

# From Function1#compose to Kleisli

Different Ways of Function Composition

© 2018 Hermann Hueck

<https://github.com/hermannhueck/composing-functions>

# Abstract (1/2)

Fine-grained composability of functions is one of the core advantages of FP.

Treating "Functions as Data" means that we can ...

- store a function in a val
- pass it as args to other (higher order) functions (HOFs)
- return a function from other functions
- process/manipulate a function like data
- organize functions in data structures like List, Option etc.
- wrap a function in a case class

In this talk I demonstrate different ways of function composition (and manipulation - what is often the same).

I only deal with *scala.Function1*, because due to currying we can regard any function (except *Function0*) as a *Function1*. Curried functions are easier to compose.

# Abstract (2/2)

I start with the methods on `Function1`: *compose* and *andThen*. Then I show how to fold a List of functions.

Then I turn to function composition with Monoids.

Functions are Functors (if *Functor* for *Function1* is defined), i.e they can be mapped over.

Functions are Monads (if *Monad* for *Function1* is defined), i.e they can be flatMapped over.

With *map* and *flatMap* we can write for-comprehensions over functions. As Monads, we can use them in any monadic context.

Next I implement my own *mycats.Kleisli* similar to *cats.data.Kleisli* and show its usage with *flatMap*, *flatMapF*, *andThen* and *compose*.

Finally I show the *Reader* Monad, a *Kleisli* simplified with *Id*.

# Agenda

1. Preliminaries
  - Kind Projector
  - Partial Unification
2. Functions as Data
3. Curried Functions
4. *Function1#compose* and *Function1#andThen*
5. Monoidal Function Composition
6. *Function1* as Functor
7. *Function1* as Monad
8. Kleisli Composition done manually
9. *case class Kleisli*
10. Reader Monad
11. Resources

# 1. Preliminaries

1.1 Kind Projector

1.2 Partial Unification

# 1.1 Kind Projector - Compiler Plugin

Enable *kind-projector* in *build.sbt*:

```
addCompilerPlugin("org.spire-math" %% "kind-projector" % "0.9.7")
```

Kind projector on Github:

<https://github.com/non/kind-projector>

See: *demo.Demo01aKindProjector*

# The Problem:

How to define a Functor or a Monad for an ADT with two type parameters?

# The Problem:

How to define a Functor or a Monad for an ADT with two type parameters?

A Functor is defined like this:

```
trait Functor[F[_]] {  
  def map[A, B](fa: F[A])(f: A => B): F[B]  
}
```

It is parameterized with a generic type constructor `F[_]` which "has one hole", i.e. `F[_]` needs another type parameter when reified.



# The Problem:

How to define a Functor or a Monad for an ADT with two type parameters?

A Functor is defined like this:

```
trait Functor[F[_]] {  
  def map[A, B](fa: F[A])(f: A => B): F[B]  
}
```

It is parameterized with a generic type constructor `F[_]` which "has one hole", i.e. `F[_]` needs another type parameter when reified.

Hence it is easy to define a Functor instance for *List*, *Option* or *Future*, ADTs which expect exactly one type parameter that "fills that hole".

# The Problem:

How to define a Functor or a Monad for an ADT with two type parameters?

A Functor is defined like this:

```
trait Functor[F[_]] {  
  def map[A, B](fa: F[A])(f: A => B): F[B]  
}
```

It is parameterized with a generic type constructor `F[_]` which "has one hole", i.e. `F[_]` needs another type parameter when reified.

Hence it is easy to define a Functor instance for *List*, *Option* or *Future*, ADTs which expect exactly one type parameter that "fills that hole".

```
implicit val listFunctor: Functor[List] = new Functor[List] {  
  override def map[A, B](fa: List[A])(f: A => B): List[B] = fa map f  
}  
  
implicit val optionFunctor: Functor[Option] = new Functor[Option] {  
  override def map[A, B](fa: Option[A])(f: A => B): Option[B] = fa map f  
}
```

# The Problem:

How to define a Functor or a Monad for an ADT with two type parameters?

A Functor is defined like this:

```
trait Functor[F[_]] {  
  def map[A, B](fa: F[A])(f: A => B): F[B]  
}
```

It is parameterized with a generic type constructor `F[_]` which "has one hole", i.e. `F[_]` needs another type parameter when reified.

Hence it is easy to define a Functor instance for *List*, *Option* or *Future*, ADTs which expect exactly one type parameter that "fills that hole".

```
implicit val listFunctor: Functor[List] = new Functor[List] {  
  override def map[A, B](fa: List[A])(f: A => B): List[B] = fa map f  
}  
  
implicit val optionFunctor: Functor[Option] = new Functor[Option] {  
  override def map[A, B](fa: Option[A])(f: A => B): Option[B] = fa map f  
}
```

What if we want to define a Functor for *Either*?

## What if we want to define a Functor for *Either*?

*Either* has kind:  $* \rightarrow * \rightarrow *$ . It has two holes and hence cannot be used easily to define a Functor or a Monad instance.

## What if we want to define a Functor for *Either*?

*Either* has kind:  $* \rightarrow * \rightarrow *$ . It has two holes and hence cannot be used easily to define a Functor or a Monad instance.

But we can fix one of the type parameters and leave the other one open.

```
// Code compiles without kind-projector.  
// It uses a type alias within a structural type.  
implicit def eitherFunctor[L]: Functor[({type f[x] = Either[L, x]})#f] =  
  new Functor[({type f[x] = Either[L, x]})#f] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```

## What if we want to define a Functor for *Either*?

*Either* has kind: `* --> * --> *`. It has two holes and hence cannot be used easily to define a Functor or a Monad instance.

But we can fix one of the type parameters and leave the other one open.

```
// Code compiles without kind-projector.  
// It uses a type alias within a structural type.  
implicit def eitherFunctor[L]: Functor[({type f[x] = Either[L, x]})#f] =  
  new Functor[({type f[x] = Either[L, x]})#f] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```

This is a type lambda (analogous to a partially applied function on the value level).

The type alias must be defined inside *def eitherFunctor[L]* because the type parameter *L* is used in the type alias. This is done inside a structural type where *f* is returned through a type projection.

```
Functor[({type f[x] = Either[L, x]})#f]
```

This code is ugly but can be improved if *kind-projector* is enabled.

Without *kind-projector*:

```
implicit def eitherFunctor[L]: Functor[({type f[x] = Either[L, x]})#f] =  
  new Functor[({type f[x] = Either[L, x]})#f] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```



Without *kind-projector*:

```
implicit def eitherFunctor[L]: Functor[({type f[x] = Either[L, x]})#f] =  
  new Functor[({type f[x] = Either[L, x]})#f] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```

With *kind-projector*:

```
implicit def eitherFunctor[L]: Functor[Lambda[x => Either[L, x]]] =  
  new Functor[Lambda[x => Either[L, x]]] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```

Without *kind-projector*:

```
implicit def eitherFunctor[L]: Functor[({type f[x] = Either[L, x]})#f] =  
  new Functor[({type f[x] = Either[L, x]})#f] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```

With *kind-projector*:

```
implicit def eitherFunctor[L]: Functor[Lambda[x => Either[L, x]]] =  
  new Functor[Lambda[x => Either[L, x]]] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```

```
implicit def eitherFunctor[L]: Functor[λ[x => Either[L, x]]] =  
  new Functor[λ[x => Either[L, x]]] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```

## Without *kind-projector*:

```
implicit def eitherFunctor[L]: Functor[({type f[x] = Either[L, x]})#f] =  
  new Functor[({type f[x] = Either[L, x]})#f] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```

## With *kind-projector*:

```
implicit def eitherFunctor[L]: Functor[Lambda[x => Either[L, x]]] =  
  new Functor[Lambda[x => Either[L, x]]] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```

```
implicit def eitherFunctor[L]: Functor[λ[x => Either[L, x]]] =  
  new Functor[λ[x => Either[L, x]]] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```

```
implicit def eitherFunctor[L]: Functor[Either[L, ?]] =  
  new Functor[Either[L, ?]] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] = fa map f  
  }
```

## 1.2 Compiler Flag *-Ypartial-unification*

Enable partial unification in *build.sbt*:

```
scalacOptions += "-Ypartial-unification"
```

See: *demo.Demo01bPartialUnification*

# The Problem:

```
def foo[F[_], A](fa: F[A]): String = fa.toString
```

```
foo { x: Int => x * 2 }
```

```
-----
```

```
[error] no type parameters for method foo: (fa: F[A])String exist  
so that it can be applied to arguments (Int => Int)
```

```
[error] --- because ---
```

```
[error] argument expression's type is not compatible with formal parameter type;
```

