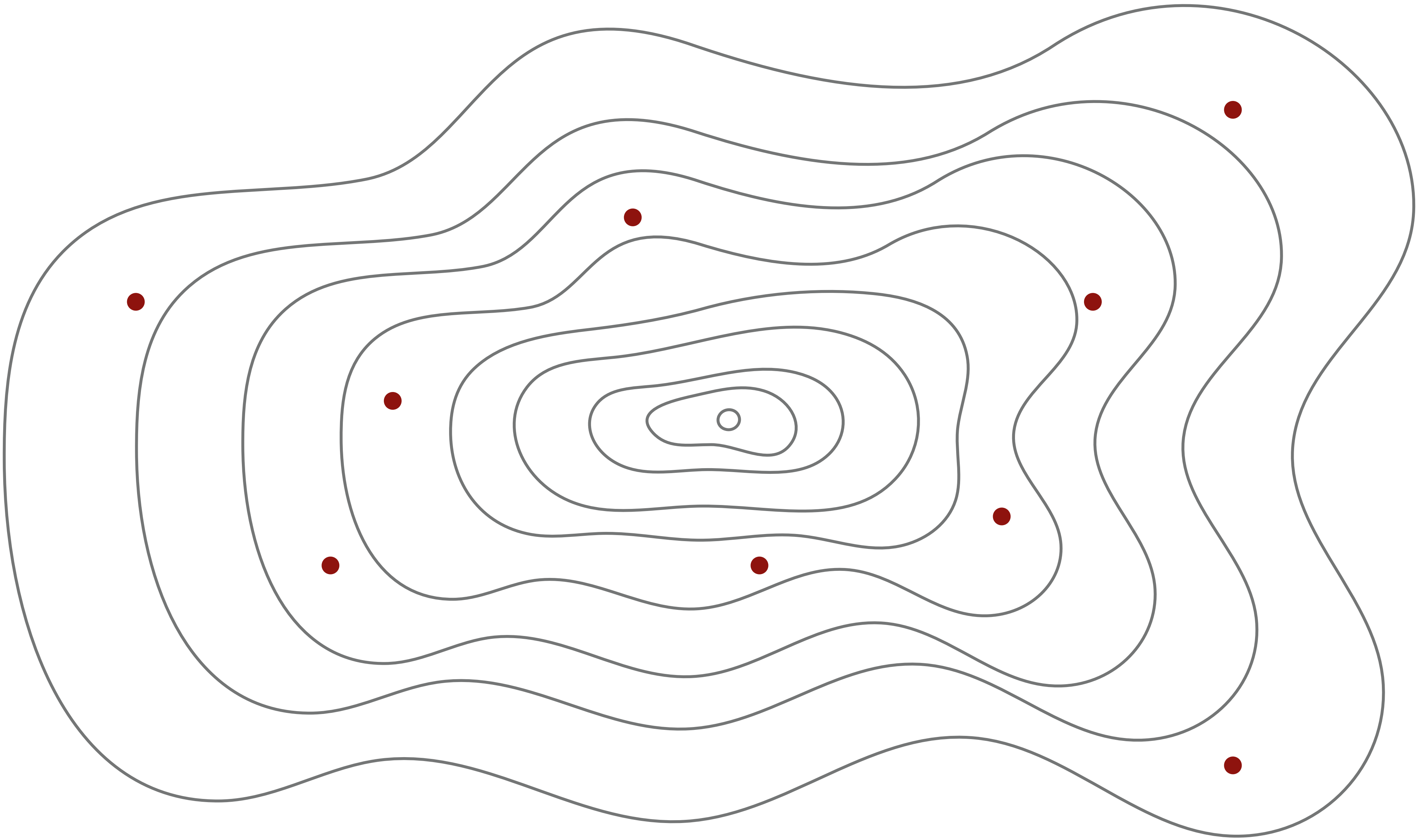
POPULATION BASED METAHEURISTICS

EVOLUTIVE ALGORITHMS

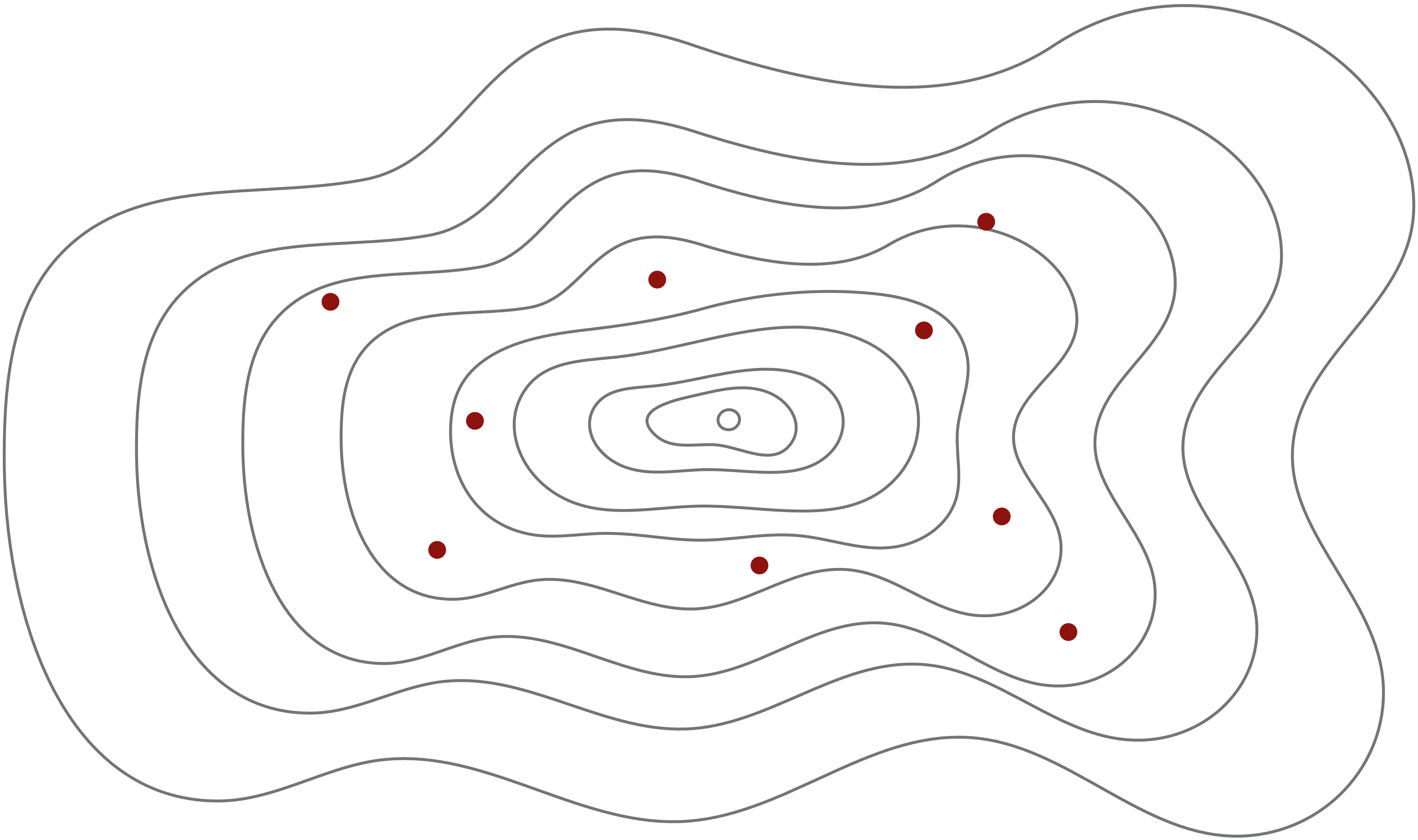
Prof. Eduardo Pécora

SURVIVAL OF THE FITTEST

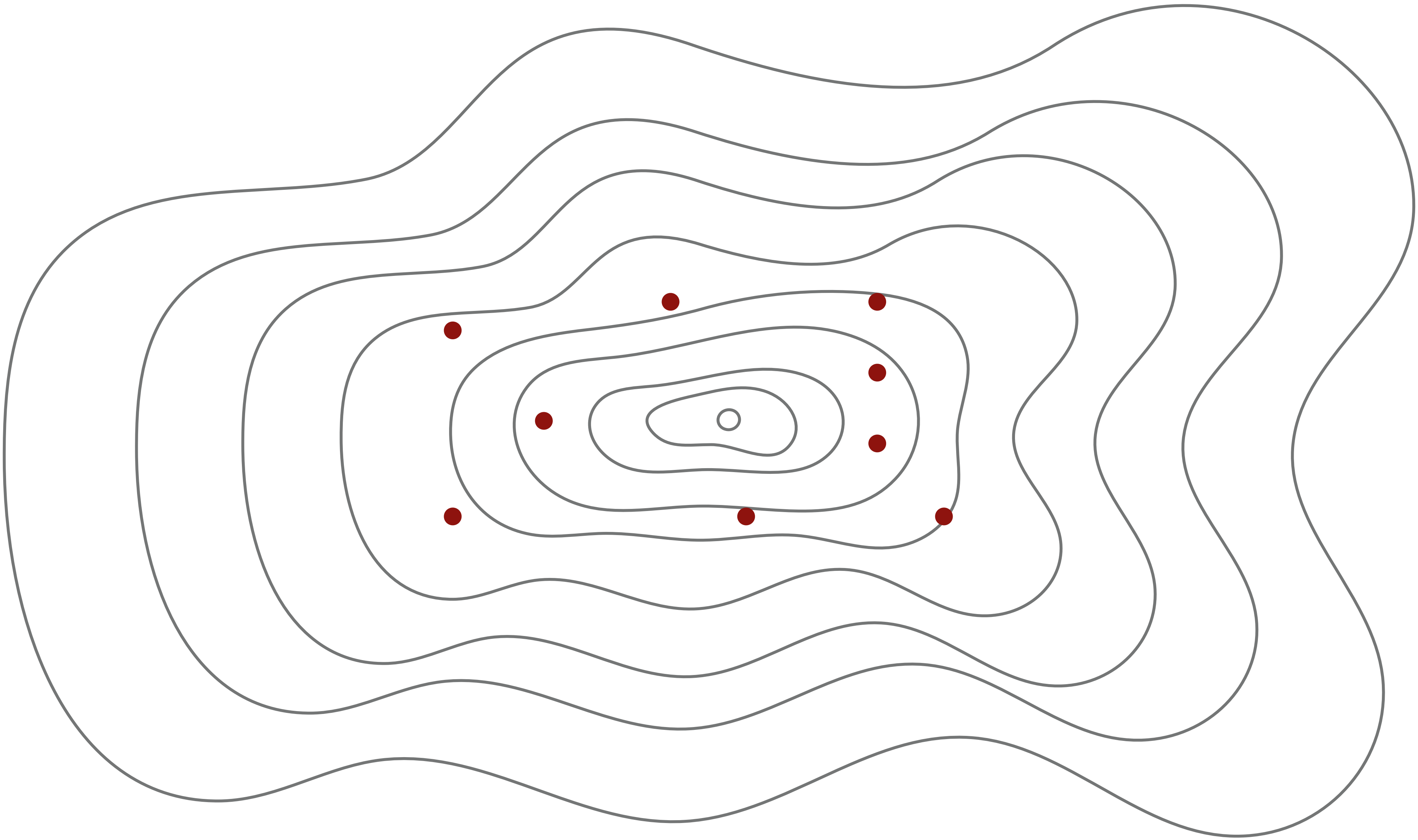
POPULATION BASED EXPLORATION



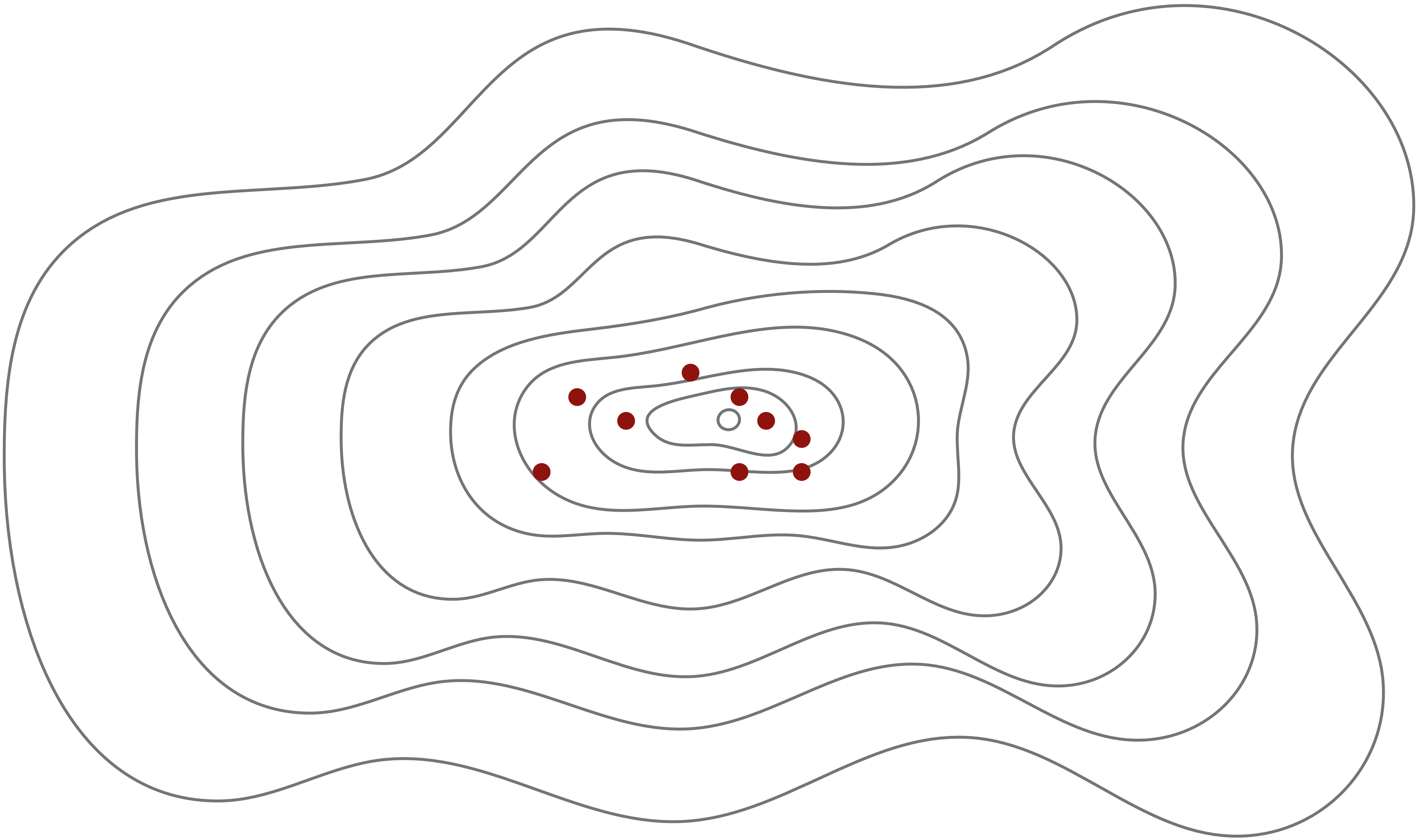
POPULATION BASED EXPLORATION



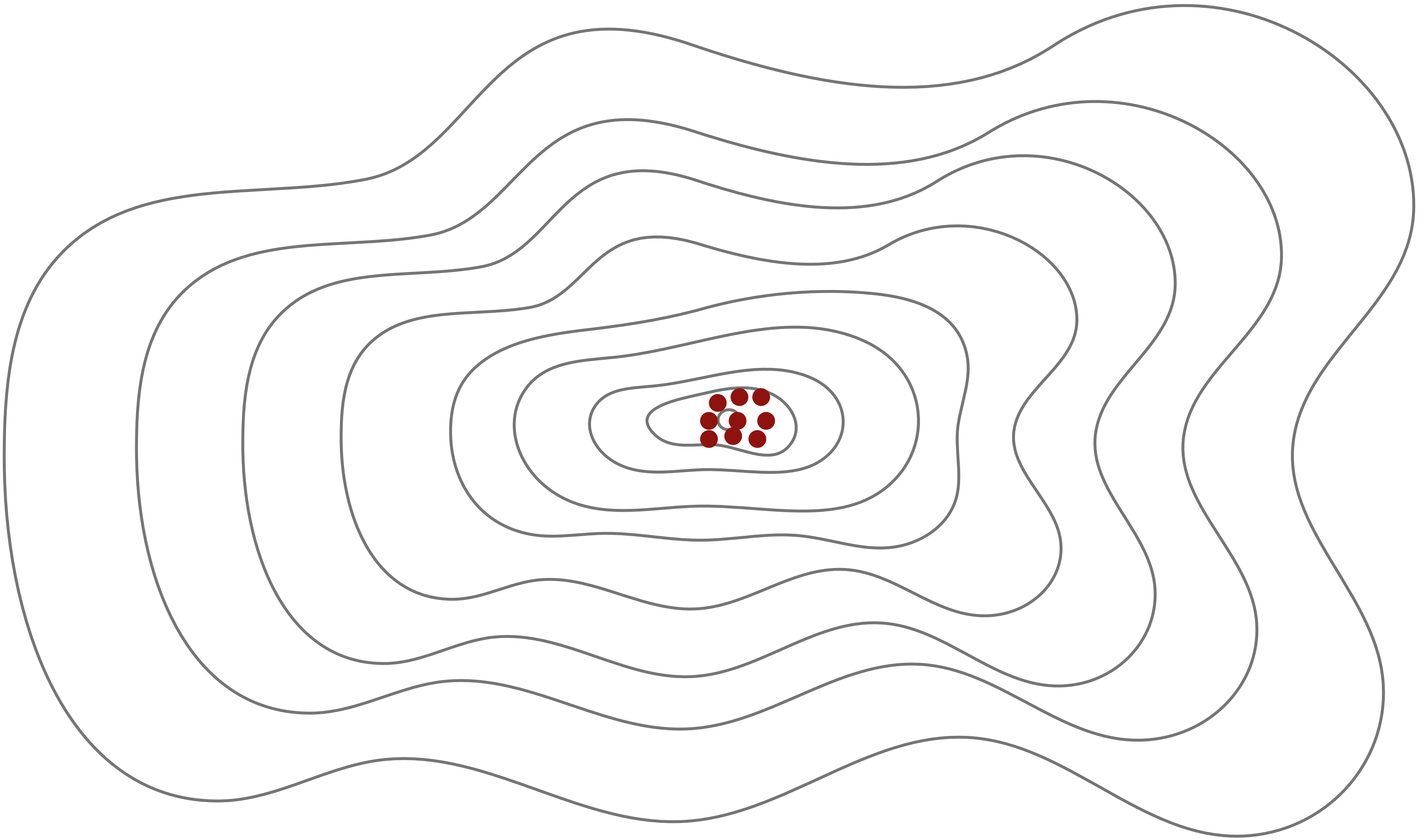
