

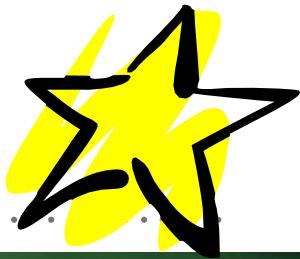


POPULATION BASED METAHEURISTICS

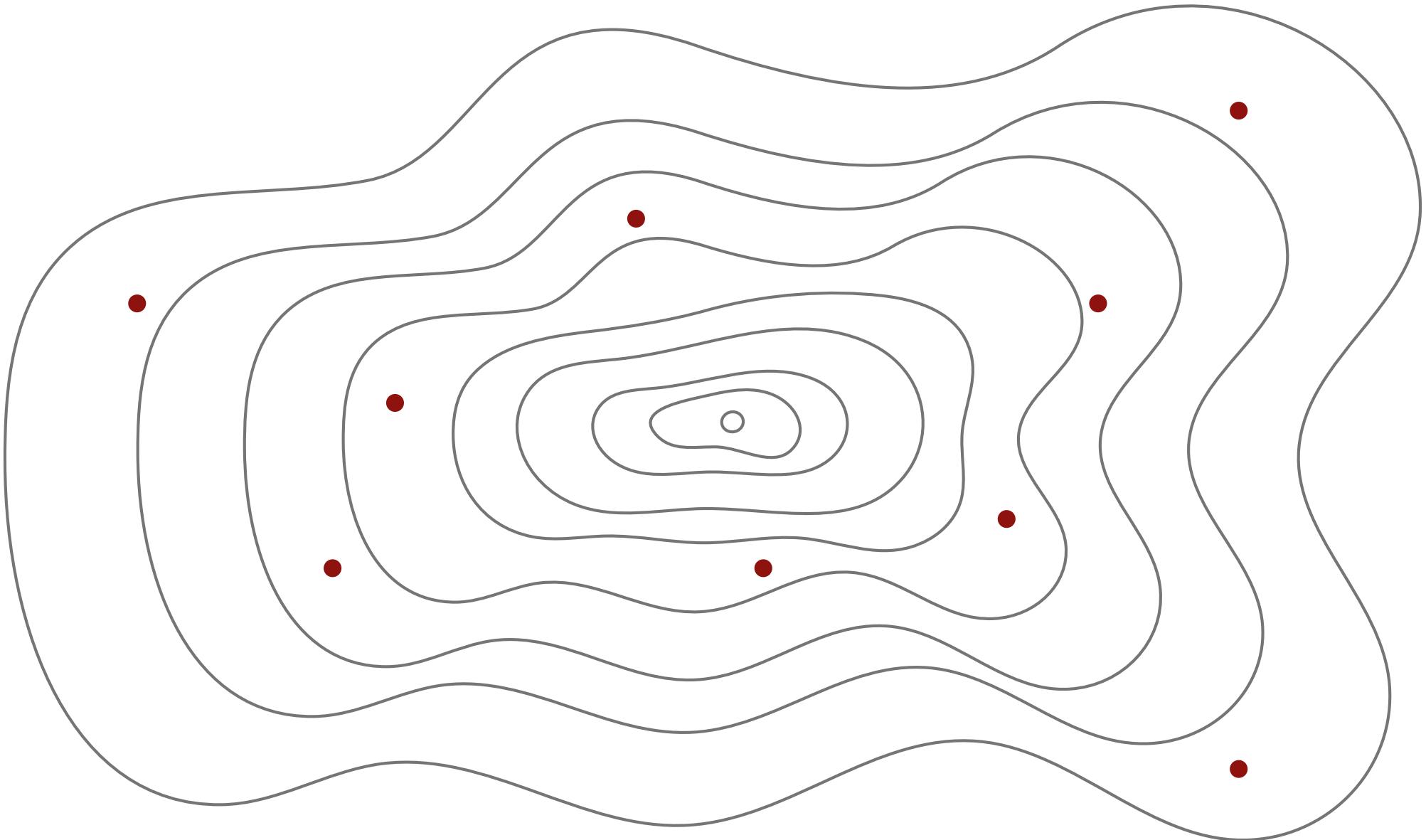
EVOLUTIVE ALGORITHMS

Prof. Eduardo Pécora

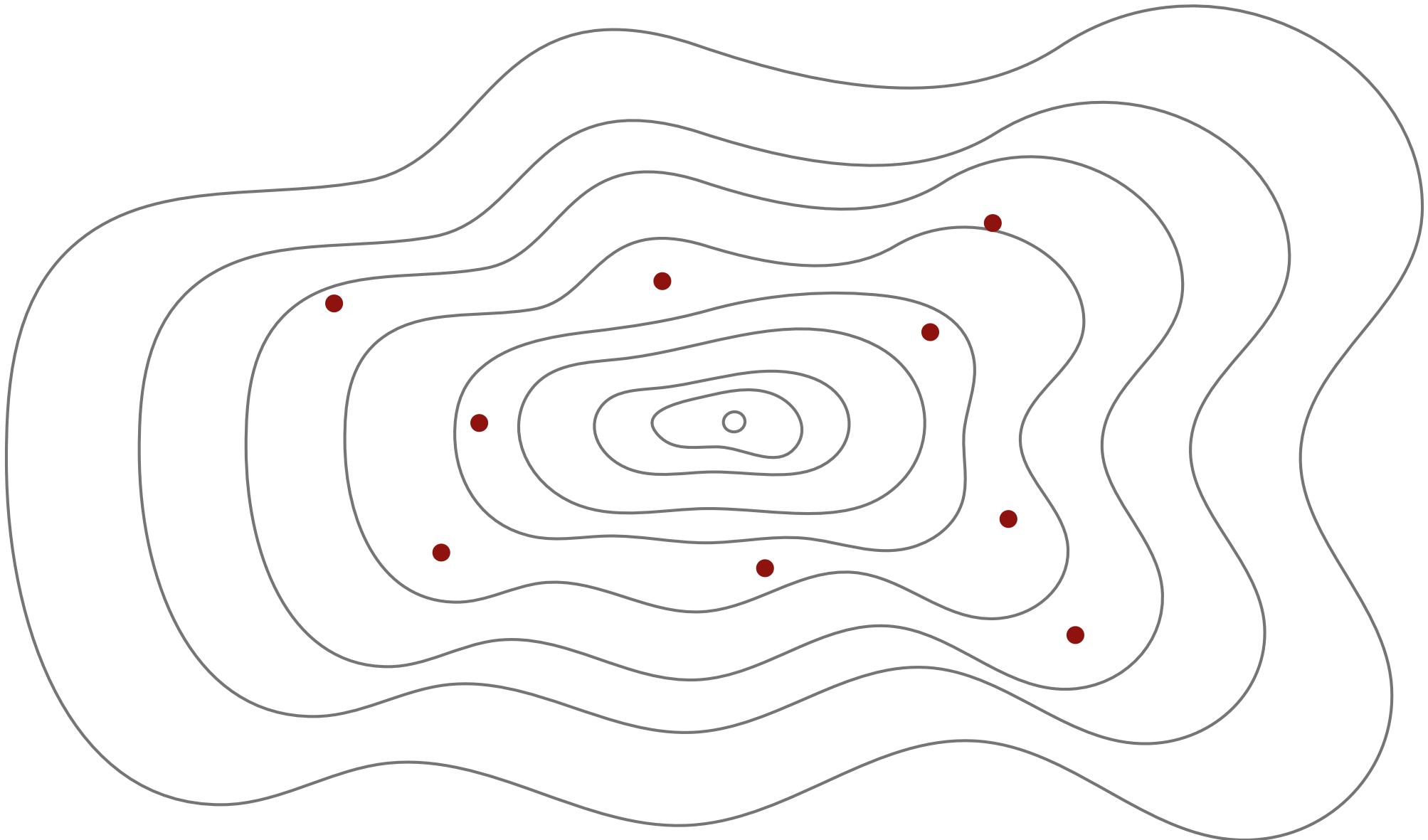
SURVIVAL OF THE FITTEST



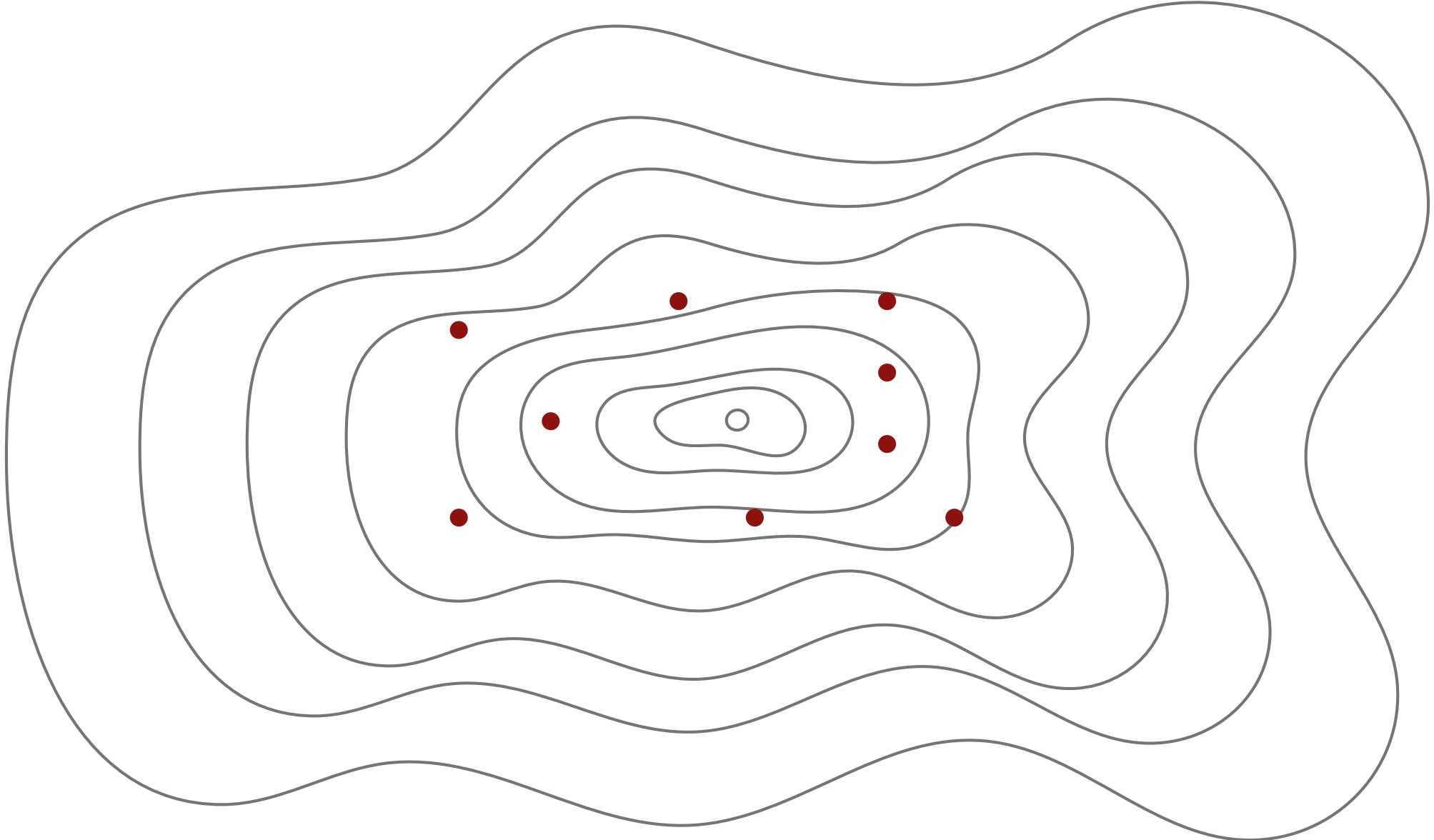
POPULATION BASED EXPLORATION



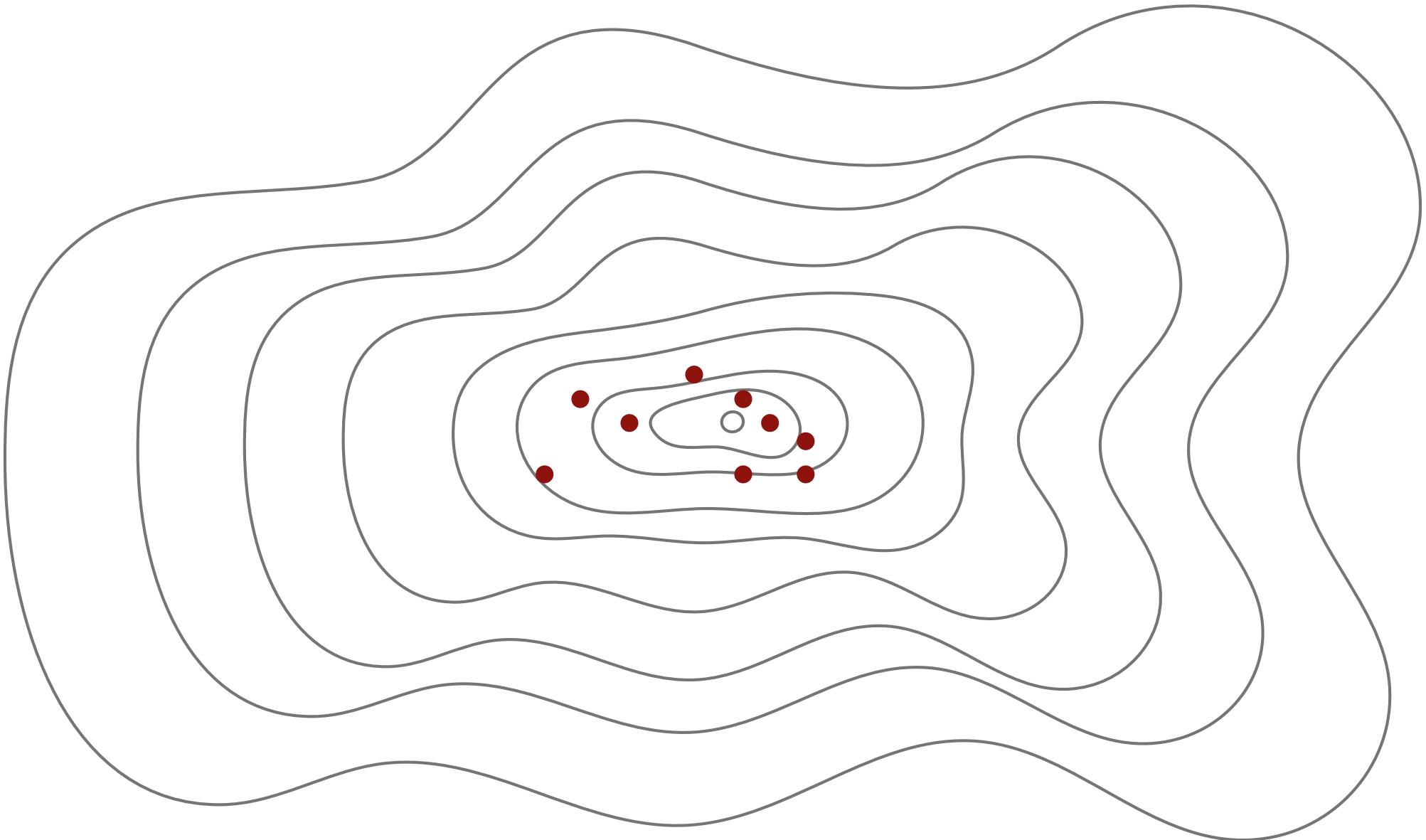
POPULATION BASED EXPLORATION



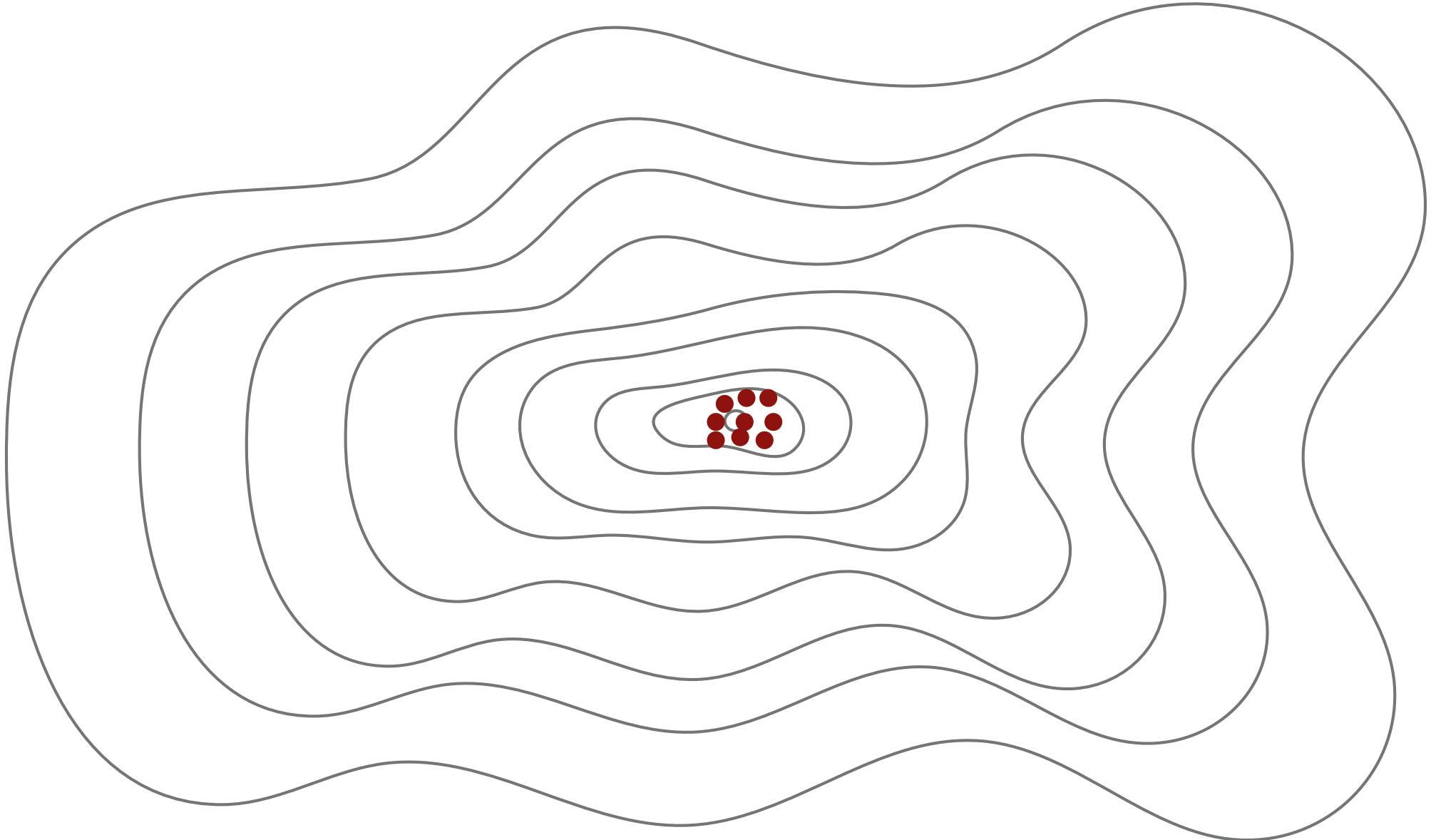
POPULATION BASED EXPLORATION



