Effects of Overfishing on Coral Reef Ecosystems

Coral Reefsearchers

Aaron Bumagat, Michelle Luces, Henry Song

University of Guam

July 16, 2021

Table of Contents

1 Background

- General Background
- General Question
- Definitions

Background

- Coral Reefs are large underwater structures composed of the skeletons of colonial marine invertebrates known as coral[13].

Background

- Coral Reefs are large underwater structures composed of the skeletons of colonial marine invertebrates known as coral[13].
- They play a crucial role in the marine ecosystem's biodiversity along with other functions, such as coastal defense from storms and economic benefits from tourism or local fisheries[9].

Background

- Coral Reefs are large underwater structures composed of the skeletons of colonial marine invertebrates known as coral[13].
- They play a crucial role in the marine ecosystem's biodiversity along with other functions, such as coastal defense from storms and economic benefits from tourism or local fisheries[9].
- According to the 2008 State of the Coral Reef Ecosystems of Guam report, Guam's coral reef resources are both economically and culturally important, providing numerous goods and services for the residents of Guam, including cultural/traditional use, tourism, recreation, fisheries, and shoreline/infrastructure protection [8].

Background

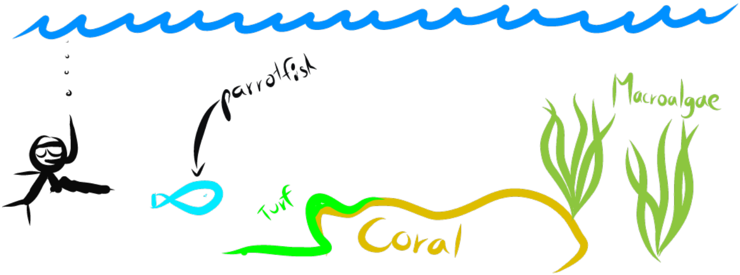
- Coral Reefs are large underwater structures composed of the skeletons of colonial marine invertebrates known as coral[13].
- They play a crucial role in the marine ecosystem's biodiversity along with other functions, such as coastal defense from storms and economic benefits from tourism or local fisheries[9].
- According to the 2008 State of the Coral Reef Ecosystems of Guam report, Guam's coral reef resources are both economically and culturally important, providing numerous goods and services for the residents of Guam, including cultural/traditional use, tourism, recreation, fisheries, and shoreline/infrastructure protection [8].
- Factors that affect coral reefs include climate change, coral reef resilience[12], and exploitative fishing practices[11].

Our Question

- General Question: How will Guam's reef ecosystem change over the coming decades?

$$\Downarrow$$

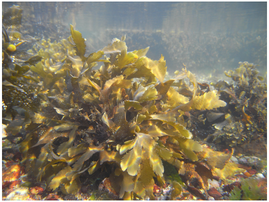
- Specific Question: How will overfishing affect Guam's coral reef ecosystem in the upcoming decades?



Identify Our Terms

(a) Corals[1]

(b) Algal Turfs[2]



(c) Macroalgae[3]

Figure 1: Images of Ecosystem

Identify Our Terms (Cont.)

Figure 2: Parrot Fish[4]

- Parrot fish are common reef fish found in many tropical reefs[5] and are known to feed on algal turfs and macroalgae.
- Their bites on corals have been shown to improve and promote coral growth.
- Parrot fish are one of the most overfished reef fish in the Caribbean, and potentially on Guam as well[5].

2 Mathematical Model

- Assumptions
- Compartments
- Parameters
- Compartment Model
- Differential Equations
- Plots

Assumptions

- Ecosystem:
    - is closed (i.e. no migration).
    - consists of only corals (C), algal turfs (T), and macroalgae (M).
    - supports maximum carrying capacity of parrotfish.
- Macroalgae is the only predator of corals.
- Coral recruit to and overgrow algal turfs[9].
- Corals are overgrown by macroalgae[9].
- Macroalgae colonize dead coral by spreading vegetative over algal turfs[9].
- Corals do not naturally die.

