Effects of Overfishing on Coral Reef Ecosystems

Coral Reefsearchers

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1 Background

- General Background
- General Question
- Definitions

Background

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- According to the 2008 State of the Coral Reef Ecosystems of Guam report, Guam's coral reef resources are both economically and culturally important, providing numerous goods and services for the residents of Guam, including cultural/traditional use, tourism, recreation, fisheries, and shoreline/infrastructure protection [6].

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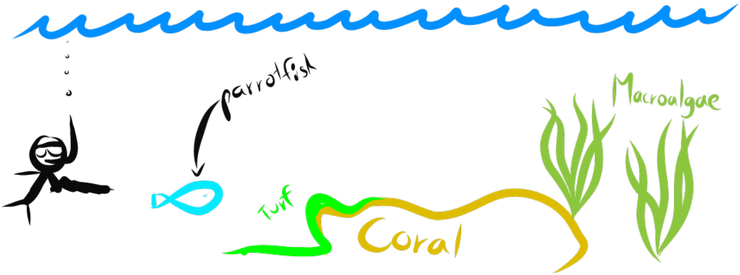
- Factors that affect coral reefs include climate change, coral reef resilience[10], and exploitative fishing practices[9].

Our Question

- General Question: How will Guam's reef ecosystem change over the coming decades?

$$\Downarrow$$

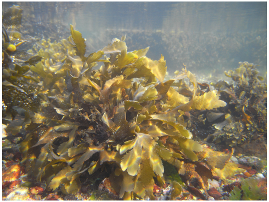
- Specific Question: How will overfishing affect Guam's coral reef ecosystem in the upcoming decades?



Identify Our Terms

(a) Corals[1]

(b) Algal Turfs[2]



(c) Macroalgae[3]

Figure 1: Images of Ecosystem

Identify Our Terms (Cont.)

Figure 2: Parrot Fish[4]

- Parrot fish are common reef fish found in many tropical reefs[5] and are known to feed on algal turfs and macroalgae.
- Their bites on corals have been shown to improve and promote coral growth.
- Parrot fish are one of the most overfished reef fish in the Caribbean, and potentially on Guam as well[5].

2 Mathematical Model
- Assumptions
- Compartments
- Parameters
- Compartment Model
- Differential Equations
- Plots

Assumptions

- Ecosystem:
    - is closed (i.e. no migration).
    - consists of only corals (C), algal turfs (T), and macroalgae (M).
    - supports maximum carrying capacity of parrotfish.
- Macroalgae is the only predator of coral.
- Coral recruit to and overgrow algal turfs[7].
- Corals are overgrown by macroalgae[7].
- Macroalgae colonize dead coral by spreading vegetative over algal turfs[7].
- Corals do not naturally die.

Coral Reef Ecosystem Model Compartments

Compartments

The Ecosystem Model consists of 4 compartments:

- C: Corals
- T: Algal Turfs
- M: Macroalgae
- P: Parrotfish

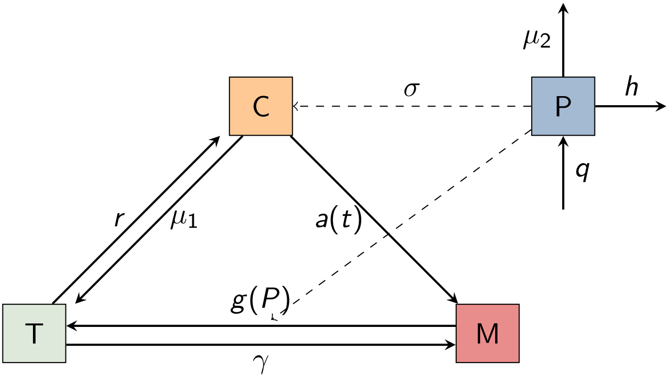
$$\text{where } C + T + M = 1.$$

Coral Reef Ecosystem Model Parameters

Parameter	Description	Rate	Units[8][7][5]
μ_1	natural death rate of coral reefs	0.15[12]	$year^{-1}$
μ_2	natural death rate of parrotfish	0.22[8]	$year^{-1}$
r	rate that coral recruit to overgrow algal turfs	0.5[12]	$year^{-1}$
γ	rate that macroalgae spread vegetative over algal turfs	0.8[13]	$year^{-1}$
q	intrinsic growth rate for parrotfish	0.47[8]	$year^{-1}$
h	harvesting rate for parrotfish	0.14[8]	$year^{-1}$
σ	rate that parrot fish bite coral	0.01*	$bites * year^{-1}$
α	maximum grazing intensity	1[5]	-
β	carrying capacity of parrotfish	1	-
a_0	control variable to simulate seasonal changes	0.99	-

$* =$ *estimated value*

Coral Reef Ecosystem Model[7]



Differential Equations

System of differential equations derived from compartment model:

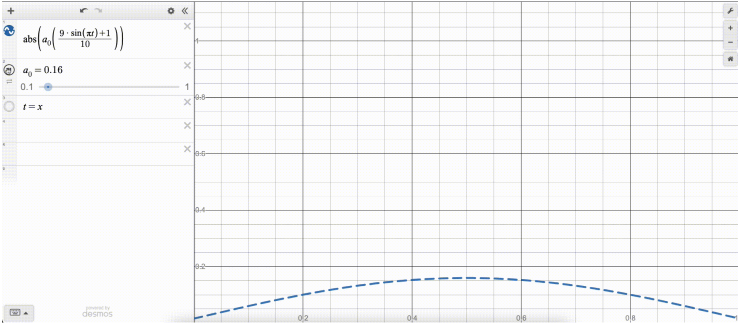
$$
\begin{aligned}
\frac{dC}{dt} &= rTC + \sigma PC - (a(t)M + \mu_1)C \\
\frac{dP}{dt} &= qP\left(1 - \frac{P}{\beta C}\right) - P\left(h + \mu_2\right) \\
\frac{dT}{dt} &= \mu_1 C + \frac{g(P)M}{M+T} - T(rC + \gamma M) \\
\frac{dM}{dt} &= (a(t)C + \gamma T)M - \frac{g(P)M}{M+T}
\end{aligned}
$$

where $g(P) = \frac{\alpha P}{\beta}$, $\quad a(t) = \left|\frac{a_0(9\sin(\pi t)+1)}{10}\right|$.

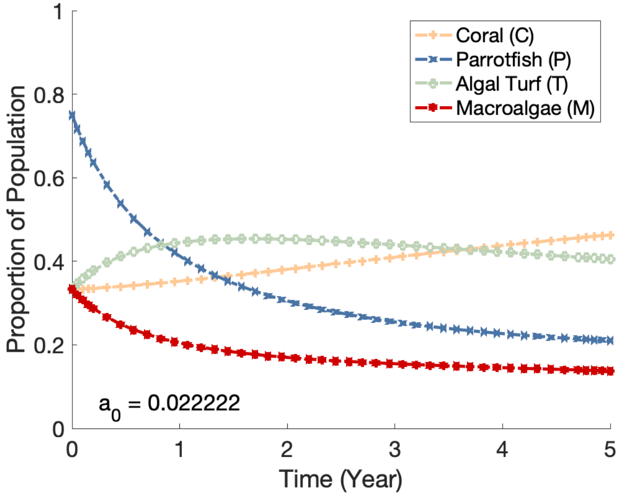
Modified from [5]