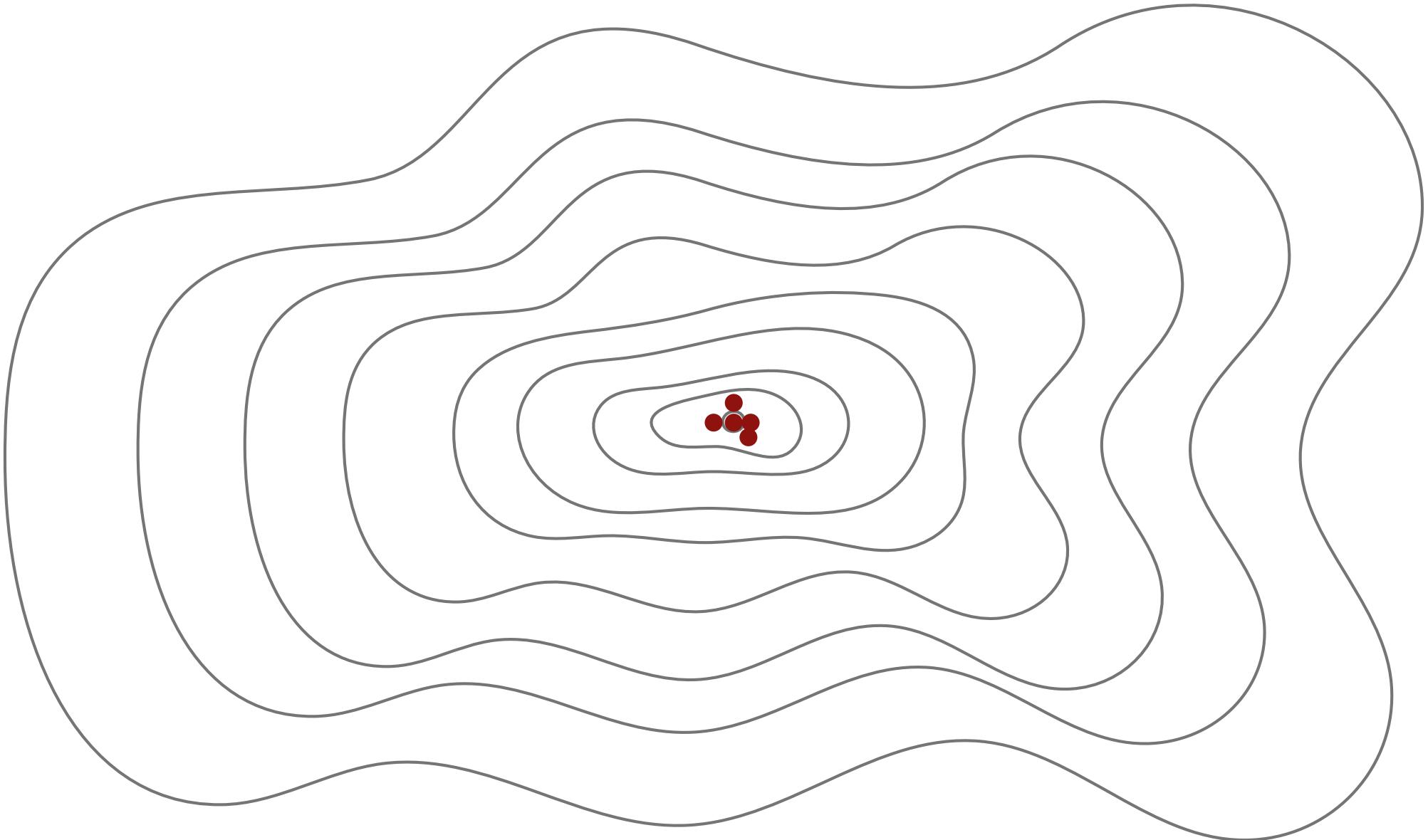
POPULATION BASED EXPLORATION



POPULATION BASED EXPLORATION

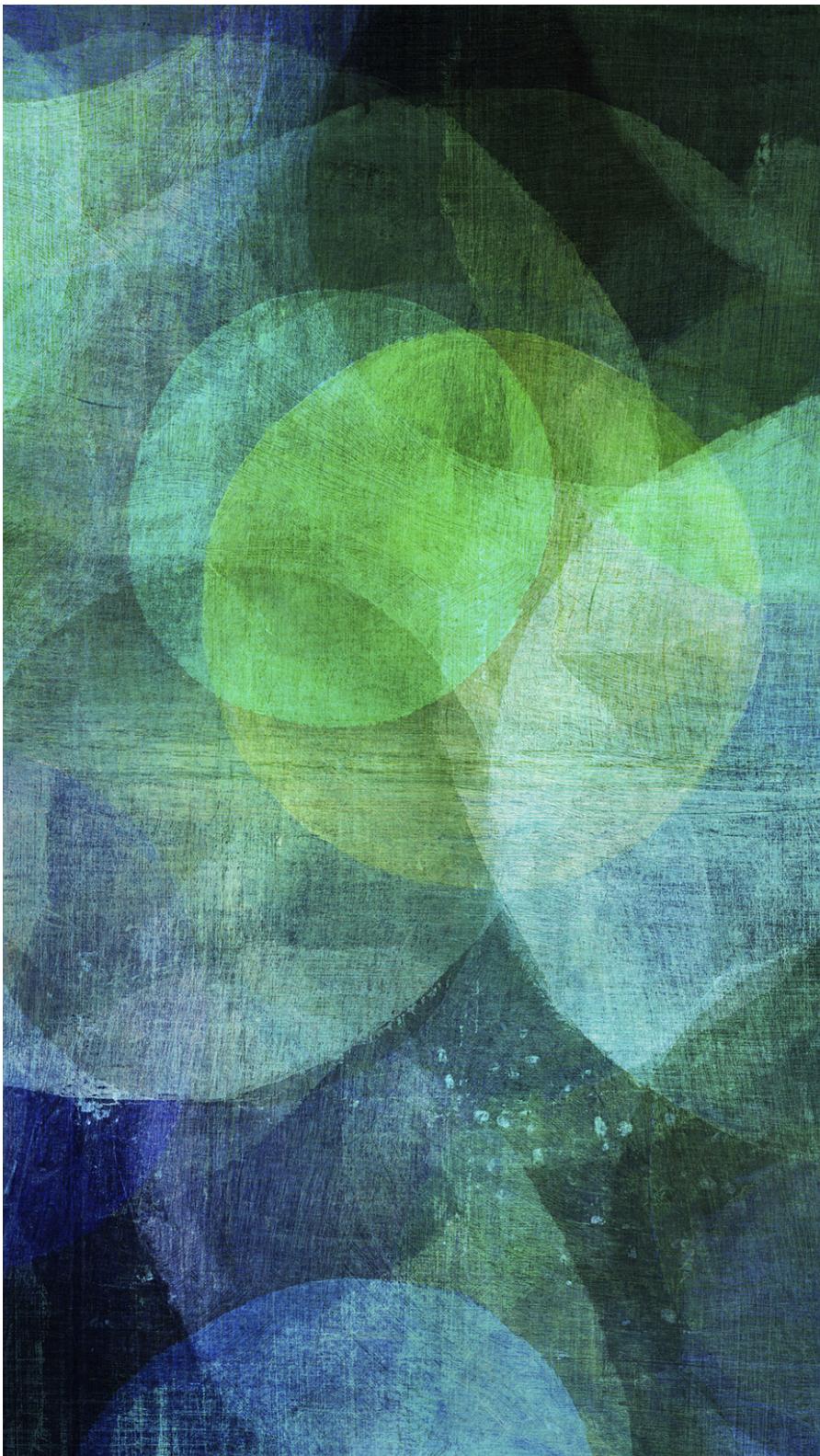


POPULATION BASED EXPLORATION



C → T

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THINK ABOUT

INTENSIFICATION

VS

DIVERSIFICATION

MAIN PRINCIPLE OF P-METAHEURISTICS

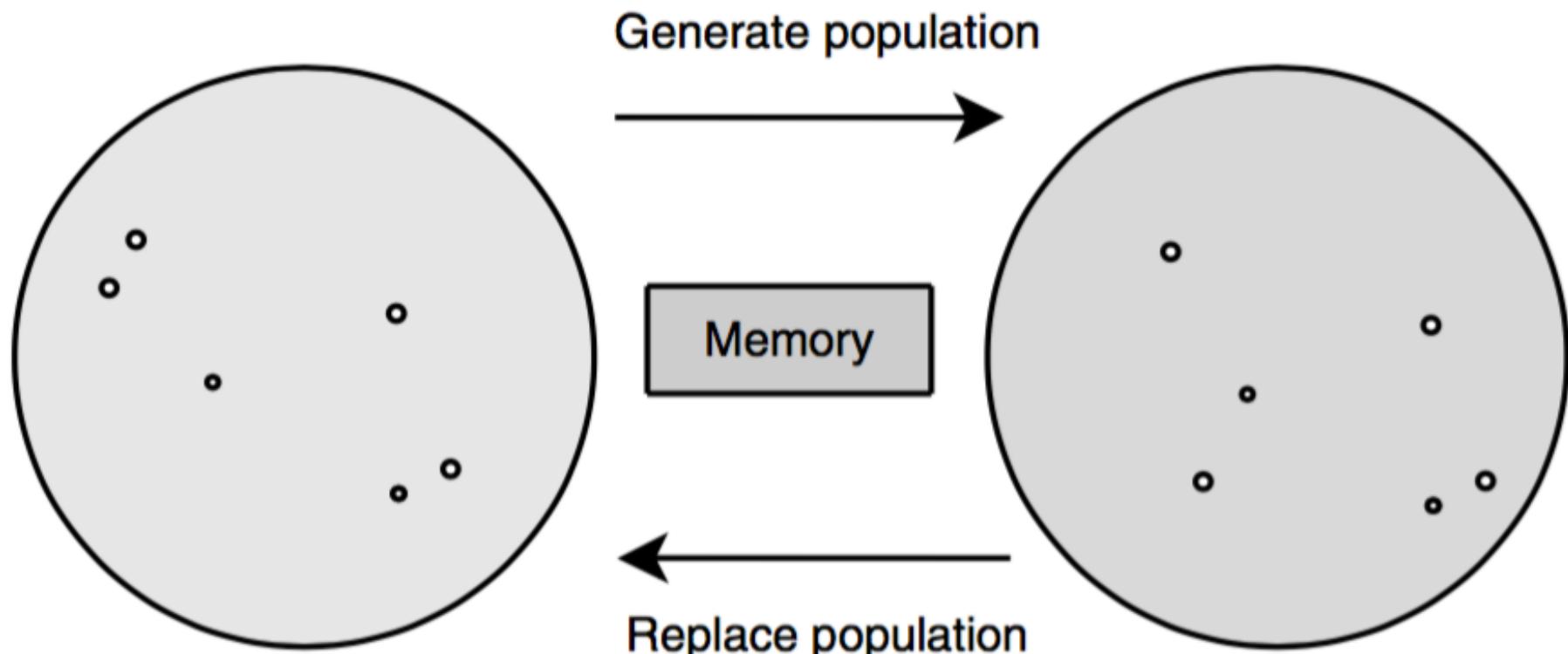
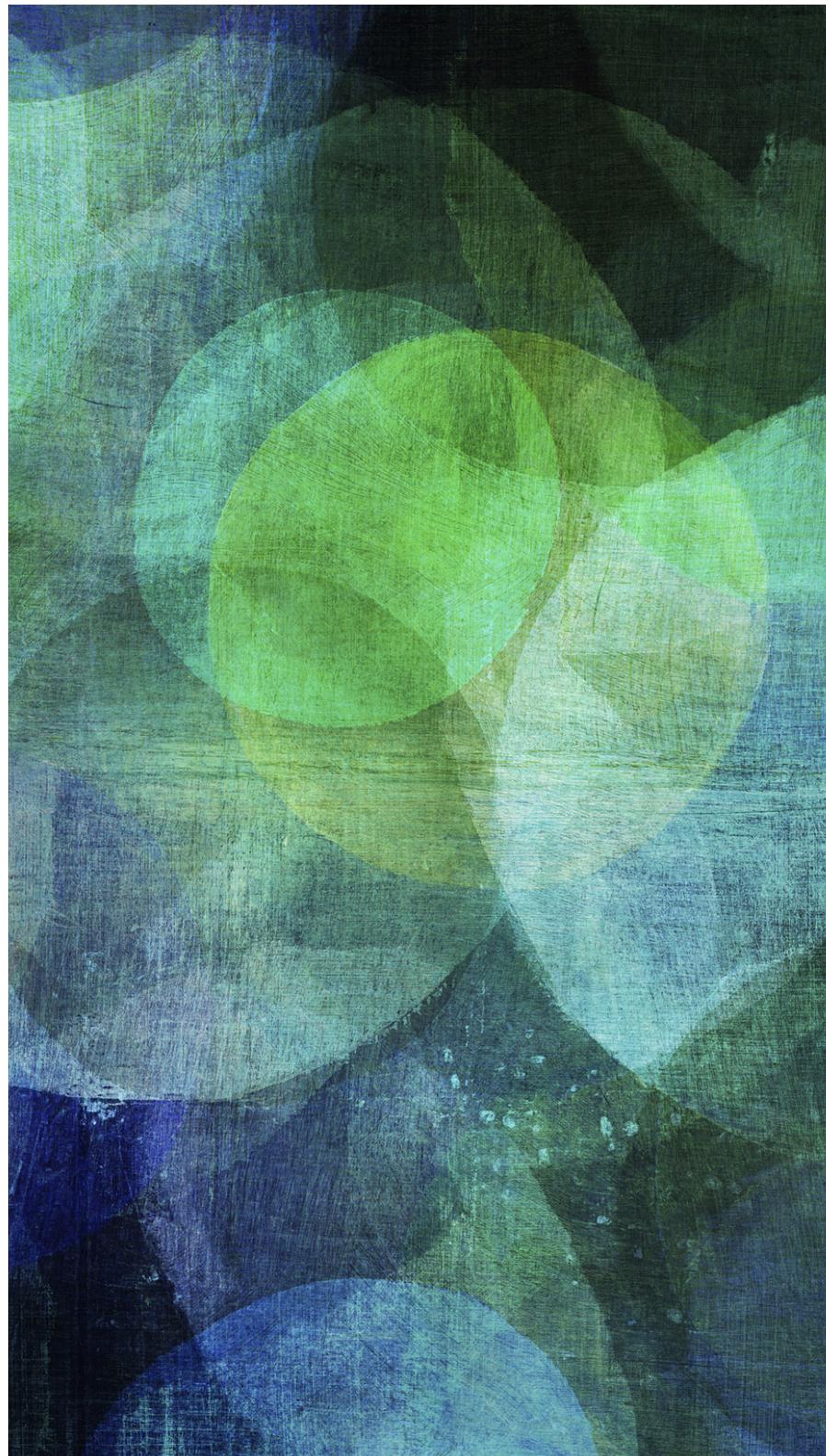


FIGURE 3.1 Main principles of P-metaheuristics.