Coral Reef Ecosystem Model Compartments

Compartments

The Ecosystem Model consists of 4 compartments:

- C: Corals
- T: Algal Turfs
- M: Macroalgae
- P: Parrotfish

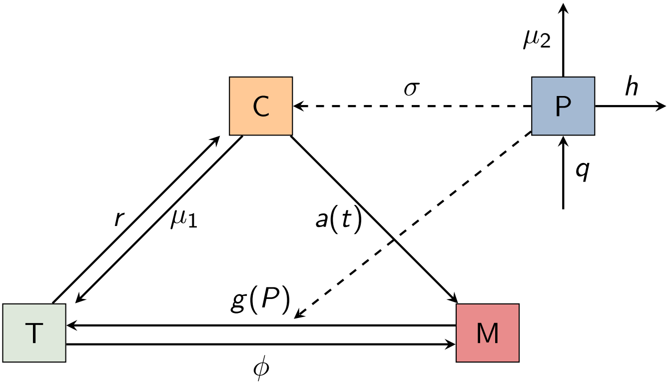
where $C + T + M = 1.$

Coral Reef Ecosystem Model Parameters

Parameter	Description	Rate	Units[10][9][5]
μ_1	natural death rate of coral reefs	0.15[14]	$year^{-1}$
μ_2	natural death rate of parrotfish	0.22[10]	$year^{-1}$
r	rate that coral recruit to overgrow algal turfs	10[14]	$year^{-1}$
ϕ	rate that macroalgae spread vegetative over algal turfs	0.8[15]	$year^{-1}$
q	intrinsic growth rate for parrotfish	0.47[10]	$year^{-1}$
h	harvesting rate for parrotfish	0.14[10]	$year^{-1}$
σ	rate that parrot fish bite coral	0.01*	$bites * year^{-1}$
ω	maximum grazing intensity	1[5]	-
β	carrying capacity of parrotfish	1	-
a_0	control variable to simulate seasonal changes	0.99	-

* $=$ *estimated value*

Coral Reef Ecosystem Model[9]



Differential Equations

System of differential equations derived from compartment model:

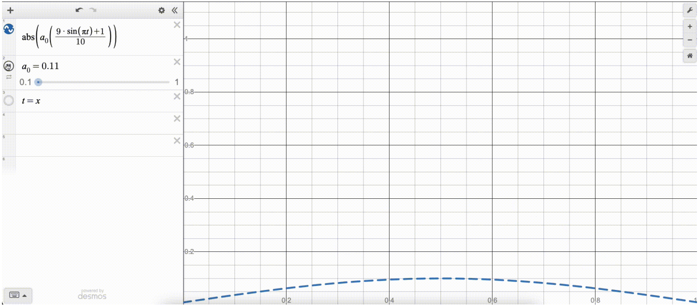
$$\frac{dC}{dt} = rTC + \sigma PC - (a(t)M + \mu_1)C$$
$$\frac{dP}{dt} = qP\left(1 - \frac{P}{\beta C}\right) - P\left(h + \mu_2\right)$$
$$\frac{dT}{dt} = \mu_1 C + \frac{g(P)M}{M+T} - T(rC + \phi M)$$
$$\frac{dM}{dt} = (a(t)C + \phi T)M - \frac{g(P)M}{M+T}$$

where $g(P) = \frac{\omega P}{\beta}$, $\quad a(t) = \left|\frac{a_0(9\sin(\pi t)+1)}{10}\right|$.