POPULATION BASED EXPLORATION



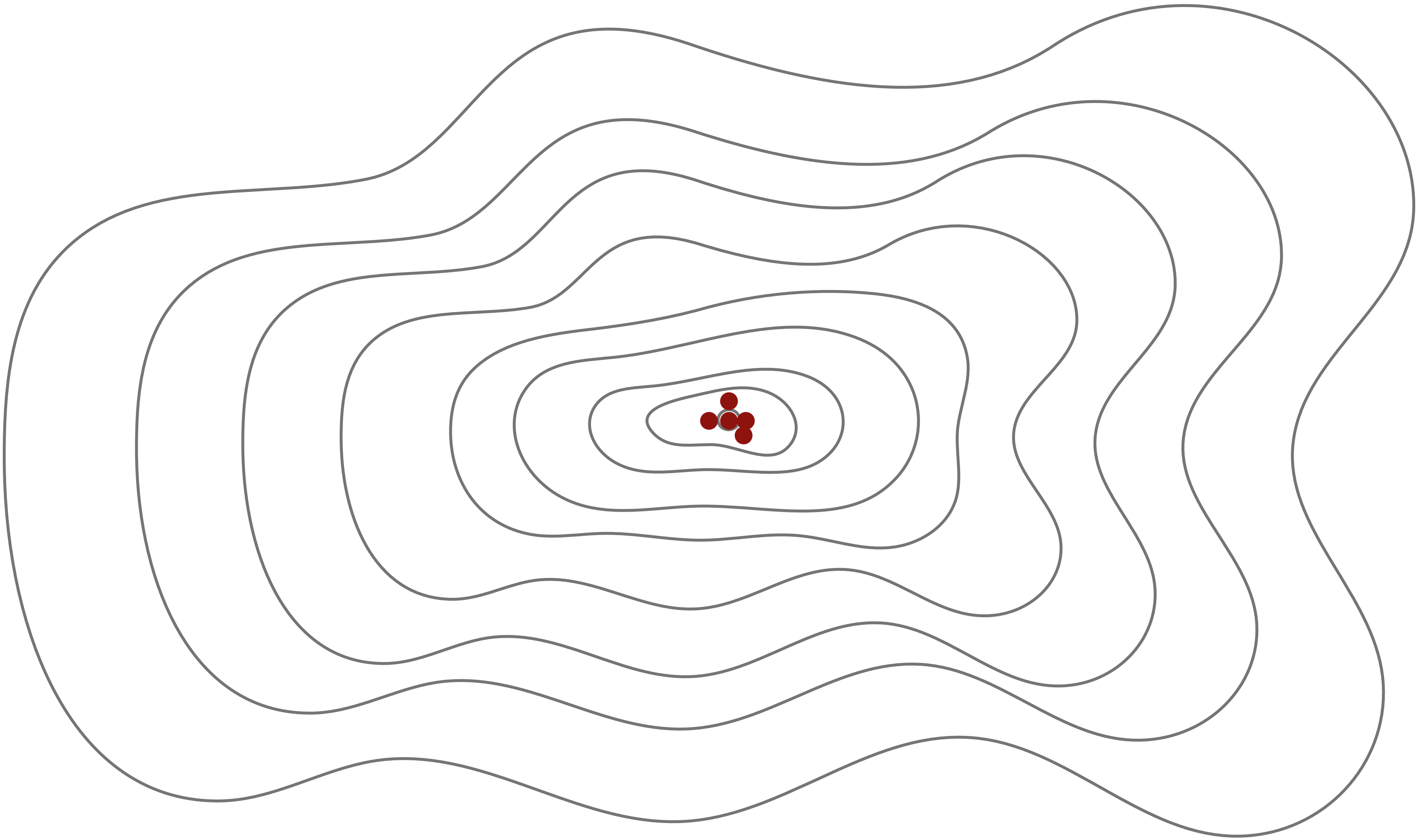
POPULATION BASED EXPLORATION



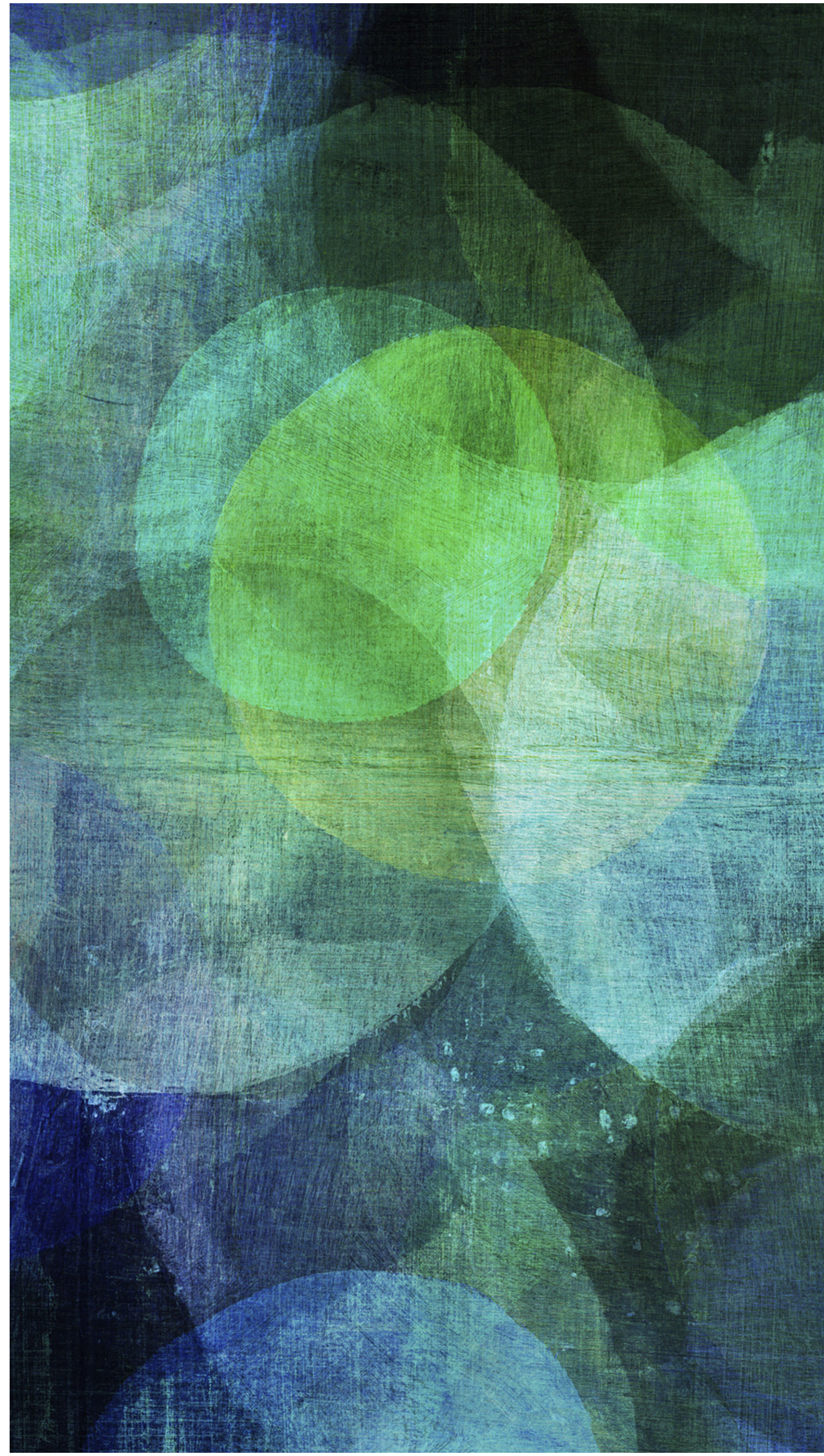
POPULATION BASED EXPLORATION



POPULATION BASED EXPLORATION



C -> T



INTENSIFICATION

VS

DIVERSIFICATION

MAIN PRINCIPLE OF P-METAHEURISTICS

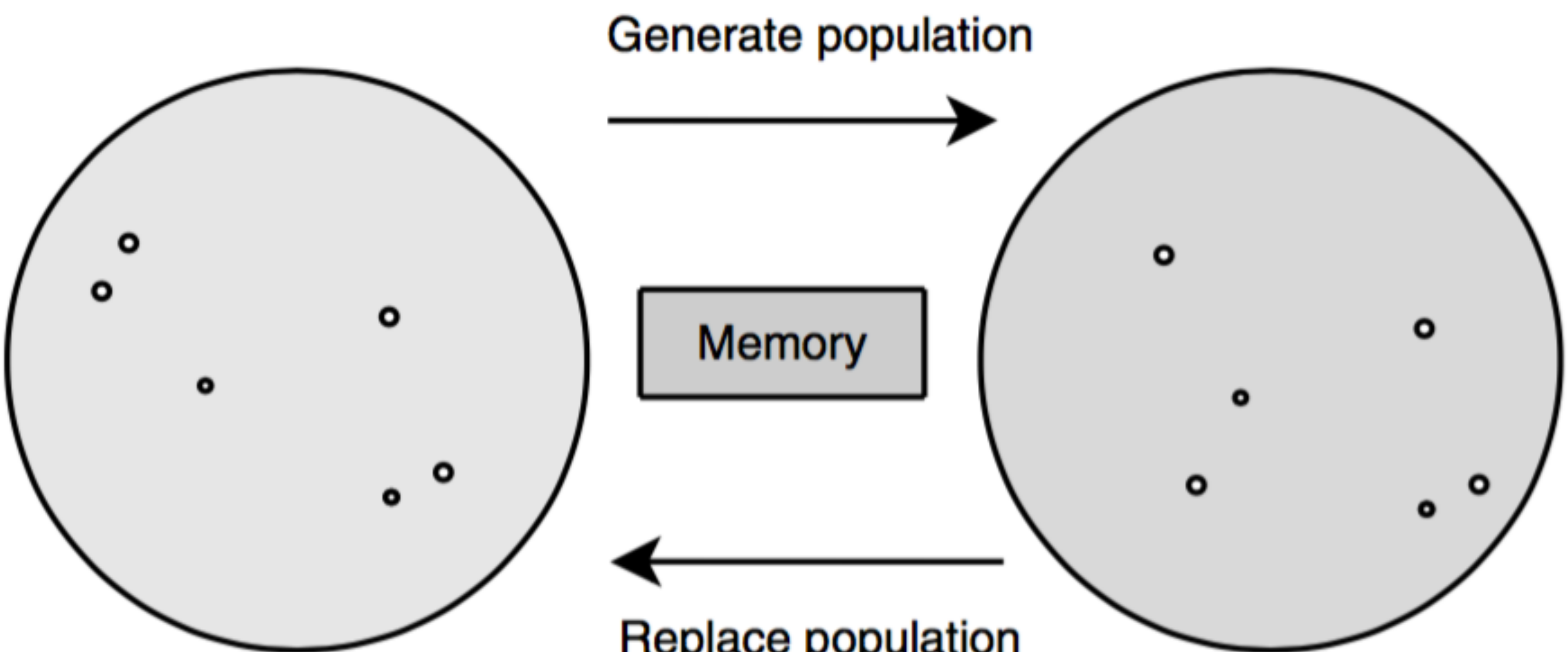


FIGURE 3.1 Main principles of P-metaheuristics.

