Model-Based Controller Design

- Direct Synthesis Method
- Internal Model Control
- Controllers With Two Degrees of Freedom

Controller Tuning

- PID controller settings can be determined by a number of alternative techniques:

1. Direct Synthesis (DS) method
2. Internal Model Control (IMC) method
3. Controller tuning relations
4. Frequency response techniques
5. Computer simulation
6. *On-line tuning* after the control system is installed.

Direct Synthesis Method

- In the Direct Synthesis (DS) method, the controller design is based on a process model and a desired closed-loop transfer function.
- The latter is usually specified for set-point changes, but responses to disturbances can also be utilized (Chen and Seborg, 2002).
- Although these feedback controllers do not always have a PID structure, the DS method does produce PI or PID controllers for common process models.

- As a starting point for the analysis, consider the block diagram of a feedback control system in Figure 12.2. The closed-loop transfer function for set-point changes was derived in Section 11.2:

$$\frac{Y}{Y_{sp}} = \frac{G_c G_v G_p}{1 + G_c G_v G_p G_m} \qquad (12\text{-}1)$$

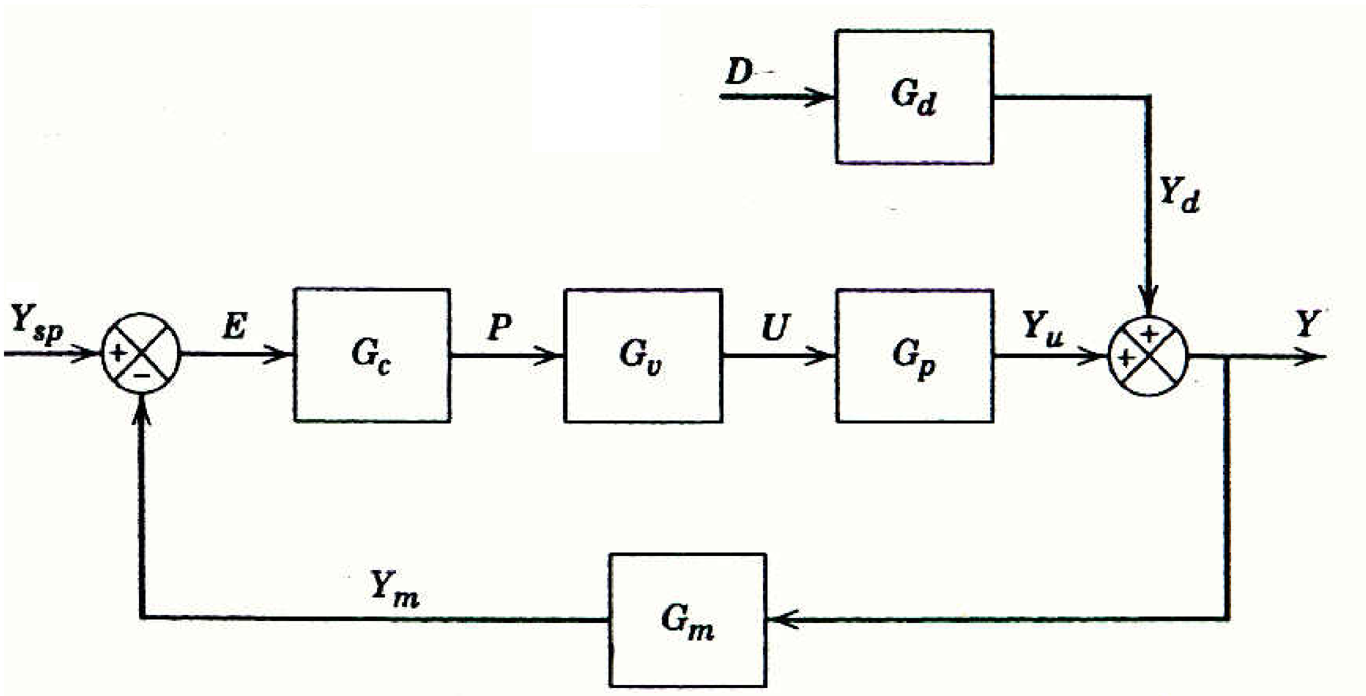

Fig. 12.2. Block diagram for a standard feedback control system.

For simplicity, let $G \triangleq G_v G_p$ and assume that $G_m = 1$. Then Eq. 12-1 reduces to

$$\frac{Y}{Y_{sp}} = \frac{G_c G}{1 + G_c G} \qquad (12\text{-}2)$$

Rearranging and solving for G_c gives an expression for the feedback controller:

$$G_c = \frac{1}{G}\left(\frac{Y / Y_{sp}}{1 - Y / Y_{sp}}\right) \qquad (12\text{-}3a)$$

- Equation 12-3a cannot be used for controller design because the closed-loop transfer function Y/Y_{sp} is not known *a priori*.
- Also, it is useful to distinguish between the actual process G and the model, $\tilde{G}$, that provides an approximation of the process behavior.

- A practical design equation can be derived by replacing the unknown G by $\tilde{G}$, and Y/Y_{sp} by a *desired closed-loop transfer function*, $(Y/Y_{sp})_d$:

$$G_c = \frac{1}{\tilde{G}}\left[\frac{\left(Y/Y_{sp}\right)_d}{1-\left(Y/Y_{sp}\right)_d}\right] \qquad (12\text{-}3b)$$

- The specification of $(Y/Y_{sp})_d$ is the key design decision and will be considered later in this section.
- Note that the controller transfer function in (12-3b) contains the inverse of the process model owing to the $1/\tilde{G}$ term.
- This feature is a distinguishing characteristic of model-based control.