C



**WITH THE CURRENT
POPULATION GENERATE A
NEW POPULATION
AND CHECK IF THE
SCORES IMPROVED!**

ALGORITHM OF P-METAHEURISTICS

Algorithm 3.1 High-level template of P-metaheuristics.

```
 $P = P_0$ ; /* Generation of the initial population */  
 $t = 0$ ;  
Repeat  
    Generate( $P'_t$ ); /* Generation a new population */  
     $P_{t+1} = \text{Select-Population}(P_t \cup P'_t)$ ; /* Select new population */  
     $t = t + 1$ ;  
Until Stopping criteria satisfied  
Output: Best solution(s) found.
```

MAIN CONCEPTS



- Generation: In this step, a new population of solutions is generated. According to the generation strategy:
 - Evolution based: In this category of P-metaheuristics, the solutions composing the population are selected and reproduced using variation operators (e.g., mutation, recombination) acting directly on their representations. A new solution is constructed from the different attributes of solutions belonging to the current population.
 - Blackboard based: Here, the solutions of the population participate in the construction of a shared memory. This shared memory will be the main input in generating the new population of solutions.

MEMORIES



Search memory: The memory of a P-metaheuristic represents the set of information extracted and memorized during the search. The content of this memory varies from a P-metaheuristic to another one.

TABLE 3.1 Search Memories of Some P-Metaheuristics

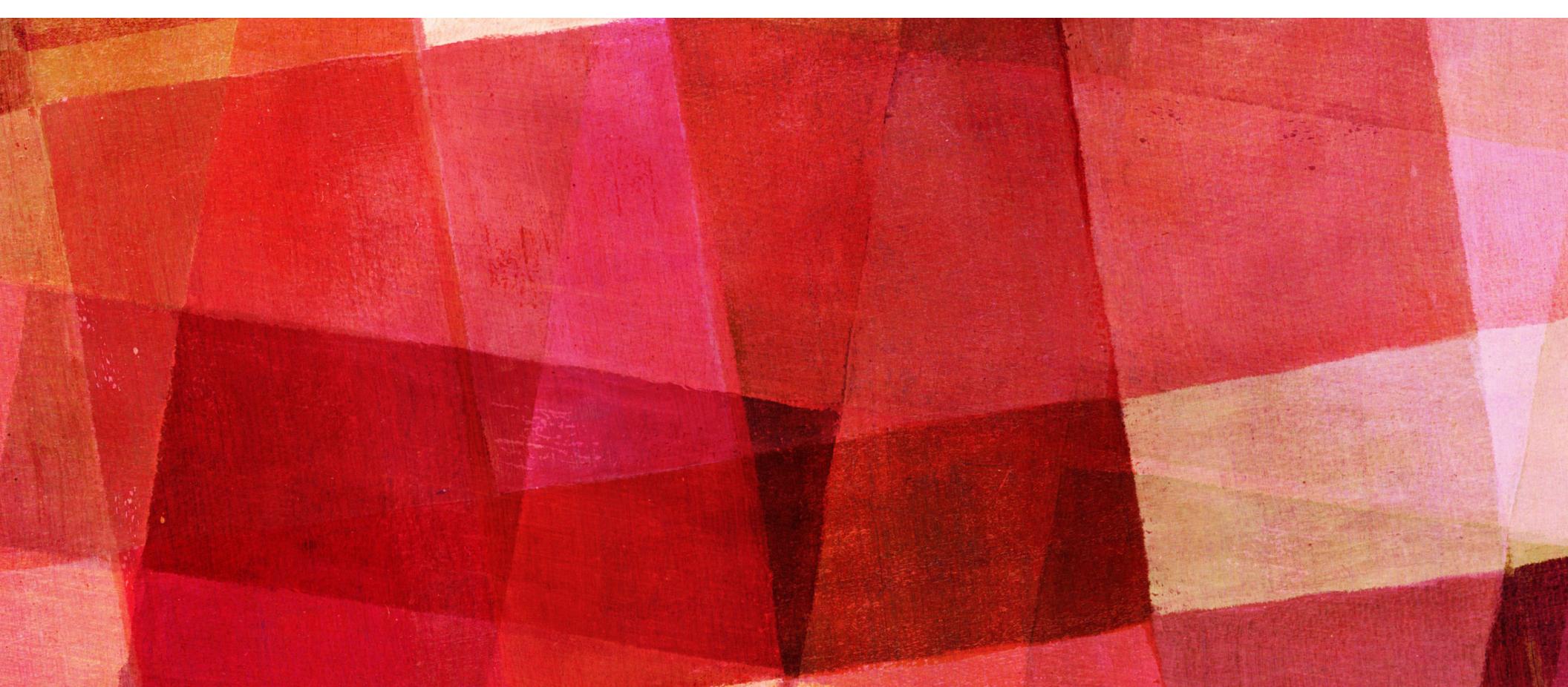
| P-metaheuristic | Search Memory |
|--|---|
| Evolutionary algorithms | Population of individuals |
| Scatter search | Population of solutions, reference set |
| Ant colonies | Pheromone matrix |
| Estimation of distribution algorithms | Probabilistic learning model |
| Particle swarm optimization | Population of particles, best global and local solutions |
| Bee colonies | Population of bees |
| Artificial immune systems: clonal selection | Population of antibodies |

MAIN CONCEPTS

SELECTION



- The last step in P-metaheuristics consists in selecting the new solutions from the union of the current population and the generated population. The traditional strategy consists in selecting the generated population as the new population. Other strategies use some elitism in the selection phase where they provide the best solutions from the two sets. In blackboard-based P-metaheuristics, there is no explicit selection.



EVOLUTIONARY ALGORITHMS

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EVOLUTIONARY ALGORITHMS

Algorithm 3.2 Template of an evolutionary algorithm.

```
Generate( $P(0)$ ) ; /* Initial population */  
 $t = 0$  ;  
While not Termination_Criterion( $P(t)$ ) Do  
    Evaluate( $P(t)$ ) ;  
     $P'(t)$       = Selection( $P(t)$ ) ;  
     $P'(t)$       = Reproduction( $P'(t)$ ); Evaluate( $P'(t)$ ) ;  
     $P(t + 1)$   = Replace( $P(t)$ ,  $P'(t)$ ) ;  
     $t = t + 1$  ;  
End While  
Output Best individual or best population found.
```

GENOTYPE AND PHENOTYPE



- In evolutionary algorithms, the genotype represents the encoding while the phenotype represents the solution. Hence, the genotype must be decoded to generate the phenotype.

| Phenotype | |
|--------------|---------------|
| Min | $x = 1$ |
| $f(x)$ | $x = 1.5$ |
| $s.t.$ | $x = 0.45$ |
| $Ax \leq b$ | $x = 3.14159$ |
| $x > 0$ | |

The diagram illustrates the relationship between a genotype and its binary encoding. On the left, a purple arrow points from the text "Binary encoding" to the sequence of bits. On the right, the genotype is shown as a set of powers of 2, and the binary encoding is shown as a sequence of 0s and 1s.

Genotype

Binary encoding

$$\{2^{-2}, 2^{-1}, 2^0, 2^1, 2^2, 2^3\}$$
$$\{0, 1, 1, 0, 1, 0\}$$

GENOTYPE AND PHENOTYPE

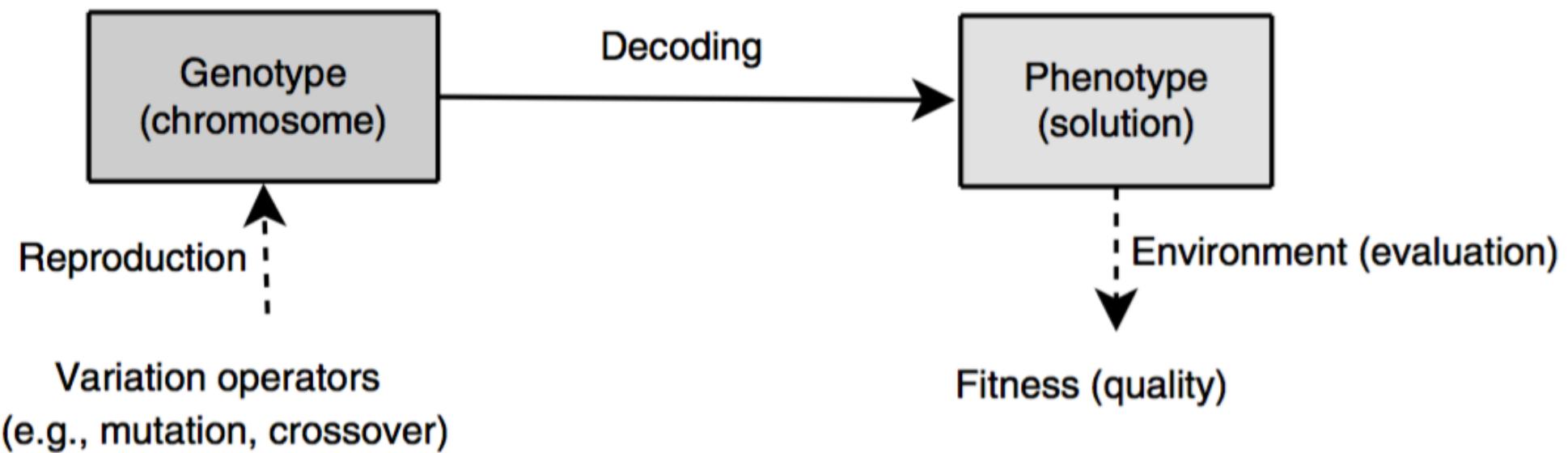
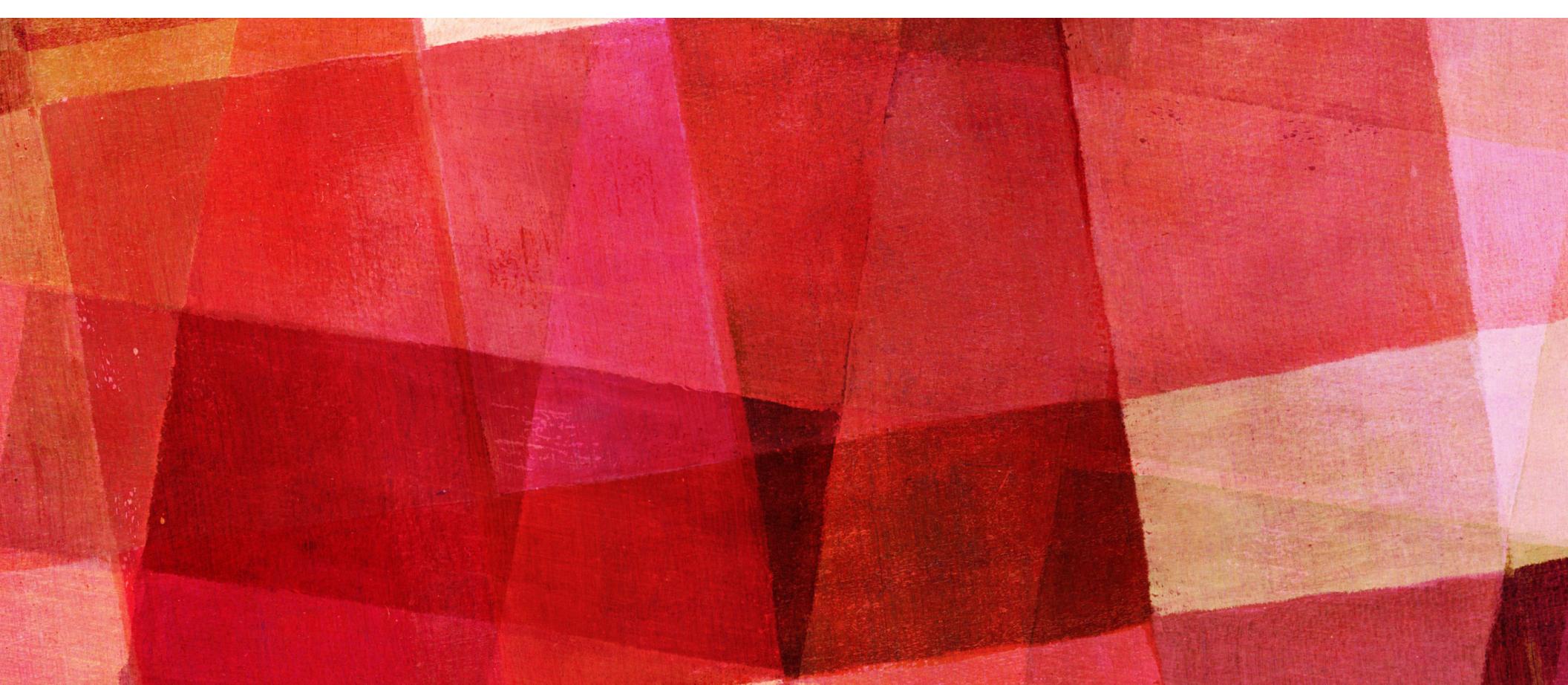


FIGURE 3.8 Genotype versus phenotype in evolutionary algorithms.

COMMON CONCEPTS FOR EVOLUTIONARY ALGORITHMS



1. Representation:
2. Population initialization:
3. Objective function:
4. Selection strategy:
5. Reproduction strategy:
6. Replacement strategy:
7. Stopping criteria:



EVOLUTIONARY ALGORITHMS

SELECTION METHODS

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SELECTION METHODS



- Roulette Wheel Selection and its variation Stochastic Universal Sampling

| | | | | | | | |
|--------------|---|---|---|-----|-----|---|---|
| Individuals: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Fitness: | 1 | 1 | 1 | 1.5 | 1.5 | 3 | 3 |

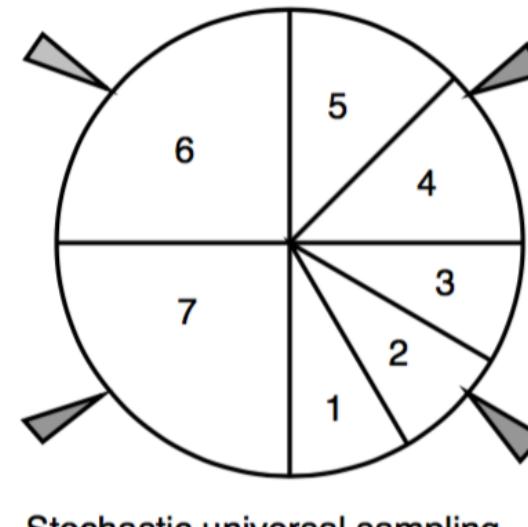
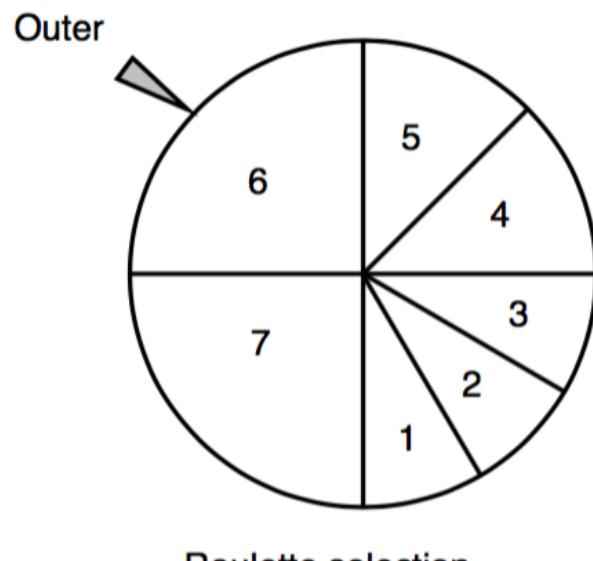


FIGURE 3.11 Roulette selection strategies. In the standard roulette selection, each spin selects a single individual. In SUS, a spin will select as individuals as outers (e.g., four individuals in the example).