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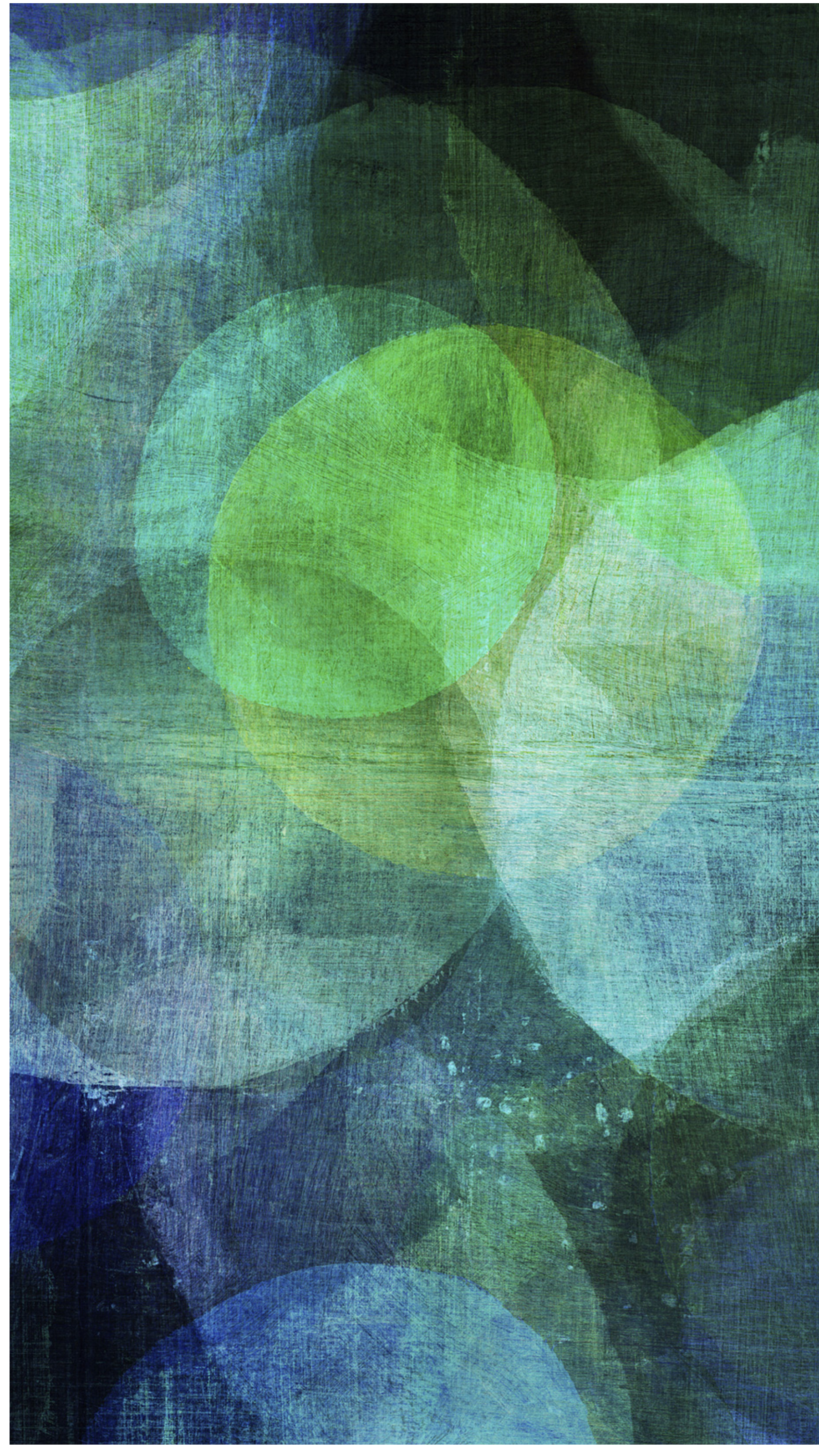

WITH THE CURRENT POPULATION GENERATE A NEW POPULATION AND CHECK IF THE SCORES IMPROVED!

ALGORITHM OF P-METAHEURISTICS

Algorithm 3.1 High-level template of P-metaheuristics.

$P = P_0$; /* Generation of the initial population */
$t = 0$;
Repeat
  Generate(P'_t); /* Generation a new population */
  P_{t+1} = Select-Population($P_t \cup P'_t$); /* Select new population */
  $t = t + 1$;
Until Stopping criteria satisfied
Output: Best solution(s) found.

MAIN CONCEPTS

- Generation: In this step, a new population of solutions is generated. According to the generation strategy:
  - Evolution based: In this category of P-metaheuristics, the solutions composing the population are selected and reproduced using variation operators (e.g., mutation, recombination) acting directly on their representations. A new solution is constructed from the different attributes of solutions belonging to the current population.
  - Blackboard based: Here, the solutions of the population participate in the construction of a shared memory. This shared memory will be the main input in generating the new population of solutions.

MEMORIES

Search memory: The memory of a P-metaheuristic represents the set of information extracted and memorized during the search. The content of this memory varies from a P-metaheuristic to another one.

TABLE 3.1 Search Memories of Some P-Metaheuristics

P-metaheuristic	Search Memory
Evolutionary algorithms	Population of individuals
Scatter search	Population of solutions, reference set
Ant colonies	Pheromone matrix
Estimation of distribution algorithms	Probabilistic learning model
Particle swarm optimization	Population of particles, best global and local solutions
Bee colonies	Population of bees
Artificial immune systems: clonal selection	Population of antibodies

MAIN CONCEPTS

SELECTION

- The last step in P-metaheuristics consists in selecting the new solutions from the union of the current population and the generated population. The traditional strategy consists in selecting the generated population as the new population. Other strategies use some elitism in the selection phase where they provide the best solutions from the two sets. In blackboard-based P-metaheuristics, there is no explicit selection.

EVOLUTIONARY ALGORITHMS

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EVOLUTIONARY ALGORITHMS

Algorithm 3.2 Template of an evolutionary algorithm.

Generate($P(0)$) ; /* Initial population */
$t = 0$;
While not Termination_Criterion($P(t)$) **Do**
    Evaluate($P(t)$) ;
    $P'(t)$ = Selection($P(t)$) ;
    $P'(t)$ = Reproduction($P'(t)$); Evaluate($P'(t)$) ;
    $P(t+1)$ = Replace($P(t)$, $P'(t)$) ;
    $t = t + 1$;
End While
Output Best individual or best population found.

GENOTYPE AND PHENOTYPE

- In evolutionary algorithms, the genotype represents the encoding while the phenotype represents the solution. Hence, the genotype must be decoded to generate the phenotype.

Min
$f(x)$
s.t.
$Ax <= b$
$x > 0$

Phenotype

$x = 1$

$x = 1.5$

$x = 0.45$

$x = 3.14159$

Binary encoding

Genotype

$$\{2^{-2}, 2^{-1}, 2^{0}, 2^{1}, 2^{2}, 2^{3}\}$$

$$\{0,1,1,0,1,0\}$$

GENOTYPE AND PHENOTYPE

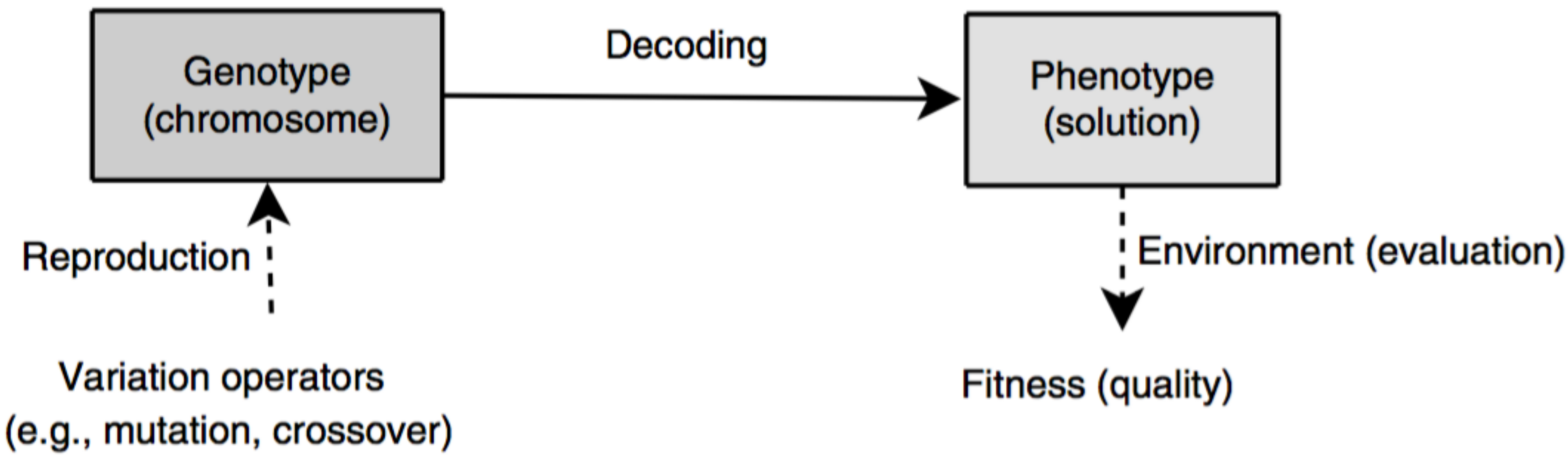

FIGURE 3.8 Genotype versus phenotype in evolutionary algorithms.

COMMON CONCEPTS FOR EVOLUTIONARY ALGORITHMS

1. Representation:
2. Population initialization:
3. Objective function:
4. **Selection strategy:**
5. **Reproduction strategy:**
6. **Replacement strategy:**
7. Stopping criteria:

EVOLUTIONARY ALGORITHMS

SELECTION METHODS

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SELECTION METHODS

➤ **Roulette Wheel Selection** and its variation **Stochastic Universal Sampling**

Individuals:	1	2	3	4	5	6	7
Fitness:	1	1	1	1.5	1.5	3	3

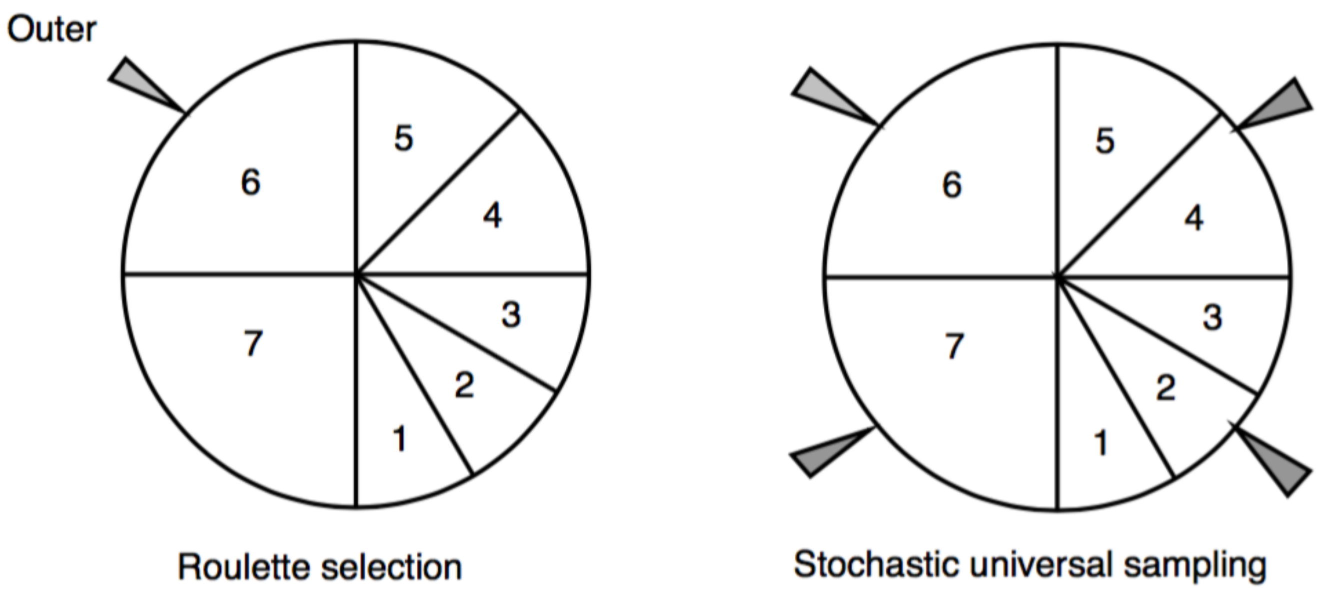


FIGURE 3.11 Roulette selection strategies. In the standard roulette selection, each spin selects a single individual. In SUS, a spin will select as individuals as outers (e.g., four individuals in the example).

SELECTION METHODS

➤ Tournament Selection

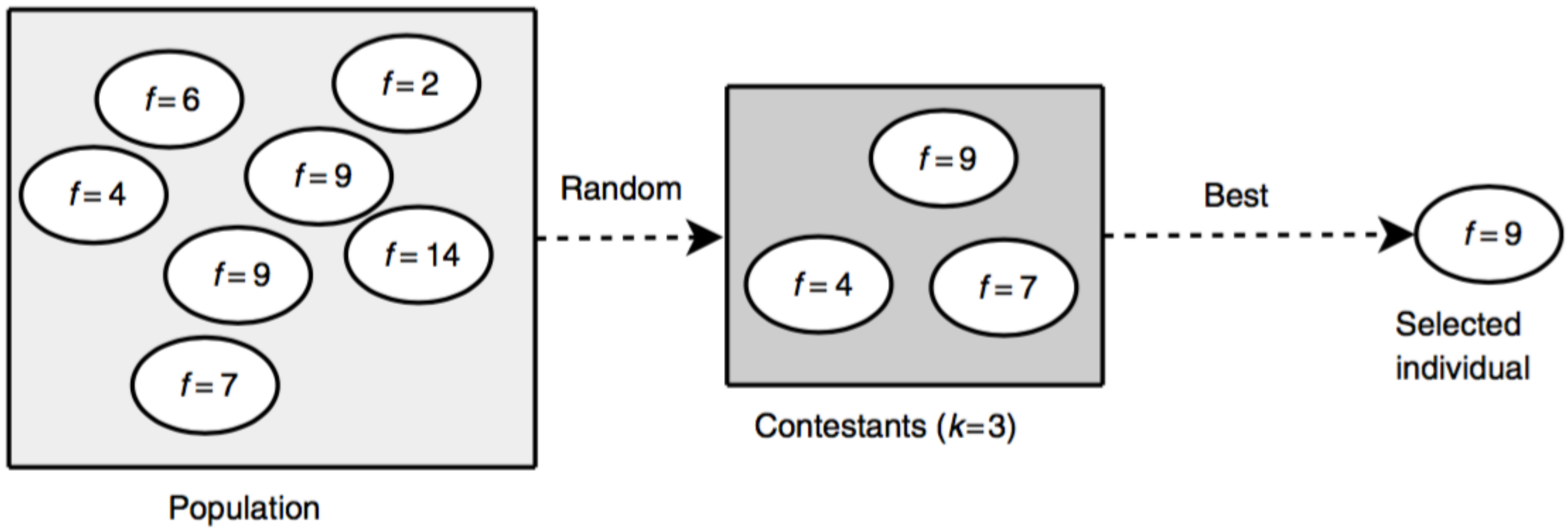


FIGURE 3.12 Tournament selection strategy. For instance, a tournament of size 3 is performed. Three solutions are picked randomly from the population. The best solution from the picked individuals is then selected.