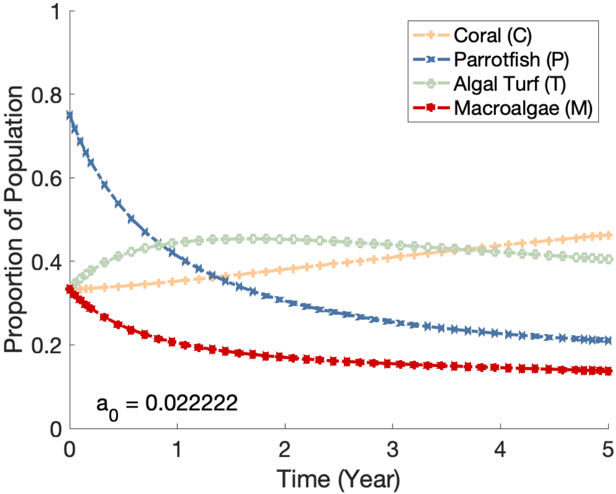
Modified from [5]

Simulating $a(t)$

$$a(t) = \left| \frac{a_0 (9 \sin(\pi t) + 1)}{10} \right|$$

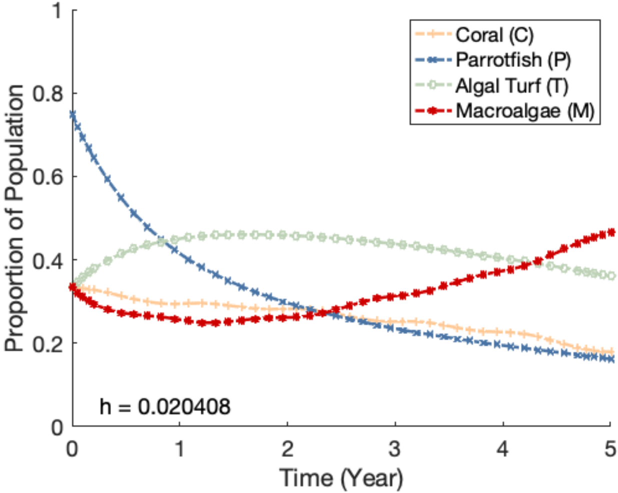


Parameter Changes: a_0



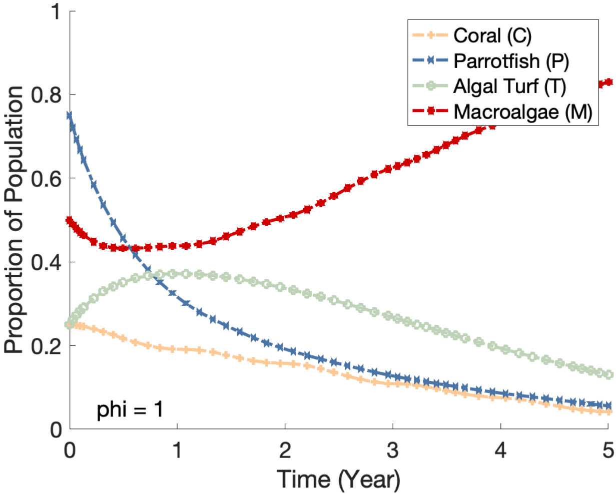
Initial Conditions: $C = T = M = \frac{1}{3}$, and $P = \frac{3}{4}$

Parameter Changes: h



Initial Conditions: $C = T = M = \frac{1}{3}$, and $P = \frac{3}{4}$

Parameter Changes: ϕ



Initial Conditions: $M = \frac{1}{2}$, $C = T = \frac{1}{4}$, and $P = \frac{3}{4}$