SELECTION METHODS

► Tournament Selection

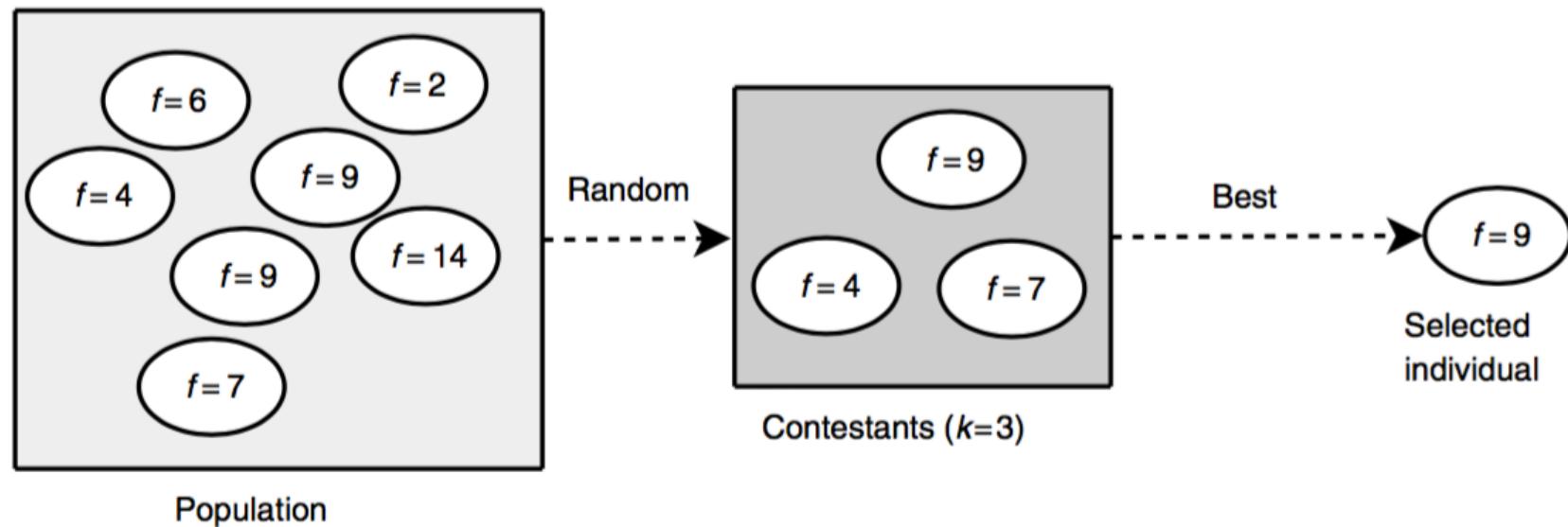


FIGURE 3.12 Tournament selection strategy. For instance, a tournament of size 3 is performed. Three solutions are picked randomly from the population. The best solution from the picked individuals is then selected.

SELECTION METHODS

- Rank-Based Selection - modified weight in roulette wheel using the rank instead of fitness.

$$P(i) = \frac{2 - s}{\mu} + \frac{2 \cdot r(i) \cdot (s - 1)}{\mu(\mu - 1)}$$

where s is the selection pressure ($1.0 < s \leq 2.0$), μ is the size of the population, and $r(i)$ is the rank associated with the individual i . Greater is the selection pressure s , more importance to better individuals is given.

SELECTION METHODS

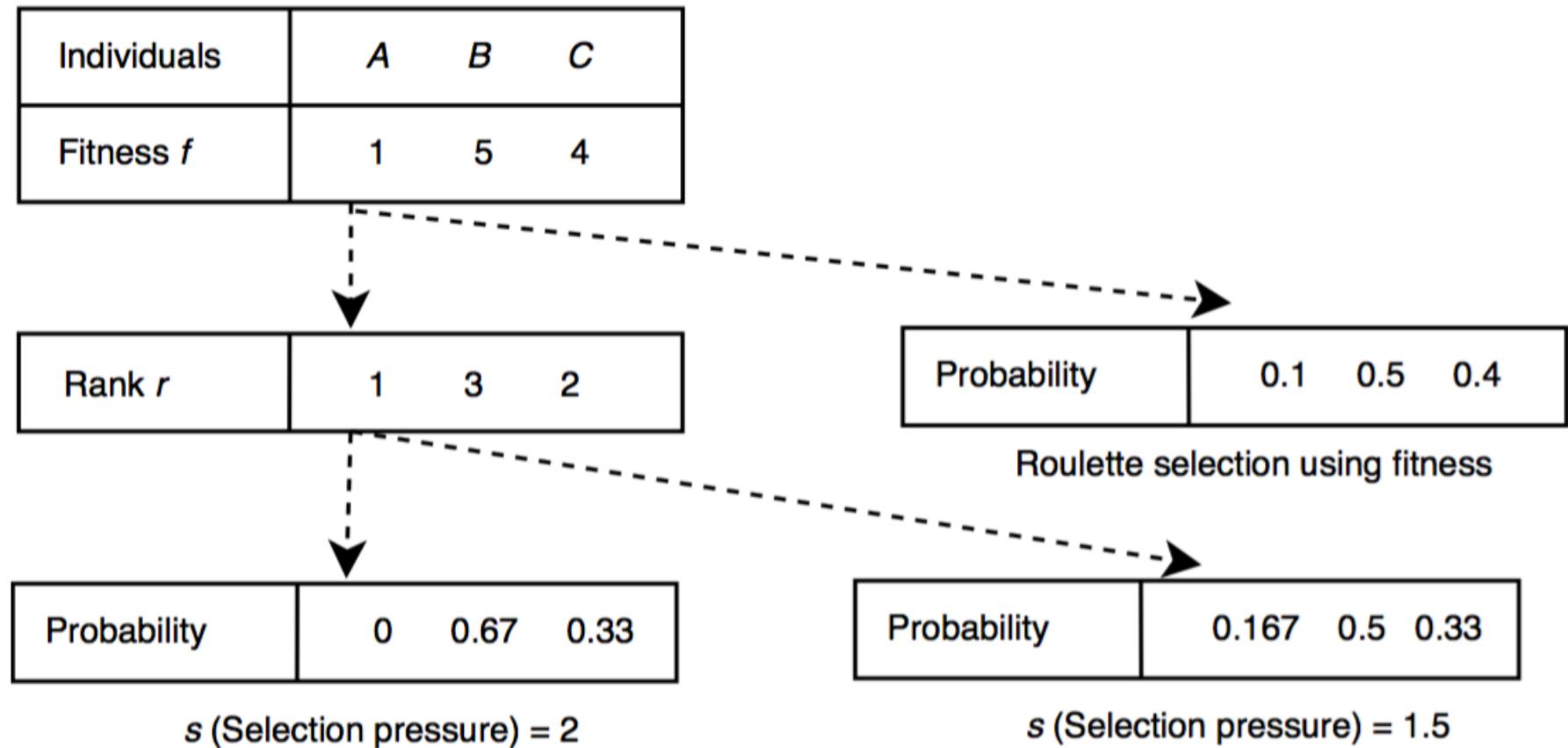
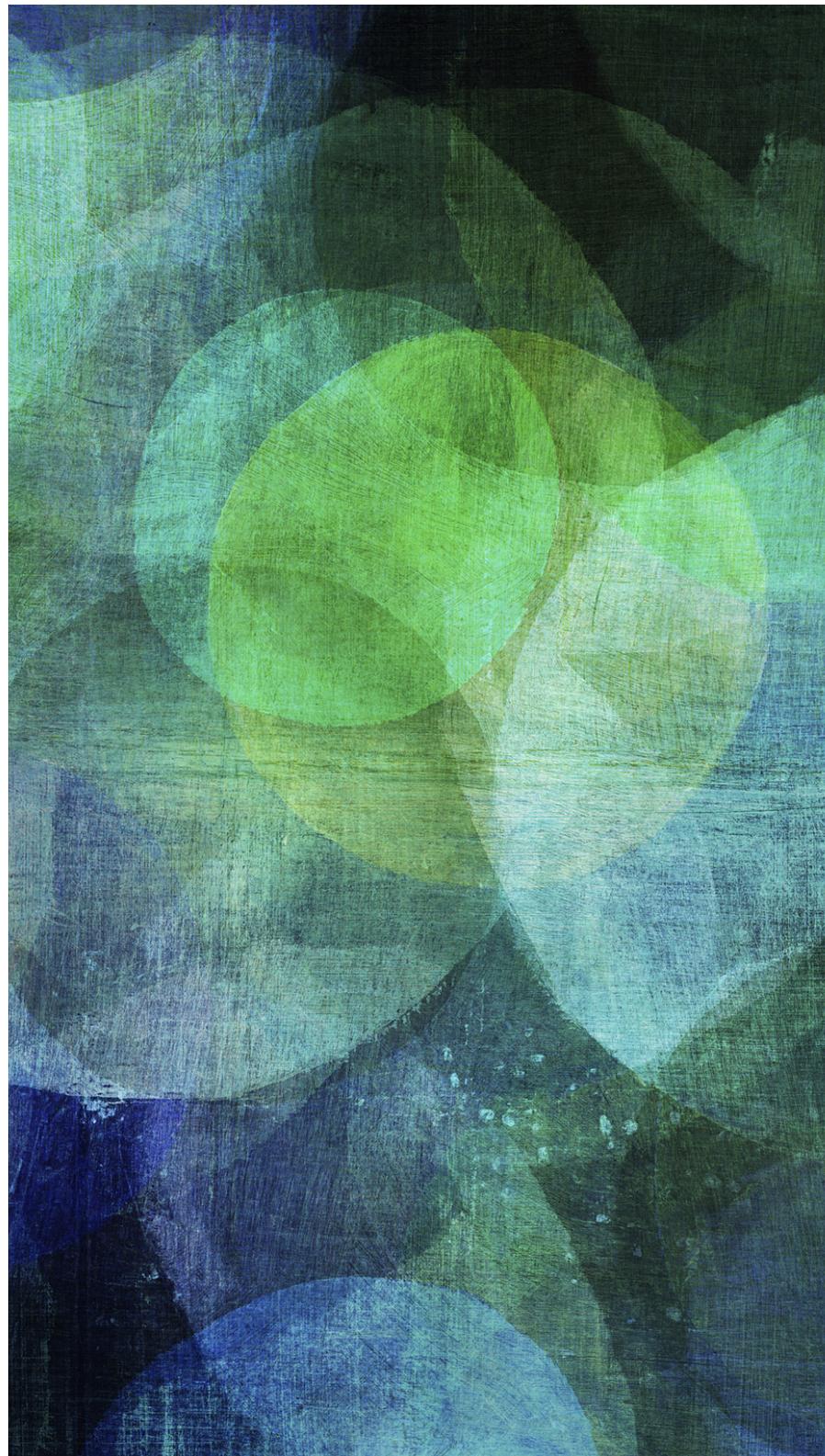


FIGURE 3.13 Rank-based selection strategy using a linear ranking.

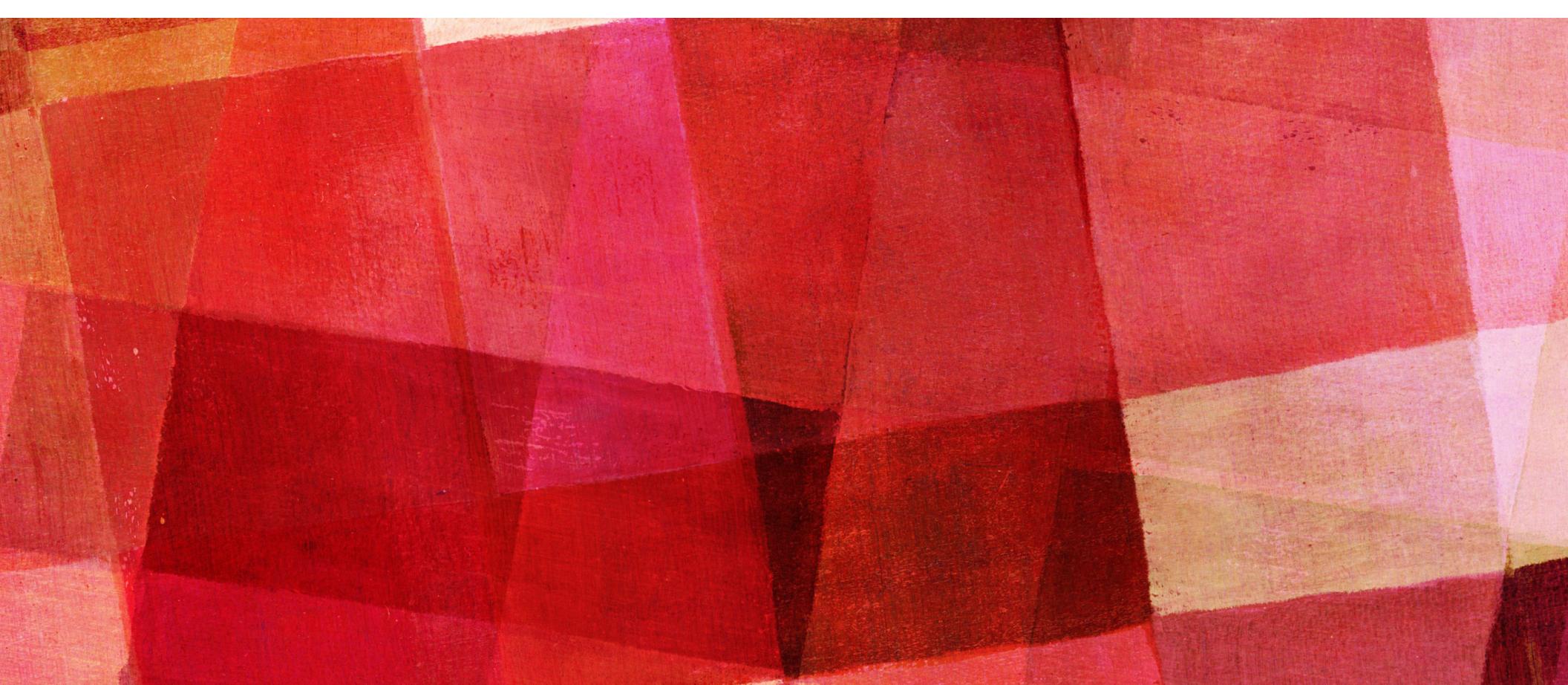
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DISCUSS

With a current
population

How will you **SELECT** the
parents to maximize the
possibility of evolution?



EVOLUTIONARY ALGORITHMS

REPRODUCTION

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REPRODUCTION



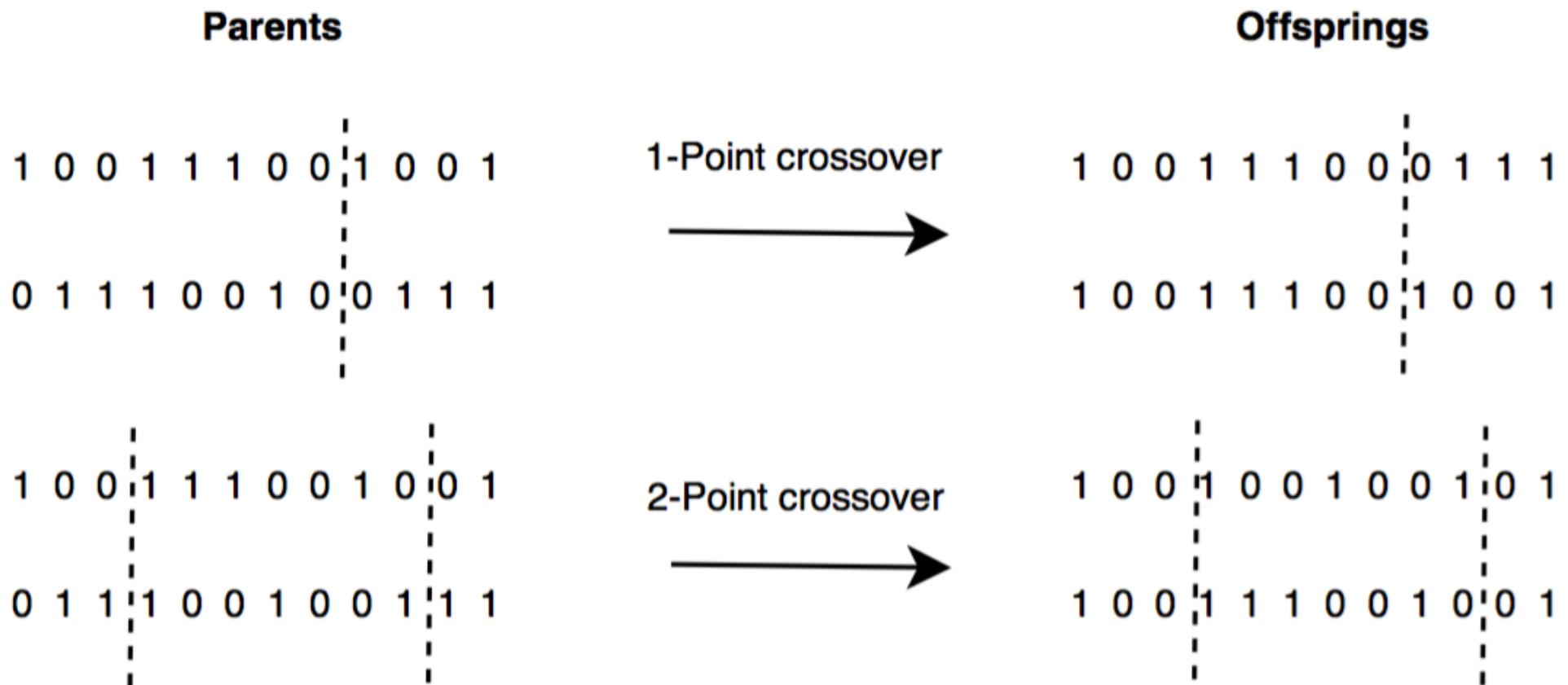
- Crossover (X-OVER) or Recombination
- Mutation