SELECTION METHODS

- **Rank-Based Selection** - modified weight in roulette wheel using the rank instead of fitness.

$$P(i) = \frac{2-s}{\mu} + \frac{2.r(i).(s-1)}{\mu(\mu-1)}$$

where s is the selection pressure ($1.0 < s \leq 2.0$), μ is the size of the population, and r(i) is the rank associated with the individual i. Greater is the selection pressure s, more importance to better individuals is given.

SELECTION METHODS

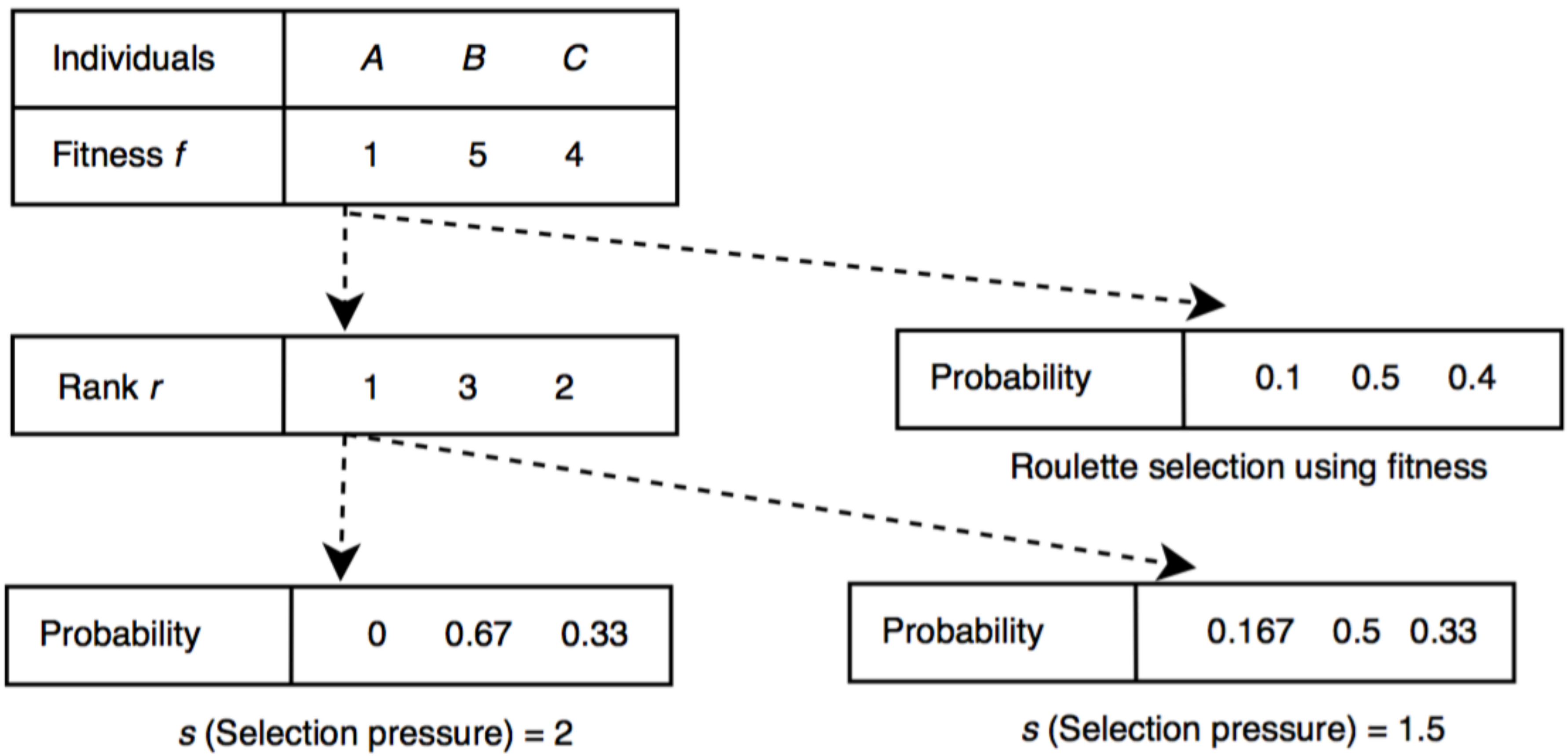
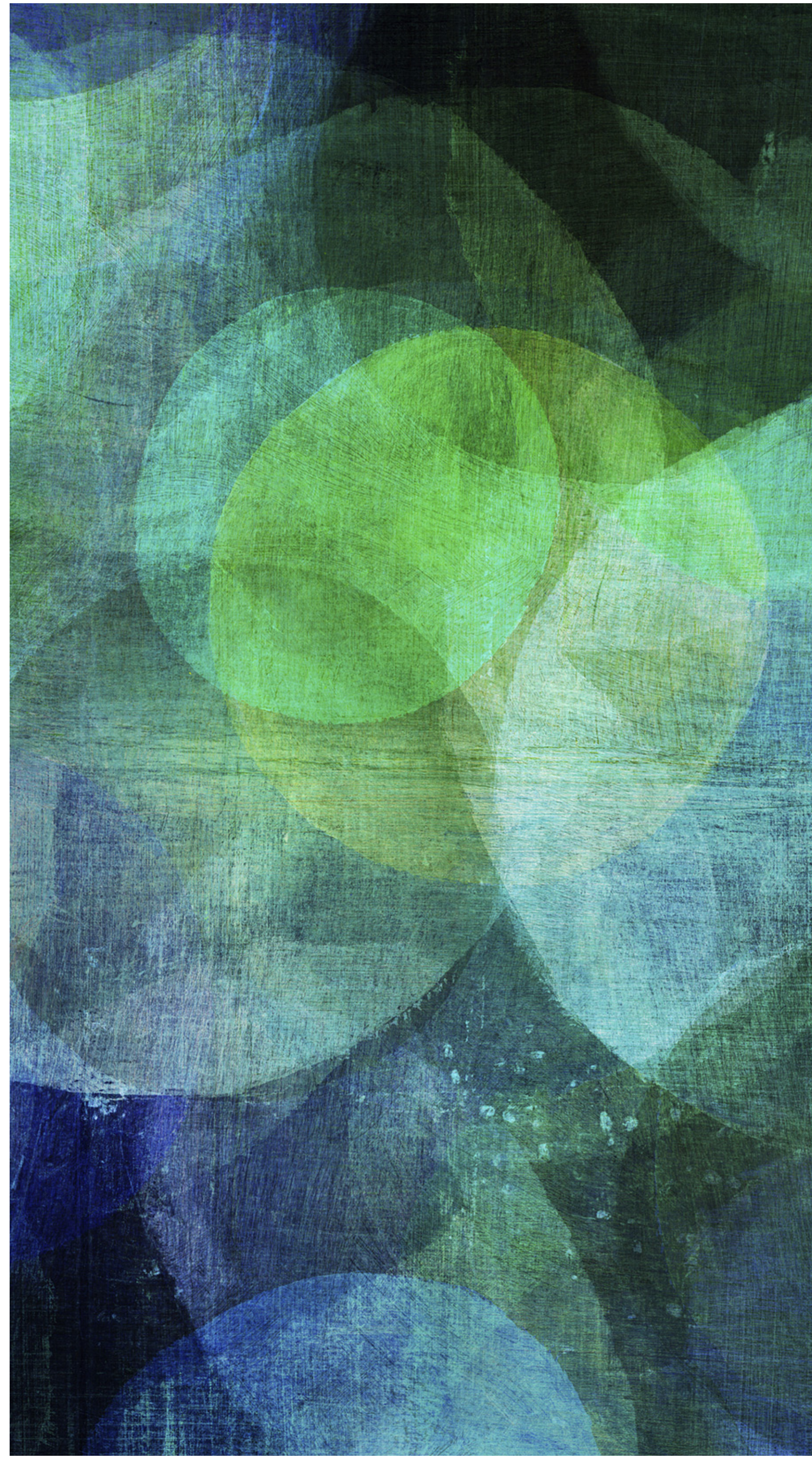


FIGURE 3.13 Rank-based selection strategy using a linear ranking.

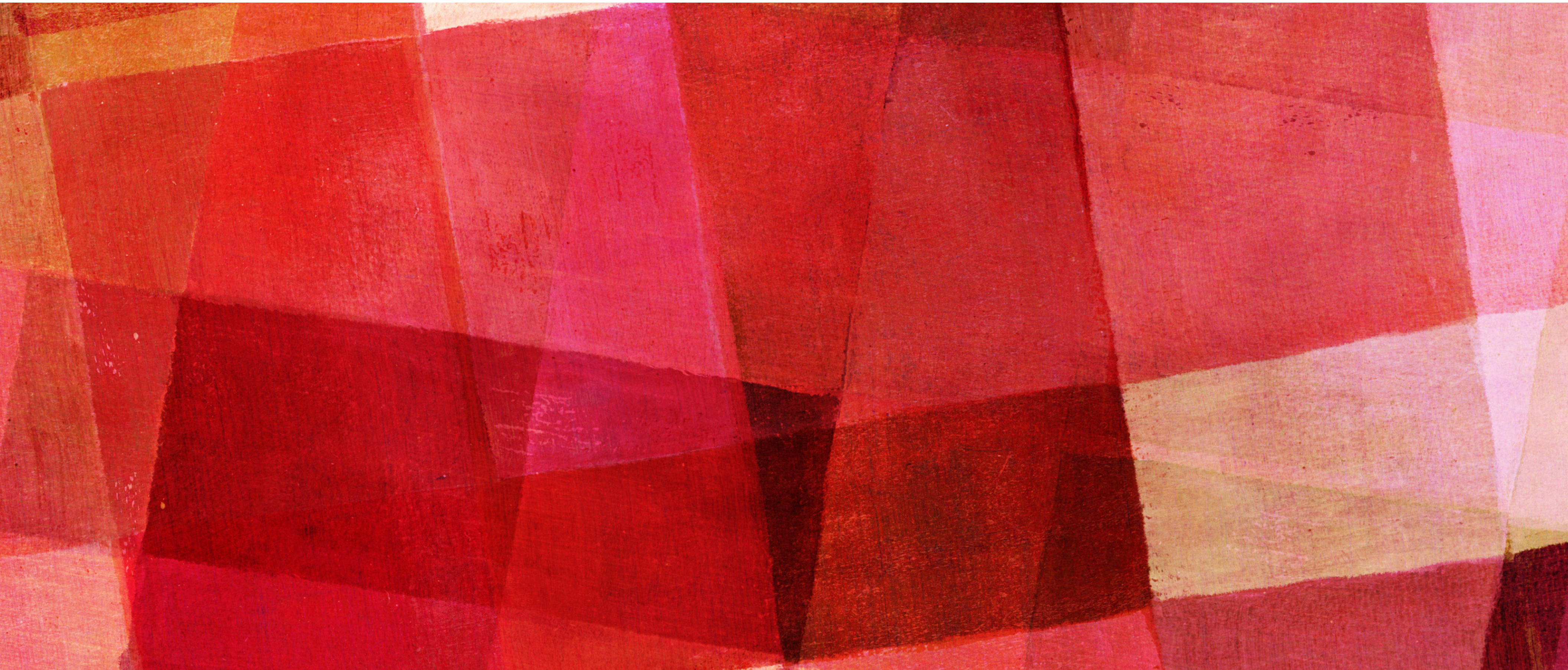
SS



DISCUSS

With a current population

How will you SELECT the parents to maximize the possibility of evolution?



EVOLUTIONARY ALGORITHMS

REPRODUCTION

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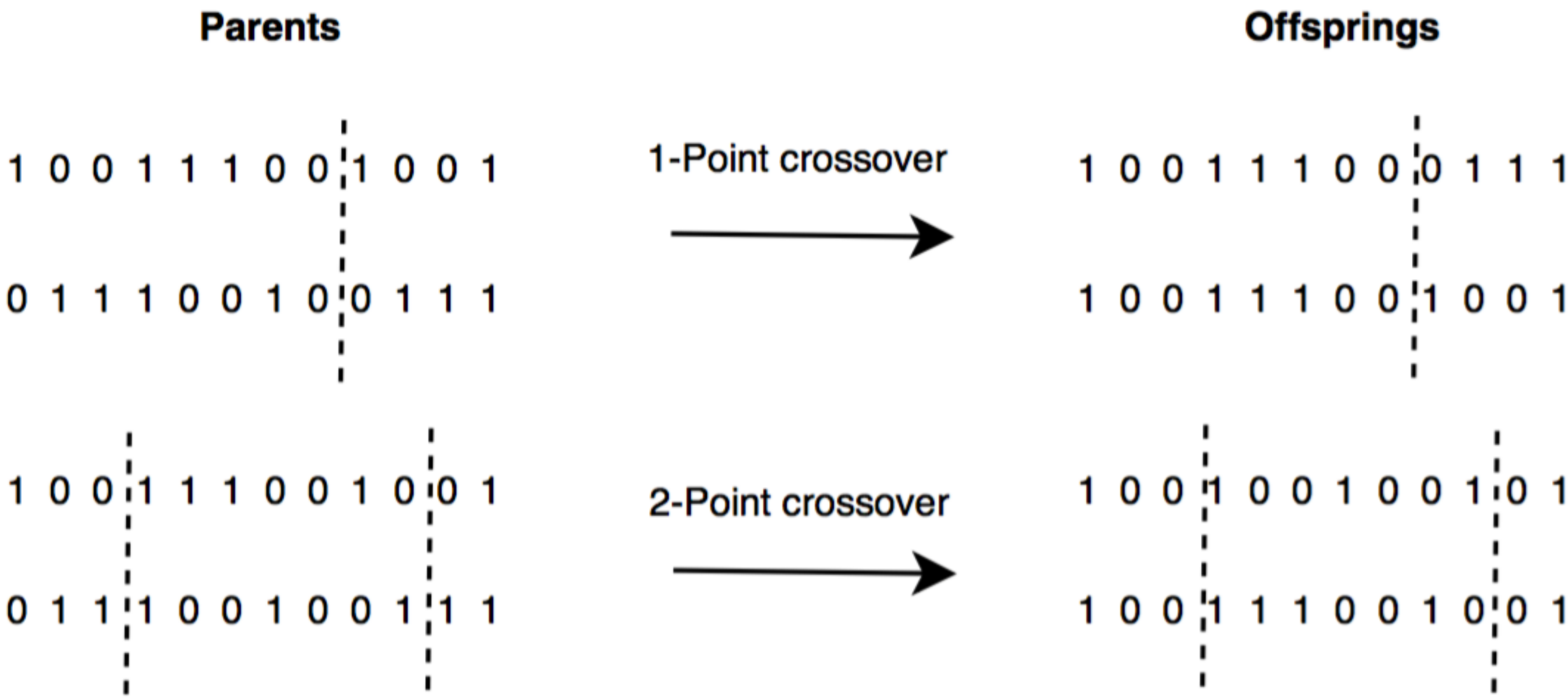
REPRODUCTION

