\}$ (INS$(a, 1)$) to the server. At the server, $o_3\{o_1\}$ was transformed with $o_2\{o_1\}$ (DEL(1)) along the remote dimension in $s2ss[c_2]$, obtaining $o_3\{o_1, o_2\}$ (INS$(a, 1)$). Besides being stored in $s2ss[c_1]$ and $s2ss[c_3]$, $o_3\{o_1, o_2\}$ (instead of $o_3\{o_1\}$ that the server receives) is redirected by the server to clients c_1 and c_3. At client c_3, the context of $o_3\{o_1, o_2\}$ matches node v_{12} in $c2ss[c_3]$. By *xForm* of *Digraph*, $o_3\{o_1, o_2\}$ should be transformed against the operations along the local dimension (in the southeast arrow "↘" direction in Fig. 6 (c)) from node v_{12} in $c2ss[c_3]$. In this example, $o_3\{o_1, o_2\}$ is transformed with $o_4\{o_1, o_2\}$ (INS$(b, 1)$), yielding v_{1234}, $o_3\{o_1, o_2, o_4\}$ (INS$(a, 2)$), and $o_4\{o_1, o_2, o_3\}$ (INS$(b, 1)$). Finally, client c_3 applies INS$(a, 2)$, obtaining the list content ba.

4.4 AJupiter

In AJupiter, each client c maintains a buffer $cbuf[c]$ for storing the operations (maybe transformed) it generates, and a counter $crec[c]$ counting the number of operations it has received from the server since the last time it generated an operation and sent a message. Similarly, the server maintains for each client c a

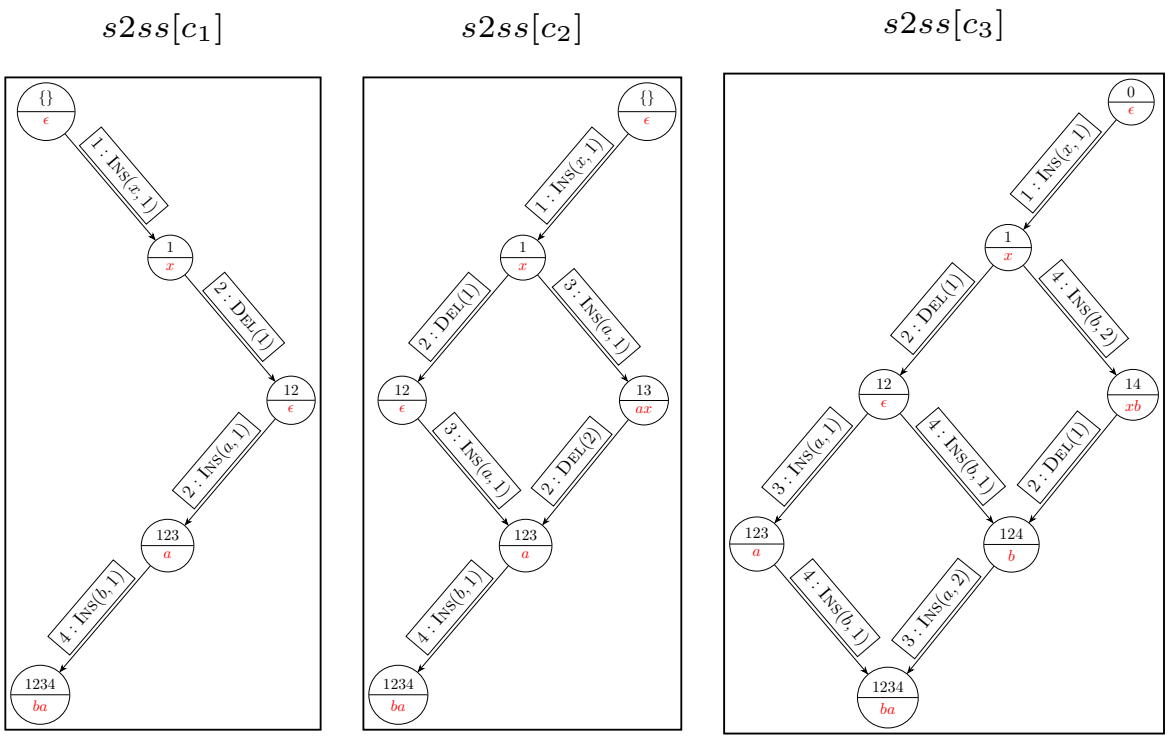


Fig.7. Illustration of the server in XJupiter under the schedule of Fig. 1. (↘: local dimension; ↙: remote dimension)

buffer $sbuf[c]$ for storing the (transformed) operations generated by other clients than c, and a counter $srec[c]$ counting the number of operations the server has received from client c since the last time an operation which was generated by other clients than c was transformed at the server and a message was broadcast.