X-OVER

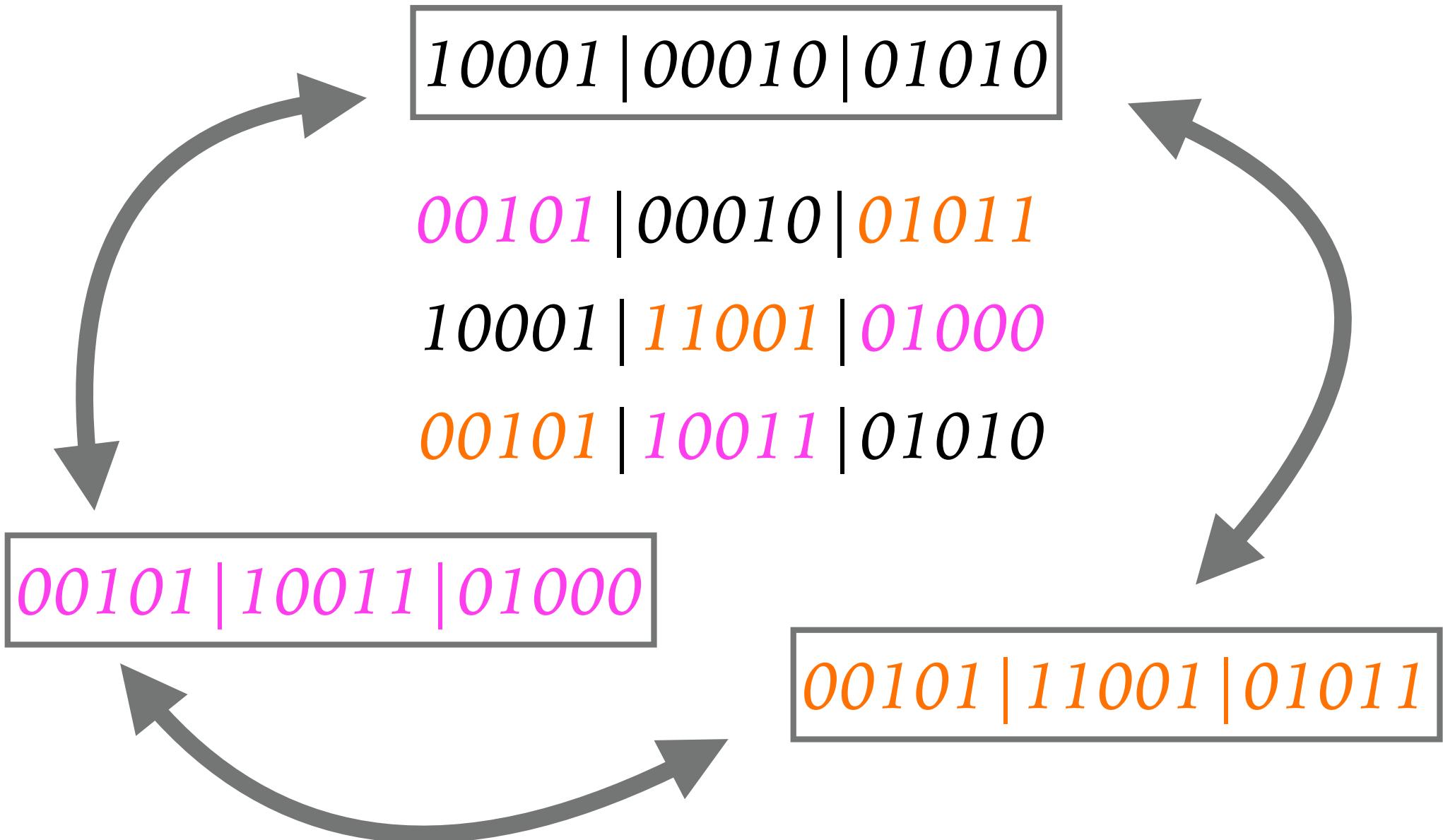


- Unlike unary operators such as mutation, the crossover operator is binary and sometimes n-ary. The role of crossover operators is to inherit some characteristics of the two parents to generate the offsprings.
- Characteristics
 - Heritability: The main characteristic of the crossover operator is heritability. The crossover operator should inherit a genetic material from both parents.
 - Validity: The crossover operator should produce valid solutions.

ONE POINT AND TWO POINT X-OVERS



MULTIPLE PARENTS – SODOMA AND GOMORA X-OVER



UNIFORM X-OVER

$P1 \ 1111111111$

$P2 \ 0000000000$

$O1 \ 10010011100$

MORE X-OVERS

- Intermediate crossover: The intermediate crossover operators attempt to average the elements of the parents. Given two parents p_1 and p_2 , the arithmetic crossover creates an offspring o using the weighted average:

$$o_i = \alpha \cdot x_{1i} + (1 - \alpha) \cdot x_{2i}$$

- Geometrical crossover: The geometrical crossover generates an offspring in The following form:

$$o_i = (x_{1i} \cdot x_{2i})^{(1/2)}$$

ORDER X-OVER

Relative order, adjacency and position preserved

Relative order preserved

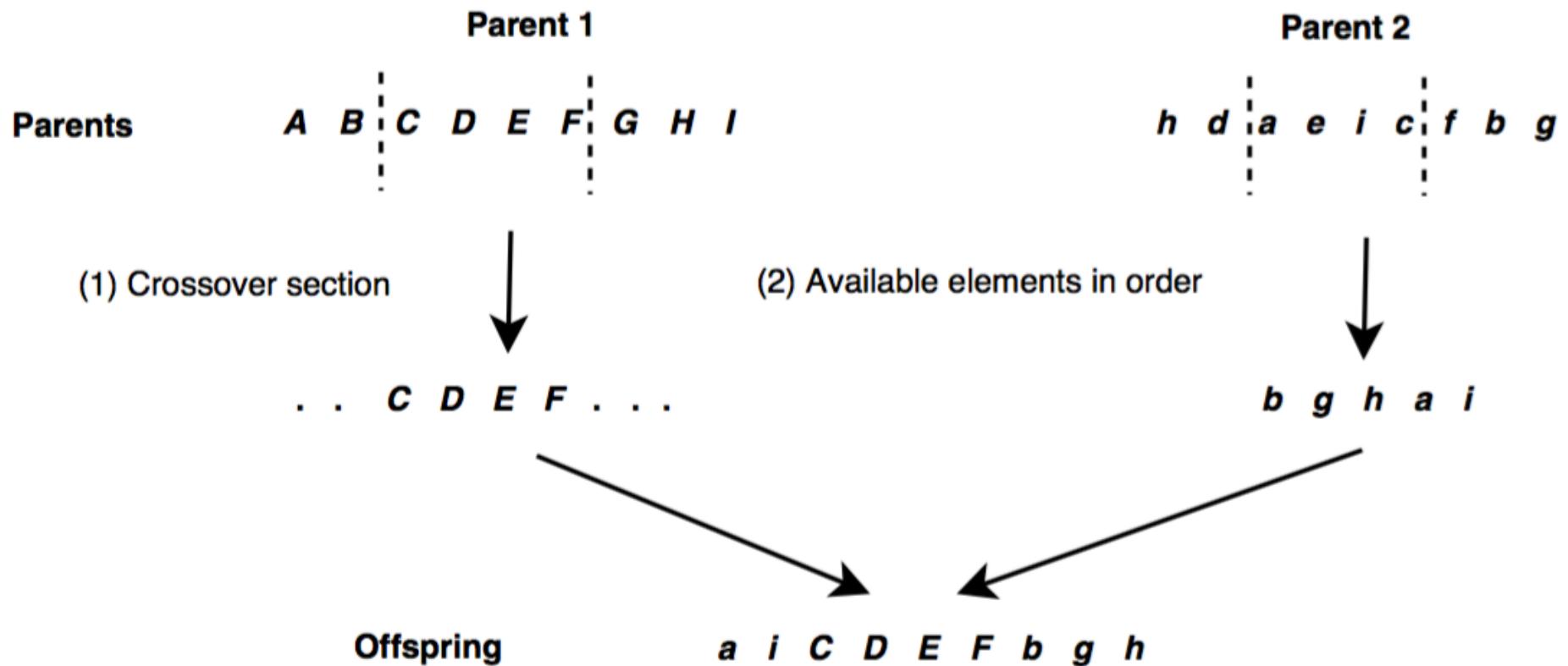
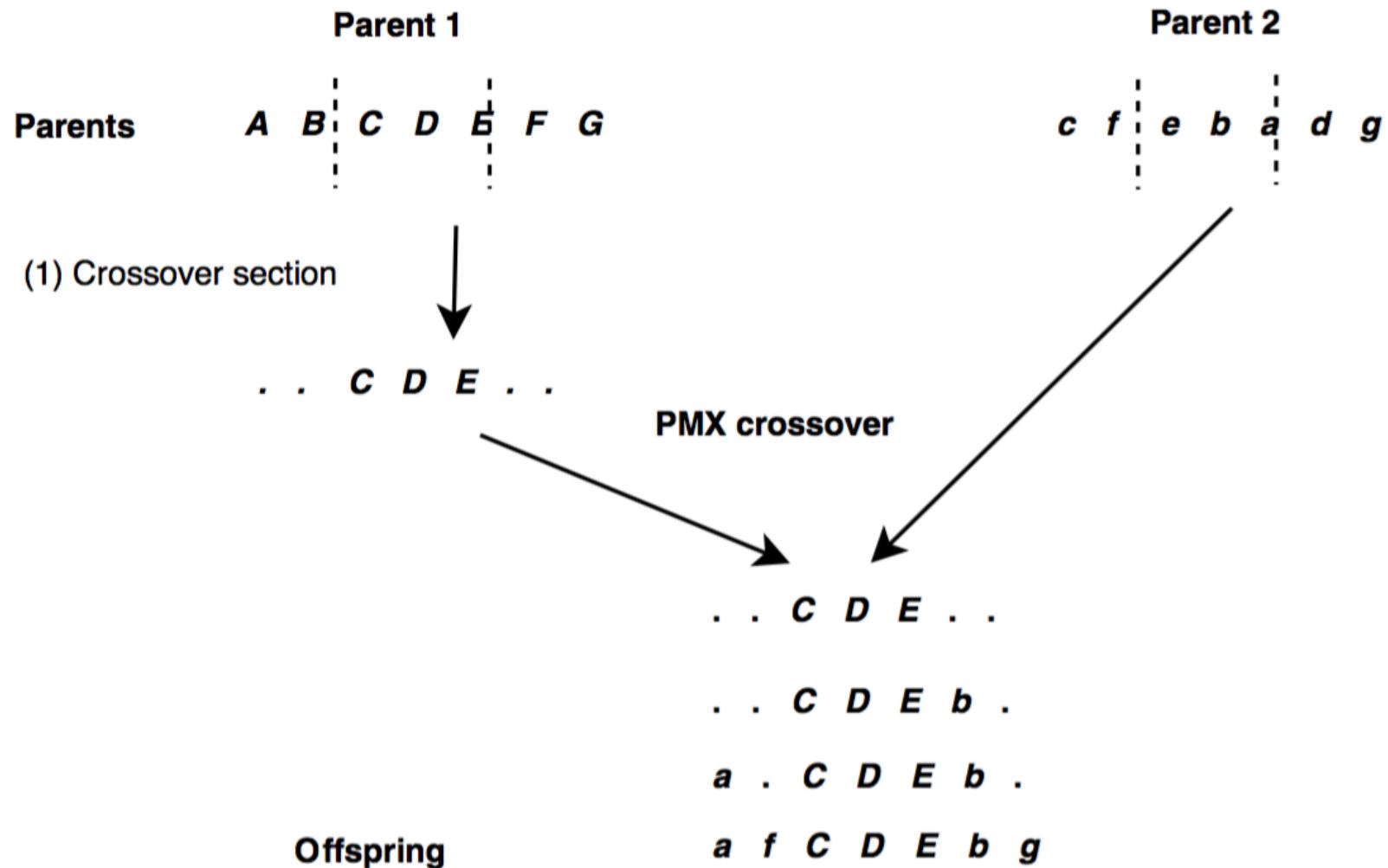


FIGURE 3.19 The order crossover for permutations.

PARTIALLY MAPPED CROSSOVER X-OVER

Relative order, adjacency and position preserved



TWO POINT X-OVER

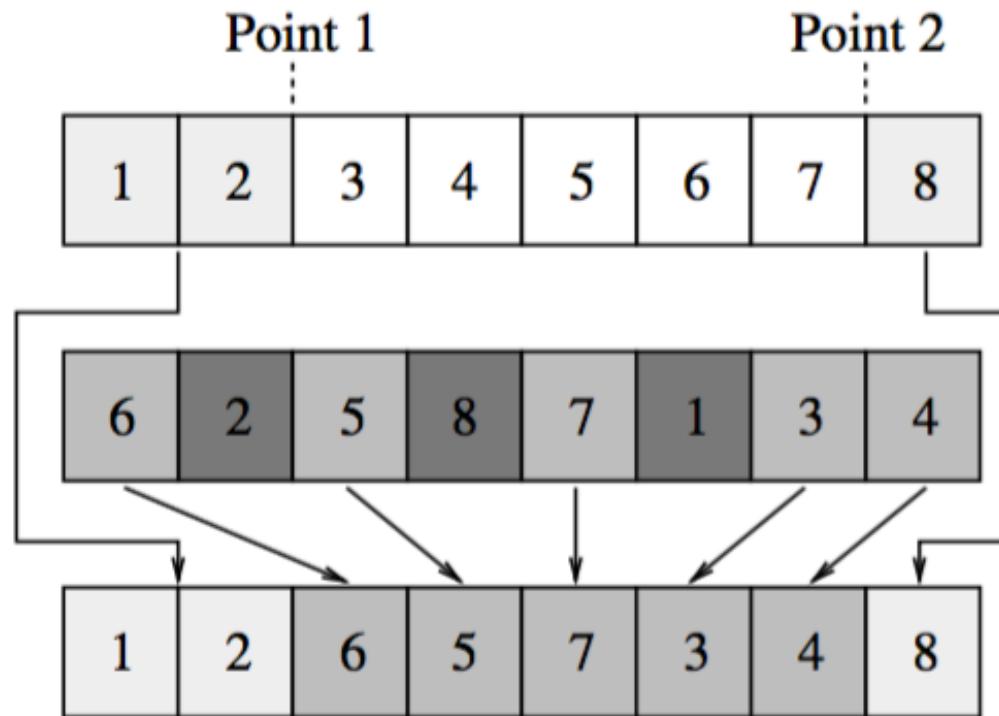
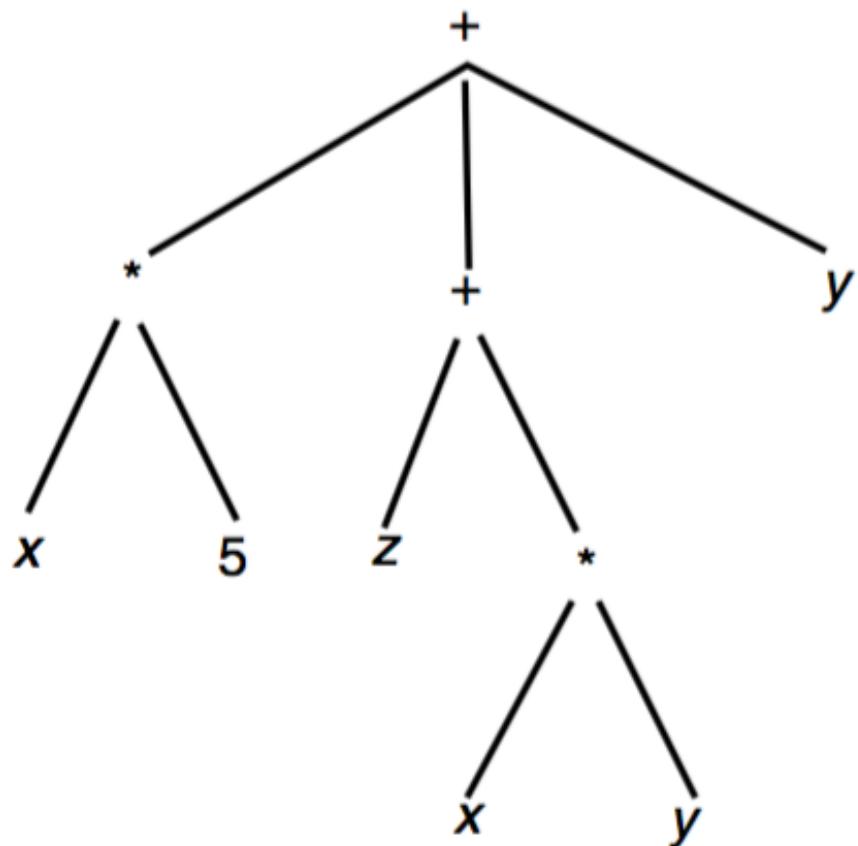


FIGURE 3.21 Two-Point crossover operator for permutations.