Desired Closed-Loop Transfer Function

For processes without time delays, the first-order model in Eq. 12-4 is a reasonable choice,

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{1}{\tau_c s + 1} \qquad (12\text{-}4)$$

- The model has a settling time of $\sim 5\tau_c$, as shown in Section 5. 2.
- Because the steady-state gain is one, no offset occurs for set-point changes.
- By substituting (12-4) into (12-3b) and solving for G_c, the controller design equation becomes:

$$G_c = \frac{1}{\tilde{G}} \frac{1}{\tau_c s} \qquad (12\text{-}5)$$

- The $1/\tau_c s$ term provides integral control action and thus eliminates offset.

- Design parameter τ_c provides a convenient controller tuning parameter that can be used to make the controller more aggressive (small τ_c) or less aggressive (large τ_c).

- If the process transfer function contains a known time delay θ, a reasonable choice for the desired closed-loop transfer function is:

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{e^{-\theta s}}{\tau_c s + 1} \qquad (12\text{-}6)$$

- The time-delay term in (12-6) is essential because it is physically impossible for the controlled variable to respond to a set-point change at $t = 0$, before $t = \theta$.

- If the time delay is unknown, θ must be replaced by an estimate.

- Combining Eqs. 12-6 and 12-3b gives:

$$G_c = \frac{1}{\tilde{G}} \frac{e^{-\theta s}}{\tau_c s + 1 - e^{-\theta s}} \qquad (12\text{-}7)$$

- Although this controller is not in a standard PID form, it is physically realizable.

- Next, we show that the design equation in Eq. 12-7 can be used to derive PID controllers for simple process models.

- The following derivation is based on approximating the time-delay term in the denominator of (12-7) with a truncated Taylor series expansion:

$$e^{-\theta s} \approx 1 - \theta s \qquad (12\text{-}8)$$

Substituting (12-8) into the denominator of Eq. 12-7 and rearranging gives

$$G_c = \frac{1}{\tilde{G}} \frac{e^{-\theta s}}{\left(\tau_c + \theta\right) s} \qquad (12\text{-}9)$$

Note that this controller also contains integral control action.

First-Order-plus-Time-Delay (FOPTD) Model

Consider the standard FOPTD model,

$$\tilde{G}(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \qquad (12\text{-}10)$$

Substituting Eq. 12-10 into Eq. 12-9 and rearranging gives a PI controller, $G_c = K_c\left(1 + 1/\tau_I s\right)$, with the following controller settings:

$$K_c = \frac{1}{K}\frac{\tau}{\theta + \tau_c}, \qquad \tau_I = \tau \qquad (12\text{-}11)$$

Second-Order-plus-Time-Delay (SOPTD) Model

Consider a SOPTD model,

$$\tilde{G}(s) = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{12-12}$$

Substitution into Eq. 12-9 and rearrangement gives a PID controller in parallel form,

$$G_c = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \tag{12-13}$$

where:

$$K_c = \frac{1}{K}\frac{\tau_1 + \tau_2}{\tau_c + \theta}, \qquad \tau_I = \tau_1 + \tau_2, \qquad \tau_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} \tag{12-14}$$

Example 12.1

Use the DS design method to calculate PID controller settings for the process:

$$G = \frac{2e^{-s}}{(10s+1)(5s+1)}$$

Consider three values of the desired closed-loop time constant: $\tau_c = 1, 3, \text{and } 10$. Evaluate the controllers for unit step changes in both the set point and the disturbance, assuming that $G_d = G$. Repeat the evaluation for two cases:

a. The process model is perfect ($\tilde{G} = G$).

b. The model gain is $\tilde{K} = 0.9$, instead of the actual value, $K = 2$. Thus,

$$\tilde{G} = \frac{0.9e^{-s}}{(10s+1)(5s+1)}$$

The controller settings for this example are:

	$\tau_c = 1$	$\tau_c = 3$	$\tau_c = 10$
$K_c\left(\tilde{K} = 2\right)$	3.75	1.88	0.682
$K_c\left(\tilde{K} = 0.9\right)$	8.33	4.17	1.51
τ_I	15	15	15
τ_D	3.33	3.33	3.33

The values of K_c decrease as τ_c increases, but the values of τ_I and τ_D do not change, as indicated by Eq. 12-14.