Simulating $a(t)$

$$a(t) = \left|\frac{a_0(9\sin(\pi t) + 1)}{10}\right|$$

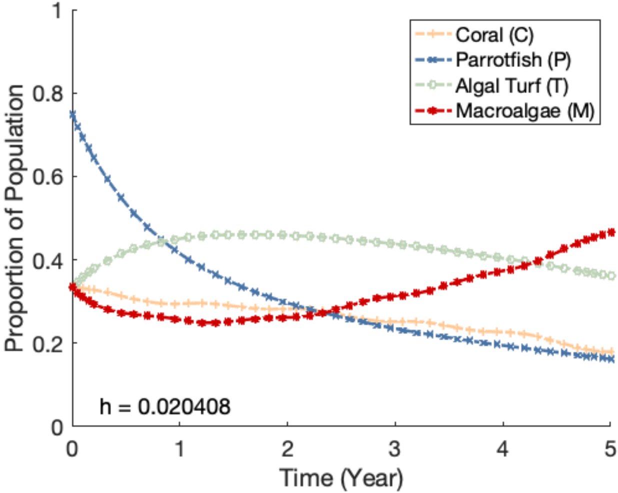


Parameter Changes: a_0



Initial Compartment Conditions: $C = T = M = \frac{1}{3}$ & $P = \frac{3}{4}$.

Parameter Changes: h



Initial Compartment Conditions: $C = T = M = \frac{1}{3}$ & $P = \frac{3}{4}$.

Compartment Initial Condition Changes

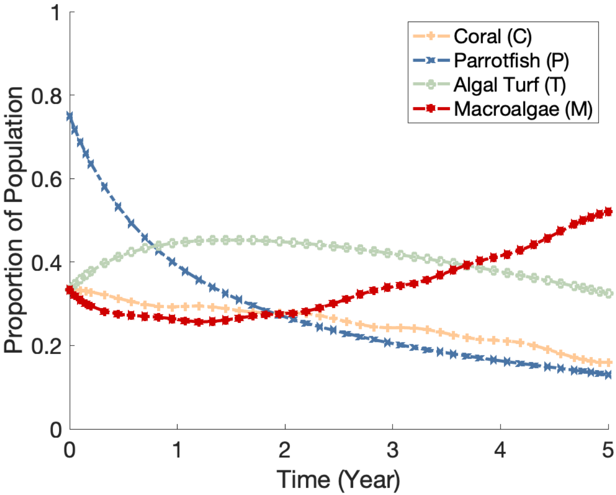


Figure 3: Initial Conditions: $C = T = M = \frac{1}{3}$, and $P = \frac{3}{4}$

Compartment Initial Condition Changes

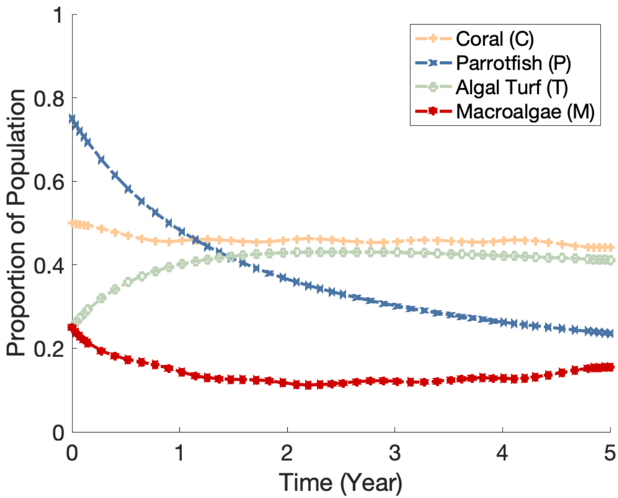


Figure 4: Initial Conditions: $C = \frac{1}{2}$, $T = M = \frac{1}{4}$, and $P = \frac{3}{4}$

Compartment Initial Condition Changes

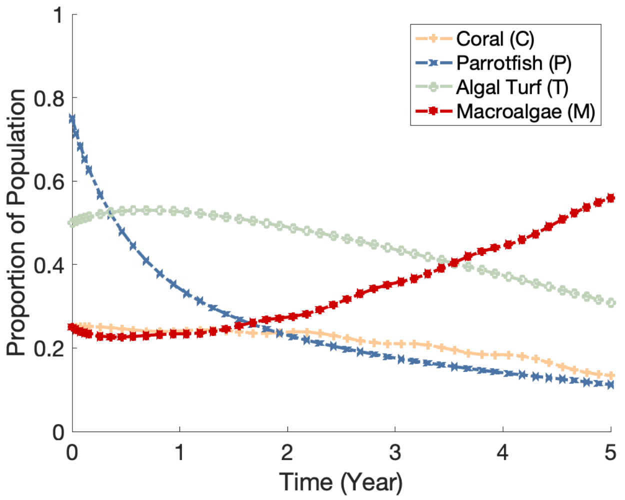


Figure 5: Initial Conditions: $T = \frac{1}{2}$, $C = M = \frac{1}{4}$, and $P = \frac{3}{4}$