- Crossover (X-OVER) or Recombination
- Mutation

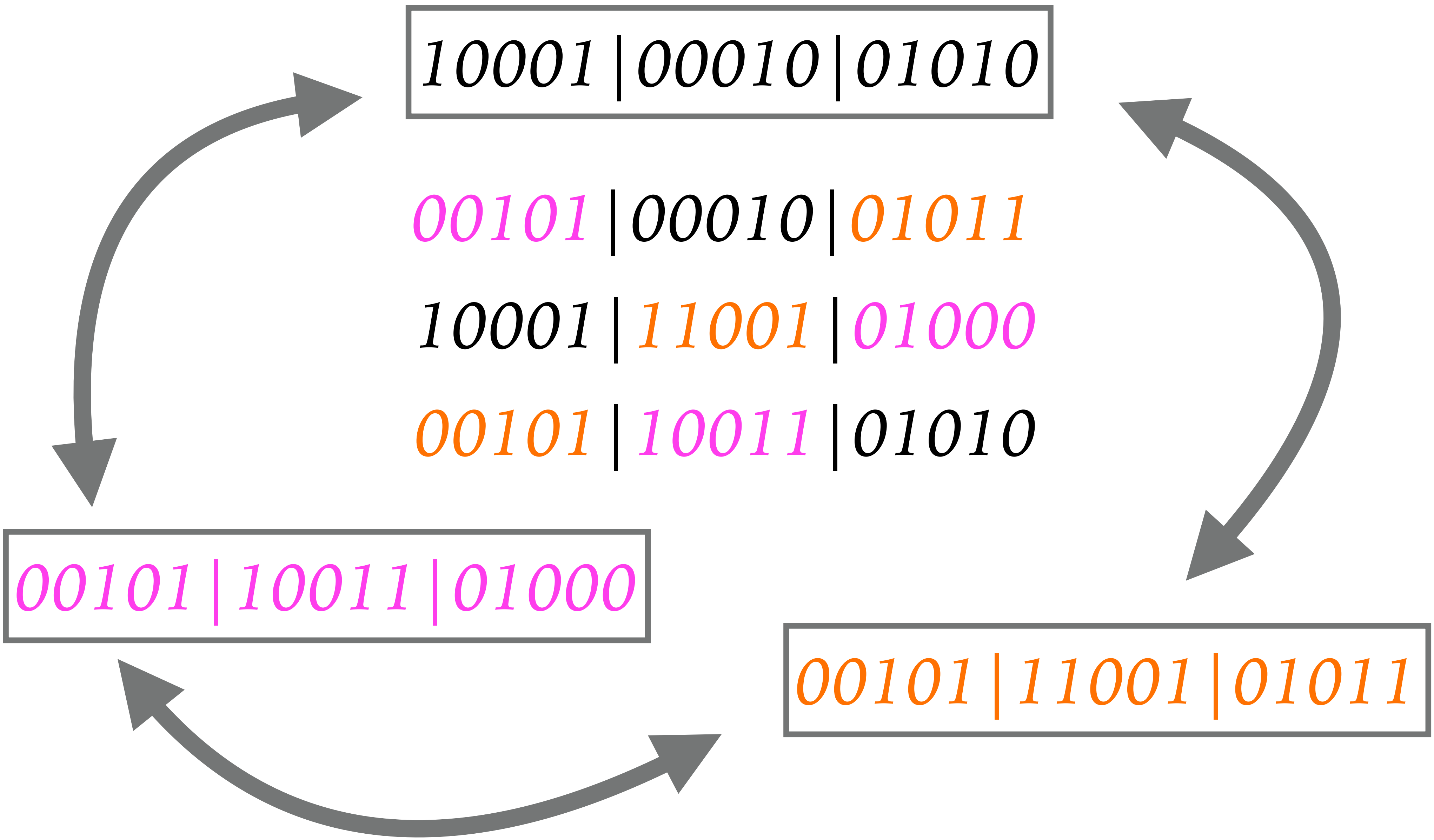
X-OVER

- Unlike unary operators such as mutation, the crossover operator is binary and sometimes n-ary. The role of crossover operators is to inherit some characteristics of the two parents to generate the offsprings.
- Characteristics
  - Heritability: The main characteristic of the crossover operator is heritability. The crossover operator should inherit a genetic material from both parents.
  - Validity: The crossover operator should produce valid solutions.

ONE POINT AND TWO POINT X-OVERS



MULTIPLE PARENTS - SODOMA AND GOMORA X-OVER



UNIFORM X-OVER

P1 11111111111
P2 00000000000

O1 10010011100

MORE X-OVERS

- Intermediate crossover: The intermediate crossover operators attempt to average the elements of the parents. Given two parents p1 and p2, the arithmetic crossover creates an offspring o using the weighted average:

$$o_i = \alpha . x_{1i} + (1 - \alpha) . x_{2i}$$

- Geometrical crossover: The geometrical crossover generates an offspring in The following form:

$$o_i = (x_{1i} . x_{2i})^{(1/2)}$$

ORDER X-OVER

Relative order, adjacency and position preserved

Relative order preserved

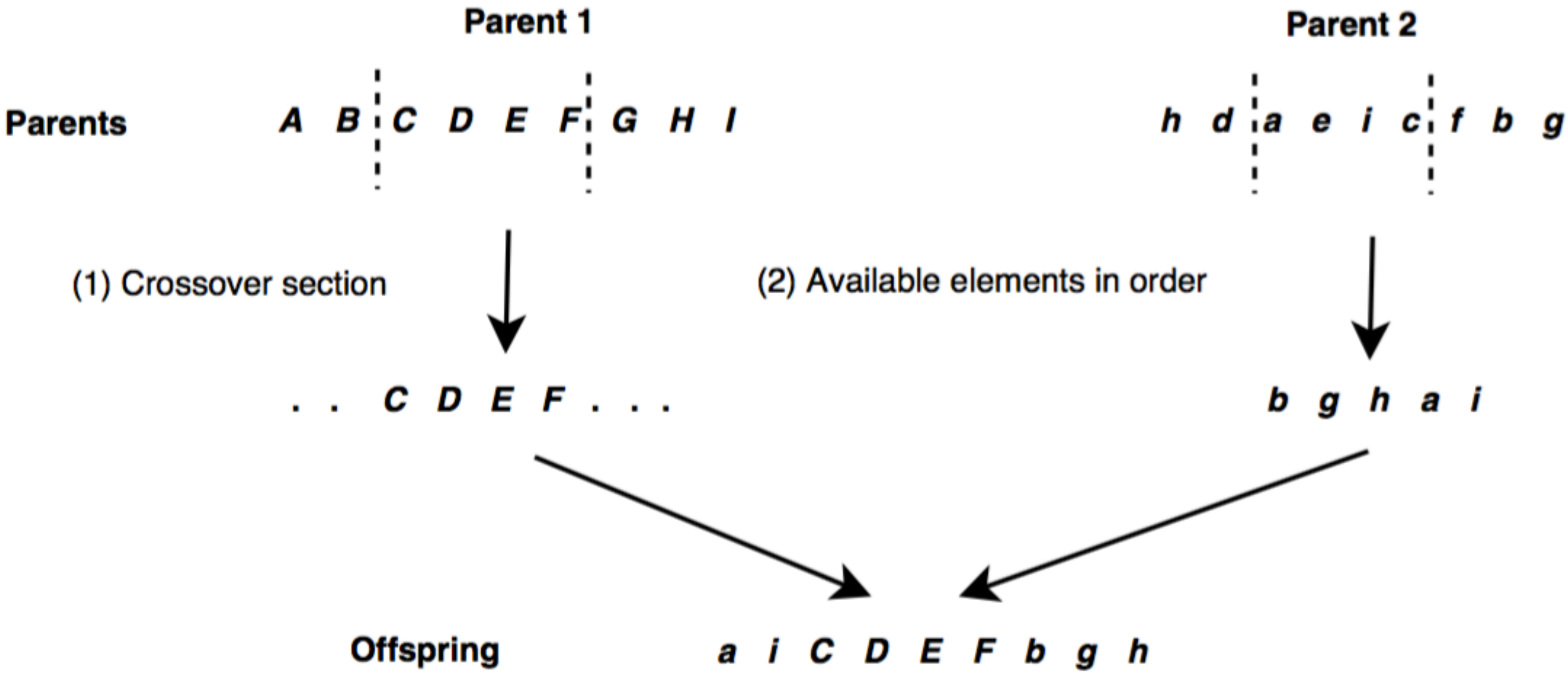
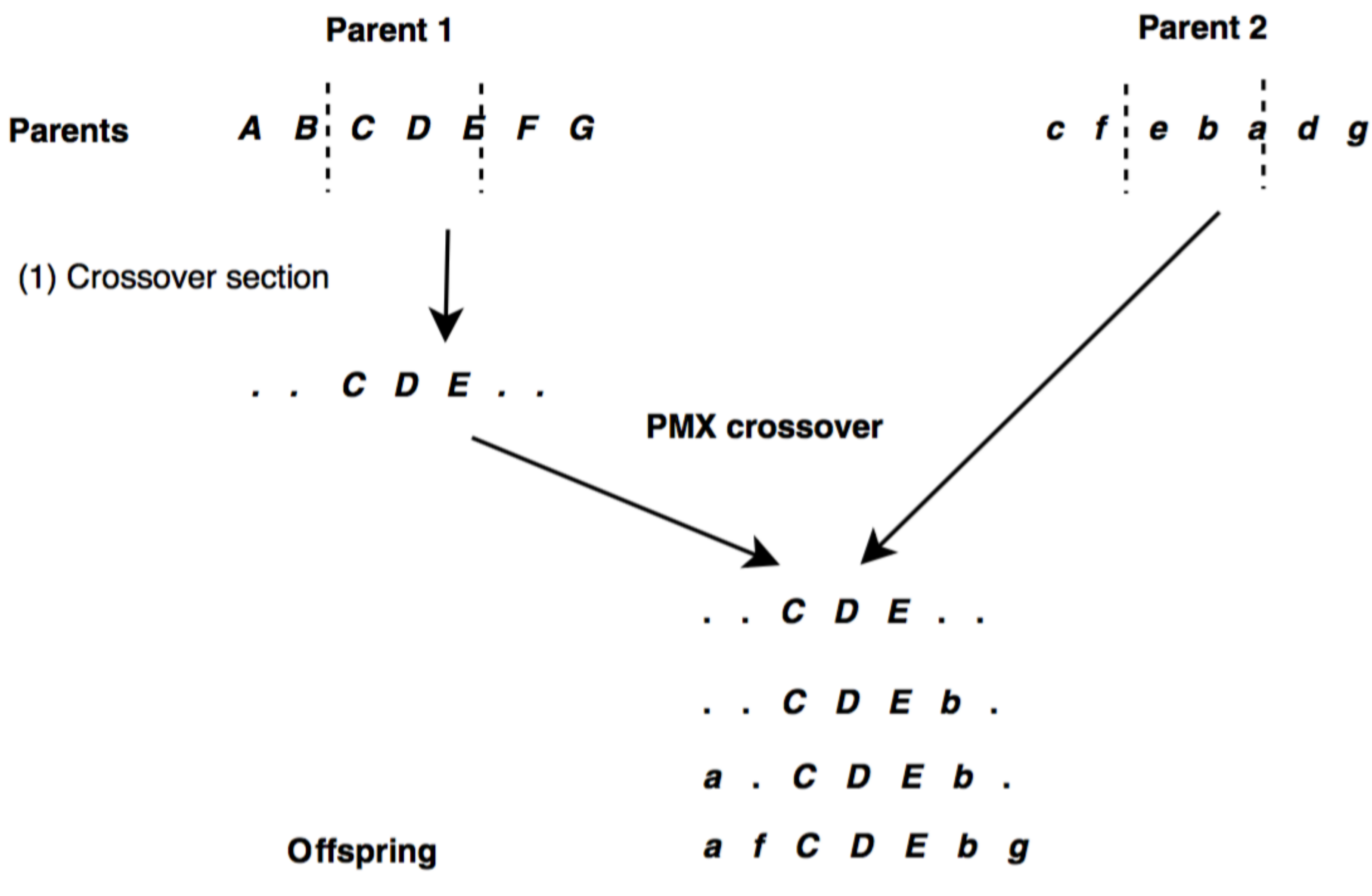


FIGURE 3.19 The order crossover for permutations.