```
[error] found    : Int => Int
```

```
[error] required: ?F[?A]
```

```
[error]   foo { x: Int => x * 2 }
```

```
[error]     ^
```

```
[error] type mismatch;
```

```
[error] found    : Int => Int
```

```
[error] required: F[A]
```

```
[error]   foo((x: Int) => x * 2)
```

# The Problem:

```
def foo[F[_], A](fa: F[A]): String = fa.toString

foo { x: Int => x * 2 }

-----
[error] no type parameters for method foo: (fa: F[A])String exist
[error] so that it can be applied to arguments (Int => Int)
[error] --- because ---
[error] argument expression's type is not compatible with formal parameter type;
[error] found    : Int => Int
[error] required: ?F[?A]
[error]   foo { x: Int => x * 2 }
[error]     ^
[error] type mismatch;
[error] found    : Int => Int
[error] required: F[A]
[error]   foo((x: Int) => x * 2)
```

This code doesn't compile without *-Ypartial-unification*. **Why?**

# The Problem:

```
def foo[F[_], A](fa: F[A]): String = fa.toString

foo { x: Int => x * 2 }

-----
[error] no type parameters for method foo: (fa: F[A])String exist
[error] so that it can be applied to arguments (Int => Int)
[error] --- because ---
[error] argument expression's type is not compatible with formal parameter type;
[error] found    : Int => Int
[error] required: ?F[?A]
[error]   foo { x: Int => x * 2 }
[error]     ^
[error] type mismatch;
[error] found    : Int => Int
[error] required: F[A]
[error]   foo((x: Int) => x * 2)
```

This code doesn't compile without *-Ypartial-unification*. **Why?**

*def foo* requires a type constructor *F[\_]* with "one hole". Its kind is:  $* \multimap *$ .

*foo* is invoked with a function *Int => Int*. *Int => Int* is syntactic sugar for *Function1[Int, Int]*. *Function1* like *Either* has two holes. Its kind is:  $* \multimap * \multimap *$ .

# The Solution:

-*Ypartial-unification* solves the problem by partially fixing (unifying) the type parameters from left to right until it fits to the number of holes required by the definition of *foo*.



# The Solution:

-*Ypartial-unification* solves the problem by partially fixing (unifying) the type parameters from left to right until it fits to the number of holes required by the definition of *foo*.

Imagine this flag turns *Function1*[*A*, *B*] into *Function1Int*[*B*]. With this fix on the fly *Function1Int* has kind *\* --> \** which is the kind required by *F*[\_]. What the compiler transforms the invocation to would look something like this:

```
def foo[F[_], A](fa: F[A]): String = fa.toString  
foo[Function1Int, Int] { x: Int => x * 2 }
```

# The Solution:

-*Ypartial-unification* solves the problem by partially fixing (unifying) the type parameters from left to right until it fits to the number of holes required by the definition of *foo*.

Imagine this flag turns *Function1*[*A*, *B*] into *Function1Int*[*B*]. With this fix on the fly *Function1Int* has kind *\* --> \** which is the kind required by *F[\_]*. What the compiler transforms the invocation to would look something like this:

```
def foo[F[_], A](fa: F[A]): String = fa.toString  
foo[Function1Int, Int] { x: Int => x * 2 }
```

Note that the partial unification fixes the types always in a left-to-right order which is a good fit for most cases where we have right-biased types like *Either*, *Tuple2* or *Function1*. (It is not a good fit for the very rare cases when you use a left-biased type like Scalactic's *Or* data type (a left-biased *Either*).)

When to use *kind-projector* and *-Ypartial-unification*?

# When to use *kind-projector* and *-Ypartial-unification*?

Enable plugin and the flag when using a Functor, Applicative or a Monad instance for higher-kinded types which take more than one type parameter. *Either*, *Tuple2* and *Function1* are the best known representatives of this kind of types.

When programming on the type level regard both as your friends and keep them enabled.

# When to use *kind-projector* and *-Ypartial-unification*?

Enable plugin and the flag when using a Functor, Applicative or a Monad instance for higher-kinded types which take more than one type parameter. *Either*, *Tuple2* and *Function1* are the best known representatives of this kind of types.

When programming on the type level regard both as your friends and keep them enabled.

Very good explanations of partial unification by Miles Sabin and Daniel Spiewak can be found at these links:

<https://github.com/scala/scala/pull/5102>

<https://gist.github.com/djspiewak/7a81a395c461fd3a09a6941d4cd040f2>

## 2. Functions as Data

# Treating Functions as Data (1/2)

allows us to ...

# Treating Functions as Data (1/2)

allows us to ...

- store a function in a val

```
val str2Int: String => Int = str => str.toInt
```



# Treating Functions as Data (1/2)

allows us to ...

- store a function in a val

```
val str2Int: String => Int = str => str.toInt
```

- pass it as arg to other (higher order) functions (HOFs)

```
val mapped = List("1", "2", "3").map(str2Int)
```

# Treating Functions as Data (1/2)

allows us to ...

- store a function in a val

```
val str2Int: String => Int = str => str.toInt
```

- pass it as arg to other (higher order) functions (HOFs)

```
val mapped = List("1", "2", "3").map(str2Int)
```

- return a function from other functions

```
val plus100: Int => Int = { i =>  
  val j = manipulate(i)  
  j + 100  
}
```

# Treating Functions as Data (1/2)

allows us to ...

- store a function in a val

```
val str2Int: String => Int = str => str.toInt
```

- pass it as arg to other (higher order) functions (HOFs)

```
val mapped = List("1", "2", "3").map(str2Int)
```

- return a function from other functions

```
val plus100: Int => Int = { i =>
  val j = manipulate(i)
  j + 100
}
```

- process/manipulate a function like data

```
str2Int map plus100    // Functor instance for Function1 must be defined
```

# Treating Functions as Data (2/2)

allows us to ...

# Treating Functions as Data (2/2)

allows us to ...

- organize functions (like data) in data structures like List, Option etc.

```
val functions: List[Int => Int] = List(_ + 1, _ + 2, _ + 3)
val f[Int => Int] = functions.foldRight(...)(...)
```

# Treating Functions as Data (2/2)

allows us to ...

- organize functions (like data) in data structures like List, Option etc.

```
val functions: List[Int => Int] = List(_ + 1, _ + 2, _ + 3)
val f[Int => Int] = functions.foldRight(...)(...)
```

- wrap a function in a case class *MyWrapper*

We can define methods on *MyWrapper* which manipulate the wrapped function and return the manipulation result again wrapped in a new instance of case class *MyWrapper*.

```
case class MyWrapper[A, B](run: A => B) {
  def map[C](f: B => C): MyWrapper[A, C] = ???
  def flatMap[C](f: B => MyWrapper[A, C]): MyWrapper[A, C] = ???
  // other methods ...
}
```

# 3. Curried functions

See: *demo.Demo03CurriedFunctions*

Every function is a *Function1* ...



# Every function is a *Function1* ...

```
val sum3Ints : (Int, Int, Int) => Int      = _ + _ + _  
val sum3Ints2: Function3[Int, Int, Int] = _ + _ + _
```

# Every function is a *Function1* ...

```
val sum3Ints : (Int, Int, Int) => Int      = _ + _ + _  
val sum3Ints2: Function3[Int, Int, Int] = _ + _ + _
```

... if you curry it.

# Every function is a *Function1* ...

```
val sum3Ints : (Int, Int, Int) => Int      = _ + _ + _  
val sum3Ints2: Function3[Int, Int, Int] = _ + _ + _
```

... if you curry it.

```
val sumCurried: Int => Int => Int => Int = sum3Ints.curried  
// sumCurried: Int => (Int => (Int => Int)) = scala.Function3$$Lambda$4346/9914305
```

# Every function is a *Function1* ...

```
val sum3Ints : (Int, Int, Int) => Int      = _ + _ + _  
val sum3Ints2: Function3[Int, Int, Int, Int] = _ + _ + _
```

... if you curry it.

```
val sumCurried: Int => Int => Int => Int = sum3Ints.curried  
// sumCurried: Int => (Int => (Int => Int)) = scala.Function3$$Lambda$4346/9914305
```

The type arrow (=>) is right associative. Hence we can omit the parentheses.

# Every function is a *Function1* ...

```
val sum3Ints : (Int, Int, Int) => Int      = _ + _ + _  
val sum3Ints2: Function3[Int, Int, Int, Int] = _ + _ + _
```

... if you curry it.

```
val sumCurried: Int => Int => Int => Int = sum3Ints.curried  
// sumCurried: Int => (Int => (Int => Int)) = scala.Function3$$Lambda$4346/9914305
```

The type arrow ( `=>` ) is right associative. Hence we can omit the parentheses.

The type arrow ( `=>` ) is syntactic sugar for *Function1*.

$A \Rightarrow B$  is equivalent to *Function1*[*A*, *B*].

# Every function is a *Function1* ...