PARSE TREE REPRESENTATION FOR A FUNCTION



$$(x * 5) + (z + (x * y)) + y$$

X-OVER FOR PARSE TREE

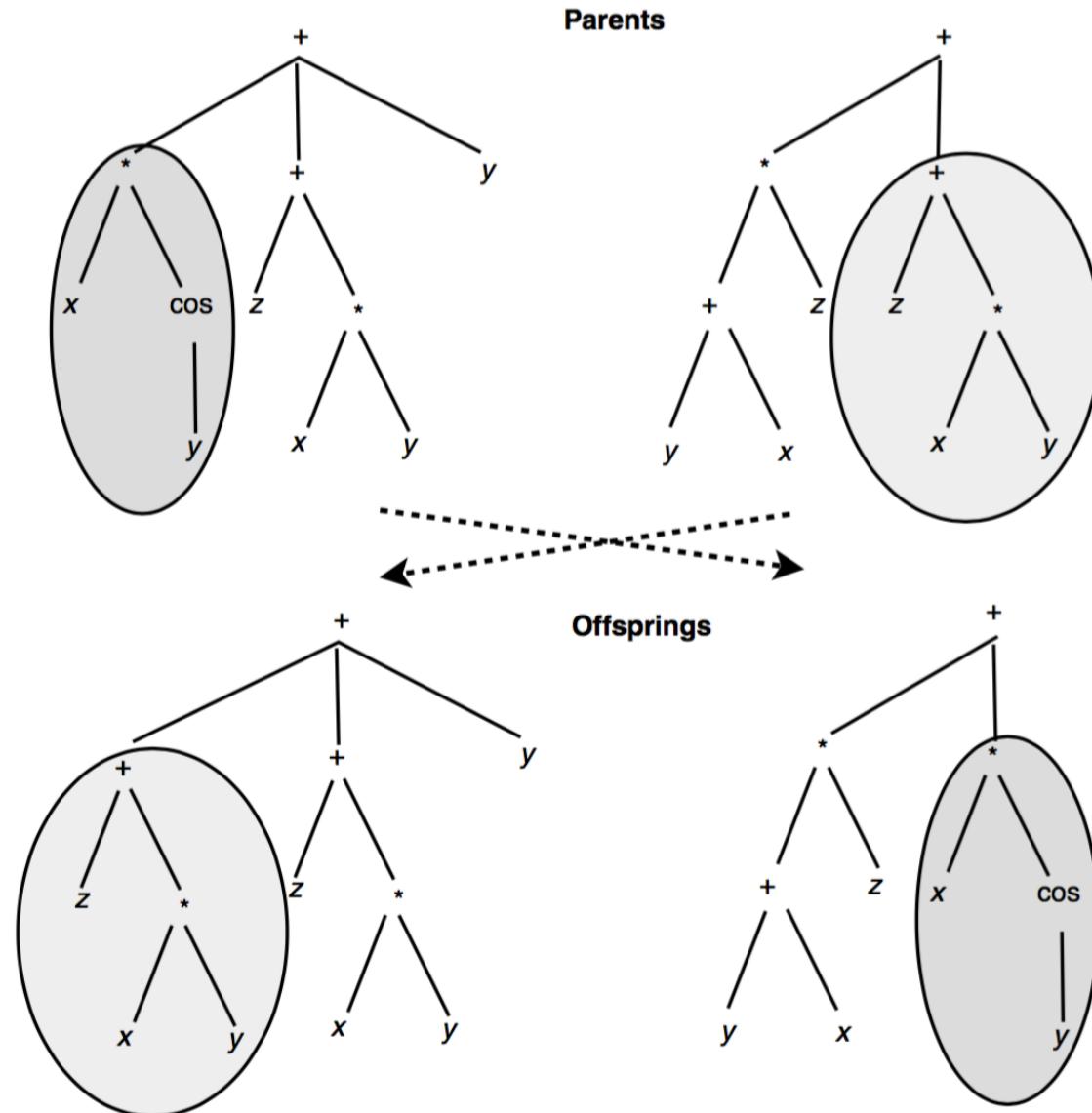
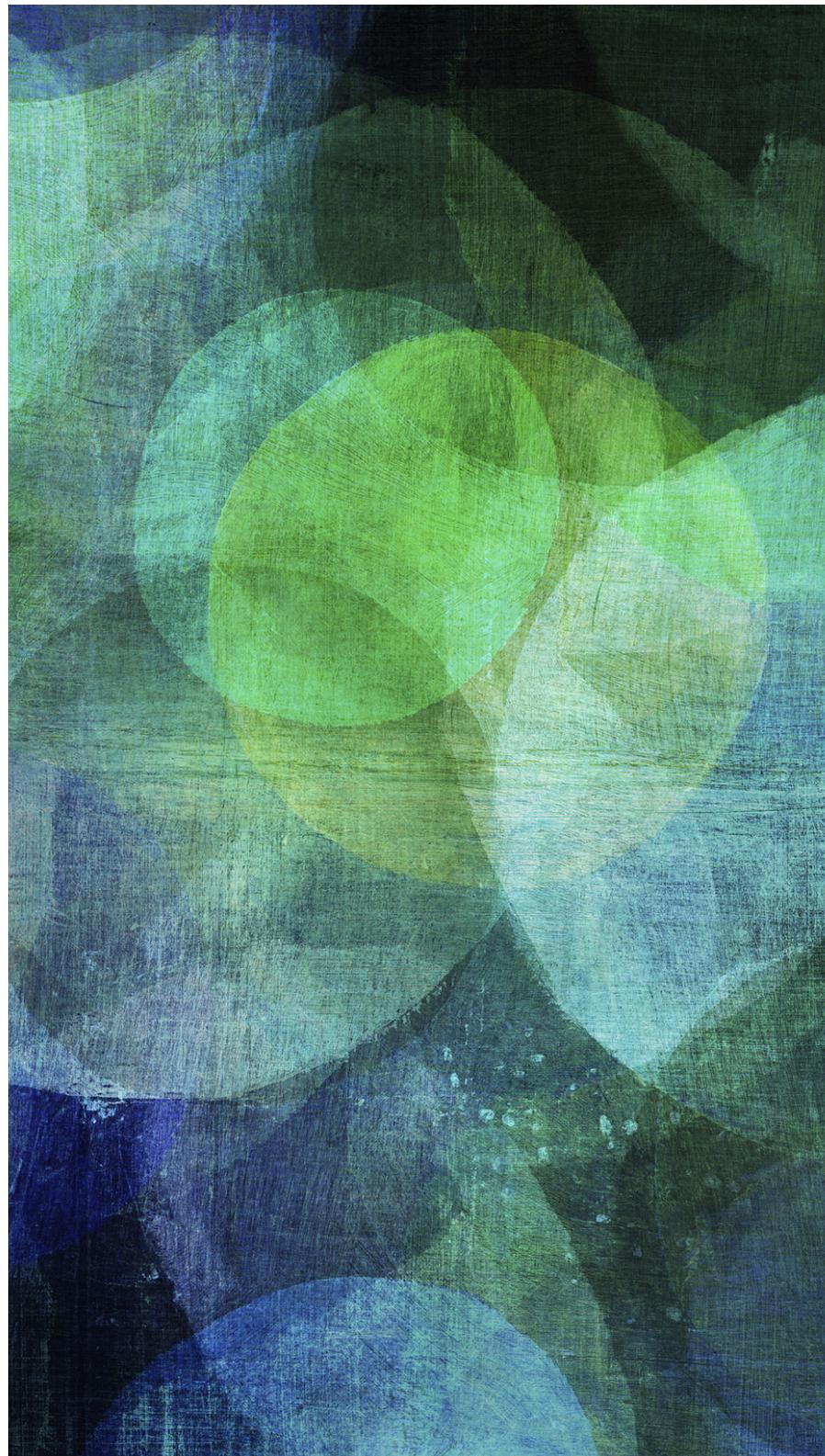


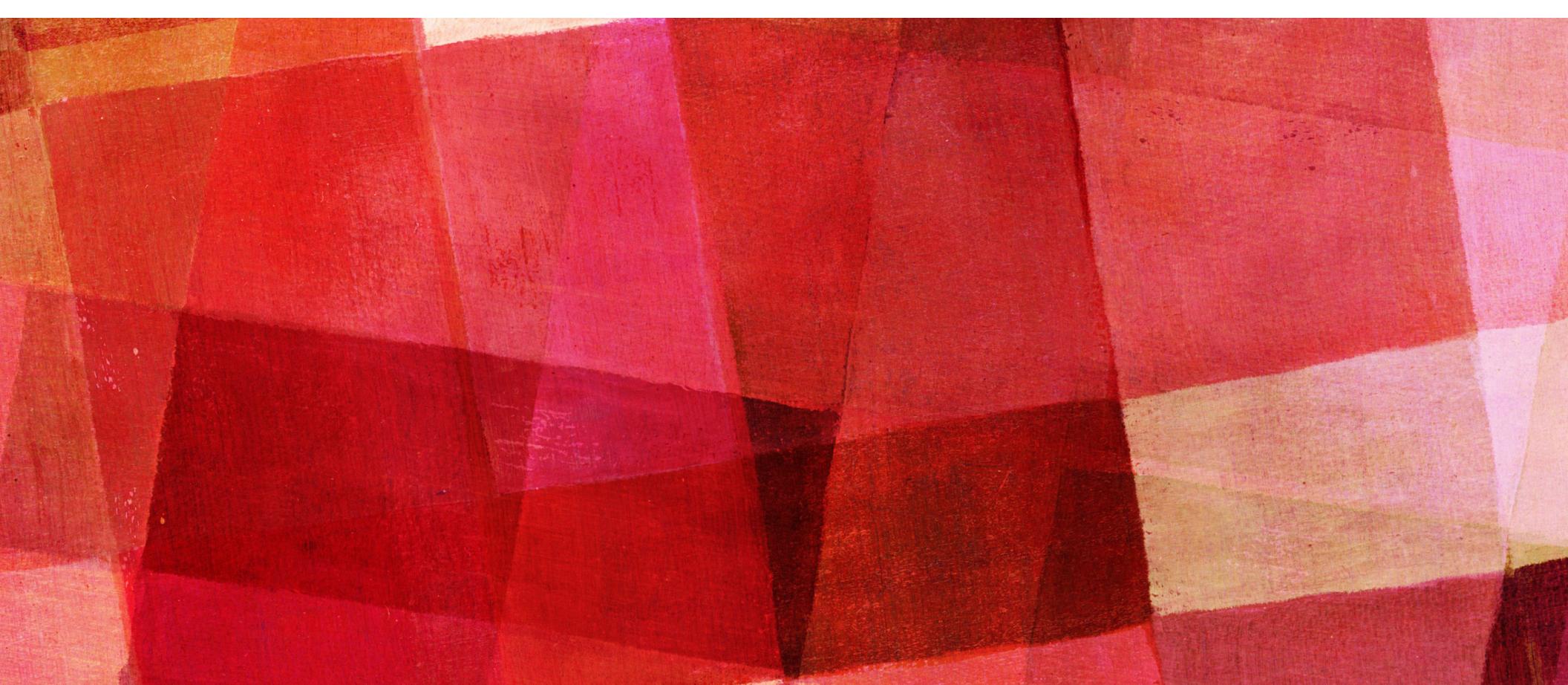
FIGURE 3.22 A crossover operator for parse tree representations.

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DISCUSSION

With a current population
How will you **REPRODUCE**
the parents to maximize
the possibility of
evolution?

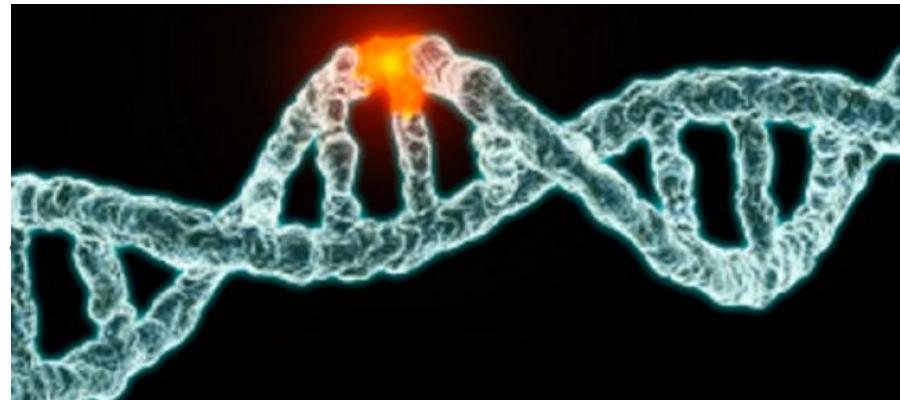


EVOLUTIONARY ALGORITHMS

MUTATION

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MUTATION



- Mutation operators are unary operators acting on a single individual.
- Mutations represent small changes of selected individuals of the population.
- Characteristics
 - Ergodicity: The mutation operator should allow every solution of the search space to be reached
 - Validity: The mutation operator should produce valid solutions.
 - Locality: The mutation should produce a minimal change in solution.

GROUPING PROBLEM

AAAABBBBBB

No disruptive mutation

AABBBBBB

Keeps the solution structure

AAAABBBBBB

Highly disruptive mutation

AACBBBBB

**It creates a new group, changing
the solution structure**

BINARY REPRESENTATION

100010001001010

100010101001010

INTEGER REPRESENTATION

143251756329015

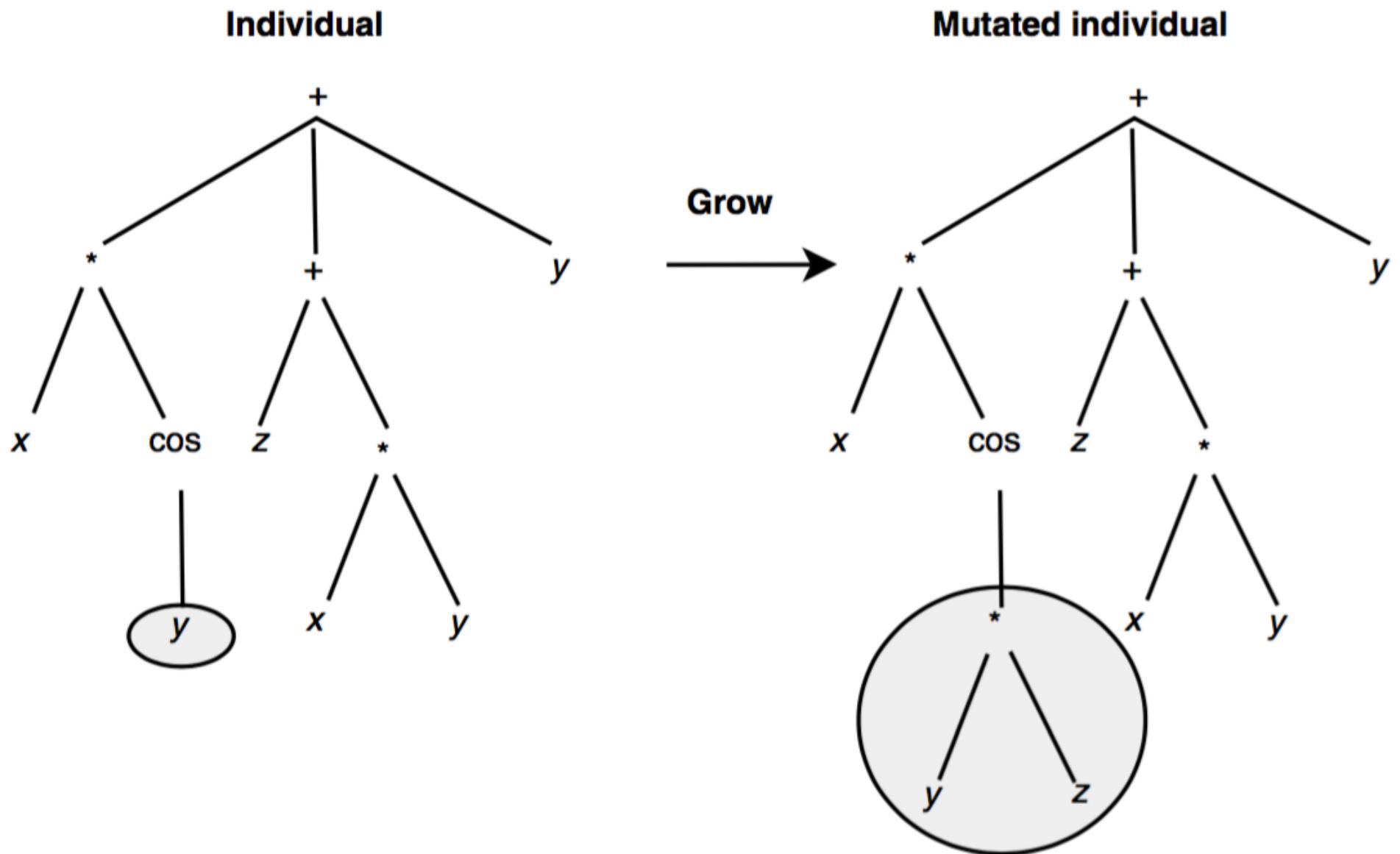
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PERMUTATION REPRESENTATION