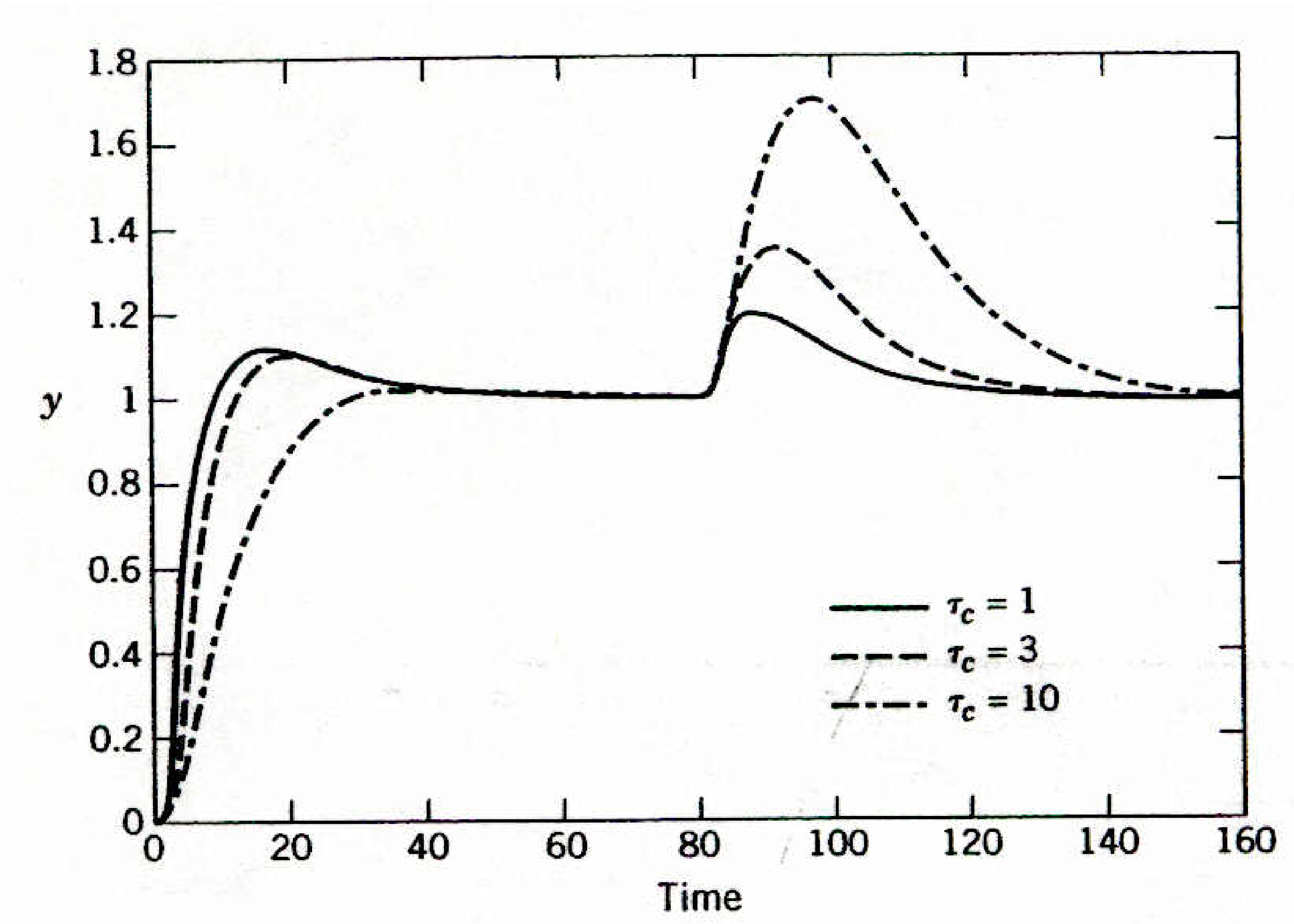

Figure 12.3 Simulation results for Example 12.1 (a): correct model gain.