The counters (i.e., $crec[c]$ and $srec[c]$) are piggybacked in the *ack* field in messages *AJMsg* telling the other side how many new messages have been received since the last time a message was sent; see module *AJupiter*. When a client c receives a message m of form $[ack \mapsto srec[c], op \mapsto xop]$ broadcast by the *Server*, it knows that op is generated by another client and more importantly that the set of operations against which op has been transformed at the *Server* contains the first ack operations in $cbuf[c]$. Thus, in $Rev(c, m)$, client c calls $xFormShift(m.op, cbuf[c], m.ack)$ of module *Buffer* to transform op against the subsequence of operations obtained by shifting the first $m.ack$ operations out of $cbuf[c]$. Similarly, when the *Server* receives a message m of form $[c \mapsto c, ack \mapsto crec[c], op \mapsto op]$ from client c, it knows that among the (transformed) operations in $sbuf[c]$ generated by other clients than c, the first ack operations have been broadcast to c and have been transformed at c before op was generated. Thus, in $SRev(m)$, the *Server* calls $xFormShift(m.op, sbuf[c], m.ack)$ of module *Buffer* to transform op against the subsequence of operations obtained by shifting the first $m.ack$ operations out of $sbuf[c]$. The transformed operation xop will be appended to other $sbuf[cl]$ for clients $cl \neq c$. Finally, the *Server* sends the transformed operation xop along with $srec[cl]$ to client $cl \neq c$.

By maintaining only $1D$-buffers and discarding/shifting obsolete operations whenever possible, AJupiter is the most efficient one among these four Jupiter protocols.

MODULE *AJupiter*

VARIABLES $cbuf$, $crec$, $sbuf$, $srec$

$AJMsg \triangleq [c : Client,\ ack : Nat,\ op : Op \cup \{Nop\}]$ ∪ from client c to *Server*
$[ack : Nat,\ op : Op \cup \{Nop\}]$ from *Server* to clients

$Do(c,\ op) \triangleq \land\ cbuf' = [cbuf$ EXCEPT $![c] = Append(@,\ op)]$
$\land\ crec' = [crec$ EXCEPT $![c] = 0]$
$\land$ apply op to $list[c]$
$\land$ send $[c \mapsto c,\ ack \mapsto crec[c],\ op \mapsto op]$ to the *Server*

$Rev(c,\ m) \triangleq$ LET $xform \triangleq xFormShift(m.op,\ cbuf[c],\ m.ack)$
$xform : [xop, xops]$
IN $\land\ cbuf' = [cbuf$ EXCEPT $![c] = xform.xops]$
$\land\ crec' = [crec$ EXCEPT $![c] = @ + 1]$
$\land$ apply $xform.xop$ to $list[c]$

$SRev(m) \triangleq$ LET $c \triangleq m.c$
$xform \triangleq xFormShift(m.op,\ sbuf[c],\ m.ack)$ $xform : [xop, xops]$
$xop \triangleq xform.xop$
IN $\land\ srec' = [cl \in Client \mapsto$
IF $cl = c$ THEN $srec[cl] + 1$ ELSE $0]$
$\land\ sbuf' = [cl \in Client \mapsto$
IF $cl = c$ THEN $xform.xops$
ELSE $Append(sbuf[cl],\ xop)]$
$\land$ apply xop to $list[Server]$
$\land$ send $[ack \mapsto srec[cl],\ op \mapsto xop]$ to client $cl \neq c$

Example 7 *(Illustration of AJupiter).* We illustrate client c_3 in AJupiter under the schedule of Fig. 1; see also Fig. 6 (d).

First, when client c_3 receives o_1 (Ins$(x, 1)$) of client c_1 from the server, its buffer $cbuf[c_3]$ is empty. Therefore, in *Rec*, it simply increases $crec[c_3]$ by 1 and applies Ins$(x, 1)$ locally.

Next, client c_3 generates o_4 (Ins$(b, 2)$). In *Do*, it appends o_4 to its currently empty buffer $cbuf[c_3]$, resets $crec[c_3]$ to 0, applies o_4 locally, and sends o_4 with $ack = 1$ to the server.

