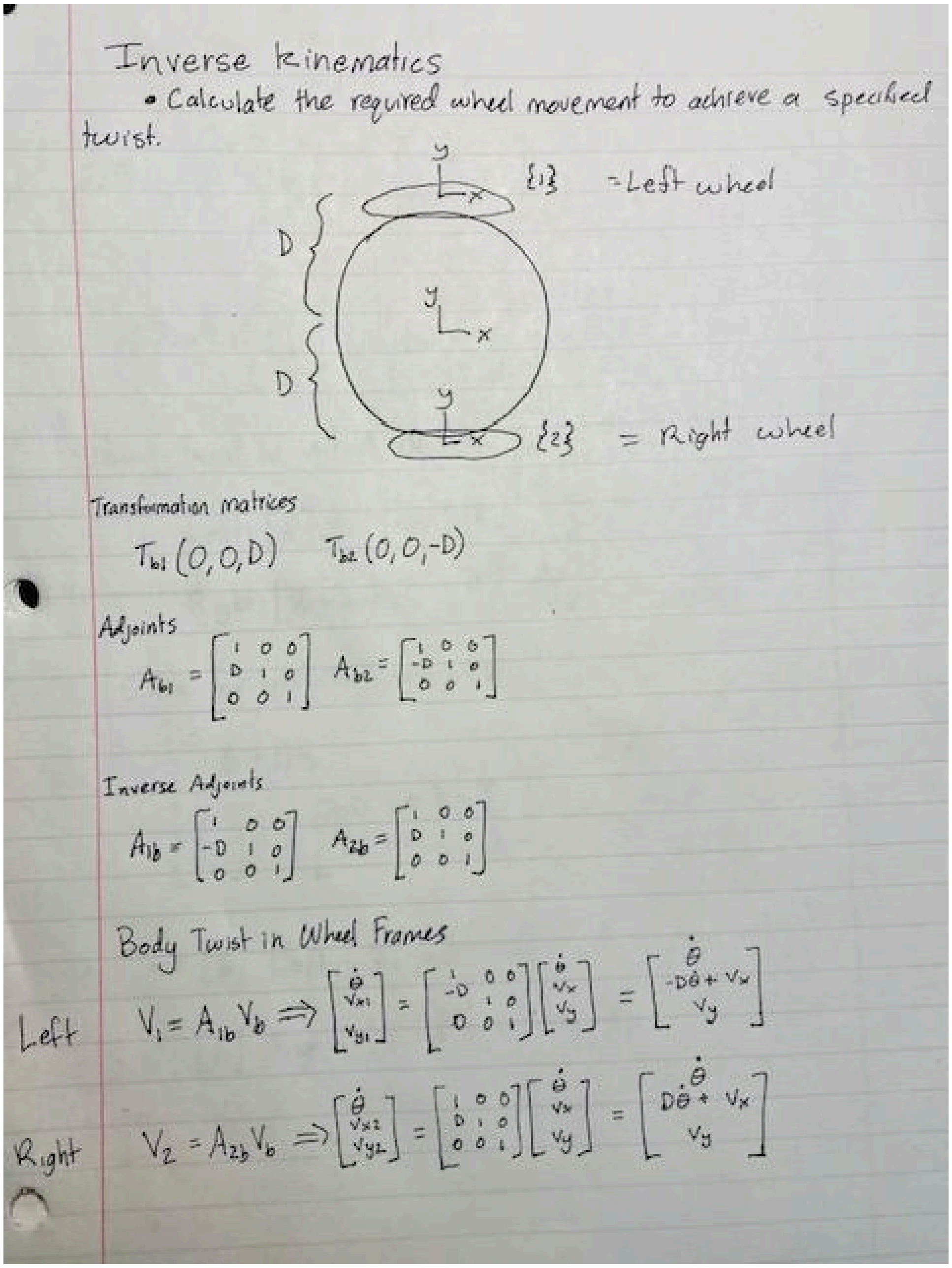
Diff Drive Kinematics
Important equations and steps are marked with [#] here and in code.