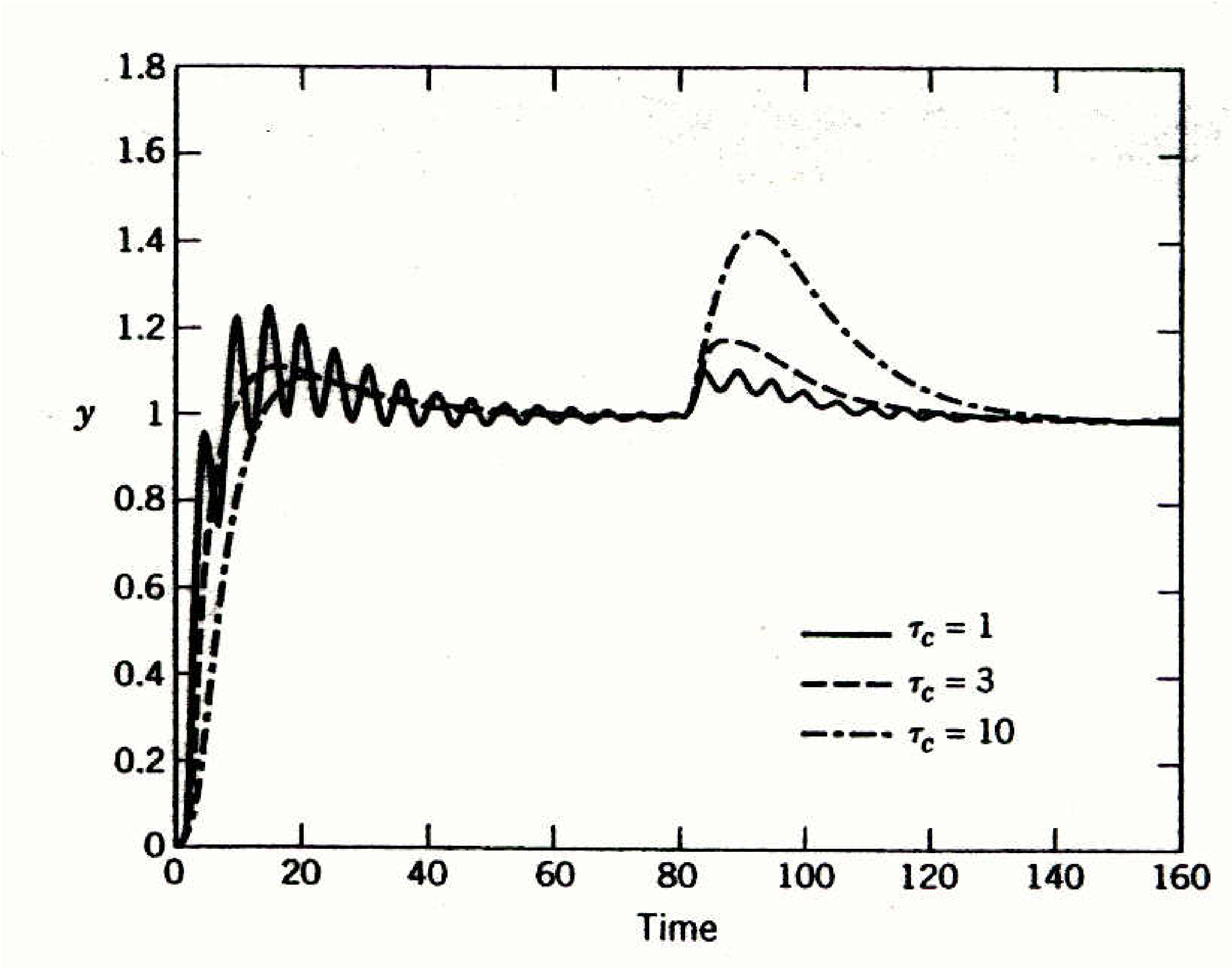

Fig. 12.4 Simulation results for Example 12.1 (b): incorrect model gain.

DS - Remark

- The specification of the desired closed-loop transfer function, $\left(Y/Y_{sp}\right)_d$, should be based on the assumed process model, as well as the desired set-point response.
  - The FOPTD model is a reasonable choice for many processes but not all.
  - For example, if the process model contains a RHP zero $\left(1-\tau_a s\right)$, we must specify

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{\left(1-\tau_a s\right)e^{-\theta s}}{\tau_c s+1} \qquad (12\text{-}15)$$

- The DS approach should not be used directly for process models with unstable poles.

Internal Model Control (IMC)

- A more comprehensive model-based design method, *Internal Model Control* (*IMC*), was developed by Morari and coworkers (Garcia and Morari, 1982; Rivera et al., 1986).
- The IMC method, like the DS method, is based on an assumed process model and leads to analytical expressions for the controller settings.
- These two design methods are closely related and produce identical controllers if the design parameters are specified in a consistent manner.
- The IMC method is based on the simplified block diagram shown in Fig. 12.6b. A process model $\tilde{G}$ and the controller output P are used to calculate the model response, $\tilde{Y}$.

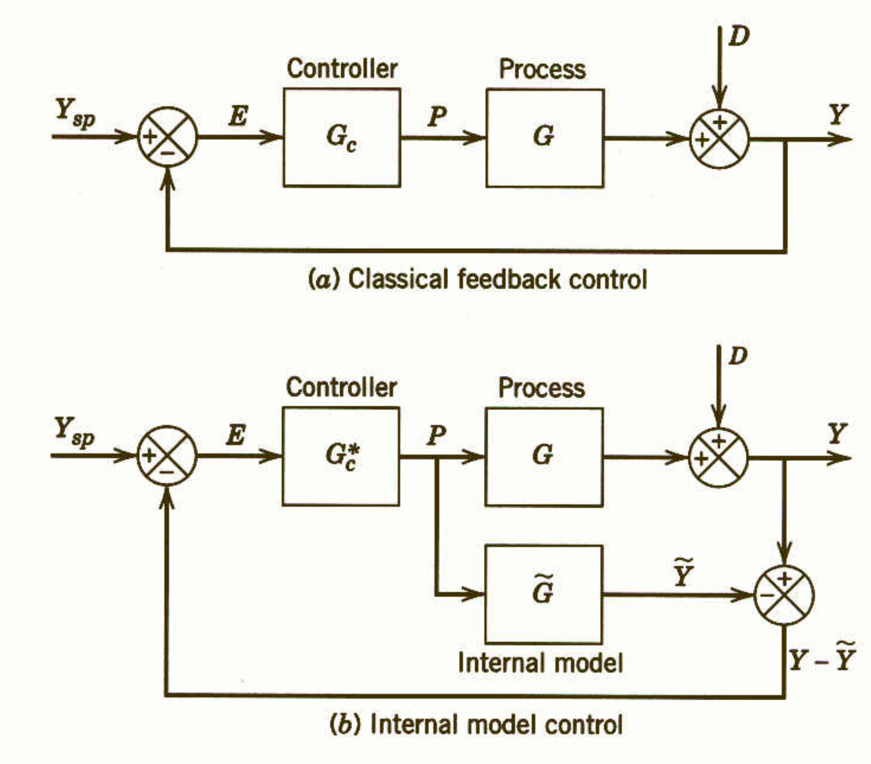

Figure 12.6. Feedback control strategies

- The model response is subtracted from the actual response Y, and the difference, $Y - \tilde{Y}$ is used as the input signal to the IMC controller, G_c^*.

- In general, $Y \neq \tilde{Y}$ due to modeling errors $\left(\tilde{G} \neq G\right)$ and unknown disturbances $\left(D \neq 0\right)$ that are not accounted for in the model.

- The block diagrams for conventional feedback control and IMC are compared in Fig. 12.6.

- It can be shown that the two block diagrams are identical if controllers G_c and G_c^* satisfy the relation

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}} \qquad (12\text{-}16)$$

- Thus, any IMC controller G_c^* is equivalent to a standard feedback controller G_c, and vice versa.

- The following closed-loop relation for IMC can be derived from Fig. 12.6b using the block diagram algebra:

$$Y = \frac{G_c^* G}{1 + G_c^* \left(G - \tilde{G}\right)} Y_{sp} + \frac{1 - G_c^* \tilde{G}}{1 + G_c^* \left(G - \tilde{G}\right)} D \qquad (12\text{-}17)$$

For the special case of a perfect model, $\tilde{G} = G$, (12-17) reduces to

$$Y = G_c^* G Y_{sp} + \left(1 - G_c^* G\right) D \qquad (12\text{-}18)$$

The IMC controller is designed in two steps:

Step 1. The process model is factored as

$$\tilde{G} = \tilde{G}_+ \tilde{G}_- \qquad (12\text{-}19)$$

where $\tilde{G}_+$ contains any time delays and right-half plane zeros.