Then, client c_3 receives o_2 (Del(1)) with $ack = 0$ of client c_1 from the server. By *xForm* of *Buffer*, o_2 (Del(1)) is transformed against o_4 (Ins$(b, 2)$) in buffer $cbuf[c_3]$. The transformed operation $OT(o_2, o_4) =$ Del(1) is applied locally, and o_4 in buffer $cbuf[c_3]$ is transformed into $OT(o_4, o_2) =$ Ins$(b, 1)$.

Finally, client c_3 receives transformed o_3 (Ins$(a, 1)$ which happens to be unchanged) with $ack = 0$ of client c_2 from the server. By *xForm* of *Buffer*, o_3 (Del(1)) is transformed against o_4 (which is now Ins$(b, 1)$) in buffer $cbuf[c_3]$. The transformed operation $OT(o_3, o_4) =$ Del(2) is applied locally, obtaining the list content ba. Meanwhile, o_4 in buffer $cbuf[c_3]$ is transformed into $OT(o_4, o_3) =$ Ins$(b, 1)$.

5 Refinement

The OT behaviors (namely, when and how to perform OTs) of four Jupiter protocols are essentially the same under the same schedule of actions of *Do*, *Rev*, and *SRev*. The main difference lies in the data structures they use to support OTs; see Fig. 8. Specifically, AbsJupiter maintains sets of context-based operations. CJupiter organizes these context-based operations into n-ary digraphs, by grouping the ones with the same context. Since the transformed context-based opera-