Compartment Initial Condition Changes

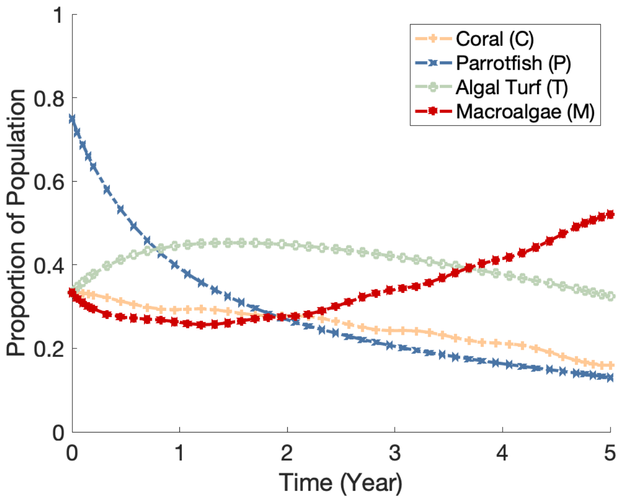


Figure 3: Initial Conditions: $C = T = M = \frac{1}{3}$, and $P = \frac{3}{4}$

Compartment Initial Condition Changes

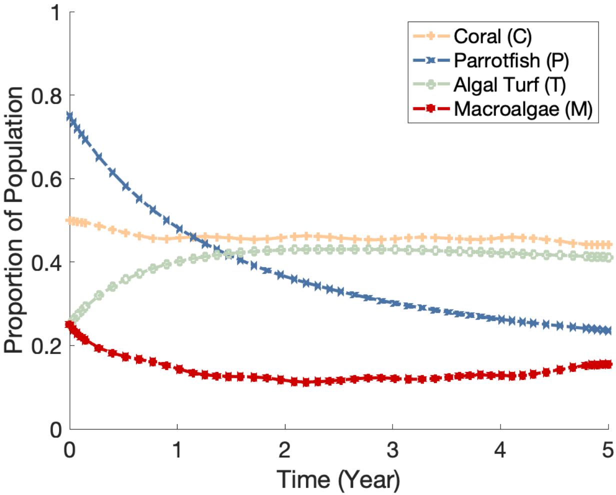


Figure 4: Initial Conditions: $C = \frac{1}{2}$, $T = M = \frac{1}{4}$, and $P = \frac{3}{4}$

Compartment Initial Condition Changes

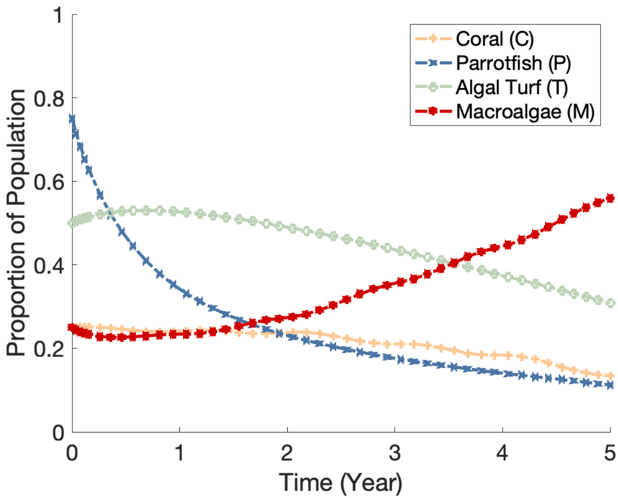


Figure 5: Initial Conditions: $T = \frac{1}{2}$, $C = M = \frac{1}{4}$, and $P = \frac{3}{4}$

Compartment Initial Condition Changes

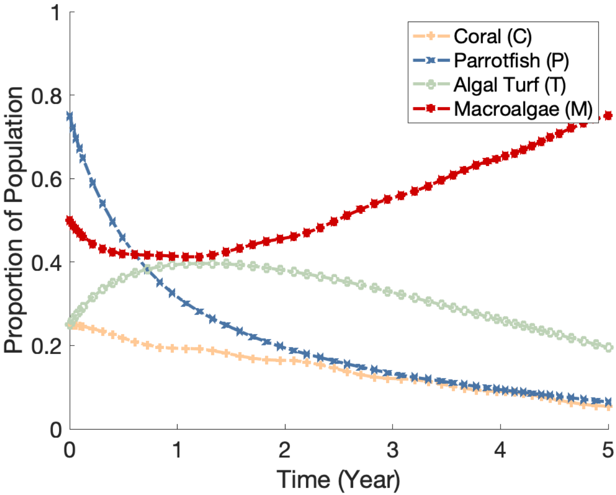


Figure 6: Initial Conditions: $M = \frac{1}{2}$, $C = T = \frac{1}{4}$, and $P = \frac{3}{4}$

3 Equilibria

- Disease Free Equilibrium (DFE)
- Endemic Equilibrium
- Basic Reproduction Number
- Sensitivity Analysis

Disease Free Equilibrium

$$C^0 = 1 - \frac{\mu_1}{r}$$

$$P^0 = -\frac{\beta(1 - \frac{\mu_1}{r})(h - \mu_2 - q)}{q}$$

$$T^0 = \frac{\mu_1}{r}$$

$$M^0 = 0 \Longleftarrow \text{our "disease" compartment}$$

Note

Since C + T + M = 1, if $M^0 = 0$, then $C^0 + T^0 = 1$.