- In addition, $\tilde{G}_+$ is required to have a steady-state gain equal to one in order to ensure that the two factors in Eq. 12-19 are unique.

Step 2. The controller is specified as

$$G_c^* = \frac{1}{\tilde{G}_-} f \qquad (12\text{-}20)$$

where *f* is a *low-pass filter* with a steady-state gain of one. It typically has the form:

$$f = \frac{1}{\left(\tau_c s + 1\right)^r} \qquad (12\text{-}21)$$

In analogy with the DS method, τ_c is the desired closed-loop time constant. Parameter *r* is a positive integer. The usual choice is $r = 1$.

For the ideal situation where the process model is perfect $\left(\tilde{G} = G\right)$, substituting Eq. 12-20 into (12-18) gives the closed-loop expression

$$Y = \tilde{G}_+ f Y_{sp} + \left(1 - f\tilde{G}_+\right) D \qquad (12\text{-}22)$$

Thus, the closed-loop transfer function for set-point changes is

$$\frac{Y}{Y_{sp}} = \tilde{G}_+ f \qquad (12\text{-}23)$$

Example 12.2

Use the IMC design method to design two controllers for the FOPDT model. Consider two approximations for the time delay term:

(a) 1/1 Pade approximation: $e^{-\theta s} \cong \dfrac{1-0.5\theta s}{1+0.5\theta s}$

(b) 1st-order Taylor series approximation: $e^{-\theta s} \cong 1-\theta s$

Solution:

(a) $$\tilde{G} = \frac{K\left(1-0.5\theta s\right)}{\left(1+0.5\theta s\right)\left(\tau s+1\right)}$$

Factor this model as $\tilde{G} = \tilde{G}_+ \tilde{G}_-$ where

$$\tilde{G}_+ = \left(1-0.5\theta s\right)$$

$$\tilde{G}_- = \frac{K}{\left(1+0.5\theta s\right)\left(\tau s+1\right)}$$

Setting $r = 1$ gives

$$G_c^* = \frac{(1+0.5\theta s)(\tau s+1)}{K(\tau_c s+1)}$$

The equivalent controller Gc can be obtained from Eq. 12-16

$$G_c = \frac{(1+0.5\theta s)(\tau s+1)}{K(\tau_c+0.5\theta)s}$$

And rearranged into the PID controller of (12-13) with:

$$K_c = \frac{1}{K}\frac{2\left(\dfrac{\tau}{\theta}\right)+1}{2\left(\dfrac{\tau_c}{\theta}\right)+1}, \qquad \tau_I = \frac{\theta}{2}+\tau, \qquad \tau_D = \frac{\tau}{2\left(\dfrac{\tau}{\theta}\right)+1}$$

(b) The IMC controller is identical to the DS controller for a FOPTD model

Selection of τ_c

- The choice of design parameter τ_c is a key decision in both the DS and IMC design methods.

- In general, increasing τ_c produces a more conservative controller because K_c decreases while τ_I increases.

- Several IMC guidelines for τ_c have been published for the FOPDT model in Eq. 12-10:

1. $\tau_c / \theta > 0.8$ and $\tau_c > 0.1\tau$ (Rivera et al., 1986)
2. $\tau > \tau_c > \theta$ (Chien and Fruehauf, 1990)
3. $\tau_c = \theta$ (Skogestad, 2003)

IMC Tuning Relations

The IMC method can be used to derive PID controller settings for a variety of transfer function models.

Table 12.1 IMC-Based PID (**parallel form**) Controller Settings for $G_c(s)$ (Chien and Fruehauf, 1990).

Case	Model	$K_c K$	τ_I	τ_D
A	$\dfrac{K}{\tau s+1}$	$\dfrac{\tau}{\tau_c}$	τ	—
B	$\dfrac{K}{(\tau_1 s+1)(\tau_2 s+1)}$	$\dfrac{\tau_1+\tau_2}{\tau_c}$	$\tau_1+\tau_2$	$\dfrac{\tau_1\tau_2}{\tau_1+\tau_2}$
C	$\dfrac{K}{\tau^2 s^2+2\zeta\tau s+1}$	$\dfrac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\dfrac{\tau}{2\zeta}$
D	$\dfrac{K(-\beta s+1)}{\tau^2 s^2+2\zeta\tau s+1}$, $\beta>0$	$\dfrac{2\zeta\tau}{\tau_c+\beta}$	$2\zeta\tau$	$\dfrac{\tau}{2\zeta}$
E	$\dfrac{K}{s}$	$\dfrac{2}{\tau_c}$	$2\tau_c$	—
F	$\dfrac{K}{s(\tau s+1)}$	$\dfrac{2\tau_c+\tau}{\tau_c^2}$	$2\tau_c+\tau$	$\dfrac{2\tau_c\tau}{2\tau_c+\tau}$
G	$\dfrac{Ke^{-\theta s}}{\tau s+1}$	$\dfrac{\tau}{\tau_c+\theta}$	τ	—
H	$\dfrac{Ke^{-\theta s}}{\tau s+1}$	$\dfrac{\tau+\frac{\theta}{2}}{\tau_c+\frac{\theta}{2}}$	$\tau+\dfrac{\theta}{2}$	$\dfrac{\tau\theta}{2\tau+\theta}$

Table 12.1 (Continued).