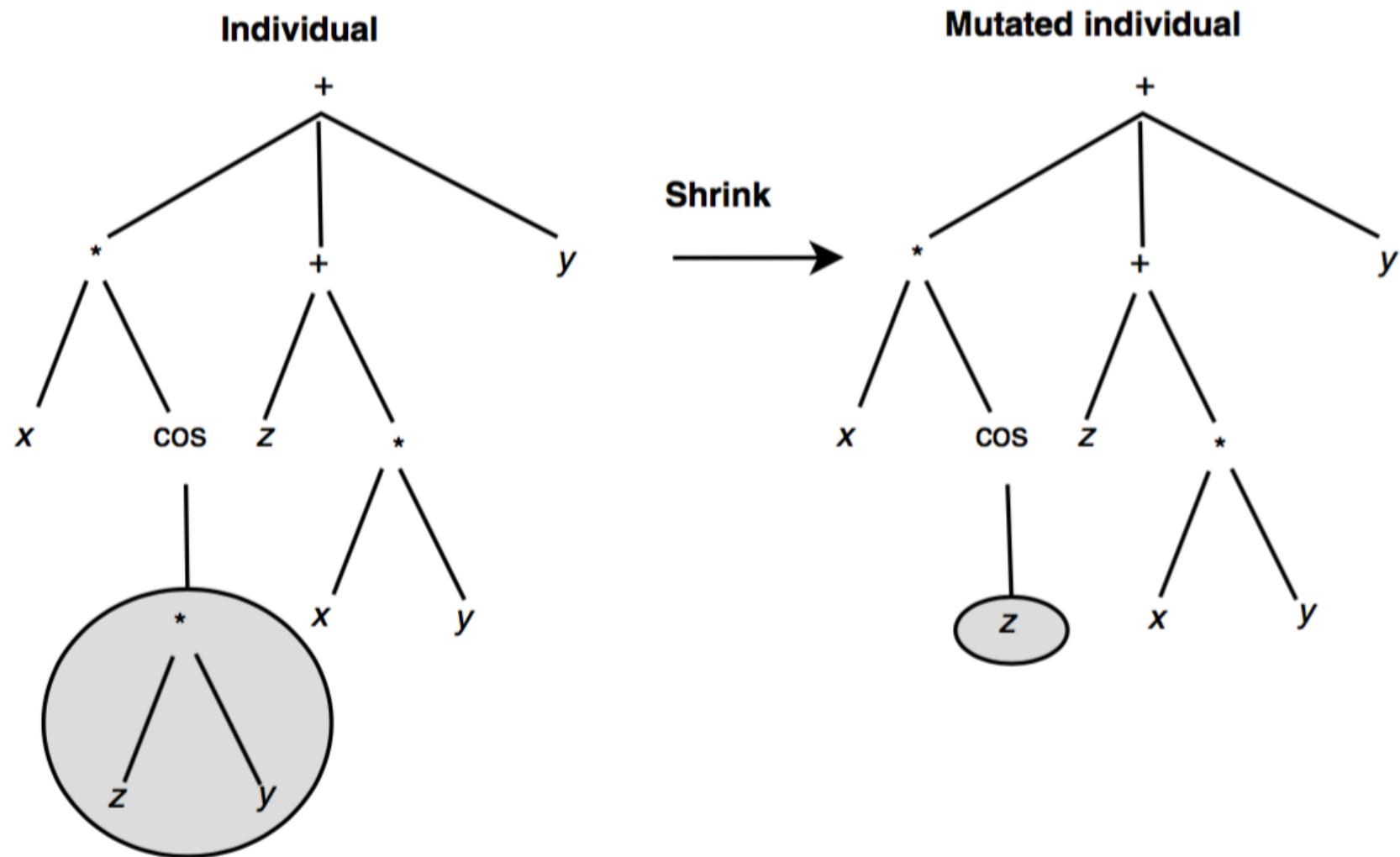
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1 $\color{orange}5$ 32517 $\color{orange}4$ 6329015