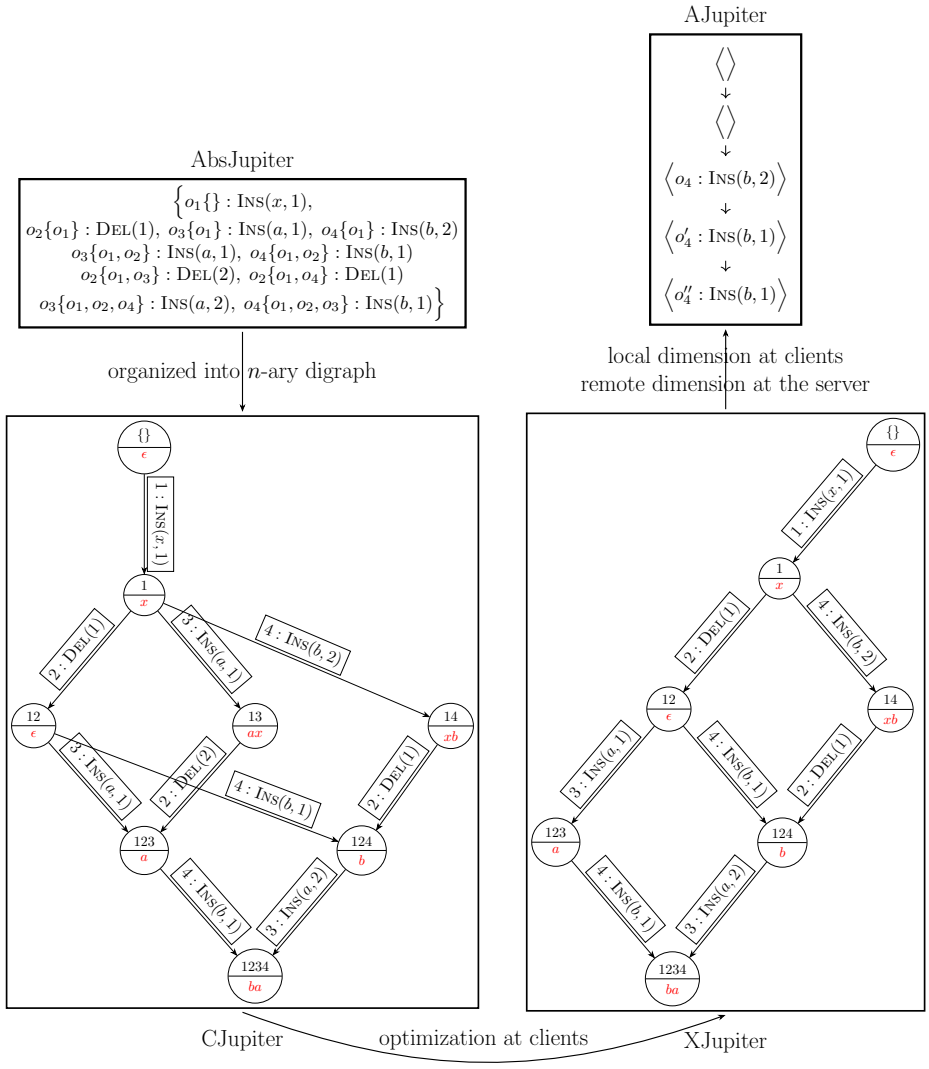


Fig.8. Illustration of the data refinement relation among Jupiter protocols (taking client c_3 in Fig. 1 as an example). First, context-based operations with the same context of AbsJupiter are connected to the same node in the digraph of CJupiter. Second, the redundant OTs performed at the server have been optimized away from the digraph of XJupiter. Finally, only the transformed operations along the local dimension of the digraph of XJupiter are kept in the buffer of AJupiter.

tions are broadcast by the server in XJupiter, XJupiter is slightly optimized in implementation at clients by eliminating redundant OTs that have already been performed at the server [6]. XJupiter synchronizes each client with its counterpart at the server, where $2D$ digraphs that distinguish the local dimension from the remote dimension are sufficient. In AJupiter, each client maintains only the local dimension for operations it generates, and the remote dimension for operations generated by other clients are maintained by its counterpart at the server. Thus, $2D$ digraphs can be reduced to $1D$ buffers. In this section, we establish the (data) refinement relation [12–14] among these Jupiter protocols. Specifically, we show that AJupiter is a refinement of XJupiter, which is a refinement of CJupiter, which is a refinement of AbsJupiter, by defining (data) refinement mappings to simulate the data structure of one Jupiter protocol using that of another one. In the following, we focus on the refinement mappings for data structures mentioned above, and omit details for other

```
MODULE CJupiterImplAbsJupiter
EXTENDS CJupiter
AbsJ ≜ INSTANCE AbsJupiter
          WITH copss ← [r ∈ Replica ↦ {e.cop : e ∈ css[r].edge}]
```

```
MODULE XJupiterImplCJupiter
EXTENDS XJupiter
 We have omitted the history variables for recording serial views.
VARIABLES op2ss,  a function mapping an operation (identifier)
                  to the 2D digraph produced during its transformation at the server
          c2ssX   c2ssX[c]: 2D digraph that has been skipped by client c
-----------------------------------------------------------------------------
InitImpl ≜ ∧ Init
           ∧ on history variables for serial views
           ∧ op2ss = ⟨⟩ the empty function expressed in TLA+
           ∧ c2ssX = [c ∈ Client ↦ EmptyGraph]
DoImpl(c, op) ≜ ∧ Do(c, op)
                ∧ on history variables for serial views
RevImpl(c, cop) ≜ ∧ Rev(c, cop)
                  ∧ on history variables for serial views
                  ∧ c2ssX' = [c2ssX EXCEPT ![c] = @ ⊕ op2ss[cop.oid]]
SRevImpl(cop) ≜ ∧ SRev(cop)
                ∧ on history variables for serial views
                ∧ LET xform ≜ xForm(NextEdge, Server, cop,
                                    s2ss[ClientOf(cop)])
                  IN  op2ss' = op2ss @@ (cop.oid :> xform.xg)
-----------------------------------------------------------------------------
CJ ≜ INSTANCE CJupiter WITH css ← [r ∈ Replica ↦
       IF r = Server THEN SetReduce(⊕, Range(s2ss), EmptyGraph)
                     ELSE c2ss[r] ⊕ c2ssX[r]]
```

variables.

5.1 CJupiter Refines AbsJupiter

The set $copss[r]$ of context-based operations maintained at replica r in AbsJupiter has been organized into an n-ary digraph $css[r]$ in CJupiter, by grouping the ones with the same context. Therefore, the refinement mapping from *CJupiter* to *AbsJupiter* only needs to simulate $copss[r]$ in *AbsJupiter* by extracting the context-based operations associated with the edges of $css[r]$ in *CJupiter*; see its definition in module *CJupiterImplAbsJupiter*.

5.2 XJupiter Refines CJupiter

The refinement mapping from XJupiter to CJupiter defined in module *XJupiterImplCJupiter* simulates, for each replica, the n-ary digraph in CJupiter using the $2D$ digraph(s) in XJupiter.