I	$\dfrac{K(\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\dfrac{\tau_1+\tau_2-\tau_3}{\tau_c+\theta}$	$\tau_1+\tau_2-\tau_3$	$\dfrac{\tau_1\tau_2-(\tau_1+\tau_2-\tau_3)\tau_3}{\tau_1+\tau_2-\tau_3}$
J	$\dfrac{K(\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$\dfrac{2\zeta\tau-\tau_3}{\tau_c+\theta}$	$2\zeta\tau-\tau_3$	$\dfrac{\tau^2-(2\zeta\tau-\tau_3)\tau_3}{2\zeta\tau-\tau_3}$
K	$\dfrac{K(-\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\dfrac{\tau_1+\tau_2+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_3+\theta}$	$\tau_1+\tau_2+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\dfrac{\tau_1\tau_2}{\tau_1+\tau_2+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
L	$\dfrac{K(-\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$\dfrac{2\zeta\tau+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_e+\theta}$	$2\zeta\tau+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\dfrac{\tau^2}{2\zeta\tau+\dfrac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
M	$\dfrac{Ke^{-\theta s}}{s}$	$\dfrac{2\tau_c+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\theta$	—
N	$\dfrac{Ke^{-\theta s}}{s}$	$\dfrac{2\tau_c+\theta}{\left(\tau_c+\dfrac{\theta}{2}\right)^2}$	$2\tau_c+\theta$	$\dfrac{\tau_c\theta+\dfrac{\theta^2}{4}}{2\tau_c+\theta}$
O	$\dfrac{Ke^{-\theta s}}{s(\tau s+1)}$	$\dfrac{2\tau_c+\tau+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\tau+\theta$	$\dfrac{(2\tau_c+\theta)\tau}{2\tau_c+\tau+\theta}$

Table 12.2 Equivalent PID Controller Settings for the Parallel and Series Forms

Parallel Form	Series Form
$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$	$G_c(s) = K'_c\left(1 + \frac{1}{\tau'_I s}\right)(1 + \tau'_D s)^\dagger$
$K_c = K'_c\left(1 + \frac{\tau'_D}{\tau'_I}\right)$	$K'_c = \frac{K_c}{2}\left(1 + \sqrt{1 - 4\tau_D/\tau_I}\right)$
$\tau_I = \tau'_I + \tau'_D$	$\tau'_I = \frac{\tau_I}{2}\left(1 + \sqrt{1 - 4\tau_D/\tau_I}\right)$
$\tau_D = \frac{\tau'_D \tau'_I}{\tau'_I + \tau'_D}$	$\tau'_D = \frac{\tau_I}{2}\left(1 - \sqrt{1 - 4\tau_D/\tau_I}\right)$

†These conversion equations are only valid if $\tau_D/\tau_I \le 0.25$.

Tuning for Lag-Dominant Models

- First- or second-order models with relatively small time delays $(\theta/\tau \ll 1)$ are referred to as *lag-dominant models*.

- The IMC and DS methods provide satisfactory set-point responses, but very slow disturbance responses, because the value of τ_I is very large.

- Fortunately, this problem can be solved in three different ways.

Method 1: Integrator approximation

$$\text{Approximate } \tilde{G}(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \text{ by } \tilde{G}(s) = \frac{K^* e^{-\theta s}}{s}$$

where $K^* \triangleq K/\tau$.

- Then can use the IMC tuning rules (Rule M or N) to specify the controller settings.

Method 2. Limit the value of τ_I

- For lag-dominant models, the standard IMC controllers for first-order and second-order models provide sluggish disturbance responses because τ_I is very large.
- For example, controller *G* in Table 12.1 has $\tau_I = \tau$ where τ is very large.
- As a remedy, Skogestad (2003) has proposed limiting the value of τ_I:

$$\tau_I = \min\left\{\tau_1, 4\left(\tau_c + \theta\right)\right\} \qquad (12\text{-}34)$$

where τ_1 is the largest time constant (if there are two).

Method 3. Design the controller for disturbances, rather than set-point changes

- The desired CLTF is expressed in terms of $(Y/D)_{\text{des}}$, rather than $(Y/Y_{sp})_{\text{des}}$
- *Reference*: Chen & Seborg (2002)

Example 12.4

Consider a lag-dominant model with $\theta / \tau = 0.01$:

$$\tilde{G}(s) = \frac{100}{100s+1} e^{-s}$$

Design four PI controllers:

a) IMC $(\tau_c = 1)$

b) IMC $(\tau_c = 2)$ based on the integrator approximation

c) IMC $(\tau_c = 1)$ with Skogestad's modification (Eq. 12-34)

d) Direct Synthesis method for disturbance rejection (Chen and Seborg, 2002): The controller settings are $K_c = 0.551$ and $\tau_I = 4.91$.

Evaluate the four controllers by comparing their performance for unit step changes in both set point and disturbance. Assume that the model is perfect and that $G_d(s) = G(s)$.