PARTIALLY MAPPED CROSSOVER X-OVER

Relative order, adjacency and position preserved



TWO POINT X-OVER

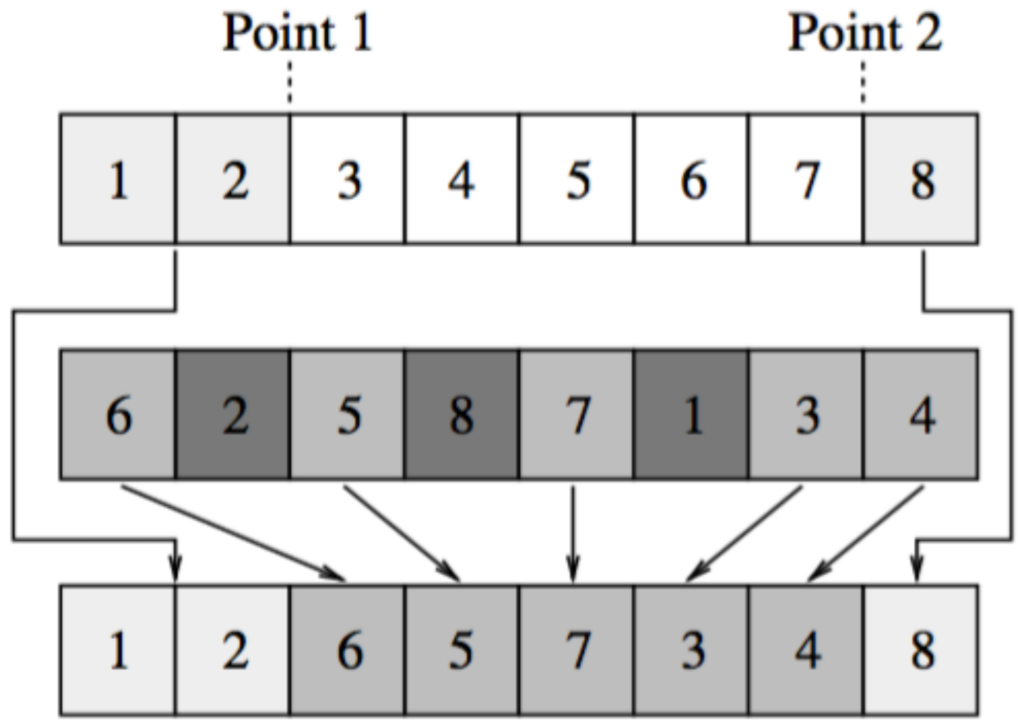
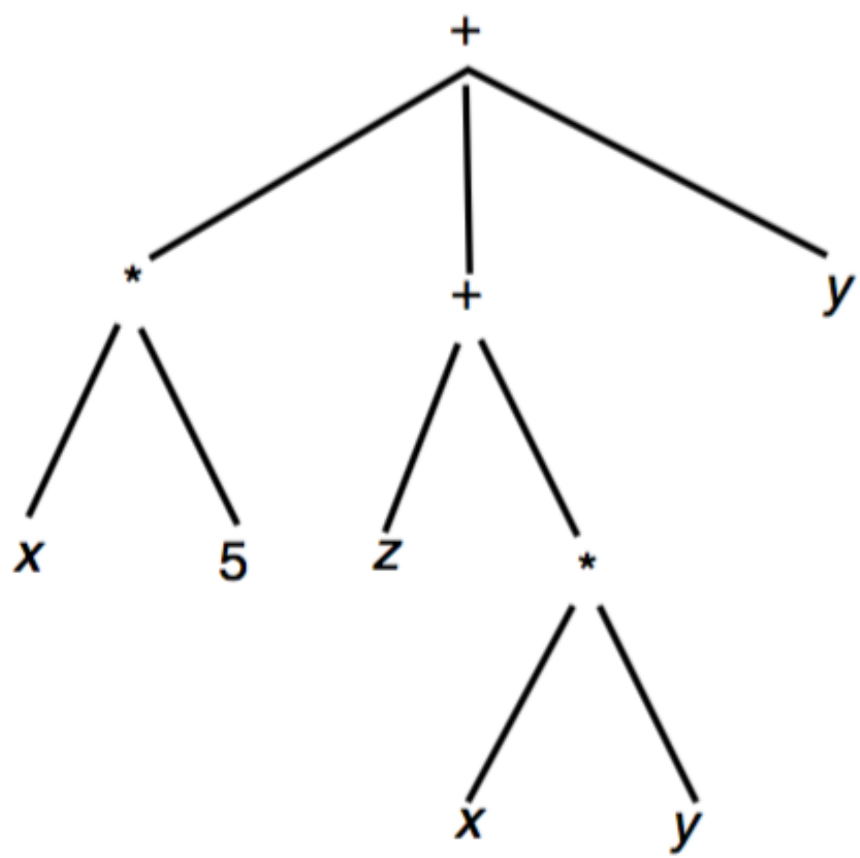


FIGURE 3.21 Two-Point crossover operator for permutations.

PARSE TREE REPRESENTATION FOR A FUNCTION



$(x*5)+(z+(x*y))+y$

X-OVER FOR PARSE TREE

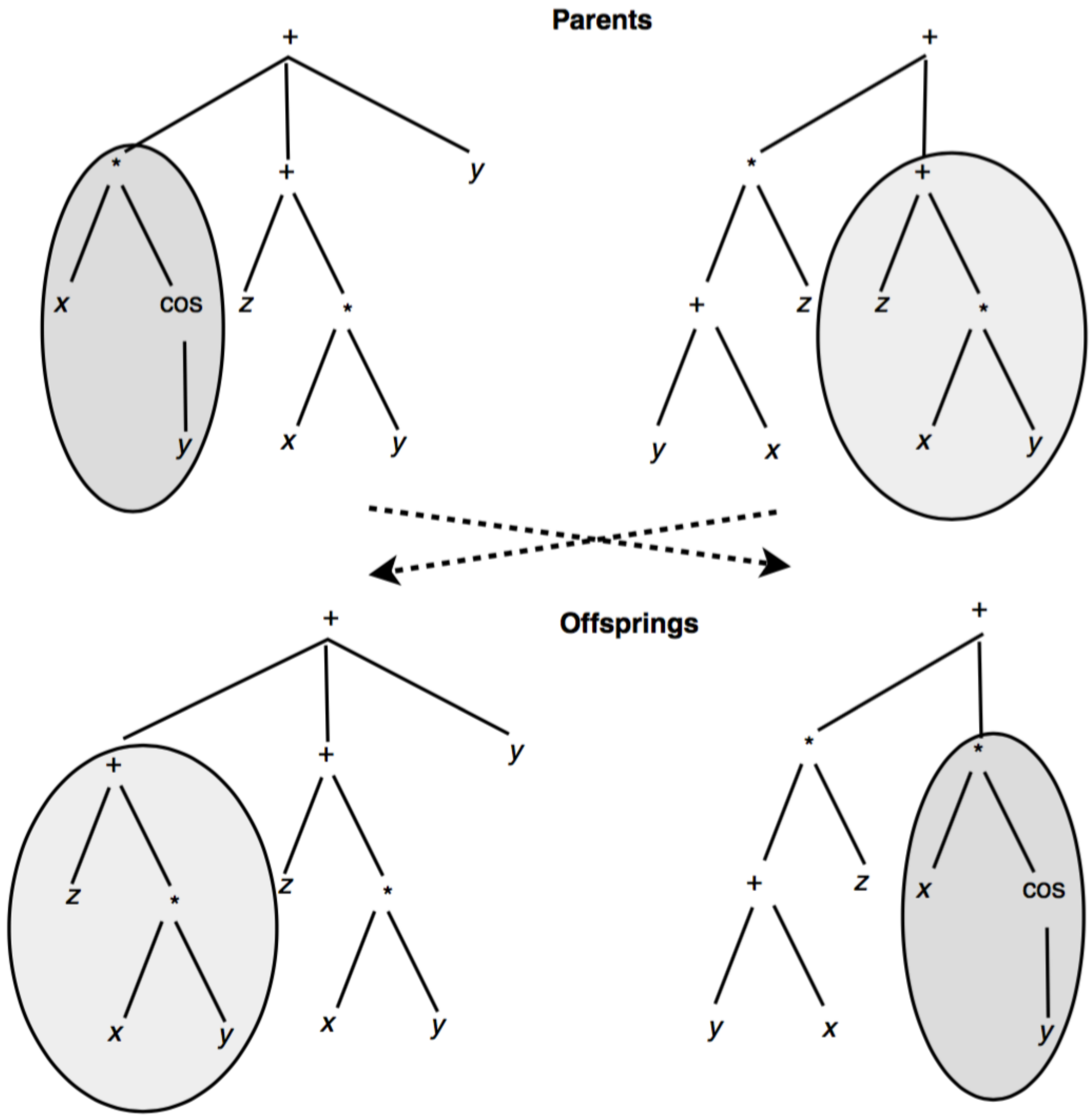
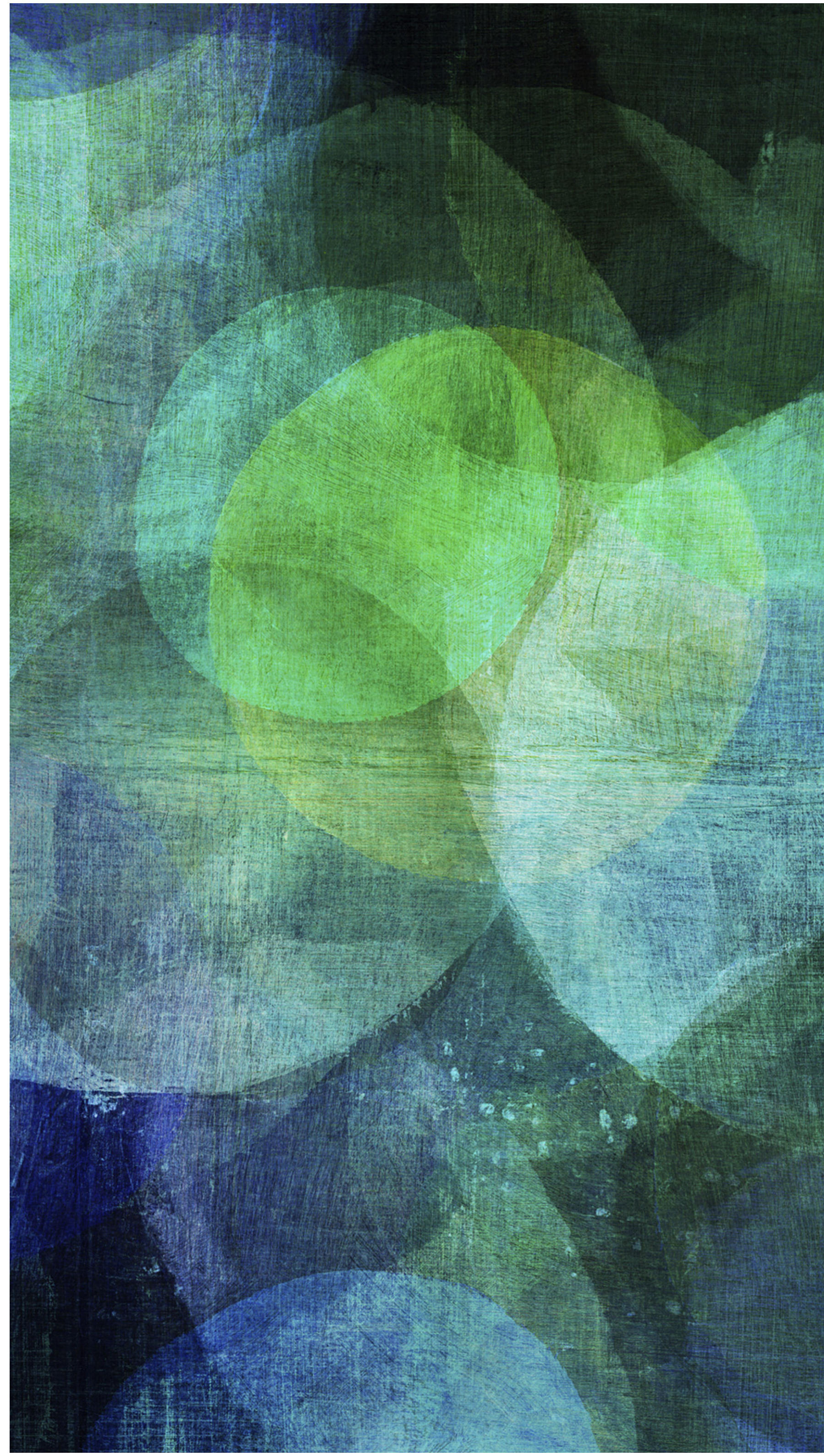


FIGURE 3.22 A crossover operator for parse tree representations.

SS



DISCUSSION

With a current population How will you REPRODUCE the parents to maximize the possibility of evolution?

EVOLUTIONARY ALGORITHMS

MUTATION

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MUTATION

- Mutation operators are unary operators acting on a single individual.
- Mutations represent small changes of selected individuals of the population.
- Characteristics
  - Ergodicity: The mutation operator should allow every solution of the search space to be reached
  - Validity: The mutation operator should produce valid solutions.
  - Locality: The mutation should produce a minimal change in solution.