At the server side, XJupiter has decomposed the single n-ary digraph $css[Server]$ in CJupiter into n $2D$ digraphs, one $s2ss[c]$ for each client c. Thus, the refinement mapping simulates $css[Server]$ by taking the union of these $s2ss[c]$ for all $c \in Client$. Conceptually, this can be expressed in TLA$^+$ as (not syntactically correct)

$$css[Server] \leftarrow$$

MODULE *AJupiterImplXJupiter*

EXTENDS *AJupiter*

We have omitted the history variables for recording operation contexts.

VARIABLES $c2ss$, $s2ss$

$$
\begin{array}{l}
\mathit{InitImpl} \triangleq \land\ \mathit{Init} \\
\qquad\qquad \land\ \text{on history variables for operation contexts} \\
\qquad\qquad \land\ c2ss = [c \in \mathit{Client} \mapsto \mathit{EmptyGraph}] \\
\qquad\qquad \land\ s2ss = [c \in \mathit{Client} \mapsto \mathit{EmptyGraph}] \\
\mathit{DoImpl}(c,\ op) \triangleq \land\ \mathit{Do}(c,\ op) \\
\qquad\qquad\qquad \land\ \text{on history variables for operation contexts} \\
\qquad\qquad\qquad \land \text{LET } \mathit{cop} \triangleq [op \mapsto op, \\
\qquad\qquad\qquad\qquad\qquad\qquad oid \mapsto [c \mapsto c,\ seq \mapsto cseq[c]],\ ctx \mapsto ds[c]] \\
\qquad\qquad\qquad\quad \text{IN} \quad c2ss' = [c2ss \text{ EXCEPT } ![c] = \\
\qquad\qquad\qquad\qquad @ \oplus [node \mapsto \{ds'[c]\}, \\
\qquad\qquad\qquad\qquad\qquad\quad edge \mapsto \{[from \mapsto ds[c],\ to \mapsto ds'[c],\ cop \mapsto cop]\}]] \\
\mathit{RevImpl}(c,\ m) \triangleq \land\ \mathit{Rev}(c,\ m) \\
\qquad\qquad\qquad \land\ \text{on history variables for operation contexts} \\
\qquad\qquad\qquad \land \text{LET } \mathit{xform} \triangleq \mathit{xFormCopCopsShift}(m.cop,\ cbuf[c],\ m.ack) \\
\qquad\qquad\qquad\quad \text{IN} \quad c2ss' = [c2ss \text{ EXCEPT } ![c] = @ \oplus \mathit{xform}.xg] \\
\mathit{SRevImpl}(m) \triangleq \land\ \mathit{SRev}(m) \\
\qquad\qquad\qquad \land\ \text{on history variables for operation contexts} \\
\qquad\qquad\qquad \land \text{LET } c \triangleq \mathit{ClientOf}(m.cop) \\
\qquad\qquad\qquad\quad \mathit{xform} \triangleq \mathit{xFormCopCopsShift}(m.cop,\ sbuf[c],\ m.ack) \\
\qquad\qquad\qquad\quad \text{IN} \quad s2ss' = [cl \in \mathit{Client} \mapsto \\
\qquad\qquad\qquad\qquad\qquad\qquad \text{IF } cl = c \text{ THEN } s2ss[cl] \oplus \mathit{xform}.xg \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad \text{ELSE } s2ss[cl] \oplus \mathit{xform}.lg]
\end{array}
$$

$XJ \triangleq$ INSTANCE *XJupiter* WITH $c2ss \leftarrow c2ss$, $s2ss \leftarrow s2ss$

$$\mathit{SetReduce}(\oplus, \mathit{Range}(s2ss), \mathit{EmptyGraph}),$$

where $\mathit{Range}(s2ss)$ is the set of $s2ss[c]$ for all c, and *SetReduce* combines $\mathit{Range}(s2ss)$ into one using $\oplus$ with an empty digraph as initial value.

The server in XJupiter broadcasts the transformed operation $\mathit{xform}.xcop$ (instead of cop that it receives) to clients. Thus, the clients can skip the OTs transforming cop to $\mathit{xform}.xcop$ performed at the server. To simulate the n-ary digraph $css[c]$ at client c in CJupiter using the $2D$ digraph $c2ss[c]$ in XJupiter, we need to complement $c2ss[c]$ with those OTs skipped by XJupiter. To this end, we introduce two history variables in *XJupiterImplCJupiter* to record OTs. The variable $op2ss$ is a function mapping an operation (identifier) to the extra $2D$ digraph produced during its transformation at the server. When an operation cop is transformed at the server, the new mapping $cop.oid :> \mathit{xform}.xg$ is added to $op2ss$; see $\mathit{SRevImpl}(cop)$. When a client c receives the transformed operation $\mathit{xform}.xcop$ broadcast by the server, it accumulates this extra $2D$ digraph $op2ss[cop.oid]$ into $c2ssX[c]$, the overall $2D$ digraph that has been skipped by client c; see $\mathit{RevImpl}(c, cop)$. Thus, for client c, the simulation between $css[c]$ and $c2ss[c]$ can be (conceptually) expressed as $css[c] \leftarrow c2ss[c] \oplus c2ssX[c]$.