Solution

The PI controller settings are:

Controller	K_c	τ_I
(a) IMC	0.5	100
(b) Integrator approximation	0.556	5
(c) Skogestad	0.5	8
(d) DS-d	0.551	4.91

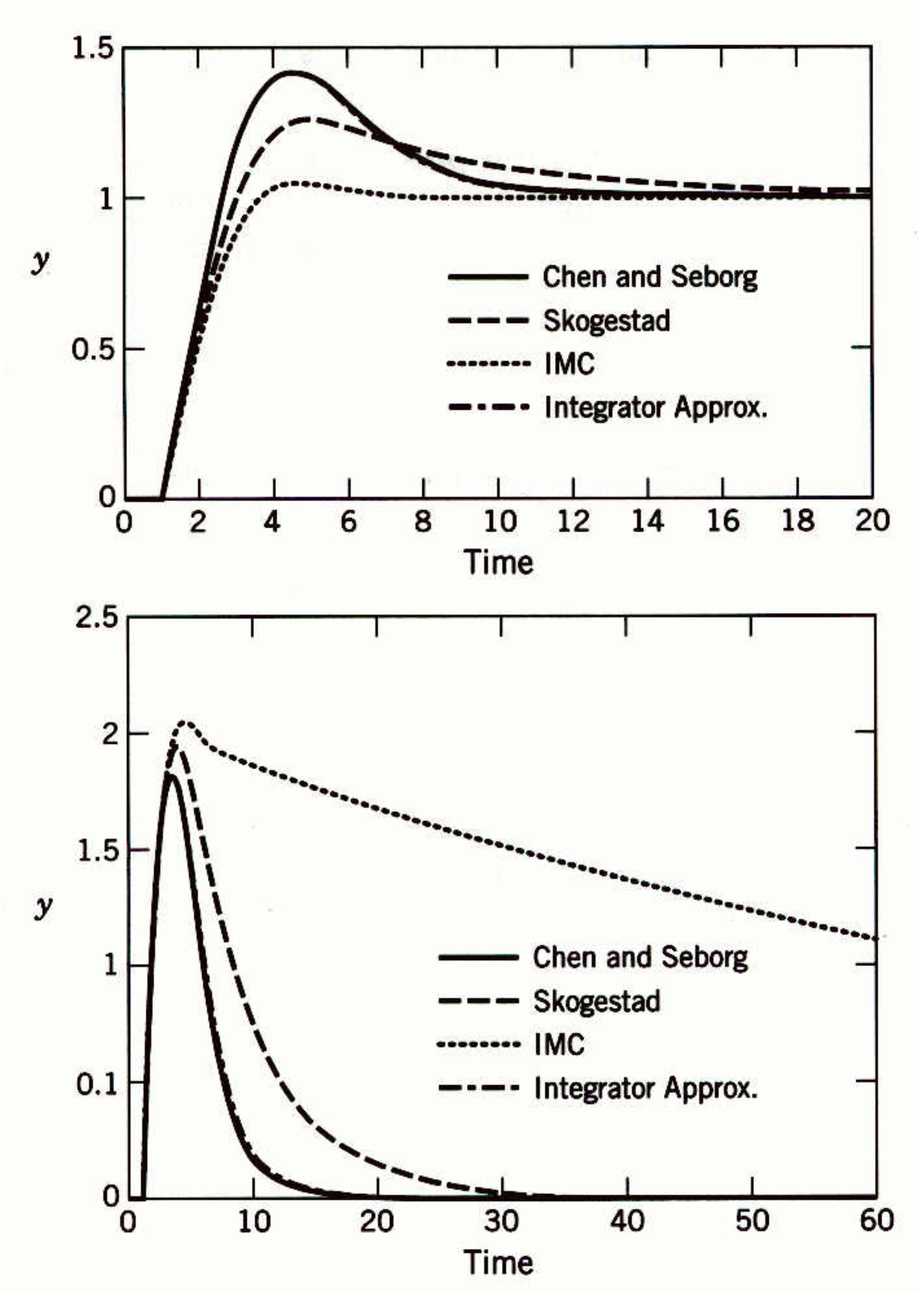

Figure 12.8. Comparison of set-point responses (top) and disturbance responses (bottom) for Example 12.4. The responses for the Chen and Seborg and integrator approximation methods are essentially identical.

Controllers With Two Degrees of Freedom

- The specification of controller settings for a standard PID controller typically requires a tradeoff between set-point tracking and disturbance rejection.
- The strategies which can be used to adjust the set-point and disturbance independently are referred to as *controllers with two-degrees-of-freedom.*
- The first strategy is very simple. Set-point changes are introduced gradually rather than as abrupt step changes.
- For example, the set point can be ramped as shown in Fig. 12.10 or "filtered" by passing it through a first-order transfer function,

$$\frac{Y_{sp}^{*}}{Y_{sp}} = \frac{1}{\tau_f s + 1} \qquad (12\text{-}38)$$

where Y_{sp}^{*} denotes the *filtered set point* that is used in the control calculations.

- The filter time constant, τ_f determines how quickly the filtered set point will attain the new value, as shown in Fig. 12.10.
- This strategy can significantly reduce overshoot for set-point changes.

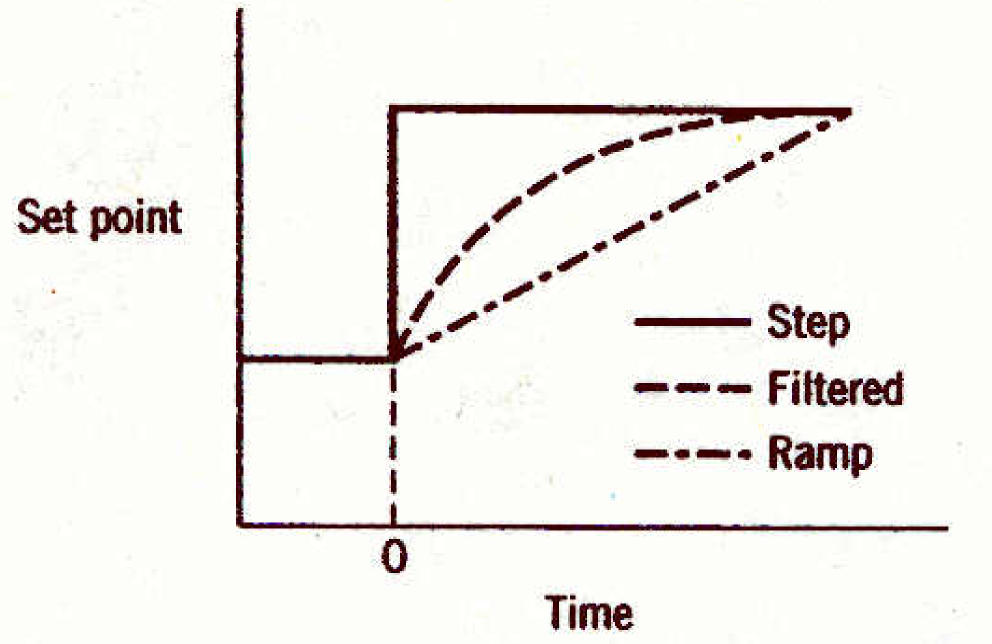

Figure 12.10 Implementation of set-point changes.