GROUPING PROBLEM

AAAABBBBB

AABABBBBB

No disruptive mutation

Keeps the solution structure

AAAABBBBB

AACABBBBB

Highly disruptive mutation

It creates a new group, changing the solution structure

BINARY REPRESENTATION

100010001001010

100010101001010

INTEGER REPRESENTATION

1432517563290 15

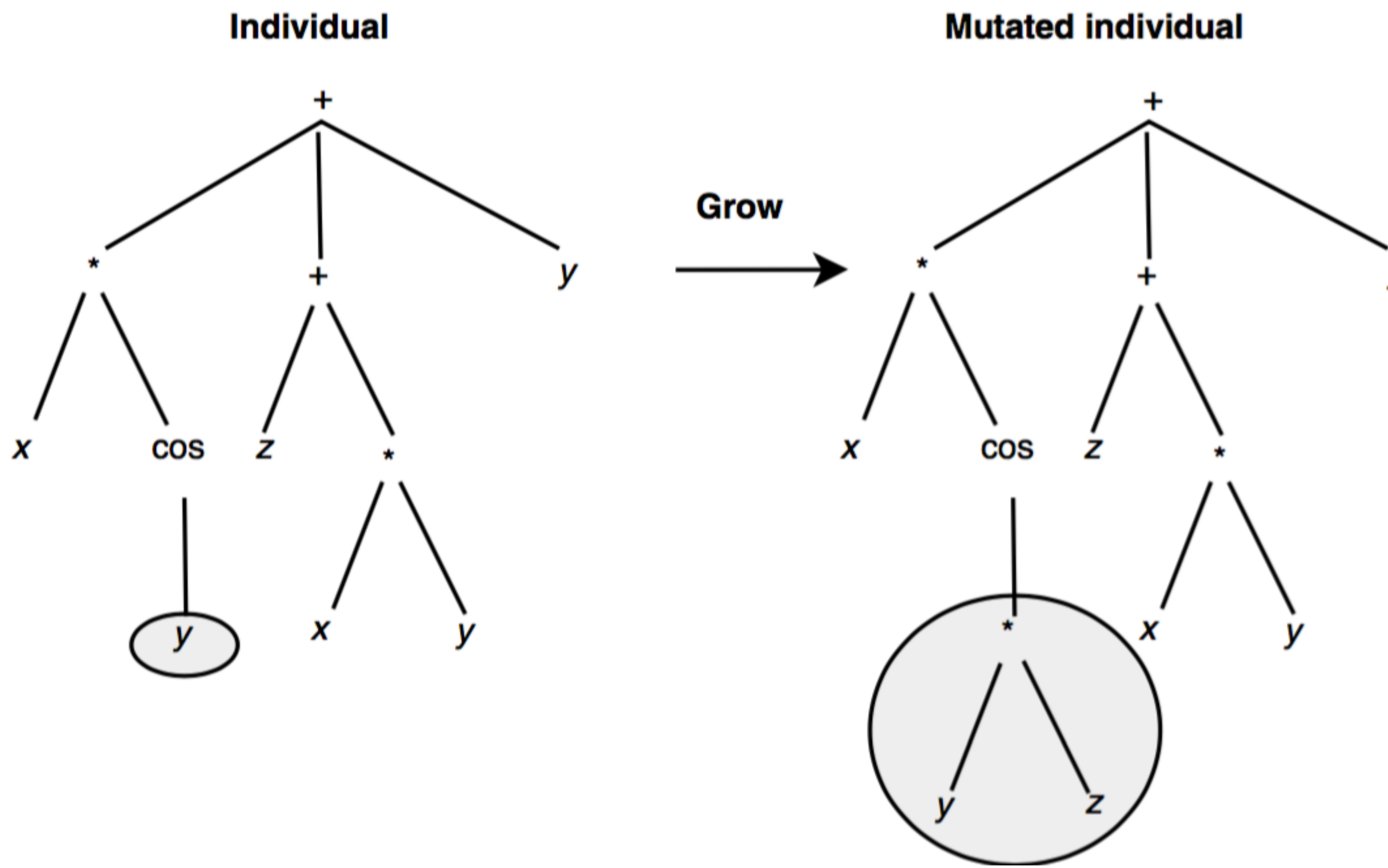
1432519563290 15

PERMUTATION REPRESENTATION

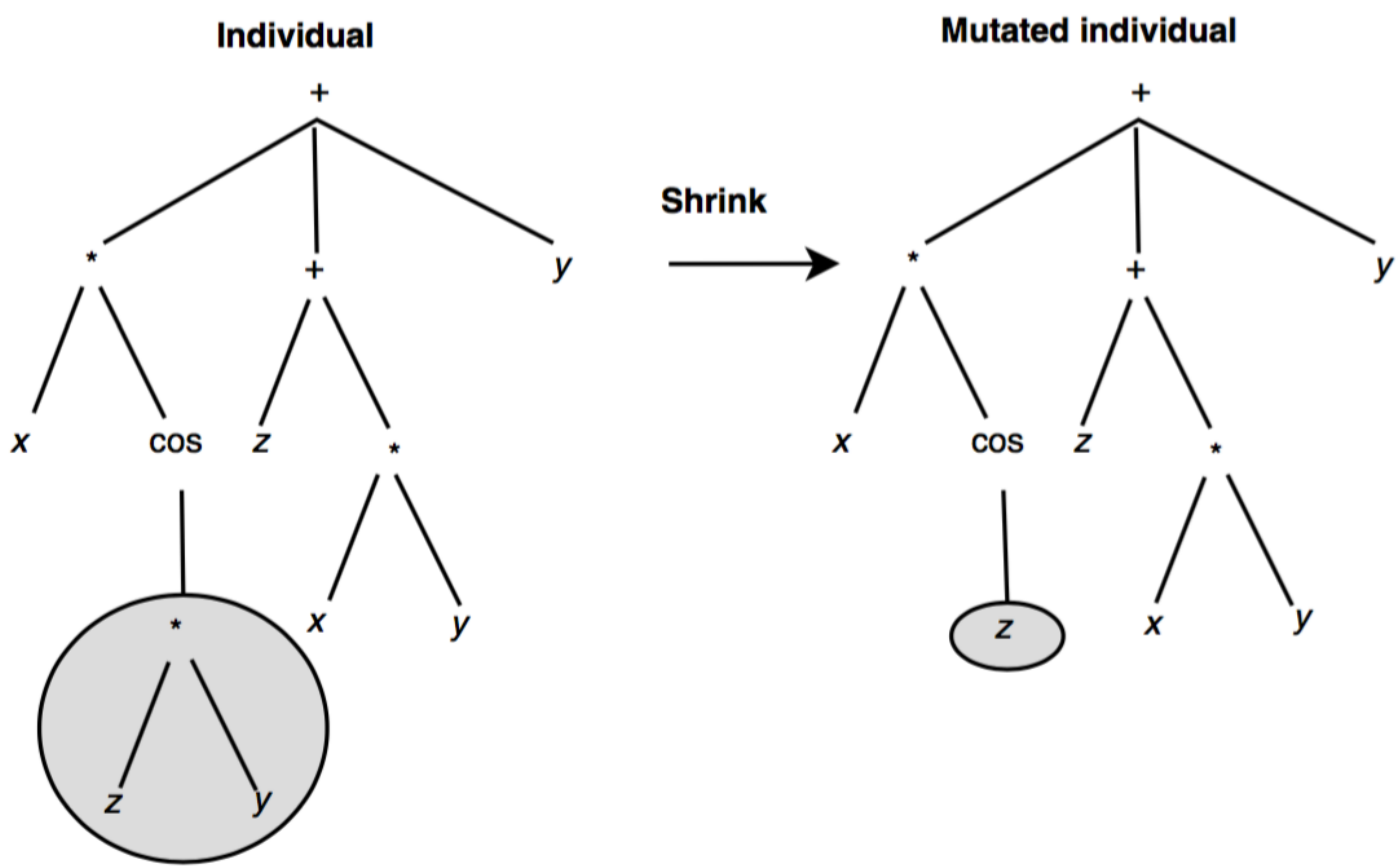
143251756329015

153251746329015

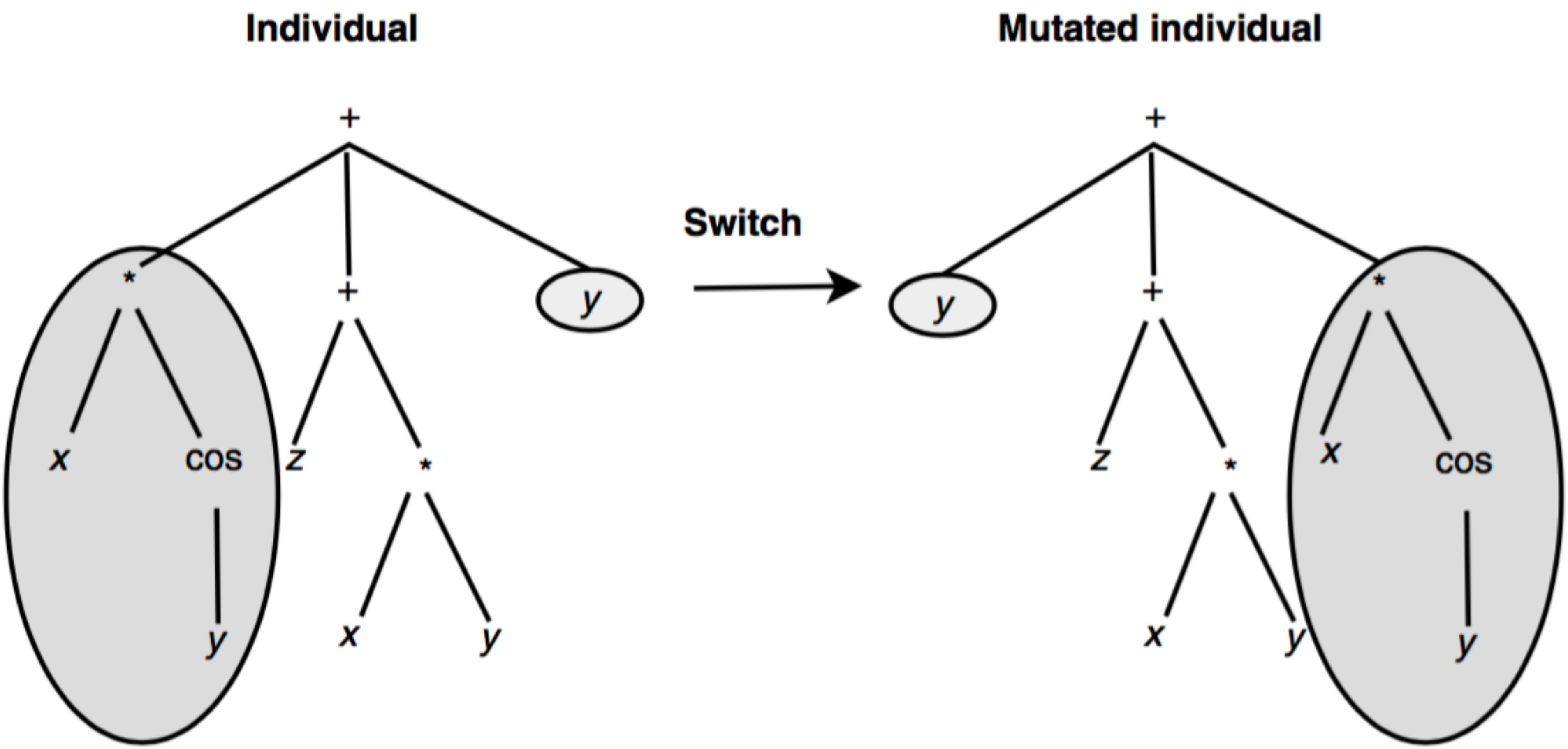
PARSE TREE MUTATION: GROW



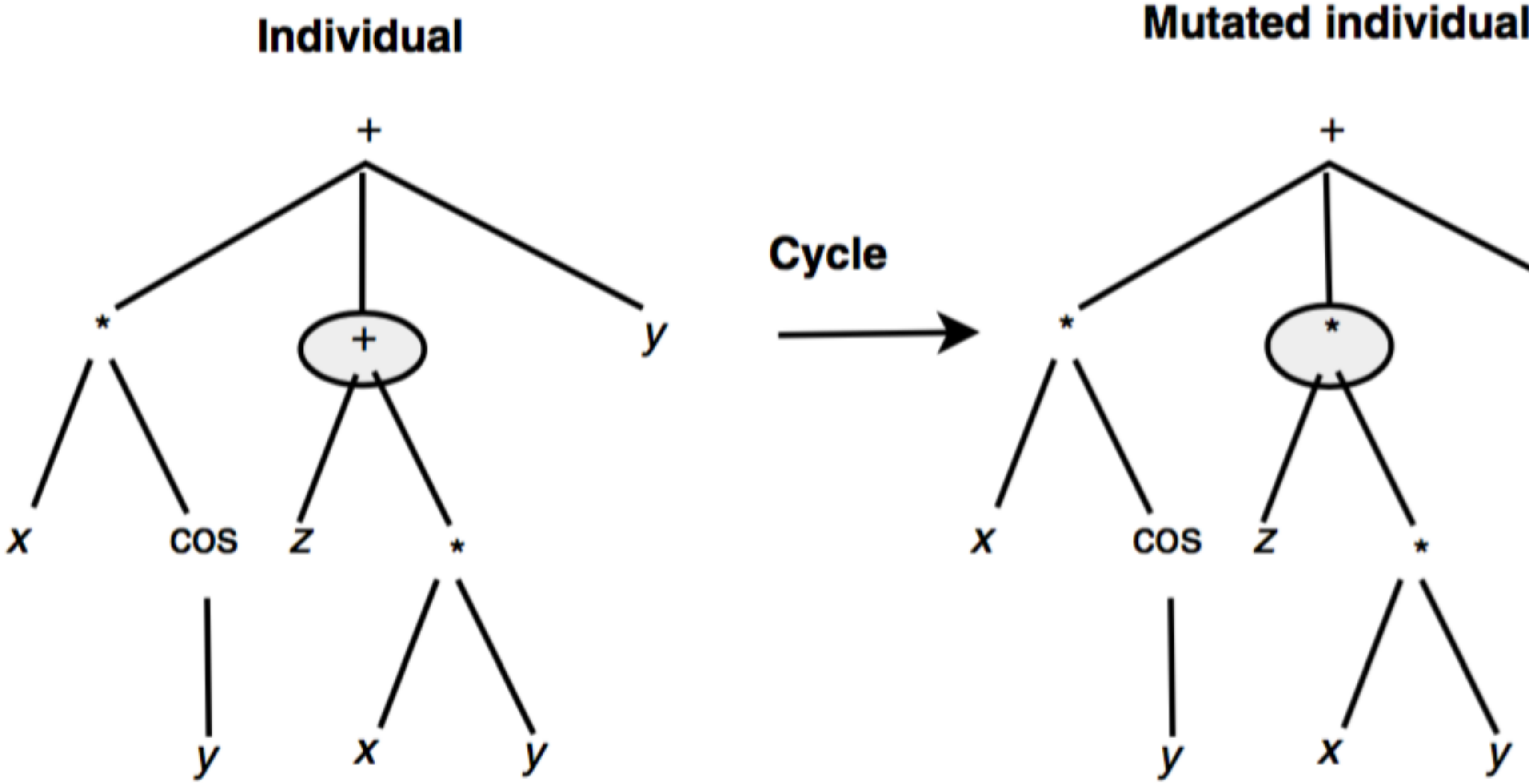
PARSE TREE MUTATION: SHRINK



PARSE TREE MUTATION: SWITCH



PARSE TREE MUTATION: CYCLE



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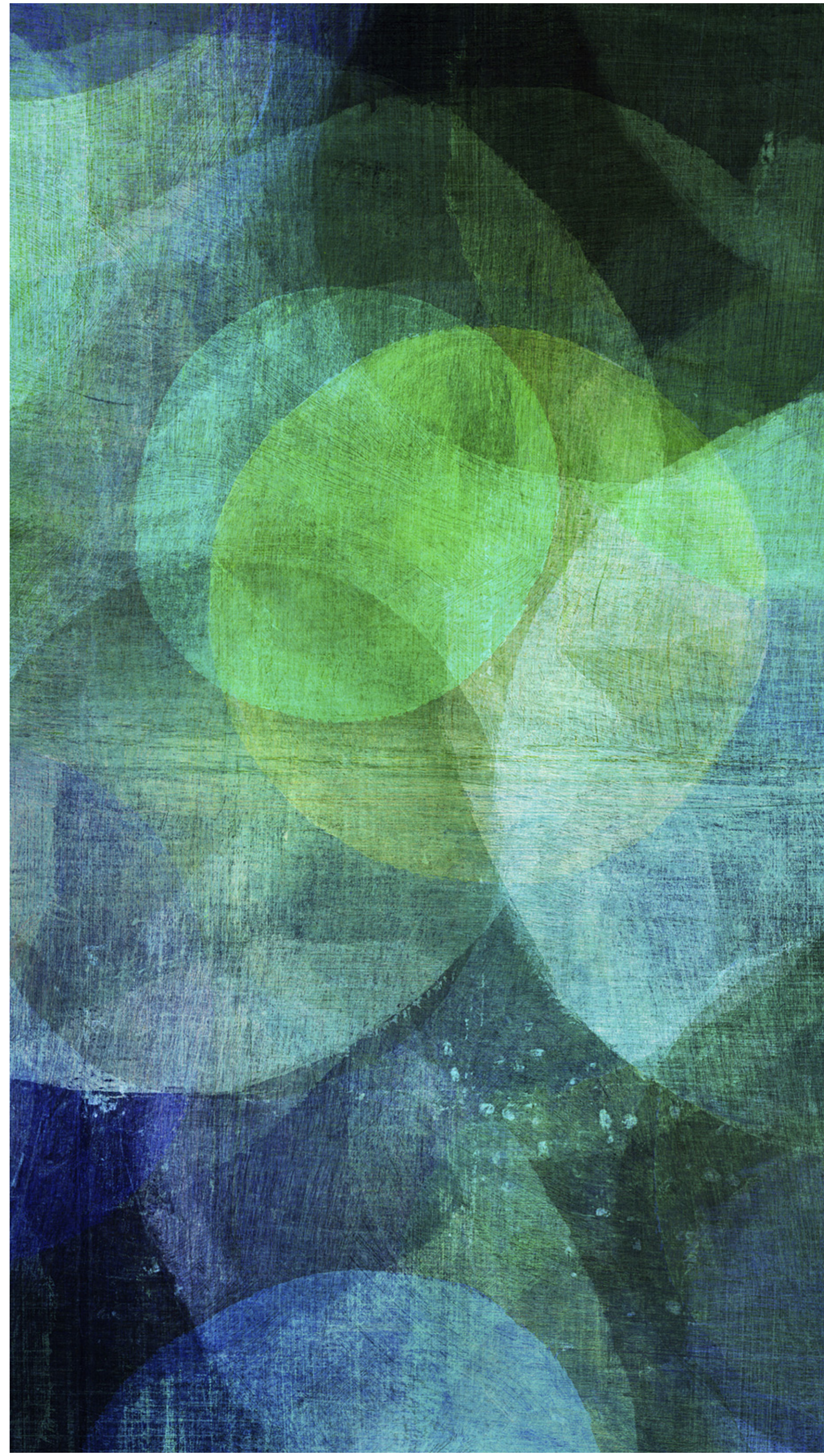

DISCUSSION

With the current population
How will you MUTATE the parents to maximize the possibility of evolution?