```
val sum3Ints : (Int, Int, Int) => Int      = _ + _ + _  
val sum3Ints2: Function3[Int, Int, Int, Int] = _ + _ + _
```

... if you curry it.

```
val sumCurried: Int => Int => Int => Int = sum3Ints.curried  
// sumCurried: Int => (Int => (Int => Int)) = scala.Function3$$Lambda$4346/9914305
```

The type arrow (=>) is right associative. Hence we can omit the parentheses.

The type arrow (=>) is syntactic sugar for *Function1*.

$A \Rightarrow B$  is equivalent to *Function1*[*A*, *B*].

```
val sumCurried2: Function1[Int, Function1[Int, Function1[Int, Int]]] = sumCurried  
// sumCurried2: Int => (Int => (Int => Int)) = scala.Function3$$Lambda$4346/991430
```

# Every function is a *Function1* ...

```
val sum3Ints : (Int, Int, Int) => Int      = _ + _ + _  
val sum3Ints2: Function3[Int, Int, Int, Int] = _ + _ + _
```

... if you curry it.

```
val sumCurried: Int => Int => Int => Int = sum3Ints.curried  
// sumCurried: Int => (Int => (Int => Int)) = scala.Function3$$Lambda$4346/9914305
```

The type arrow (=>) is right associative. Hence we can omit the parentheses.

The type arrow (=>) is syntactic sugar for *Function1*.

$A \Rightarrow B$  is equivalent to *Function1*[*A*, *B*].

```
val sumCurried2: Function1[Int, Function1[Int, Function1[Int, Int]]] = sumCurried  
// sumCurried2: Int => (Int => (Int => Int)) = scala.Function3$$Lambda$4346/991430
```

If uncurried again, we get the original function back.

```
val sumUncurried: (Int, Int, Int) => Int = Function.uncurried(sumCurried)  
// sumUncurried: (Int, Int, Int) => Int = scala.Function$$$Lambda$6605/301079867@1
```

# Partial application of curried functions



# Partial application of curried functions

```
val sum3Ints: (Int, Int, Int) => Int = _ + _ + _  
val sumCurried: Int => Int => Int => Int = sum3Ints.curried
```

# Partial application of curried functions

```
val sum3Ints: (Int, Int, Int) => Int = _ + _ + _  
val sumCurried: Int => Int => Int => Int = sum3Ints.curried
```

```
val applied1st: Int => Int => Int = sumCurried(1)  
// applied1st: Int => (Int => Int) = scala.Function3$$Lambda$4348/1531035406@5a231
```

# Partial application of curried functions

```
val sum3Ints: (Int, Int, Int) => Int = _ + _ + _  
val sumCurried: Int => Int => Int => Int = sum3Ints.curried
```

```
val applied1st: Int => Int => Int = sumCurried(1)  
// applied1st: Int => (Int => Int) = scala.Function3$$Lambda$4348/1531035406@5a231
```

```
val applied2nd: Int => Int = applied1st(2)  
// applied2nd: Int => Int = scala.Function3$$Lambda$4349/402963549@117e96fb
```

# Partial application of curried functions

```
val sum3Ints: (Int, Int, Int) => Int = _ + _ + _  
val sumCurried: Int => Int => Int => Int = sum3Ints.curried
```

```
val applied1st: Int => Int => Int = sumCurried(1)  
// applied1st: Int => (Int => Int) = scala.Function3$$Lambda$4348/1531035406@5a231
```

```
val applied2nd: Int => Int = applied1st(2)  
// applied2nd: Int => Int = scala.Function3$$Lambda$4349/402963549@117e96fb
```

```
val applied3rd: Int = applied2nd(3)  
// applied3rd: Int = 6
```

# Partial application of curried functions

```
val sum3Ints: (Int, Int, Int) => Int = _ + _ + _  
val sumCurried: Int => Int => Int => Int = sum3Ints.curried
```

```
val applied1st: Int => Int => Int = sumCurried(1)  
// applied1st: Int => (Int => Int) = scala.Function3$$Lambda$4348/1531035406@5a231
```

```
val applied2nd: Int => Int = applied1st(2)  
// applied2nd: Int => Int = scala.Function3$$Lambda$4349/402963549@117e96fb
```

```
val applied3rd: Int = applied2nd(3)  
// applied3rd: Int = 6
```

```
val appliedAllAtOnce: Int = sumCurried(1)(2)(3)  
// appliedAllAtOnce: Int = 6
```

# Advantages of curried functions

# Advantages of curried functions

1. Curried functions can be partially applied.  
They are better composable than their uncurried counterparts.

# Advantages of curried functions

1. Curried functions can be partially applied.  
They are better composable than their uncurried counterparts.
2. Curried functions (and methods) help the compiler with type inference.  
The compiler infers types by argument lists from left to right.



# Advantages of curried functions

1. Curried functions can be partially applied.  
They are better composable than their uncurried counterparts.
2. Curried functions (and methods) help the compiler with type inference.  
The compiler infers types by argument lists from left to right.

```
def filter1[A](la: List[A], p: A => Boolean) = ??? // uncurried

scala> filter1(List(0,1,2), _ < 2)
<console>:50: error: missing parameter type for expanded function ((x$1: <error>)
    filter1(List(0,1,2), _ < 2)
                      ^
```

# Advantages of curried functions

1. Curried functions can be partially applied.  
They are better composable than their uncurried counterparts.
2. Curried functions (and methods) help the compiler with type inference.  
The compiler infers types by argument lists from left to right.

```
def filter1[A](la: List[A], p: A => Boolean) = ??? // uncurried
```

```
scala> filter1(List(0,1,2), _ < 2)
<console>:50: error: missing parameter type for expanded function ((x$1: <error>)
    filter1(List(0,1,2), _ < 2)
                      ^
```

```
def filter2[A](la: List[A])(p: A => Boolean) = ??? // curried
```

```
scala> filter2(List(0,1,2))(_ < 2)
res5: List[Int] = List(0,1)
```

## 4. *Function1#compose* and Function1#apply

See: *demo.Demo04ComposingFunctions*

# Trait *Function1*

```
trait Function1[-T1, +R] {  
  def apply(a: T1): R  
  def compose[A](g: A => T1): A => R = { x => apply(g(x)) }  
  def andThen[A](g: R => A): T1 => A = { x => g(apply(x)) }  
  override def toString = "<function1>"  
}
```

# Trait *Function1*

```
trait Function1[-T1, +R] {  
  def apply(a: T1): R  
  def compose[A](g: A => T1): A => R = { x => apply(g(x)) }  
  def andThen[A](g: R => A): T1 => A = { x => g(apply(x)) }  
  override def toString = "<function1>"  
}
```

```
scala> val f: Int => Int = _ + 10  
f: Int => Int = $$Lambda$4204/320646851@733dc5b7  
  
scala> val g: Int => Int = _ * 2  
g: Int => Int = $$Lambda$4205/1093482910@3a179009  
  
scala> (f compose g apply 1) == f(g(1))  
res6: Boolean = true  
  
scala> (f andThen g apply 1) == g(f(1))  
res7: Boolean = true
```

# A pipeline of functions

# A pipeline of functions

```
val s2i: String => Int = _.toInt  
val plus2: Int => Int = _ + 2  
val div10By: Int => Double = 10.0 / _  
val d2s: Double => String = _.toString
```

# A pipeline of functions

```
val s2i: String => Int = _.toInt
val plus2: Int => Int = _ + 2
val div10By: Int => Double = 10.0 / _
val d2s: Double => String = _.toString
```

```
// Function1#compose
val fComposed1: String => String = d2s compose div10By compose plus2 compose s2i
val res1 = fComposed1("3") // 2.0 !!!
```



# A pipeline of functions

```
val s2i: String => Int = _.toInt
val plus2: Int => Int = _ + 2
val div10By: Int => Double = 10.0 / _
val d2s: Double => String = _.toString
```

```
// Function1#compose
val fComposed1: String => String = d2s compose div10By compose plus2 compose s2i
val res1 = fComposed1("3") // 2.0 !!!
```

```
// Function1#andThen
val fComposed2: String => String = s2i andThen plus2 andThen div10By andThen d2s
val res2 = fComposed2("3") // 2.0 !!!
```

# Folding a List of Functions

# Folding a List of Functions

```
val lf: List[Int => Int] = List(_*2, _+10, _+100)
```

# Folding a List of Functions

```
val lf: List[Int => Int] = List(_*2, _+10, _+100)
```

```
val lfFoldedRight = lf.foldRight(identity[Int] _) {(f, acc) => acc andThen f}  
val resLfFoldedRight = lfFoldedRight(1)  
// 222
```

# Folding a List of Functions