- A second strategy for independently adjusting the set-point response is based on a simple modification of the PID control law,

$$p(t) = \overline{p} + K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(t^*)\, dt^* + \tau_D \frac{de(t)}{dt} \right]$$

where y_m is the measured value of y and e is the error signal.
$e \triangleq y_{sp} - y_m$

- The control law modification consists of multiplying the set point in the proportional term by a *set-point weighting factor*, β :

$$p(t) = \overline{p} + K_c \left[\beta y_{sp}(t) - y_m(t) \right] + K_c \left[\frac{1}{\tau_I} \int_0^t e(t^*)\, dt^* + \tau_D \frac{de(t)}{dt} \right] \qquad (12\text{-}39)$$

The set-point weighting factor is bounded, $0 < \beta < 1$, and serves as a convenient tuning factor.

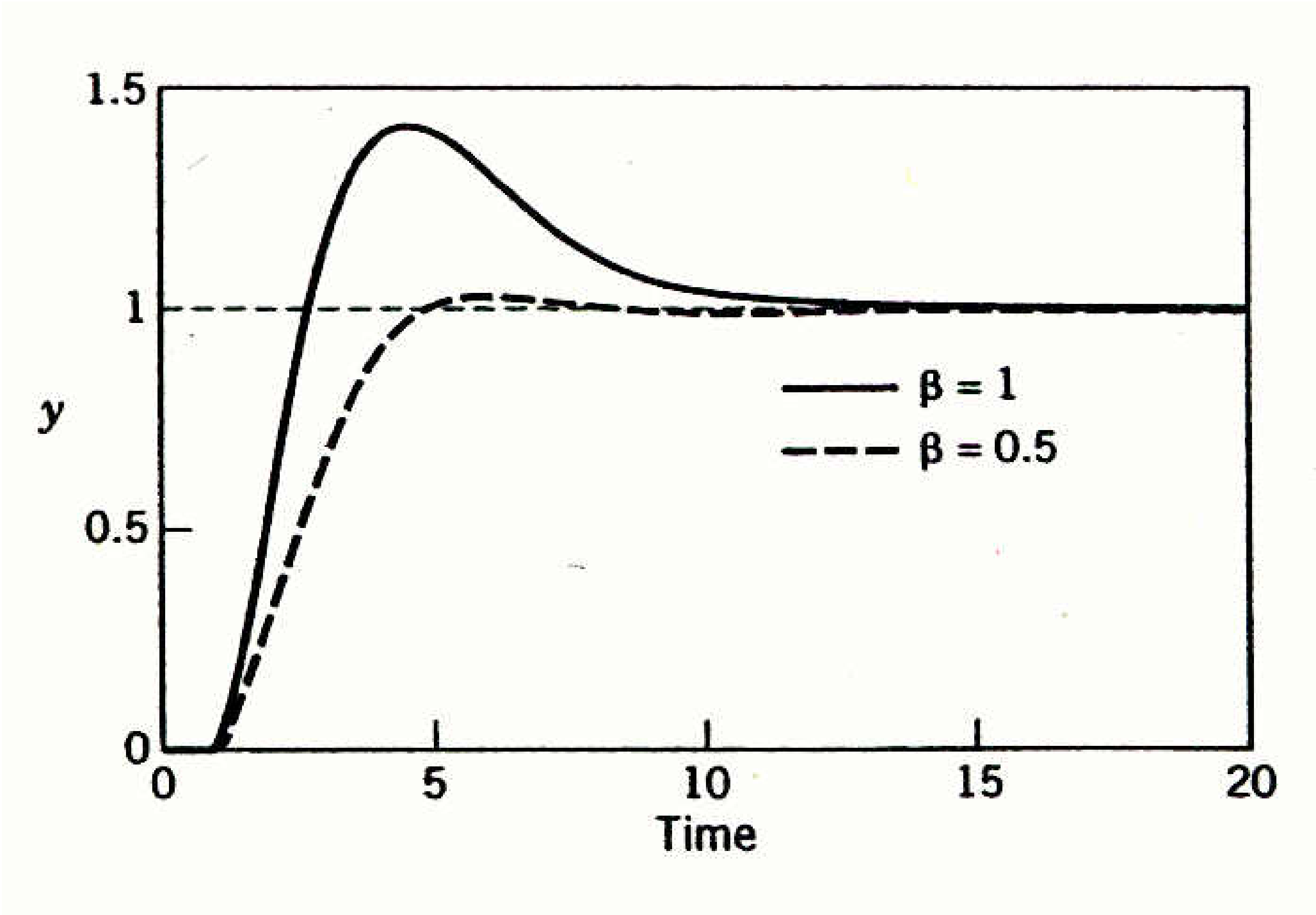

Figure 12.11 Influence of set-point weighting on closed-loop responses for Example 12.6.