Compartment Initial Condition Changes

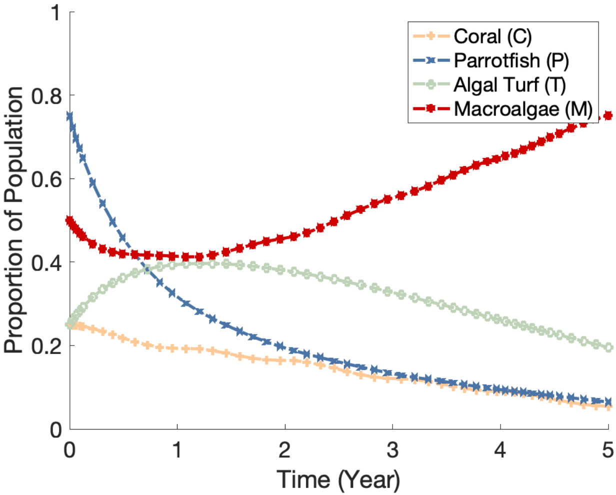


Figure 6: Initial Conditions: $M = \frac{1}{2}$, $C = T = \frac{1}{4}$, and $P = \frac{3}{4}$

3 Equilibria

- Disease Free Equilibrium (DFE)
- Endemic Equilibrium
- Basic Reproduction Number

Disease Free Equilibrium

$$C^0 = 1 - \frac{\mu_1}{r}$$

$$P^0 = -\frac{\beta(1 - \frac{\mu_1}{r})(h - \mu_2 - q)}{q}$$

$$T^0 = \frac{\mu_1}{r}$$

$$M^0 = 0 \Longleftarrow \text{our "disease" compartment}$$

Note

Since C + T + M = 1, if $M^0 = 0$, then $C^0 + T^0 = 1$.

Endemic Equilibrium

$$P^* = \beta C \left(\frac{q - (h + \mu_2)}{q} \right)$$

$$C^* = rTC + \sigma PC - (a(t)M + \mu_1)C$$

$$T^* = T^2(rC + \gamma M) + T(MrC + M^2 + \mu_1 C) - \left(\frac{\alpha P}{\beta} M + \mu_1 CM \right)$$

$$M^* = \frac{\alpha P}{\beta(aC + \gamma T)} - T$$

Basic Reproduction Number: $\mathscr{R}_0$

$$\mathscr{F} = [a(t)CM + \gamma MT] \qquad \mathscr{V} = \left[\frac{g(P)M}{M+T}\right]$$

$$\downarrow \qquad \downarrow$$

$$F = [a(t)C + \gamma T^0] \qquad V = \left[\frac{g(P)T^0}{(M+T)^2}\right]$$

$$\mathscr{R}_0 = \rho(\det(FV^{-1} - \lambda I))$$

$$\downarrow$$

$$\mathscr{R}_0 = \frac{\mu_1(a(t)r - a(t)\mu_1 + \gamma\mu_1)}{r^2 g(P)}$$

4 Future Plans

Continued Work

- Finish endemic equilibrium calculations.
- Perform sensitivity analysis on parameters.
- Application of Education Game Theory on the harvest rate parameter (h) to quantify human behavior and the best strategy to protect coral reef sustainability. These tasks include:
  - Payoff Matrix
  - Dominant and/or mixed equilibrium equations
  - Nash Equilibrium/Evolutionarily Stable Strategy
  - Replicator Equation

Questions?

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Thank you!