```
val lf: List[Int => Int] = List(_*2, _+10, _+100)
```

```
val lfFoldedRight = lf.foldRight(identity[Int] _) {(f, acc) => acc andThen f}  
val resLfFoldedRight = lfFoldedRight(1)  
// 222
```

```
val lfFoldedLeft = lf.foldLeft(identity[Int] _) {(acc, f) => acc andThen f}  
val resLfFoldedLeft = lfFoldedLeft(1)  
// 112
```

# Folding a List of Functions

```
val lf: List[Int => Int] = List(_*2, _+10, _+100)
```

```
val lfFoldedRight = lf.foldRight(identity[Int] _) {(f, acc) => acc andThen f}  
val resLfFoldedRight = lfFoldedRight(1)  
// 222
```

```
val lfFoldedLeft = lf.foldLeft(identity[Int] _) {(acc, f) => acc andThen f}  
val resLfFoldedLeft = lfFoldedLeft(1)  
// 112
```

The *identity* function is the no-op of function composition.

# Folding a List of Functions

```
val lf: List[Int => Int] = List(_*2, _+10, _+100)
```

```
val lfFoldedRight = lf.foldRight(identity[Int] _) {(f, acc) => acc andThen f}  
val resLfFoldedRight = lfFoldedRight(1)  
// 222
```

```
val lfFoldedLeft = lf.foldLeft(identity[Int] _) {(acc, f) => acc andThen f}  
val resLfFoldedLeft = lfFoldedLeft(1)  
// 112
```

The *identity* function is the no-op of function composition.

Folding functions is not a commutative operation!  
Hence *foldRight* and *foldLeft* return different results.

# 5. Monoidal Function Composition

See: *demo.Demo05ComposingWithMonoid*



# Trait Monoid

```
trait Monoid[A] {  
  def empty: A  
  def combine(x: A, y: A): A  
  
  def combineAll(as: List[A]): A = // combines all functions in a List of funct  
    as.foldLeft(empty)(combine)  
}
```

# Trait Monoid

```
trait Monoid[A] {  
  def empty: A  
  def combine(x: A, y: A): A  
  
  def combineAll(as: List[A]): A = // combines all functions in a List of funct  
    as.foldLeft(empty)(combine)  
}
```

When composing functions with Monoid the functions must have the same input and output type. You cannot compose functions of type  $A \Rightarrow B$ , only functions of type  $A \Rightarrow A$ .

# Default Monoid Instance for *Function1*

```
// This one is the default Function1-Monoid in Cats
// It requires A to be a Monoid too.
//
implicit def function1Monoid[A: Monoid]: Monoid[A => A] = new Monoid[A => A] {
  override def empty: A => A =
    _ => Monoid[A].empty
  override def combine(f: A => A, g: A => A): A => A =
    a => Monoid[A].combine(f(a), g(a))
}
```

# Default Monoid Instance for *Function1*

```
// This one is the default Function1-Monoid in Cats
// It requires A to be a Monoid too.
//
implicit def function1Monoid[A: Monoid]: Monoid[A => A] = new Monoid[A => A] {
  override def empty: A => A =
    _ => Monoid[A].empty
  override def combine(f: A => A, g: A => A): A => A =
    a => Monoid[A].combine(f(a), g(a))
}
```

- This instance defines a Monoid for Functions of type  $A \Rightarrow A$ .
- It requires type  $A$  to be a Monoid too.
- *empty* defines a function which ignores its input and returns *Monoid[A].empty*.
- *combine* takes two functions  $f$  and  $g$ , invokes  $f(a)$  and  $g(a)$  on its input and returns the combined result.

# Composition with Monoid

```
val f: Int => Int = _ + 1  
val g: Int => Int = _ * 2  
val h: Int => Int = _ + 100
```

# Composition with Monoid

```
val f: Int => Int = _ + 1
val g: Int => Int = _ * 2
val h: Int => Int = _ + 100
```

```
import Monoid.function1Monoid

(f combine g)(4)           // 13
(f |+| g)(4)              // 13

(f |+| g |+| h)(4)        // 117
Monoid[Int => Int].combineAll(List(f, g, h))(4) // 117
```

# Composition with Monoid

```
val f: Int => Int = _ + 1
val g: Int => Int = _ * 2
val h: Int => Int = _ + 100
```

```
import Monoid.function1Monoid

(f combine g)(4)           // 13
(f |+| g)(4)              // 13

(f |+| g |+| h)(4)        // 117
Monoid[Int => Int].combineAll(List(f, g, h))(4) // 117
```

Another Monoid instance yields a different result.

# Instance for *function1ComposeMonoid*

```
implicit def function1ComposeMonoid[A]: Monoid[A => A] = new Monoid[A => A] {  
  override def empty: A => A = identity  
  override def combine(f: A => A, g: A => A): A => A = f compose g  
}
```



# Instance for *function1ComposeMonoid*

```
implicit def function1ComposeMonoid[A]: Monoid[A => A] = new Monoid[A => A] {  
  override def empty: A => A = identity  
  override def combine(f: A => A, g: A => A): A => A = f compose g  
}
```

- This instance composes the Functions with *Function1#compose*.

# Instance for *function1ComposeMonoid*

```
implicit def function1ComposeMonoid[A]: Monoid[A => A] = new Monoid[A => A] {  
  override def empty: A => A = identity  
  override def combine(f: A => A, g: A => A): A => A = f compose g  
}
```

- This instance composes the Functions with *Function1#compose*.

```
import Monoid.function1ComposeMonoid  
  
(f combine g)(4)           // 9  
(f |+| g)(4)              // 9  
  
(f |+| g |+| h)(4)        // 209  
Monoid[Int => Int].combineAll(List(f, g, h))(4) // 209
```

# Instance for *function1AndThenMonoid*

```
implicit def function1AndThenMonoid[A]: Monoid[A => A] = new Monoid[A => A] {  
  override def empty: A => A = identity  
  override def combine(f: A => A, g: A => A): A => A = f andThen g  
}
```

# Instance for *function1AndThenMonoid*

```
implicit def function1AndThenMonoid[A]: Monoid[A => A] = new Monoid[A => A] {  
  override def empty: A => A = identity  
  override def combine(f: A => A, g: A => A): A => A = f andThen g  
}
```

- This instance composes the Functions with *Function1#andThen*.

# Instance for *function1AndThenMonoid*

```
implicit def function1AndThenMonoid[A]: Monoid[A => A] = new Monoid[A => A] {  
  override def empty: A => A = identity  
  override def combine(f: A => A, g: A => A): A => A = f andThen g  
}
```

- This instance composes the Functions with *Function1#andThen*.

```
import Monoid.function1AndThenMonoid  
  
(f combine g)(4)           // 10  
(f |+| g)(4)              // 10  
  
(f |+| g |+| h)(4)        // 110  
Monoid[Int => Int].combineAll(List(f, g, h))(4) // 110
```

## 6. *Function1* as Functor

See: *demo.Demo06Functor*

# Functor

A Functor is any Context  $F[_]$  that provides a function *map* ... and abides by the Functor laws (which are not presented here).

```
trait Functor[F[_]] {  
  def map[A, B](fa: F[A])(f: A => B): F[B]  
}
```

# Functor instance

A Functor instance for *Functions* (found automatically by the compiler if defined in implicit scope, i.e inside the *Functor* companion object):

```
implicit def function1Functor[P]: Functor[Function1[P, ?]] =  
  new Functor[Function1[P, ?]] {  
    override def map[A, B](f: Function1[P, A])(g: A => B): Function1[P, B] =  
      f andThen g  
  }
```



# Functor instance

A Functor instance for *Functions* (found automatically by the compiler if defined in implicit scope, i.e inside the *Functor* companion object):

```
implicit def function1Functor[P]: Functor[Function1[P, ?]] =  
  new Functor[Function1[P, ?]] {  
    override def map[A, B](f: Function1[P, A])(g: A => B): Function1[P, B] =  
      f andThen g  
  }
```

A Functor instance for *Either* (just for comparison):

```
implicit def eitherFunctor[L]: Functor[Either[L, ?]] =  
  new Functor[Either[L, ?]] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] =  
      fa map f  
  }
```

# Functor instance

A Functor instance for *Functions* (found automatically by the compiler if defined in implicit scope, i.e inside the *Functor* companion object):

```
implicit def function1Functor[P]: Functor[Function1[P, ?]] =  
  new Functor[Function1[P, ?]] {  
    override def map[A, B](f: Function1[P, A])(g: A => B): Function1[P, B] =  
      f andThen g  
  }
```

A Functor instance for *Either* (just for comparison):

```
implicit def eitherFunctor[L]: Functor[Either[L, ?]] =  
  new Functor[Either[L, ?]] {  
    override def map[A, B](fa: Either[L, A])(f: A => B): Either[L, B] =  
      fa map f  
  }
```

Using the *Function1* Functor:

```
val f: Int => Int = _ + 3  
val g: Int => Int = _ * 2  
val h = Functor[Function1[Int, ?]].map(f)(g)  
val res = h(2) // 10
```

# Functor syntax

defined as implicit conversion ...

# Functor syntax

defined as implicit conversion ...

in a specific way for *Functor[Function1]*:

```
implicit class FunctorSyntaxFunction1[P, A](fa: Function1[P, A]) {  
  def map[B](f: A => B): Function1[P, B] = Functor[Function1[P, ?]].map(fa)(f)  
}
```

# Functor syntax

defined as implicit conversion ...

in a specific way for *Functor[Function1]*:

```
implicit class FunctorSyntaxFunction1[P, A](fa: Function1[P, A]) {  
  def map[B](f: A => B): Function1[P, B] = Functor[Function1[P, ?]].map(fa)(f)  
}
```

or in a generic way for any *Functor[F[\_]]*:

```
implicit class FunctorSyntax[F[_]: Functor, A](fa: F[A]) {  
  def map[B](f: A => B): F[B] = Functor[F].map(fa)(f)  
}
```

# Functor syntax

defined as implicit conversion ...

in a specific way for *Functor[Function1]*:

```
implicit class FunctorSyntaxFunction1[P, A](fa: Function1[P, A]) {  
  def map[B](f: A => B): Function1[P, B] = Functor[Function1[P, ?]].map(fa)(f)  
}
```

or in a generic way for any *Functor[F[\_]]*:

```
implicit class FunctorSyntax[F[_]: Functor, A](fa: F[A]) {  
  def map[B](f: A => B): F[B] = Functor[F].map(fa)(f)  
}
```

This allows for convenient invocation of *map*  
as if *map* were a method of *Function1*:

```
val f: Int => Int = _ + 3  
val g: Int => Int = _ * 2  
val h = f map g  
val res = h(2) // 10
```

# A pipeline of functions

# A pipeline of functions

```
val s2i: String => Int = _.toInt  
val plus2: Int => Int = _ + 2  
val div10By: Int => Double = 10.0 / _  
val d2s: Double => String = _.toString
```



# A pipeline of functions

```
val s2i: String => Int = _.toInt
val plus2: Int => Int = _ + 2
val div10By: Int => Double = 10.0 / _
val d2s: Double => String = _.toString
```

composed with *map*:

```
val fMapped = s2i map plus2 map div10By map d2s // requires -Ypartial-unification
val res1 = fMapped("3") // 2.0 !!!
```

# A pipeline of functions

```
val s2i: String => Int = _.toInt
val plus2: Int => Int = _ + 2
val div10By: Int => Double = 10.0 / _
val d2s: Double => String = _.toString
```

composed with *map*:

```
val fMapped = s2i map plus2 map div10By map d2s // requires -Ypartial-unification
val res1 = fMapped("3") // 2.0 !!!
```

*Function1* can also be seen as a Monad ...

# *7. Function1* as Monad

See: *demo.Demo07Monad*

# Monad

A Monad is any Context `F[_]` that provides the functions *pure* and *flatMap* ... and abides by the Monad laws (which are not presented here).

```
trait Monad[F[_]] extends Functor[F] {  
  def pure[A](a: A): F[A]  
  def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]  
}
```

# Monad instance

A Monad instance for *Functions* (found automatically by the compiler if defined in implicit scope, i.e inside the *Monad* companion object):

```
implicit def function1Monad[P]: Monad[P => ?] = new Monad[P => ?] {  
  override def pure[A](r: A): P => A = _ => r  
  override def flatMap[A, B](f: P => A)(g: A => P => B)  
    : P => B = p => g(f(p))(p)  
}
```

# Monad instance

A Monad instance for *Functions* (found automatically by the compiler if defined in implicit scope, i.e inside the *Monad* companion object):

```
implicit def function1Monad[P]: Monad[P => ?] = new Monad[P => ?] {  
  override def pure[A](r: A): P => A = _ => r  
  override def flatMap[A, B](f: P => A)(g: A => P => B)  
    : P => B = p => g(f(p))(p)  
}
```

Alternative instance definition:

```
implicit def function1Monad[P]: Monad[Function1[P, ?]] = new Monad[Function1[P, ?]]  
  override def pure[A](r: A): Function1[P, A] = _ => r  
  override def flatMap[A, B](f: Function1[P, A])(g: A => Function1[P, B])  
    : Function1[P, B] = p => g(f(p))(p)  
}
```

# Monad instance

A Monad instance for *Functions* (found automatically by the compiler if defined in implicit scope, i.e inside the *Monad* companion object):

```
implicit def function1Monad[P]: Monad[P => ?] = new Monad[P => ?] {  
  override def pure[A](r: A): P => A = _ => r  
  override def flatMap[A, B](f: P => A)(g: A => P => B)  
    : P => B = p => g(f(p))(p)  
}
```

Alternative instance definition:

```
implicit def function1Monad[P]: Monad[Function1[P, ?]] = new Monad[Function1[P, ?]] {  
  override def pure[A](r: A): Function1[P, A] = _ => r  
  override def flatMap[A, B](f: Function1[P, A])(g: A => Function1[P, B])  
    : Function1[P, B] = p => g(f(p))(p)  
}
```

A Monad instance for *Either* (just for comparison):

```
implicit def eitherMonad[L]: Monad[Either[L, ?]] = new Monad[Either[L, ?]] {  
  override def pure[A](r: A): Either[L, A] = Right(r)  
  override def flatMap[A, B](fa: Either[L, A])(f: A => Either[L, B])  
    : Either[L, B] = fa flatMap f  
}
```

# Monad syntax

defined as implicit conversion ...



# Monad syntax

defined as implicit conversion ...

in a specific way for *Monad[Function1]*:

```
implicit class MonadSyntaxFunction1[P, A](f: Function1[P, A]) {  
  def flatMap[B](g: A => P => B): P => B = Monad[Function1[P, ?]].flatMap(f)(g)  
}
```

# Monad syntax

defined as implicit conversion ...

in a specific way for *Monad[Function1]*:

```
implicit class MonadSyntaxFunction1[P, A](f: Function1[P, A]) {  
  def flatMap[B](g: A => P => B): P => B = Monad[Function1[P, ?]].flatMap(f)(g)  
}
```

or in a generic way for any *Monad[F[\_]]*:

```
implicit class MonadSyntax[F[_]: Monad, A](fa: F[A]) {  
  def flatMap[B](f: A => F[B]): F[B] = Monad[F].flatMap(fa)(f)  
}
```

# Monad syntax

defined as implicit conversion ...

in a specific way for *Monad[Function1]*:

```
implicit class MonadSyntaxFunction1[P, A](f: Function1[P, A]) {  
  def flatMap[B](g: A => P => B): P => B = Monad[Function1[P, ?]].flatMap(f)(g)  
}
```

or in a generic way for any *Monad[F[\_]]*:

```
implicit class MonadSyntax[F[_]: Monad, A](fa: F[A]) {  
  def flatMap[B](f: A => F[B]): F[B] = Monad[F].flatMap(fa)(f)  
}
```

This allows for convenient invocation of *flatMap*  
as if *flatMap* were a method of *Function1*:

```
val h = f flatMap g
```

instead of:

```
val h = Functor[Function1[Int, ?]].flatMap(f)(g)
```

# A pipeline of functions (Reader Monad)

# A pipeline of functions (Reader Monad)

```
val countLines: String => Int = text => text.split("\n").length
val countWords: String => Int = text => text.split("\\W+").length
val countChars: String => Int = text => text.length
```

# A pipeline of functions (Reader Monad)

```
val countLines: String => Int = text => text.split("\n").length
val countWords: String => Int = text => text.split("\\W+").length
val countChars: String => Int = text => text.length
```

FlatMapping over *Function1*:

```
val computeStatistics1: String => (Int, Int, Int) =
  countLines flatMap { nLines =>           // define a pure program which does nothing
    countWords flatMap { nWords =>
      countChars map { nChars =>
        (nLines, nWords, nChars)
      } } }
val stat1: (Int, Int, Int) = computeStatistics1(getInput) // exec program (impure)
```

# A pipeline of functions (Reader Monad)

```
val countLines: String => Int = text => text.split("\n").length
val countWords: String => Int = text => text.split("\\W+").length
val countChars: String => Int = text => text.length
```

FlatMapping over *Function1*:

```
val computeStatistics1: String => (Int, Int, Int) =
  countLines flatMap { nLines =>           // define a pure program which does nothing
    countWords flatMap { nWords =>
      countChars map { nChars =>
        (nLines, nWords, nChars)
      } } }
val stat1: (Int, Int, Int) = computeStatistics1(getInput) // exec program (impure)
```

alternatively with a for-comprehension:

```
val computeStatistics2: String => (Int, Int, Int) =
  for {
    nLines <- countLines // uses Function1#flatMap
    nWords <- countWords
    nChars <- countChars
  } yield (nLines, nWords, nChars)
val stat2: (Int, Int, Int) = computeStatistics2(getInput) // exec program (impure)
```

## Another Reader Monad example - a bit more realistic ...