EVOLUTIONARY ALGORITHMS

MUTATION

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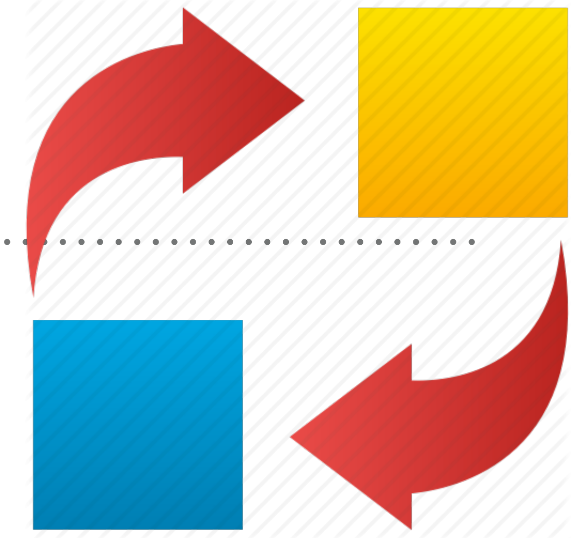
REPLACENT

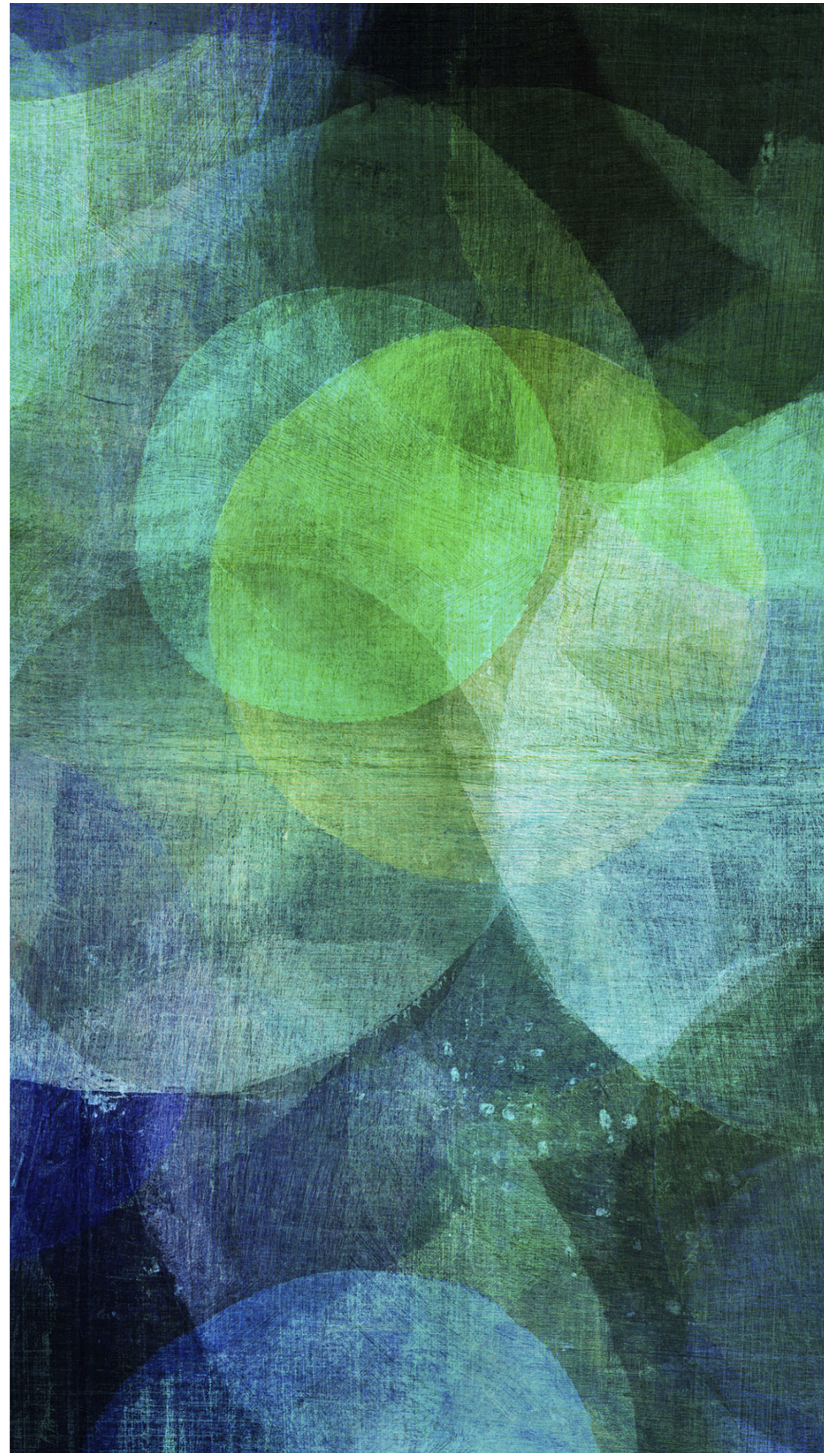


- The replacement phase concerns the survivor selection of both the parent and the offspring populations.
  - Generational replacement: The replacement will concern the whole population of size μ.
  - Steady-state replacement: At each generation of an EA, only one offspring is generated.

PARAMETERS

- Mutation probability: A large mutation probability will disrupt a given individual and the search is more likely random.
- Crossover probability pc: The crossover probability is generally set from medium to large values
- Population size: The larger is the size of the population, the better is the convergence toward “good” solutions. Sampling errors are more important in smaller populations.

SS

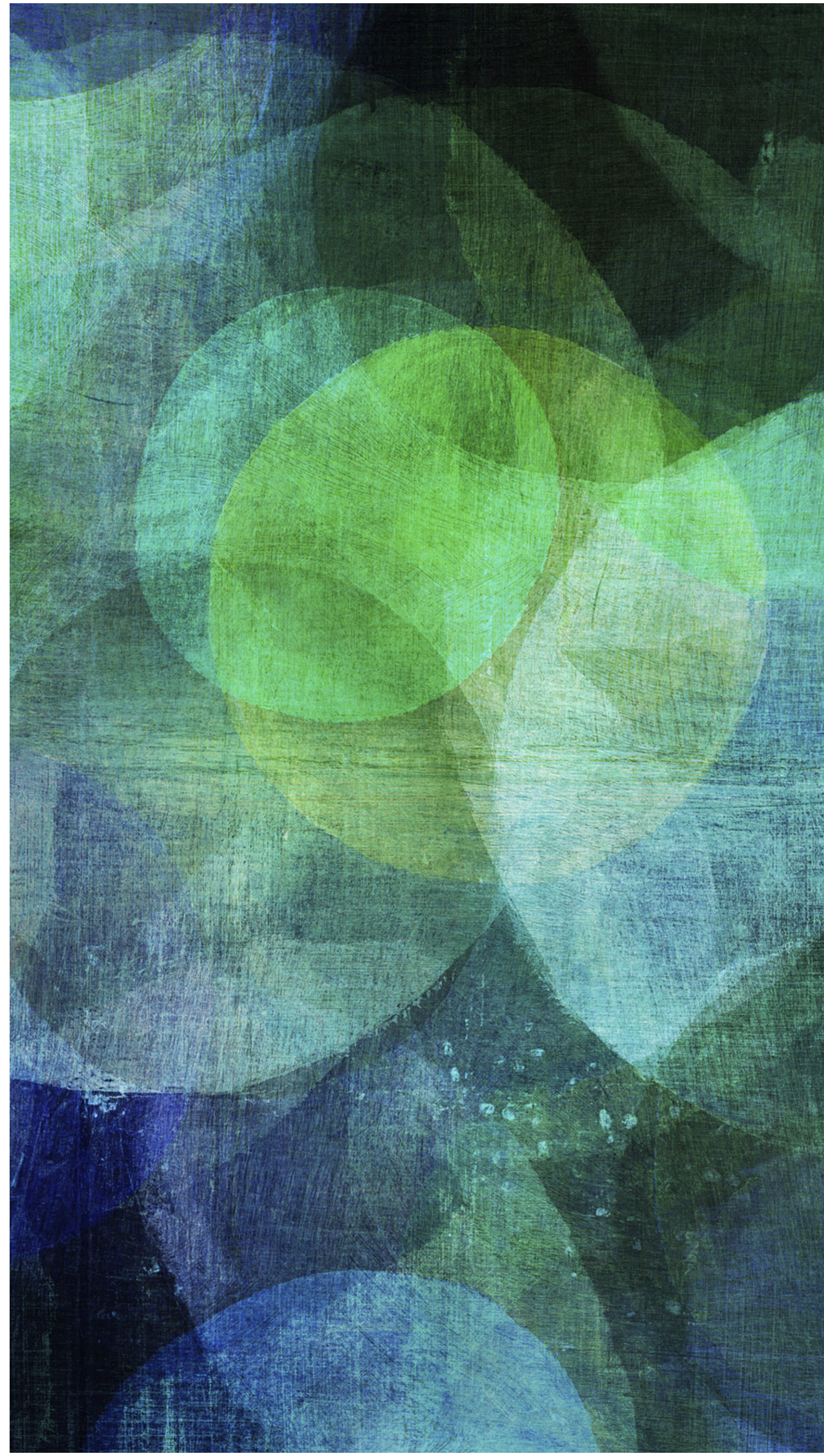


DISCUSSION

With the current population

How will you REPLACE THE CURRENT POPULATION to maximize the possibility of evolution?

SS
SS -> C



WORK, WORK, WORK!

With the WORKSHEET perform at least 3 generations of the method you just designed.

PRESENT YOUR RESULTS TO THE CLASS

CODING TIPS

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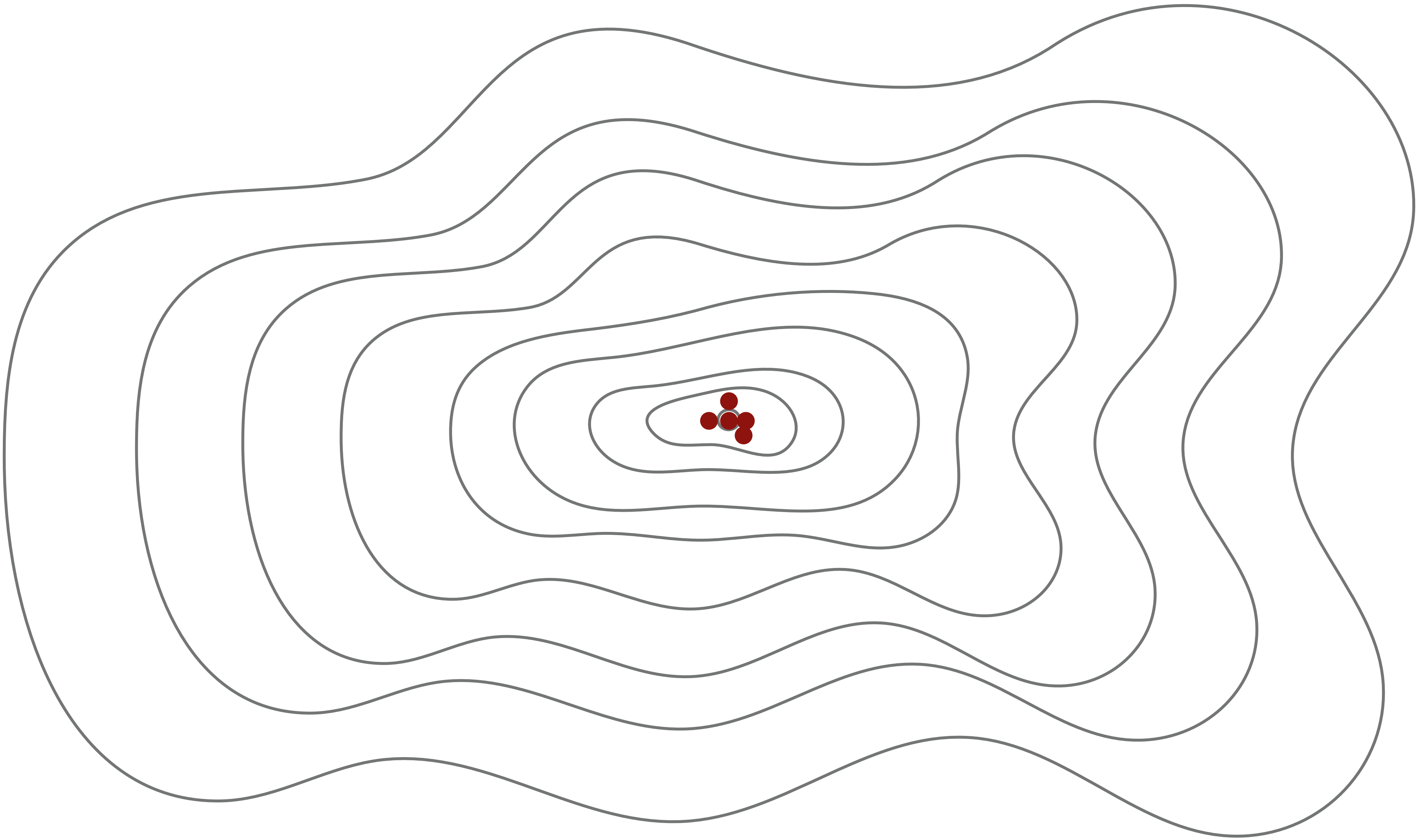
LET'S TALK ABOUT VARIABILITY LOSS



INTENSIFICATION

VS

DIVERSIFICATION

INITIAL POPULATION

- In the generation of the initial population, the main criterion to deal with is diversification.
- If the initial population is not well diversified, a premature convergence can occur for any P-metaheuristic.
- The determination of the initial population is often disregarded in the design of a P-metaheuristic.
- This step plays a crucial role in the effectiveness of the algorithm and its efficiency.

INITIAL POPULATION

TABLE 3.2 Analysis of the Different Initialization Strategies

Strategy	Diversity	Computational Cost	Quality of Initial Solutions
Pseudo-random	++	+++	+
Quasi-random	+++	+++	+
Sequential diversification	++++	++	+
Parallel diversification	++++	+++	+
Heuristic	+	+	+++

More generally, the generation of the initial population in a way to optimize a given diversification criterion can be defined as an optimization problem. This problem may be as (or more) difficult as the original problem to solve.

STOP CRITERIA

- Static procedure: In a static procedure, the end of the search may be known a priori. For instance, one can use a fixed number of iterations (generations), a limit on CPU resources, limited time or a maximum number of objective function evaluations.
- Adaptive procedure: In an adaptive procedure, the end of the search cannot be known a priori. One can use a fixed number of iterations (generations) without improvement, when an optimum or a satisfactory solution is reached, one measure of diversity is too low, etc.