MODULE *Order*

Order related operators.

See https://*github.com*/jameshfisher/tlaplus/blob/master/examples/*TransitiveClosure*/*TransitiveClosure.tla*

Support of relation R.

$Support(R) \triangleq \{r[1] : r \in R\} \cup \{r[2] : r \in R\}$

Is R a reflexive relation on set S?

$Reflexive(S, R) \triangleq \forall\, a \in S : \langle a, a\rangle \in R$

Is R a transitive relation (on its support set)?

$$
\begin{array}{l}
Transitive(R) \triangleq \\
\quad \text{LET } S \triangleq Support(R) \\
\quad \text{IN } \forall\, a, b, c \in S : \\
\qquad (\langle a, b\rangle \in R \land \langle b, c\rangle \in R) \Rightarrow \langle a, c\rangle \in R
\end{array}
$$

Composition of two relations R and T.

$$
\begin{array}{l}
R ** T \triangleq \\
\quad \text{LET } SR \triangleq Support(R) \\
\qquad ST \triangleq Support(T) \\
\quad \text{IN } \{\langle r, t\rangle \in SR \times ST : \\
\qquad \exists\, s \in SR \cap ST : (\langle r, s\rangle \in R) \land (\langle s, t\rangle \in T)\}
\end{array}
$$

Transitive closure of relation R.

$$
\begin{array}{l}
\text{RECURSIVE } TC(\_) \\
TC(R) \triangleq \\
\quad \text{LET } RR \triangleq R ** R \\
\quad \text{IN IF } RR \subseteq R \text{ THEN } R \text{ ELSE } TC(R \cup RR)
\end{array}
$$

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