5.3 AJupiter Refines XJupiter

AJupiter uses $1D$ buffers to replace $2D$ digraphs in XJupiter, by keeping only the latest operation se-

Table 3. Model Checking Results of Verifying that CJupiter Refines AbsJupiter

TLC Model (No. of Clients, No. of Chars)	**Diameter**	**No. of States**	**No. of Distinct States**	**Checking Time** ($hh:mm:ss$)
$(1,1)$	5	7	6	0 : 00 : 00
$(1,2)$	9	86	57	0 : 00 : 00
$(1,3)$	13	1,696	1,014	0 : 00 : 01
$(1,4)$	17	53,273	30,393	0 : 00 : 06
$(2,1)$	10	71	53	0 : 00 : 01
$(2,2)$	19	50,215	28,307	0 : 00 : 05
$(2,3)$	28	150,627,005	75,726,121	4 : 37 : 36
$(2,4)$	18	121,964,031	$\theta = 80,000,000^{\star}$	5 : 21 : 04
$(3,1)$	17	2,785	1,288	0 : 00 : 01
$(3,2)$	33	206,726,218	74,737,027	5 : 43 : 26
$(3,3)$	18	139,943,577	$\theta = 80,000,000^{\star}$	5 : 18 : 57
$(4,1)$	26	194,877	61,117	0 : 00 : 18
$(4,2)$	21	177,451,069	$\theta = 80,000,000^{\star}$	6 : 12 : 48

quences that should participate in further OTs and discarding the old ones and intermediate transformed operations. Therefore, the refinement mapping needs to reconstruct these $2D$ digraphs in XJupiter from the OTs performed on $1D$ buffers in AJupiter. To this end, we introduce two history variables $c2ss$ and $s2ss$ in *AJupiterImplXJupiter* which are to simulate $c2ss$ and $s2ss$ in *XJupiter*, respectively. They are supposed to be updated in accordance with *cbuf* and *sbuf* of *AJupiter*. Specifically, in $DoImpl(c, op)$, the generated operation *op* is wrapped as a context-based operation *cop* and added to $c2ss[c]$ as in *XJupiter*, besides it is stored in $cbuf[c]$ as in *AJupiter* (not shown here). In $RevImpl(c, m)$ and $SRevImpl(m)$, *xFormCopCopsShift* behaves as *xFormShift* and *xFormOpOps* used in *AJupiter*, except that the former performs *COT*s on context-based operations and stores intermediate nodes and edges produced during *COT*s into $c2ss[c]$ and $s2ss$ as in *XJupiter*, respectively.

6 Model Checking Results

In this section, we first present the model checking results of verifying the refinement relation among Jupiter protocols defined in Section 5. Thanks to the refinement relation, we then only need to verify AbsJupiter with respect to desired properties to ensure the correctness of all Jupiter protocols.

Verification by model checking is conducted by TLC [16] (implemented in the TLA$^+$ Toolbox of version 1.5.7), a model checker for TLA$^+$, on a 2.40GHz 6-core machine with 64GB RAM. For each group of model checking experiments, we vary the number of clients and the number of characters allowed to be inserted. [9] We use symmetry set [15] for the set *Char* of characters. The initial lists on all replicas are empty. We use 10 threads and report the following statistics: the diameter of the reachable-state graph (i.e., the length of the longest behavior of protocol), the number of states TLC examines, the number of distinct states, and the checking time in $hh:mm:ss$.

[9]The positive model checking results help to gain great confidence in the correctness of these Jupiter protocols and the refinement relation among them, given the empirical study [21] that "almost all failures (of 198 production failures in distributed data-intensive systems) require only 3 or fewer nodes to reproduce." In our experiments, with some configurations such as $(3, 2)$, we are able to explore the behaviors of the protocol with diameter of length greater than 30 and with more than 200 million states.