Endemic Equilibrium

$$T^* = \frac{\mu_1 + a(t)M^*}{r}$$

$$C^* = 1 - \left(\frac{\mu_1 + a(t)M^*}{r} + M^*\right)$$

$$P^* = \beta\left(1 - \left(\frac{\mu_1 + a(t)M^*}{r} + M^*\right)\right)\left(\frac{q - (h + \mu_2)}{q}\right)$$

Endemic Equilibrium Cont.

Calculation for M^*:

$$\frac{-e \pm \sqrt{e^2 - 4df}}{2d}$$

, where

$$d = (a(t)q - 2a(t)r + r\phi) - q(r^2 + a(t)^3)$$

$$e = a(t)q(r(r - 2\mu_1 + a(t)) - 2\mu_1(a(t) + \phi)) + r(q(\phi\mu_1 + r\omega) - \omega(hr + a(t)\mu_2 + r\mu_2)$$

$$f = -qr^2\omega + qr\mu_1\omega + hr^2\omega - hr\mu_1\omega + r^2\mu_2\omega - r\mu_1\mu_2\omega + a(t)qr\mu_1 - a(t)q\mu_1^2 + q\phi\mu_1^2$$

Basic Reproduction Number: $\mathscr{R}_0$

Definition

A metric used to describe the contagiousness or transmissibility of infectious agents[7], i.e. the number of secondary infections

Basic Reproduction Number: $\mathscr{R}_0$

$$\mathscr{F} = \left[a(t)CM + \phi TM\right] \qquad \mathscr{V} = \left[\frac{g(P)M}{M+T}\right]$$

$$\Downarrow \qquad \Downarrow$$

$$F = \left[a(t)C^0 + \phi T^0\right] \qquad V = \left[\frac{g(P)T^0}{(M^0+T^0)^2}\right]$$

$$\mathscr{R}_0 = \rho(\det(FV^{-1} - \lambda I))$$

$$\Downarrow$$

$$\mathscr{R}_0 = -\frac{\beta\mu_1 q(a(t)(\frac{\mu_1}{r} - 1) - \frac{\mu_1}{r})}{\omega r(\beta h(\frac{\mu_1}{r} - 1) + \beta\mu_2(\frac{\mu_1}{r} - 1) - \beta q(\frac{\mu_1}{r} - 1))}$$

Sensitivity Analysis for $\mathscr{R}_0$

The Sensitivity Analysis for $\mathscr{R}_0$ determines the most influential parameters affecting $\mathscr{R}_0$. It is defined by

$$S_\lambda = \frac{\frac{\Delta \mathscr{R}_0}{\mathscr{R}_0}}{\frac{\Delta x}{x}} = \frac{\lambda}{\mathscr{R}_0} \cdot \frac{\partial \mathscr{R}_0}{\partial \lambda}$$

where λ is a parameter in the quantity $\mathscr{R}_0$

λ	S_λ
μ_1	**8.3513**
μ_2	**5.2819**
q	-3.5962
ω	-0.7923
σ	0
r	-2.5054
ϕ	0.4029
β	0
h	**5.2819**
$a(t)$	0.94

Table 1: Sensitivity Analysis

4 Harvesting Game Theory

- Harvesting Threshold
- Expected Payoff - Harvesting
- Nash Equilibrium

Harvesting Game

Game of Harvesting

- **Player:** Individual
- **Strategy:**
    - Proportion of Individual Harvesting (h)

Harvesting Threshold

Definition (Harvesting Threshold)

The rate at which parrotfish can be harvested in order for macroalgae growth to become stable in the ecosystem.