Inverse Kinematics



Inverse Kinematics

- Calculate the required wheel movement to achieve a specified twist.

$\{1\}$ = Left wheel

$\{2\}$ = Right wheel

D

D

Transformation matrices

$T_{b1}(0,0,D)$ $T_{b2}(0,0,-D)$

Adjoints

$$A_{b1} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_{b2} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse Adjoints

$$A_{1b} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_{2b} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Body Twist in Wheel Frames

Left
$$V_1 = A_{1b} V_b \Rightarrow \begin{bmatrix} \dot\theta \\ v_{x1} \\ v_{y1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ -D\dot\theta + v_x \\ v_y \end{bmatrix}$$

Right
$$V_2 = A_{2b} V_b \Rightarrow \begin{bmatrix} \dot\theta \\ v_{x2} \\ v_{y2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ D\dot\theta + v_x \\ v_y \end{bmatrix}$$

Substitute $\begin{bmatrix} v_{x_i} \\ v_{y_i} \end{bmatrix} = \begin{bmatrix} r\dot{\phi}_i \\ 0 \end{bmatrix}$

$\Rightarrow$ Left: $\begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_i \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ -D\dot{\theta} + v_x \\ v_y \end{bmatrix}$

$\Rightarrow$ Right: $\begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_i \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ D\dot{\theta} + v_x \\ v_y \end{bmatrix}$

Body Twist to Wheel Motion

Left: $[\dot{\phi}_1] = \frac{1}{r}\begin{bmatrix} -D & 1 & 0 \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix}$

Right: $[\dot{\phi}_2] = \frac{1}{r}\begin{bmatrix} D & 1 & 0 \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix}$

Solve for inverse kinematics...

$\dot{\phi} = H\mathcal{V}_b$

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} = \frac{1}{r}\begin{bmatrix} -D & 1 & 0 \\ D & 1 & 0 \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix}$$

$\Rightarrow$ [5] Left: $\frac{1}{r}(-D\dot{\theta} + v_x) = \dot{\phi}_1$

[6] Right: $\frac{1}{r}(D\dot{\theta} + v_x) = \dot{\phi}_2$

Forward Kinematics

Forward kinematics

- Update robot configuration based on change in wheel position
- Formula from Modern Robotics, 13.4, pg 549-550

Input: $\begin{bmatrix} \Delta\theta_L \\ \Delta\theta_R \end{bmatrix}$ = increments for right and left wheel (rad)

[1] Get body twist V_b

$$V_b = r \begin{bmatrix} -1/2d & 1/2d \\ 1/2 & 1/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\theta_L \\ \Delta\theta_R \end{bmatrix}$$

$$V_b = \begin{bmatrix} \theta \\ x \\ y \end{bmatrix} = \begin{bmatrix} (r/d)(\Delta\theta_R - \Delta\theta_L) \\ (r/2)(\Delta\theta_L + \Delta\theta_R) \\ 0 \end{bmatrix}$$

[2] Find the body transformation, $T_{bb'}$, from the twist

$$T_{bb'} = e^{[V_b]}$$

$$\Rightarrow T_{bb'} = \text{integrate\_twist}(V_b)$$

And extract translation and rotation values

d_qb'_theta'

d_qb_x'

d_qb_y'

the change in coordinates relative to the body frame.

[3] Transform the change in coordinates from the body frame to the fixed frame

$$\Delta q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_k & -\sin\phi_k \\ 0 & \sin\phi_k & \cos\phi_k \end{bmatrix} \Delta q_b$$

$\Rightarrow$ $\Delta q_\theta = \Delta q_{b1}$

$\Delta q_x = \cos(\phi_k)\Delta q_{bx} - \sin(\phi_k)\Delta q_{by}$

$\Delta q_y = \sin(\phi_k)\Delta q_{bx} + \cos(\phi_k)\Delta q_{by}$

[4] Update robot configuration

$\rightarrow$ Add the coordinate changes (fixed frame) to the robot configuration