Table 4. Model Checking Results of Verifying that XJupiter Refines CJupiter

TLC Model (No. of Clients, No. of Chars)	**Diameter**	**No. of States**	**No. of Distinct States**	**Checking Time** ($hh : mm : ss$)
$(1, 1)$	5	7	6	0 : 00 : 00
$(1, 2)$	9	86	57	0 : 00 : 00
$(1, 3)$	13	1,696	1,014	0 : 00 : 01
$(1, 4)$	17	53,273	30,393	0 : 00 : 07
$(2, 1)$	10	71	53	0 : 00 : 00
$(2, 2)$	19	50,215	28,307	0 : 00 : 07
$(2, 3)$	28	150,627,005	75,726,121	5 : 38 : 00
$(2, 4)$	19	122,113,291	$\theta = 80,000,000^\star$	8 : 01 : 35
$(3, 1)$	17	2,785	1,288	0 : 00 : 02
$(3, 2)$	33	206,726,218	74,737,027	8 : 50 : 40
$(3, 3)$	20	139,577,795	$\theta = 80,000,000^\star$	8 : 59 : 52
$(4, 1)$	26	194,877	61,117	0 : 00 : 30
$(4, 2)$	19	175,896,403	$\theta = 80,000,000^\star$	11 : 40 : 50

Table 5. Model Checking Results of Verifying that AJupiter Refines XJupiter

TLC Model (No. of Clients, No. of Chars)	**Diameter**	**No. of States**	**No. of Distinct States**	**Checking Time** ($hh : mm : ss$)
$(1, 1)$	5	7	6	0 : 00 : 01
$(1, 2)$	9	86	57	0 : 00 : 01
$(1, 3)$	13	1,696	1,014	0 : 00 : 01
$(1, 4)$	17	53,273	30,393	0 : 00 : 07
$(2, 1)$	10	71	53	0 : 00 : 00
$(2, 2)$	19	50,215	28,307	0 : 00 : 05
$(2, 3)$	28	150,627,005	75,726,121	4 : 23 : 52
$(2, 4)$	18	122,137,621	$\theta = 80,000,000^\star$	3 : 52 : 46
$(3, 1)$	17	2,785	1,288	0 : 00 : 01
$(3, 2)$	33	206,726,218	74,737,027	4 : 52 : 39
$(3, 3)$	18	139,823,551	$\theta = 80,000,000^\star$	4 : 48 : 23
$(4, 1)$	26	194,877	61,117	0 : 00 : 17
$(4, 2)$	21	176,794,063	$\theta = 80,000,000^\star$	3 : 49 : 58

Table 6. Model Checking Results of Verifying that AbsJupiter Satisfies *WLSpec*

TLC Model (No. of Clients, No. of Chars)	**Diameter**	**No. of States**	**No. of Distinct States**	**Checking Time** ($hh : mm : ss$)
$(1, 1)$	5	7	6	0 : 00 : 01
$(1, 2)$	9	86	57	0 : 00 : 01
$(1, 3)$	13	1,696	1,014	0 : 00 : 00
$(1, 4)$	17	53,273	30,393	0 : 00 : 04
$(2, 1)$	10	71	53	0 : 00 : 00
$(2, 2)$	19	50,215	28,307	0 : 00 : 03
$(2, 3)$	28	150,627,005	75,726,121	1 : 54 : 46
$(2, 4)$	20	153,275,009	$\theta = 100,000,000^\star$	3 : 54 : 49
$(3, 1)$	17	2,785	1,288	0 : 00 : 01
$(3, 2)$	33	206,726,218	74,737,027	2 : 46 : 02
$(3, 3)$	25	175,457,016	$\theta = 100,000,000^\star$	2 : 59 : 29
$(4, 1)$	26	194,877	61,117	0 : 00 : 09
$(4, 2)$	22	222,738,876	$\theta = 100,000,000^\star$	3 : 16 : 45

MODULE *AbsJupiterH*

EXTENDS *AbsJupiter*
VARIABLE *hlist*

$InitH \triangleq Init \wedge hlist = \{\}$
$DoH(c) \triangleq Do(c) \wedge hlist' = hlist \cup \{list'[c]\}$
$RevH(c) \triangleq Rev(c) \wedge hlist' = hlist \cup \{list'[c]\}$
$SRevH \triangleq SRev \wedge hlist' = hlist \cup \{list'[Server]\}$

$WLSpec \triangleq \forall\, l1,\, l2 \in hlist : Compatible(l1,\, l2)$

6.1 Verifying Refinement Relation among Jupiter Protocols

We verify the refinement mapping *AbsJ* from *CJupiter* to *AbsJupiter* defined in *CJupiterImplAbsJupiter* by checking that each behavior of *CJupiter* with variables substituted by *AbsJ* is a behavior allowed by *AbsJupiter*. The model checking results are shown in Table 3. [10] Similar results on verification of the refinement mappings defined in *XJupiterImplCJupiter* and *AJupiterImplXJupiter* are shown in Table 4 and Table 5, respectively.