When $\mathscr{R}_0 = 1$,

$$h_{TH} = q - \mu_2 + \frac{\mu_1 q(a(t)\mu_1 - a(t)r - \phi\mu_1)}{\omega r(r - \mu_1)}$$

$$h_{pop} < h_{TH} : \text{Macroalgae growth is stable}$$
$$h_{pop} > h_{TH} : \text{Macroalgae growth is unstable}$$

Harvesting Threshold Graph

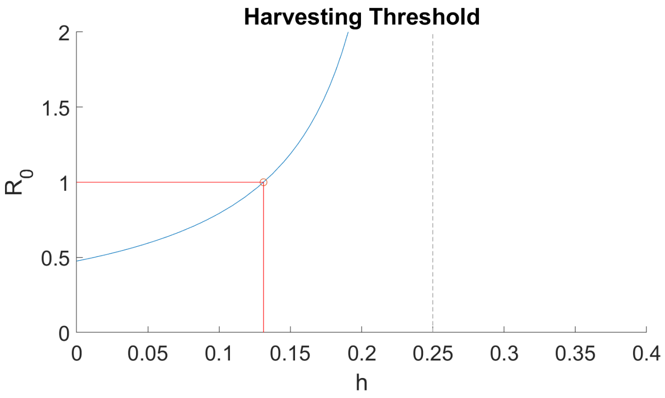


Figure 7: Threshold Analysis

When $\mathscr{R}_0 = 1$, $h = 0.1312$.

Expected Payoff - Harvesting

Let h $\in$ [0,1] be the proportion at which an individual can harvest parrotfish.

$$E(h, h_{pop}) = -hC_h - \left(\frac{h_{pop}}{h_{pop} + \mu_2} \cdot \frac{g(P^*)(1-h)M^*}{M^* + T^*}\right) C_D$$

Symbol	Definition
$E(h, h_{pop})$	Expected payoff for an individual to harvest based on the harvesting rate of the population
C_h	Cost of harvesting
C_D	Cost of coral disease

Expected Payoff - Harvesting

$$E(h, h_{pop}) = -hC^h - \frac{h_{pop}}{h_{pop} + \mu_2} \cdot \frac{g(P^*)(1-h)M^*}{M^* + T^*}$$

where $C^h = \frac{C_h}{C_D}$

Expected Payoff - Harvesting

$E(h, h_{pop})$ is a convex function since $\frac{\partial^2 E(h,h_{pop})}{\partial h^2} > 0$

$$\frac{\partial^2 E}{\partial h^2} = \frac{2\omega P\mu 2(M+T)^2(\mu_1+1)}{\beta(h(M+T)+\mu_2(M+T))^3} > 0$$

Thus, E achieves a maximum value at $h = 0$ or $h = 1$.

What is Nash Equilibrium?

Definition[6]

- The Nash equilibrium is a decision-making theorem within game theory that states a player can achieve the desired outcome by not deviating from their initial strategy.
- Each player's strategy is optimal when considering the decisions of other players. Every player wins because everyone gets the outcome they desire.

Nash Equilibrium - Harvesting

To obtain the Nash Equilibrium, we let $E(0, h_{pop}) = E(1, h_{pop})$, where

$$E(0, h_{pop}) = -\frac{h_{pop}}{h_{pop} + \mu_2} \cdot \frac{g(P^*)(1-h)M^*}{M^* + T^*}$$
$$E(1, h_{pop}) = -C^h$$

$$\Downarrow$$

$$\frac{h_{pop}}{h_{pop} + \mu_2} \cdot \frac{g(P^*)(1-h)M^*}{M^* + T^*} = C^h$$

5 Closing Remarks
- Future Plans
- Q & A
- Acknowledgements
- Bibliography

Future Plans

- Continue application of Harvesting Game Theory on the harvest rate parameter (h) to quantify human behavior and the best strategy to protect coral reef sustainability. These tasks include:
  - continue solving for the Nash Equilibrium

Questions?