```
val users: Map[Int, String] = Map(
  1 -> "dade", 2 -> "kate", 3 -> "margo")
val passwords: Map[String, String] = Map(
  "dade" -> "zerocool", "kate" -> "acidburn", "margo" -> "secret")

case class Db(usernames: Map[Int, String], passwords: Map[String, String])
val db = Db(users, passwords)

type DbReader[A] = Db => A    // ^= Function1[Db, A]

def findUsername(userId: Int): DbReader[Option[String]] =
  db => db.usernames.get(userId)

def checkPassword(username: String, password: String): DbReader[Boolean] =
  db => db.passwords.get(username).contains(password)

def checkLogin(userId: Int, password: String): DbReader[Boolean] = // ^= Function1
  for {
    optUsername <- findUsername(userId)
    passwordOk <- optUsername
      .map(name => checkPassword(name, password))
      .getOrElse(_:Db) => false
  } yield passwordOk

val login10k = checkLogin(1, "zerocool")(db) // true
val login40k = checkLogin(4, "davinci")(db) // false
```

Example taken from "Scala with Cats" (see chapter Resources for link)



# 8. Kleisli composition done manually

See: *demo.Demo08KleisliDoneManually*

# The Problem:

# The Problem:

```
// Functions: A => F[B], where F is Option in this case
val s2iOpt: String => Option[Int] = s => Option(s.toInt)
val plus2Opt: Int => Option[Int] = i => Option(i + 2)
val div10ByOpt: Int => Option[Double] = i => Option(10.0 / i)
val d2sOpt: Double => Option[String] = d => Option(d.toString + " !!!")
```

These functions take an  $A$  and return a  $B$  inside of a context  $F[_]$ :  $A \Rightarrow F[B]$   
In our case  $F[_]$  is *Option*, but could be *List*, *Future* etc.

# The Problem:

```
// Functions: A => F[B], where F is Option in this case
val s2iOpt: String => Option[Int] = s => Option(s.toInt)
val plus2Opt: Int => Option[Int] = i => Option(i + 2)
val div10ByOpt: Int => Option[Double] = i => Option(10.0 / i)
val d2sOpt: Double => Option[String] = d => Option(d.toString + " !!!")
```

These functions take an  $A$  and return a  $B$  inside of a context  $F[_]$ :  $A \Rightarrow F[B]$   
In our case  $F[_]$  is *Option*, but could be *List*, *Future* etc.

We want to compose these functions to a single function which is then fed with some input string.

# The Problem:

```
// Functions: A => F[B], where F is Option in this case
val s2iOpt: String => Option[Int] = s => Option(s.toInt)
val plus20pt: Int => Option[Int] = i => Option(i + 2)
val div10ByOpt: Int => Option[Double] = i => Option(10.0 / i)
val d2sOpt: Double => Option[String] = d => Option(d.toString + " !!!")
```

These functions take an  $A$  and return a  $B$  inside of a context  $F[_]$ :  $A \Rightarrow F[B]$   
In our case  $F[_]$  is *Option*, but could be *List*, *Future* etc.

We want to compose these functions to a single function which is then fed with some input string.

Let's try *map*.

```
val fMapped: String => Option[Option[Option[Option[String]]]] = str =>
  s2iOpt(str) map { i1 =>
    plus20pt(i1) map { i2 =>
      div10ByOpt(i2) map {
        d => d2sOpt(d)
      }}
  }
```

We get nested *Options*. So let's try *flatMap* on the *Option* context.

## FlatMapping on the *Option* context

## FlatMapping on the *Option* context

with *flatMap* (this works):

```
val flatMappedOnOpt1: String => Option[String] = input =>
  s2iOpt(input) flatMap { i1 =>
    plus20pt(i1) flatMap { i2 =>
      div10ByOpt(i2) flatMap { d =>
        d2sOpt(d)
      }}
    }
val res1: Option[String] = flatMappedOnOpt1("3") // Some(2.0)
```

## FlatMapping on the *Option* context

with *flatMap* (this works):

```
val flatMappedOnOpt1: String => Option[String] = input =>
  s2iOpt(input) flatMap { i1 =>
    plus20pt(i1) flatMap { i2 =>
      div10ByOpt(i2) flatMap { d =>
        d2sOpt(d)
      }
    }
  }
val res1: Option[String] = flatMappedOnOpt1("3") // Some(2.0)
```

or with a for-comprehension (this looks nicer):

```
val flatMappedOnOpt2: String => Option[String] = input => for {
  i1 <- s2iOpt(input)
  i2 <- plus20pt(i1)
  d <- div10ByOpt(i2)
  s <- d2sOpt(d)
} yield s
val res2: Option[String] = flatMappedOnOpt2("3") // Some(2.0)
```



## FlatMapping on the *Option* context

with *flatMap* (this works):

```
val flatMappedOnOpt1: String => Option[String] = input =>
  s2iOpt(input) flatMap { i1 =>
    plus2Opt(i1) flatMap { i2 =>
      div10ByOpt(i2) flatMap { d =>
        d2sOpt(d)
      }
    }
  }
val res1: Option[String] = flatMappedOnOpt1("3") // Some(2.0)
```

or with a for-comprehension (this looks nicer):

```
val flatMappedOnOpt2: String => Option[String] = input => for {
  i1 <- s2iOpt(input)
  i2 <- plus2Opt(i1)
  d <- div10ByOpt(i2)
  s <- d2sOpt(d)
} yield s
val res2: Option[String] = flatMappedOnOpt2("3") // Some(2.0)
```

But: We still have to bind the variables *i1*, *i2*, *d* and *s* to names.

We would like to build a function pipeline with some kind of *andThenF*.

*s2iOpt andThenF plus2Opt andThenF div10ByOpt andThenF d2sOpt*

# Kleisli Composition

Kleisli composition takes two functions  $A \Rightarrow F[B]$  and  $B \Rightarrow F[C]$  and yields a new function  $A \Rightarrow F[C]$  where the context  $F[_]$  is required to be a Monad.

# Kleisli Composition

Kleisli composition takes two functions  $A \Rightarrow F[B]$  and  $B \Rightarrow F[C]$  and yields a new function  $A \Rightarrow F[C]$  where the context  $F[_]$  is required to be a Monad.

Let's define *kleisli*:

# Kleisli Composition

Kleisli composition takes two functions  $A \Rightarrow F[B]$  and  $B \Rightarrow F[C]$  and yields a new function  $A \Rightarrow F[C]$  where the context  $F[_]$  is required to be a Monad.

Let's define *kleisli*:

```
def kleisli[F[_]: Monad, A, B, C](f: A => F[B], g: B => F[C]): A => F[C] =  
  a => Monad[F].flatMap(f(a))(g)
```

# Kleisli Composition

Kleisli composition takes two functions  $A \Rightarrow F[B]$  and  $B \Rightarrow F[C]$  and yields a new function  $A \Rightarrow F[C]$  where the context  $F[_]$  is required to be a Monad.

Let's define *kleisli*:

```
def kleisli[F[_]: Monad, A, B, C](f: A => F[B], g: B => F[C]): A => F[C] =  
  a => Monad[F].flatMap(f(a))(g)
```

Using *kleisli*:

```
val kleisliComposed1: String => Option[String] =  
  kleisli(kleisli(kleisli(s2i0pt, plus20pt), div10By0pt), d2s0pt)  
  
val resKleisli1 = kleisliComposed1("3")    // 2.0 !!!
```

# Kleisli Composition

Kleisli composition takes two functions  $A \Rightarrow F[B]$  and  $B \Rightarrow F[C]$  and yields a new function  $A \Rightarrow F[C]$  where the context  $F[_]$  is required to be a Monad.

Let's define *kleisli*:

```
def kleisli[F[_]: Monad, A, B, C](f: A => F[B], g: B => F[C]): A => F[C] =  
  a => Monad[F].flatMap(f(a))(g)
```

Using *kleisli*:

```
val kleisliComposed1: String => Option[String] =  
  kleisli(kleisli(kleisli(s2i0pt, plus20pt), div10By0pt), d2s0pt)  
  
val resKleisli1 = kleisliComposed1("3")    // 2.0 !!!
```

This works, but is still not exactly what we want.  
*kleisli* should behave like a method of *Function1*.

*kleisli* defined on *Function1*

# *kleisli* defined on *Function1*

with an implicit conversion:

```
implicit class RichFunction1[F[_]: Monad, A, B](f: A => F[B]) {  
  def kleisli[C](g: B => F[C]): A => F[C] = a => Monad[F].flatMap(f(a))(g)  
  def andThenF[C](g: B => F[C]): A => F[C] = f kleisli g  
  def >=>[C](g: B => F[C]): A => F[C] = f kleisli g    // Haskell's fish operator  
}
```



# *kleisli* defined on *Function1*

with an implicit conversion:

```
implicit class RichFunction1[F[_]: Monad, A, B](f: A => F[B]) {  
  def kleisli[C](g: B => F[C]): A => F[C] = a => Monad[F].flatMap(f(a))(g)  
  def andThenF[C](g: B => F[C]): A => F[C] = f kleisli g  
  def >=>[C](g: B => F[C]): A => F[C] = f kleisli g    // Haskell's fish operator  
}
```

Using it:

```
val kleisliComposed2: String => Option[String] =  
  s2iOpt kleisli plus20pt kleisli div10By0pt kleisli d2sOpt  
kleisliComposed2("3") foreach println    // 2.0 !!!
```

# *kleisli* defined on *Function1*

with an implicit conversion:

```
implicit class RichFunction1[F[_]: Monad, A, B](f: A => F[B]) {  
  def kleisli[C](g: B => F[C]): A => F[C] = a => Monad[F].flatMap(f(a))(g)  
  def andThenF[C](g: B => F[C]): A => F[C] = f kleisli g  
  def >=>[C](g: B => F[C]): A => F[C] = f kleisli g    // Haskell's fish operator  
}
```

Using it:

```
val kleisliComposed2: String => Option[String] =  
  s2iOpt kleisli plus20pt kleisli div10By0pt kleisli d2sOpt  
kleisliComposed2("3") foreach println    // 2.0 !!!
```

```
(s2iOpt andThenF plus20pt andThenF div10By0pt andThenF d2sOpt) foreach println
```

# *kleisli* defined on *Function1*

with an implicit conversion:

```
implicit class RichFunction1[F[_]: Monad, A, B](f: A => F[B]) {  
  def kleisli[C](g: B => F[C]): A => F[C] = a => Monad[F].flatMap(f(a))(g)  
  def andThenF[C](g: B => F[C]): A => F[C] = f kleisli g  
  def >=>[C](g: B => F[C]): A => F[C] = f kleisli g    // Haskell's fish operator  
}
```

Using it:

```
val kleisliComposed2: String => Option[String] =  
  s2iOpt kleisli plus20pt kleisli div10By0pt kleisli d2sOpt  
kleisliComposed2("3") foreach println    // 2.0 !!!
```

```
(s2iOpt andThenF plus20pt andThenF div10By0pt andThenF d2sOpt) foreach println
```

```
(s2iOpt >=> plus20pt >=> div10By0pt >=> d2sOpt) foreach println
```

## *9. case class Kleisli*

See: *demo.Demo09KleisliCaseClass*

## *case class Kleisli*

That was my artefact. Cats does not provide a *kleisli* method on *Function1*. Cats instead provides a *case class Kleisli* with the functionality shown above and more.

## *case class Kleisli*

That was my artefact. Cats does not provide a *kleisli* method on *Function1*. Cats instead provides a *case class Kleisli* with the functionality shown above and more.

I tinkered my own impl in *mycats.Kleisli* which works much like the Cats impl: see next slide.

## *case class Kleisli*

```
case class Kleisli[F[_], A, B](run: A => F[B]) {  
  def apply(a: A): F[B] = run(a)  
  
  def map[C](f: B => C)(implicit F: Functor[F]): Kleisli[F, A, C] =  
    Kleisli { a => F.map(run(a))(f) }  
  
  def flatMap[C](f: B => Kleisli[F, A, C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
    Kleisli { a => M.flatMap(run(a))(b => f(b).run(a)) }  
  
  def flatMapF[C](f: B => F[C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
    Kleisli { a => M.flatMap(run(a))(f) }  
  
  def andThen[C](f: B => F[C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
    flatMapF(f)  
  
  def andThen[C](that: Kleisli[F, B, C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
    this andThen that.run  
  
  def compose[Z](f: Z => F[A])(implicit M: Monad[F]): Kleisli[F, Z, B] =  
    Kleisli(f) andThen this.run  
  
  def compose[Z](that: Kleisli[F, Z, A])(implicit M: Monad[F]): Kleisli[F, Z, B] =  
    that andThen this  
}
```

The case class methods delegate to the wrapped *run* function and return the resulting *run* function rewrapped in a *Kleisli* instance.

## *Kleisli* companion object with Monad instance

```
object Kleisli { self =>

  def pure[F[_], A, B](b: B)(implicit F: Monad[F]): Kleisli[F, A, B] =
    Kleisli { _ => F.pure(b) }

  object ops {

    implicit def kleisliMonad[F[_] : Monad, A]: Monad[Kleisli[F, A, ?]] =
      new Monad[Kleisli[F, A, ?]] {

        override def pure[B](b: B): Kleisli[F, A, B] = self.pure(b)

        override def flatMap[B, C](kl: Kleisli[F, A, B])(f: B => Kleisli[F, A, C])
          : Kleisli[F, A, C] = kl flatMap f
      }
  }
}
```



## *Kleisli#flatMap*

```
def flatMap[C](f: B => Kleisli[F, A, C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
  M.flatMap(run(a))(b => f(b).run(a))  
}
```

*flatMap* composes *this.run: A => F[B]* with the function *f: B => Kleisli[F, A, C]* yielding a new *Kleisli[F, A, C]* wrapping a new function *run: A => F[C]*.

# *Kleisli#flatMap*

```
def flatMap[C](f: B => Kleisli[F, A, C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
  M.flatMap(run(a))(b => f(b).run(a))  
}
```

*flatMap* composes *this.run: A => F[B]* with the function *f: B => Kleisli[F, A, C]* yielding a new *Kleisli[F, A, C]* wrapping a new function *run: A => F[C]*.

```
val kleisli1: String => Option[String] = input =>  
  Kleisli(s2iOpt).run(input) flatMap { i1 =>  
    Kleisli(plus20pt).run(i1) flatMap { i2 =>  
      Kleisli(div10By0pt).run(i2) flatMap { d =>  
        Kleisli(d2sOpt).run(d)  
      } } }  
kleisli1("3") foreach println
```

# *Kleisli#flatMap*

```
def flatMap[C](f: B => Kleisli[F, A, C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
  M.flatMap(run(a))(b => f(b).run(a))  
}
```

*flatMap* composes *this.run: A => F[B]* with the function *f: B => Kleisli[F, A, C]* yielding a new *Kleisli[F, A, C]* wrapping a new function *run: A => F[C]*.

```
val kleisli1: String => Option[String] = input =>  
  Kleisli(s2iOpt).run(input) flatMap { i1 =>  
    Kleisli(plus20pt).run(i1) flatMap { i2 =>  
      Kleisli(div10By0pt).run(i2) flatMap { d =>  
        Kleisli(d2s0pt).run(d)  
      } } }  
kleisli1("3") foreach println
```

```
val kleisli2: String => Option[String] = input => for {  
  i1 <- Kleisli(s2iOpt).run(input)  
  i2 <- Kleisli(plus20pt).run(i1)  
  d <- Kleisli(div10By0pt).run(i2)  
  s <- Kleisli(d2s0pt).run(d)  
} yield s  
kleisli2("3") foreach println
```

## *Kleisli#flatMapF*

As we saw *Kleisli#flatMap* is not very convenient. We have to bind values to variables and thread them through the for-comprehension. *flatMapF* is easier to use.

# *Kleisli#flatMap*

As we saw *Kleisli#flatMap* is not very convenient. We have to bind values to variables and thread them through the for-comprehension. *flatMapF* is easier to use.

```
def flatMapF[C](f: B => F[C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
  Kleisli { a => M.flatMap(run(a))(f) }
```

*flatMapF* composes *this.run: A => F[B]* with the function *f: B => F[C]* yielding a new *Kleisli[F, A, C]* wrapping a new function *run: A => F[C]*.

## *Kleisli#flatMap*

As we saw *Kleisli#flatMap* is not very convenient. We have to bind values to variables and thread them through the for-comprehension. *flatMapF* is easier to use.

```
def flatMapF[C](f: B => F[C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
  Kleisli { a => M.flatMap(run(a))(f) }
```

*flatMapF* composes *this.run: A => F[B]* with the function *f: B => F[C]* yielding a new *Kleisli[F, A, C]* wrapping a new function *run: A => F[C]*.