6.2 Verifying Correctness of Jupiter Protocols

We present the model checking results of verifying that AbsJupiter satisfies the weak list specification *WLSpec* [2]. To express *WLSpec* in TLA$^+$, we introduce module *AbsJupiterH* which extends *AbsJupiter* with a history variable *hlist* [13]. *AbsJupiterH* behaves exactly as *AbsJupiter*, except that it collects the new list state $list'[r]$ in each action into *hlist*. We check that *WLSpec* is an invariant of *AbsJupiterH* using TLC, and the model checking results are shown in Table 6.

7 Related Work

OT was pioneered by Ellis et al. in 1989 [1]. Though the idea of OT is simple, OT-based protocols are subtle and error-prone. For example, the dOPT protocol in [1] for P2P systems did not work in all cases [7, 8]. Remarkably, after several failed attempts [8, 9, 22], it was shown impossible [10, 11] to design OT functions (and thus OT-based protocols) for P2P systems for lists with signatures of *Ins* and *Del* as described in Subsection 2.3. In other words, extra parameters are needed for *Ins* and *Del* operations [11]. On the other hand, researchers made efforts to gain a better understanding why some OT-based protocols work [4, 20]. In this paper, we identified the key OT issue in centralized settings, present a generic solution, and summarized several techniques to carry out the solution. We also proposed AbsJupiter, inspired by the COT protocol for P2P systems [20], which is abstract from implementation and captures the essence of existing Jupiter protocols.

The first Jupiter protocol appeared in 1995 [3] and is now used in many collaborative editors such as Google Docs [11], Firepad, and SubEthaEdit. However, its original description involves only a single client. Based on the notion of COT they developed before [20], Xu et

[10] In a "starred" experiment, we exit TLC when the number of distinct states it examines reaches a threshold θ. This is supported by a TLA$^+$ Toolbox nightly build as of 01-28-2019 (at 05:56).

[11] What's different about the new Google Docs: Making collaboration fast. `https://drive.googleblog.com/2010/09/whats-different-about-new-google-docs.html`, accessed in 09/2020.

al. [4] reported a multi-client version of Jupiter, which we call XJupiter. XJupiter uses $2D$ digraphs to manage COTs. Independently, Attiya et al. described another multi-client version of Jupiter, which we call AJupiter. [12] AJupiter relies on the acknowledgment mechanism and uses $1D$ buffers to manage OTs, thus reducing the metadata overhead. To facilitate the proof that XJupiter satisfies the weak list specification [2], Wei et al. [6] proposed CJupiter (Compact Jupiter), which is equivalent to XJupiter. CJupiter is compact in the sense that at a high level, it maintains only a single n-ary digraph that encompasses all replica states. In this work, we established the (data) refinement relation [12–14] among the family of Jupiter protocols, including AbsJupiter, CJupiter, XJupiter, and AJupiter.

Much work has been devoted to formal verification of OT functions for lists or trees [9, 10, 23–25]. In contrast, little has been done on formal verification of complete OT-based protocols. To our knowledge, we are the first to formally specify and verify a family of OT-based Jupiter protocols and the refinement relation among them.

8 Conclusion and Future Work

In this paper, we studied a family of OT-based Jupiter protocols for replicated lists. We proposed an implementation-independent AbsJupiter protocol and established the data refinement relation among several Jupiter protocols. These protocols and the refinement relation among them have been formally specified and verified using TLA$^+$ and TLC. Since OT-based protocols are subtle and error-prone, we hope that our work would be helpful to promote a rigorous study of them.

We will develop a mechanical correctness proof for our AbsJupiter protocol with respect to both strong eventual consistency and weak list specification using TLAPS [13], a proof system for TLA$^+$. Then we will extend our work to OT-based protocols for replicated lists for P2P systems. In particular, we will study the COT protocol [20] for P2P systems that has inspired us to propose AbsJupiter for client/server systems.

References

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