Acknowledgements

Support for the Young Scholars Research Experience in Mathematics (YSREM) is through the MAA Tensor SUMMA Program. Support for the MAA National Research Experience for Undergraduates Program (NREUP) is provided by the National Science Foundation (Grant Number DMS-1950644). Support for the NSF EPSCoR project, Guam Ecosystems Collaboratorium for Corals and Oceans (GECCO) is provided by the National Science Foundation (Grant Number DMS-1946352).

Special thanks to the UOG Marine Laboratory(Dr. Bastian Bentlage and Ms. Grace McDermott), our faculty mentors (Dr. JaeYong Choi, Dr. HyunJu Oh, & Dr. Leslie Aquino), and our Research Assistants (Jaron Bautista & Regina-Mae Dominguez).

Bibliography I

[1] URL: https://static.euronews.com/articles/stories/05/00/14/58/1440x810_cmsv2_3031d677-63ce-54f2-baa9-1deb93f016c1-5001458.jpg.

[2] URL: https://i0.wp.com/urbanmarineecology.org/wp-content/uploads/2018/03/Diversity-Algae.jpg?resize=300,225.

[3] URL: https://upload.wikimedia.org/wikipedia/commons/6/67/Fucus_serratus_2015-09-08_ag_M0010140.jpg.

[4] URL: https://seaunseen.com/wp-content/uploads/2020/12/MG_9789.jpg.

[5] J. C. Blackwood, A. Hastings, and P. J. Mumby. "The effect of fishing on hysteresis in Caribbean coral reefs". In: *5* 1 (Nov. 2010), pp. 105–114. DOI: 10.1007/s12080-010-0102-0.

[6] James Chen. *Nash Equilibrium*. May 2021. URL: https://www.investopedia.com/terms/n/nash-equilibrium.asp.

[7] Paul L. Delamater et al. *Complexity of the Basic Reproduction Number(R0) - Volume 25, Number 1-January 2019 - Emerging Infectious Diseases journal - CDC*. Jan. 2019. URL: https://wwwnc.cdc.gov/eid/article/25/1/17-1901_article.

[8] *Importance of Guam's Reefs*. URL: http://www.guamreeflife.com/reef-conservation/importance-of-guams-reefs/.

Bibliography II

[9] Xiong Li et al. "Mathematical analysis of coral reef models". In: *Journal of Mathematical Analysis and Applications* 416.1 (2014), pp. 352–373. ISSN: 0022-247X. DOI: https://doi.org/10.1016/j.jmaa.2014.02.053. URL: https://www.sciencedirect.com/science/article/pii/S0022247X14001917.

[10] Marc O. Nadon. 2019, pp. 1–100.

[11] Sarai Quintero et al. "A Mathematical Model of Coral Reef Response to Destructive Fishing Practices with Predator-Prey Interactions". In: (July 2016).

[12] Bernhard M. Riegl and Samuel J. Purkis. "Model of coral population response to accelerated bleaching and mass mortality in a changing climate". In: *Ecological Modelling* 220.2 (Jan. 2009), pp. 192–208. DOI: 10.1016/j.ecolmodel.2008.09.022.

[13] Rachel Ross. *What Are Coral Reefs?* Sept. 2018. URL: https://www.livescience.com/40276-coral-reefs.html.

[14] Eric Wolanski, Robert H. Richmond, and Laurence Mccook. "A model of the effects of land-based, human activities on the health of coral reefs in the Great Barrier Reef and in Fouha Bay, Guam, Micronesia". In: *Journal of Marine Systems* 46.1-4 (2004), pp. 133–144. DOI: 10.1016/j.jmarsys.2003.11.018.

Bibliography III

[15] R N Zikkah, N Anggriani, and A K Supriatna. "Mathematical model of coral reefs with fish harvesting effect". In: *Journal of Physics: Conference Series* 1657 (2020), pp. 012–062. DOI: 10.1088/1742-6596/1657/1/012062.

Thank you!