```
val kleisli4: Kleisli[Option, String, String] =  
  Kleisli(s2i0pt) flatMapF plus20pt flatMapF div10By0pt flatMapF d2s0pt  
  
kleisli4.run("3") foreach println
```

## *Kleisli#andThen*

The behaviour of *flatMap* is exactly what we expect from *andThen*.

# *Kleisli#andThen*

The behaviour of *flatMap* is exactly what we expect from *andThen*.

```
def andThen[C](f: B => F[C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
  flatMapF(f)
```



# *Kleisli#andThen*

The behaviour of *flatMap* is exactly what we expect from *andThen*.

```
def andThen[C](f: B => F[C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
  flatMapF(f)
```

The first version of *andThen* is an alias for *flatMapF*.

# *Kleisli#andThen*

The behaviour of *flatMap* is exactly what we expect from *andThen*.

```
def andThen[C](f: B => F[C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
  flatMapF(f)
```

The first version of *andThen* is an alias for *flatMapF*.

```
(Kleisli(s2i0pt) andThen plus20pt andThen  
  div10By0pt andThen d2s0pt) foreach println
```

*Kleisli#andThen*

# *Kleisli#andThen*

```
def andThen[C](that: Kleisli[F, B, C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
  this andThen that.run
```

This overloaded version of *andThen* doesn't take a function  $f: B \Rightarrow F[C]$ . Instead it takes a *Kleisli*[F, B, C] wrapping such a function. This version allows us to concatenate several Kleislis to a pipeline.

# *Kleisli#andThen*

```
def andThen[C](that: Kleisli[F, B, C])(implicit M: Monad[F]): Kleisli[F, A, C] =  
  this andThen that.run
```

This overloaded version of *andThen* doesn't take a function  $f: B \Rightarrow F[C]$ . Instead it takes a *Kleisli*[F, B, C] wrapping such a function. This version allows us to concatenate several Kleislis to a pipeline.

```
(Kleisli(s2i0pt) andThen Kleisli(plus20pt) andThen  
  Kleisli(div10By0pt) andThen Kleisli(d2s0pt)) foreach println
```

## *Kleisli#compose*

As with *andThen* there are two versions of *compose*. They work like *andThen* with the arguments flipped.

# *Kleisli#compose*

As with *andThen* there are two versions of *compose*. They work like *andThen* with the arguments flipped.

```
def compose[Z](f: Z => F[A])(implicit M: Monad[F]): Kleisli[F, Z, B] =  
  Kleisli(f) andThen this.run  
  
def compose[Z](that: Kleisli[F, Z, A])(implicit M: Monad[F]): Kleisli[F, Z, B] =  
  that andThen this
```

# *Kleisli#compose*

As with *andThen* there are two versions of *compose*. They work like *andThen* with the arguments flipped.

```
def compose[Z](f: Z => F[A])(implicit M: Monad[F]): Kleisli[F, Z, B] =  
  Kleisli(f) andThen this.run  
  
def compose[Z](that: Kleisli[F, Z, A])(implicit M: Monad[F]): Kleisli[F, Z, B] =  
  that andThen this
```

```
(Kleisli(d2s0pt) compose div10By0pt compose  
  plus20pt compose s2i0pt) foreach println  
  
(Kleisli(d2s0pt) compose Kleisli(div10By0pt) compose  
  Kleisli(plus20pt) compose Kleisli(s2i0pt)) foreach println
```



# 10. Reader Monad

See: *demo.Demo10Reader*

# Reader again

We already saw, that the *Function1* Monad is the *Reader* Monad.

But *Kleisli* can also be used as *Reader*, if  $F[_]$  is fixed to the *Id* context.

# Reader again

We already saw, that the *Function1* Monad is the *Reader* Monad.

But *Kleisli* can also be used as *Reader*, if  $F[_]$  is fixed to the *Id* context.

```
type Id[A] = A

type ReaderT[F[_], A, B] = Kleisli[F, A, B]
val ReaderT = Kleisli

type Reader[A, B] = Kleisli[Id, A, B]

object Reader {
  def apply[A, B](f: A => B): Reader[A, B] = ReaderT[Id, A, B](f)
}
```

*Reader#flatMap*

# *Reader#flatMap*

```
val s2i: String => Int = _.toInt
val plus2: Int => Int = _ + 2
val div10By: Int => Double = 10.0 / _
val d2s: Double => String = _.toString + " !!!"
```

# *Reader#flatMap*

```
val s2i: String => Int = _.toInt
val plus2: Int => Int = _ + 2
val div10By: Int => Double = 10.0 / _
val d2s: Double => String = _.toString + " !!!"
```

```
val reader1: String => String = input => for {
  i1 <- Reader(s2i).run(input)
  i2 <- Reader(plus2).run(i1)
  d <- Reader(div10By).run(i2)
  s <- Reader(d2s).run(d)
} yield s

println(reader1("3"))    // 2.0 !!!
```

# *Reader#flatMap*

```
val s2i: String => Int = _.toInt
val plus2: Int => Int = _ + 2
val div10By: Int => Double = 10.0 / _
val d2s: Double => String = _.toString + " !!!"
```

```
val reader1: String => String = input => for {
  i1 <- Reader(s2i).run(input)
  i2 <- Reader(plus2).run(i1)
  d <- Reader(div10By).run(i2)
  s <- Reader(d2s).run(d)
} yield s

println(reader1("3"))    // 2.0 !!!
```

Again *flatMap* is not the best choice as we have to declare all these intermediate identifiers in the for-comprehension.

## *Reader#andThen* (two versions)



## *Reader#andThen* (two versions)

```
val reader2 =  
    Reader(s2i) andThen Reader(plus2) andThen Reader(div10By) andThen Reader(d2s)  
println(reader2("3"))    // 2.0 !!!
```

## *Reader#andThen* (two versions)

```
val reader2 =  
    Reader(s2i) andThen Reader(plus2) andThen Reader(div10By) andThen Reader(d2s)  
println(reader2("3"))    // 2.0 !!!
```

```
val reader3 =  
    Reader(s2i) andThen plus2 andThen div10By andThen d2s  
println(reader3("3"))    // 2.0 !!!
```

## *Reader#andThen* (two versions)

```
val reader2 =  
  Reader(s2i) andThen Reader(plus2) andThen Reader(div10By) andThen Reader(d2s)  
println(reader2("3"))    // 2.0 !!!
```

```
val reader3 =  
  Reader(s2i) andThen plus2 andThen div10By andThen d2s  
println(reader3("3"))    // 2.0 !!!
```

All methods of *Kleisli* are available for *Reader*,  
because *Kleisli* is *Reader* with *Id*.

The Reader example from before runs with minor changes.

```
val users: Map[Int, String] = Map(
  1 -> "dade", 2 -> "kate", 3 -> "margo" )
val passwords: Map[String, String] = Map(
  "dade" -> "zerocool", "kate" -> "acidburn", "margo" -> "secret" )

case class Db(usernames: Map[Int, String], passwords: Map[String, String])
val db = Db(users, passwords)

type DbReader[A] = Reader[Db, A]      // ^= Kleisli[Id, Db, Boolean]

def findUsername(userId: Int): DbReader[Option[String]] =
  Reader { db => db.usernames.get(userId) }

def checkPassword(username: String, password: String): DbReader[Boolean] =
  Reader { db => db.passwords.get(username).contains(password) }

def checkLogin(userId: Int, password: String): DbReader[Boolean] =
  for {
    optUsername <- findUsername(userId)
    passwordOk <- optUsername
      .map(name => checkPassword(name, password))
      .getOrElse(Kleisli.pure[Id, Db, Boolean](false))
  } yield passwordOk

val loginOk1 = checkLogin(1, "zerocool").run(db) // true
val loginOk2 = checkLogin(4, "davinci").run(db) // false
```

Example taken from "Scala with Cats" (see chapter Resources for link)

# 11. Resources

# Resources (1/2)

- Code and Slides of this Talk:  
<https://github.com/hermannhueck/composing-functions>
- "Scala with Cats"  
Book by Noel Welsh and Dave Gurnell  
<https://underscore.io/books/scala-with-cats/>
- "Functional Programming in Scala"  
Book by Paul Chiusano and Dave Gurnell  
<https://www.manning.com/books/functional-programming-in-scala>
- Cats documentation for *Kleisli*:  
<https://typelevel.org/cats/datatypes/kleisli.html>

# Resources (2/2)

- Miles Sabin's pull request for partial unification:  
<https://github.com/scala/scala/pull/5102>
- "Explaining Miles's Magic:  
Gist of Daniel Spiewak on partial unification  
<https://gist.github.com/djspiewak/7a81a395c461fd3a09a6941d4cd040f2>
- "Kind Projector Compiler Plugin:  
<https://github.com/non/kind-projector>

# Thanks for Listening

## Q & A

<https://github.com/hermannhueck/use-applicative-where-applicable>