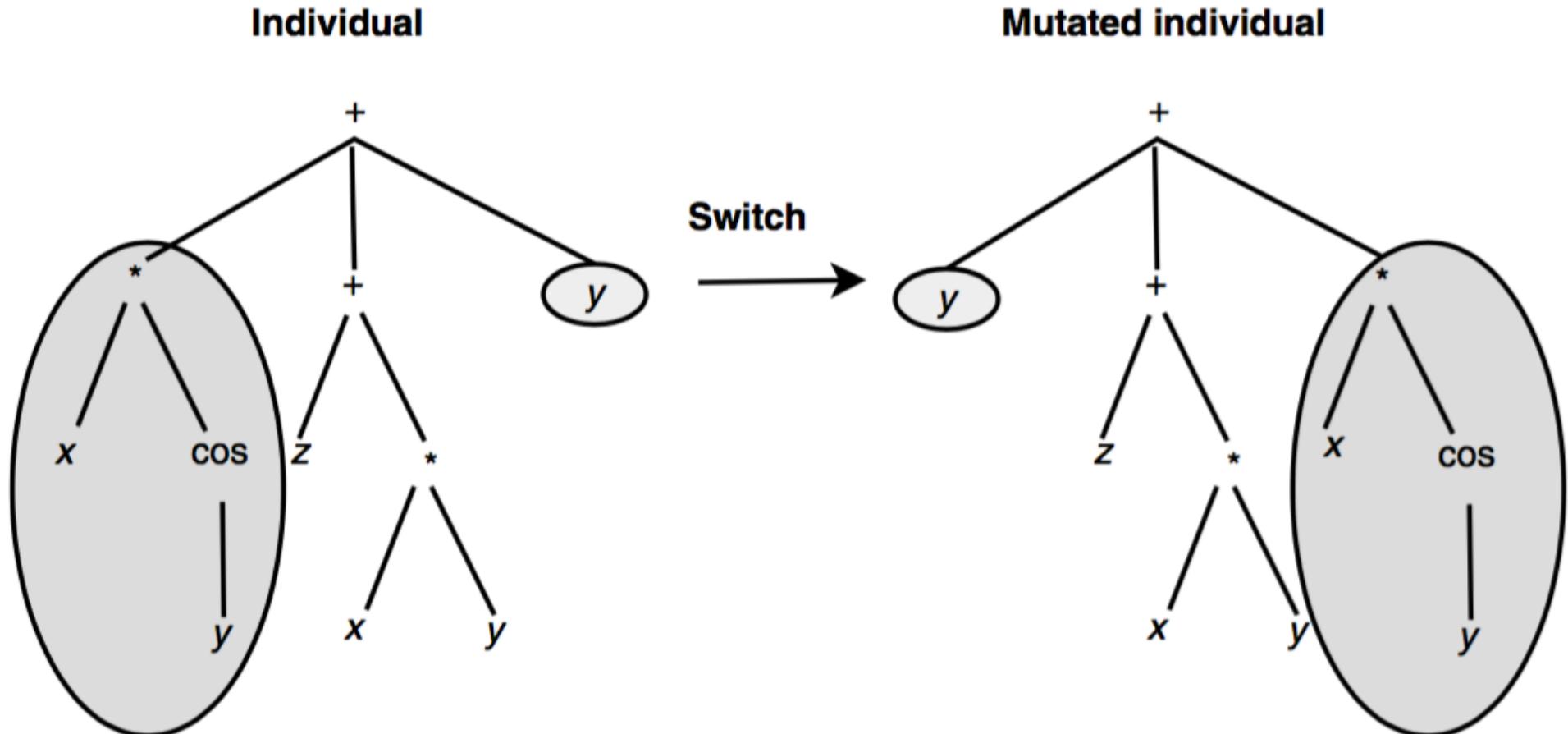
PARSE TREE MUTATION: GROW



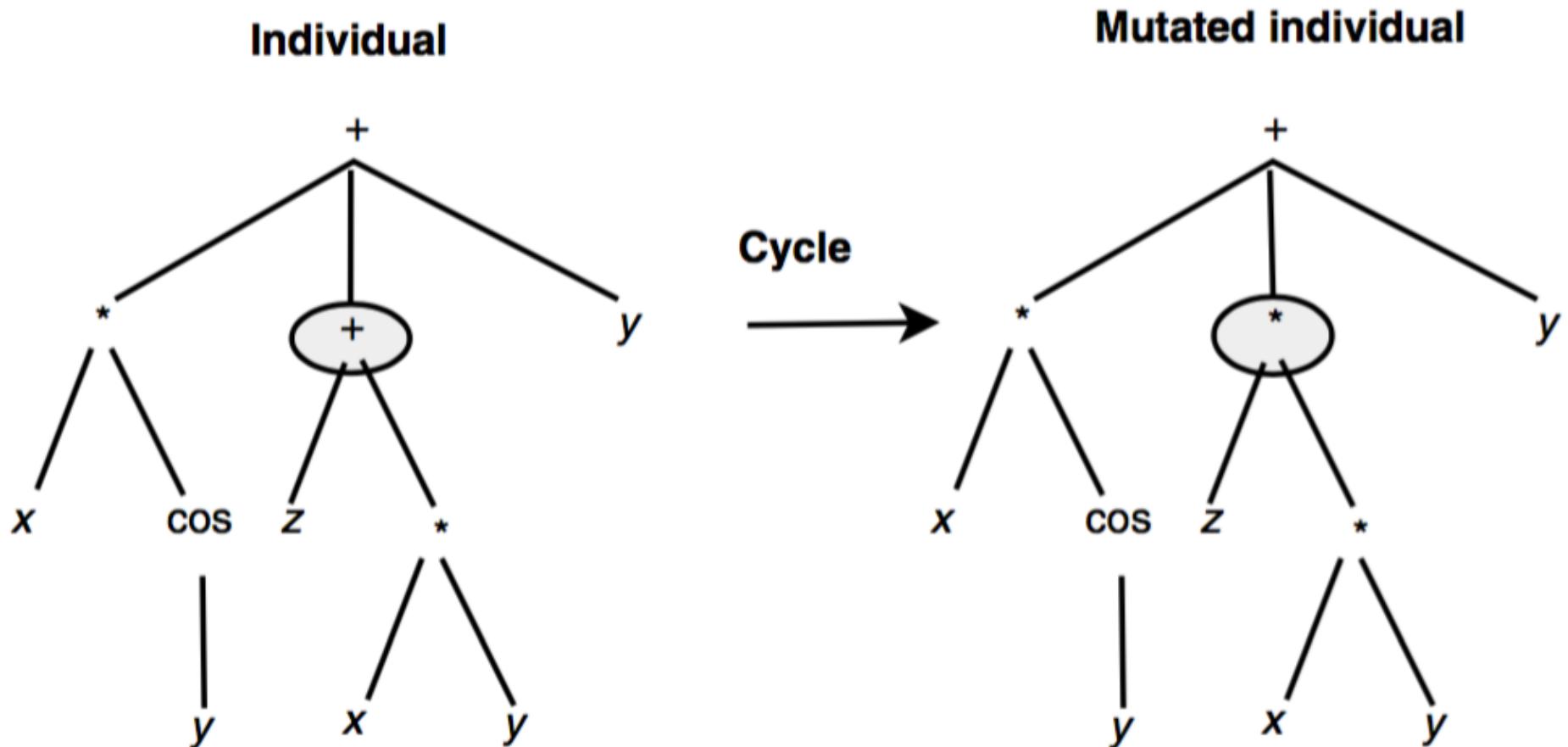
PARSE TREE MUTATION: SHRINK



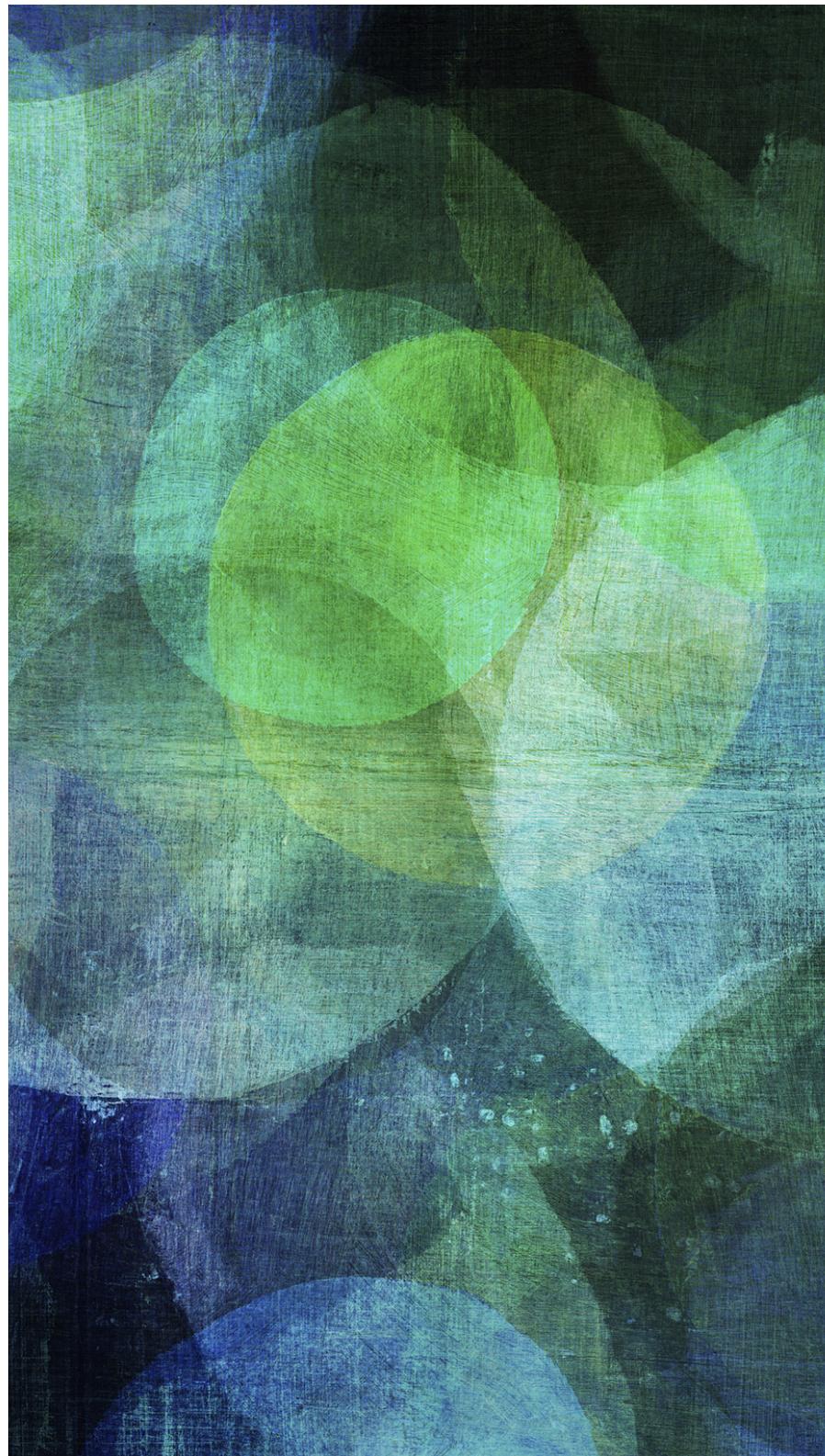
PARSE TREE MUTATION: SWITCH



PARSE TREE MUTATION: CYCLE



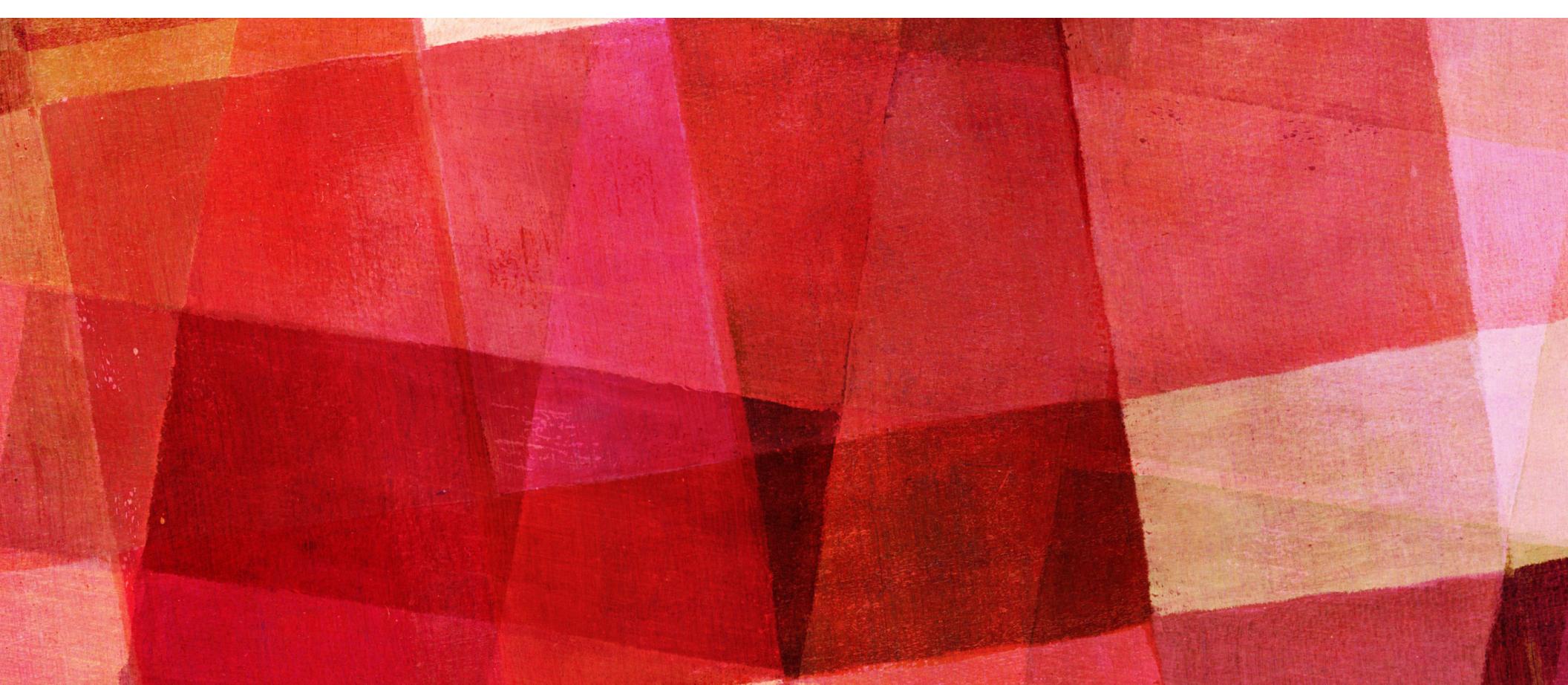
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DISCUSSION

With the current
population

How will you **MUTATE** the
parents to maximize the
possibility of evolution?



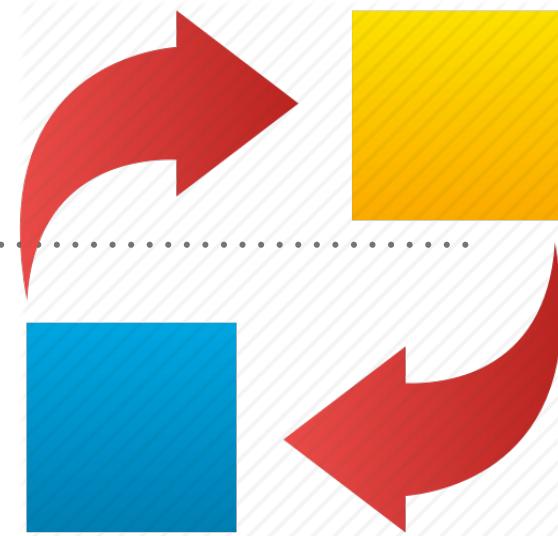
EVOLUTIONARY ALGORITHMS

MUTATION

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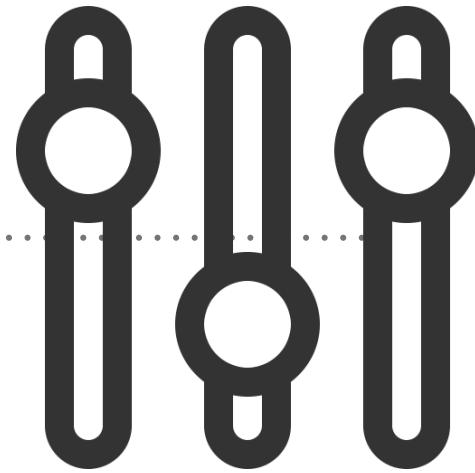
REPLACENT



- The replacement phase concerns the survivor selection of both the parent and the offspring populations.
- Generational replacement: The replacement will concern the whole population of size μ .

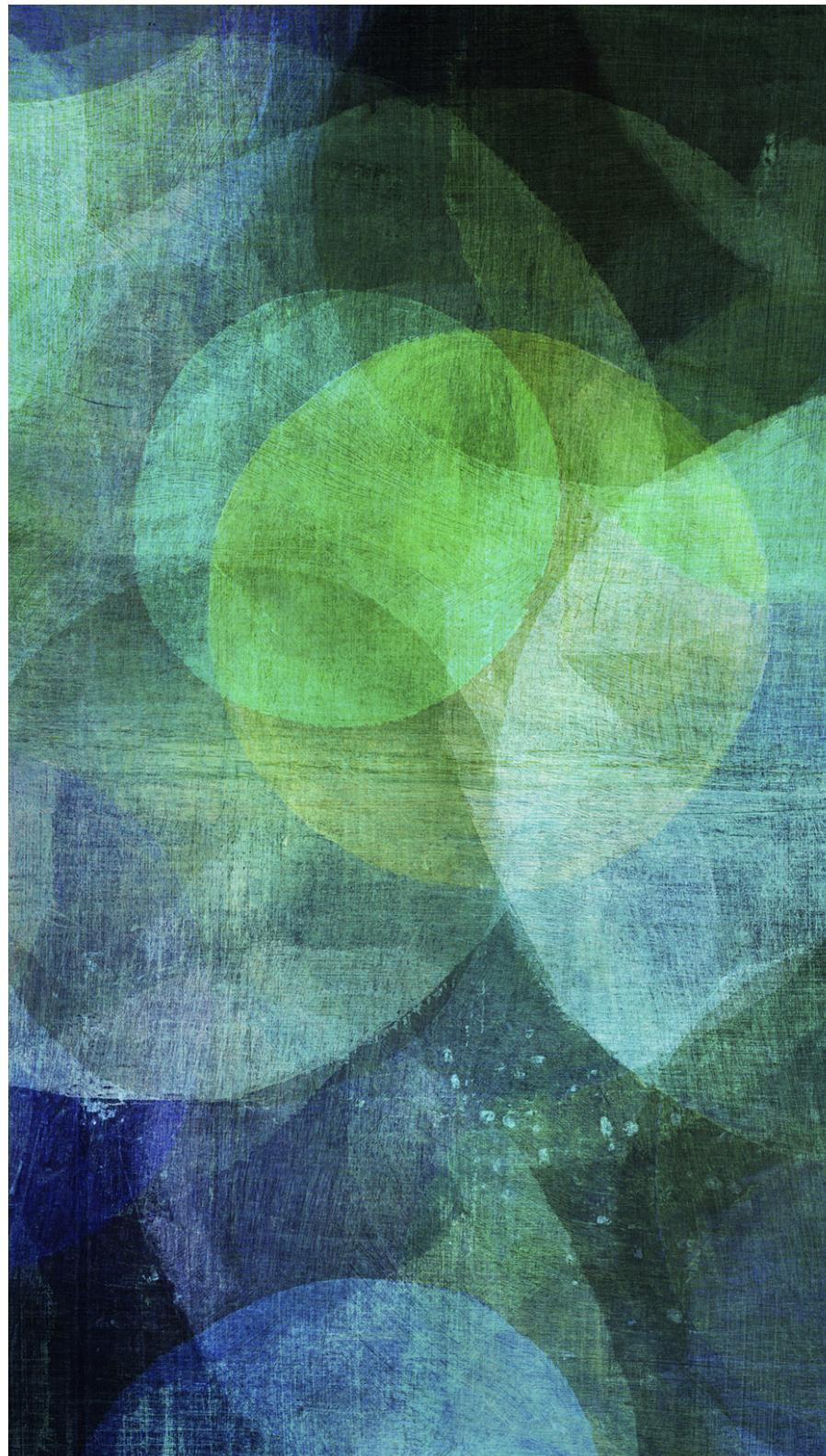
- Steady-state replacement: At each generation of an EA, only one offspring is generated.

PARAMETERS



- Mutation probability: A large mutation probability will disrupt a given individual and the search is more likely random.
- Crossover probability pc : The crossover probability is generally set from medium to large values
- Population size: The larger is the size of the population, the better is the convergence toward “good” solutions. Sampling errors are more important in smaller populations.

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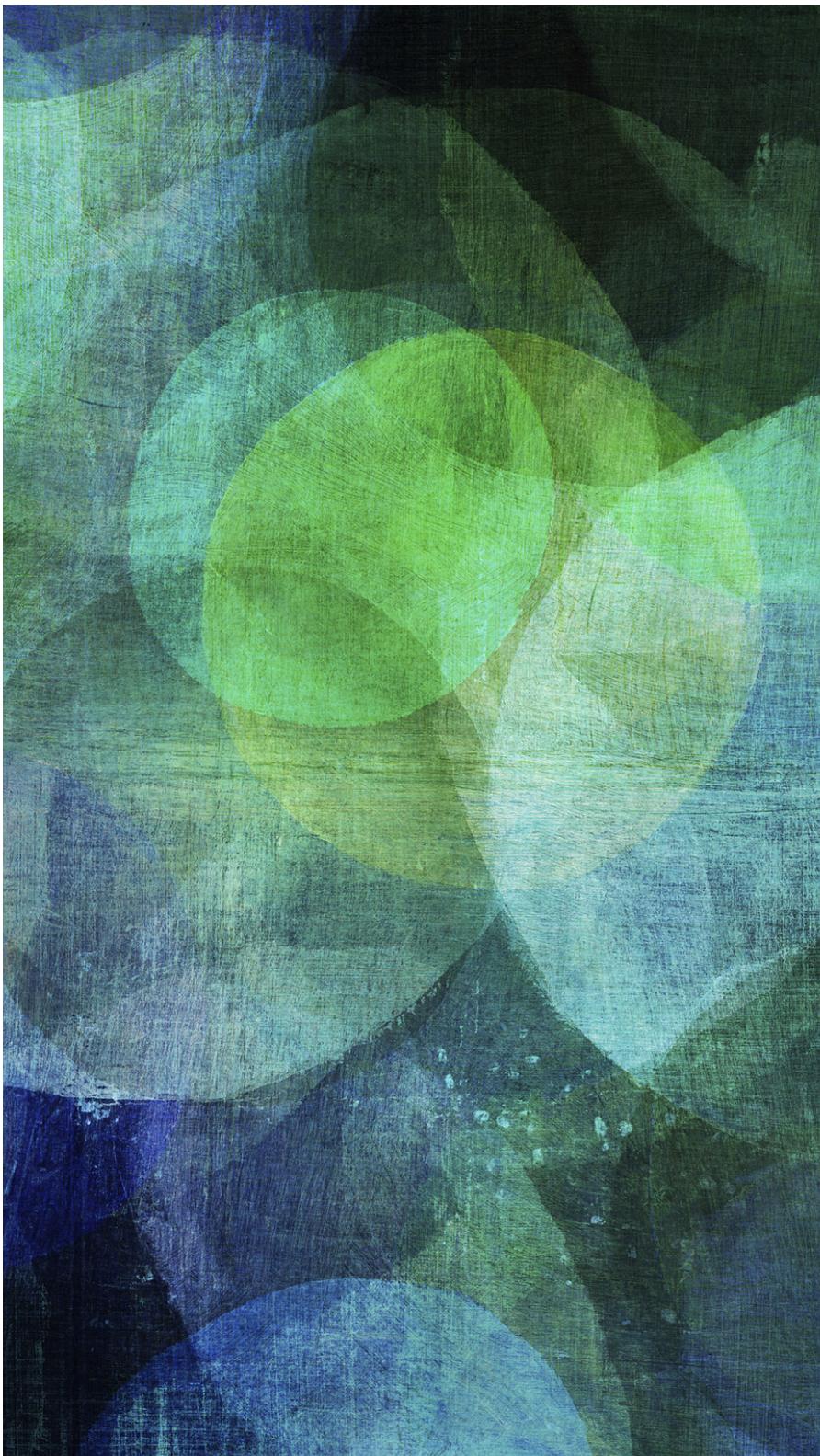


DISCUSSION

With the current
population

How will you REPLACE THE
CURRENT POPULATION to
maximize the possibility of
evolution?

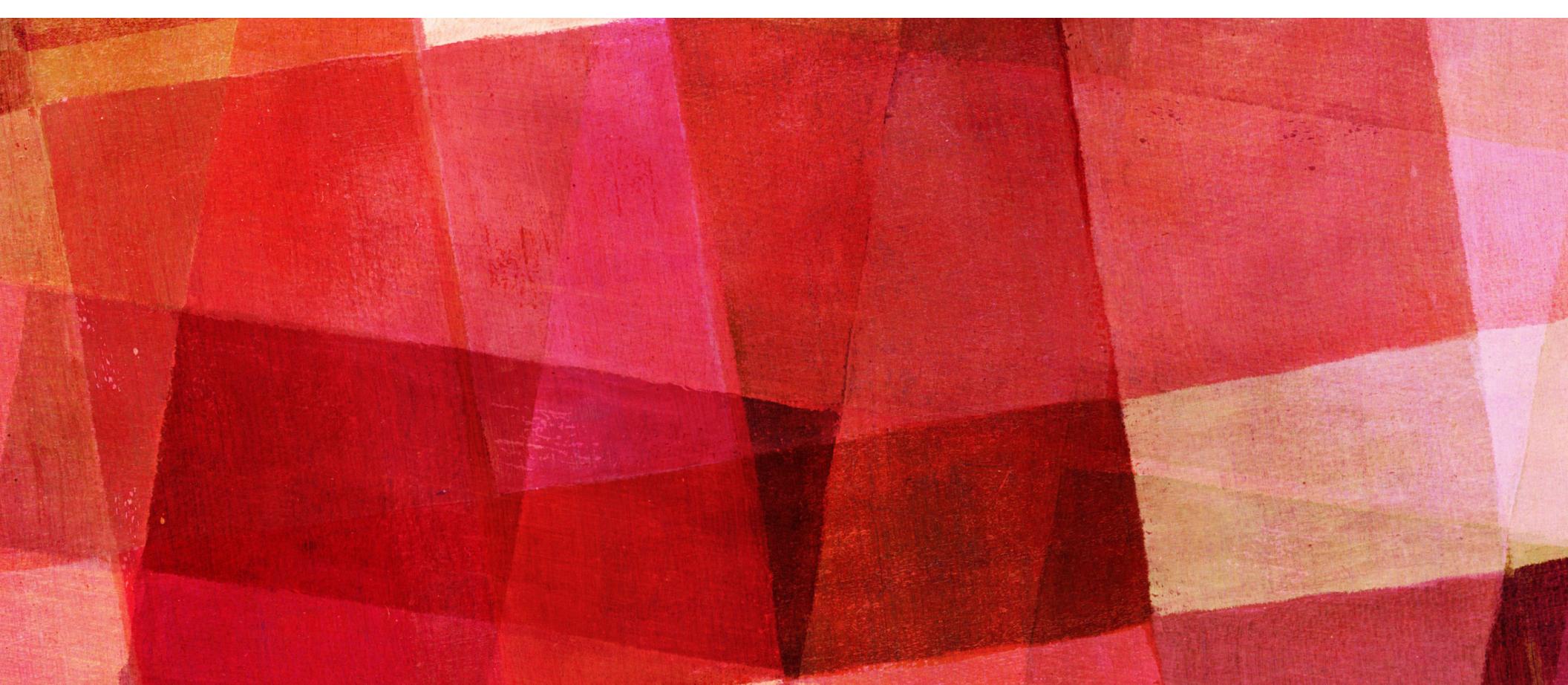
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WORK, WORK, WORK!

With the
WORKSHEET perform
at least 3 generations
of the method you just
designed.

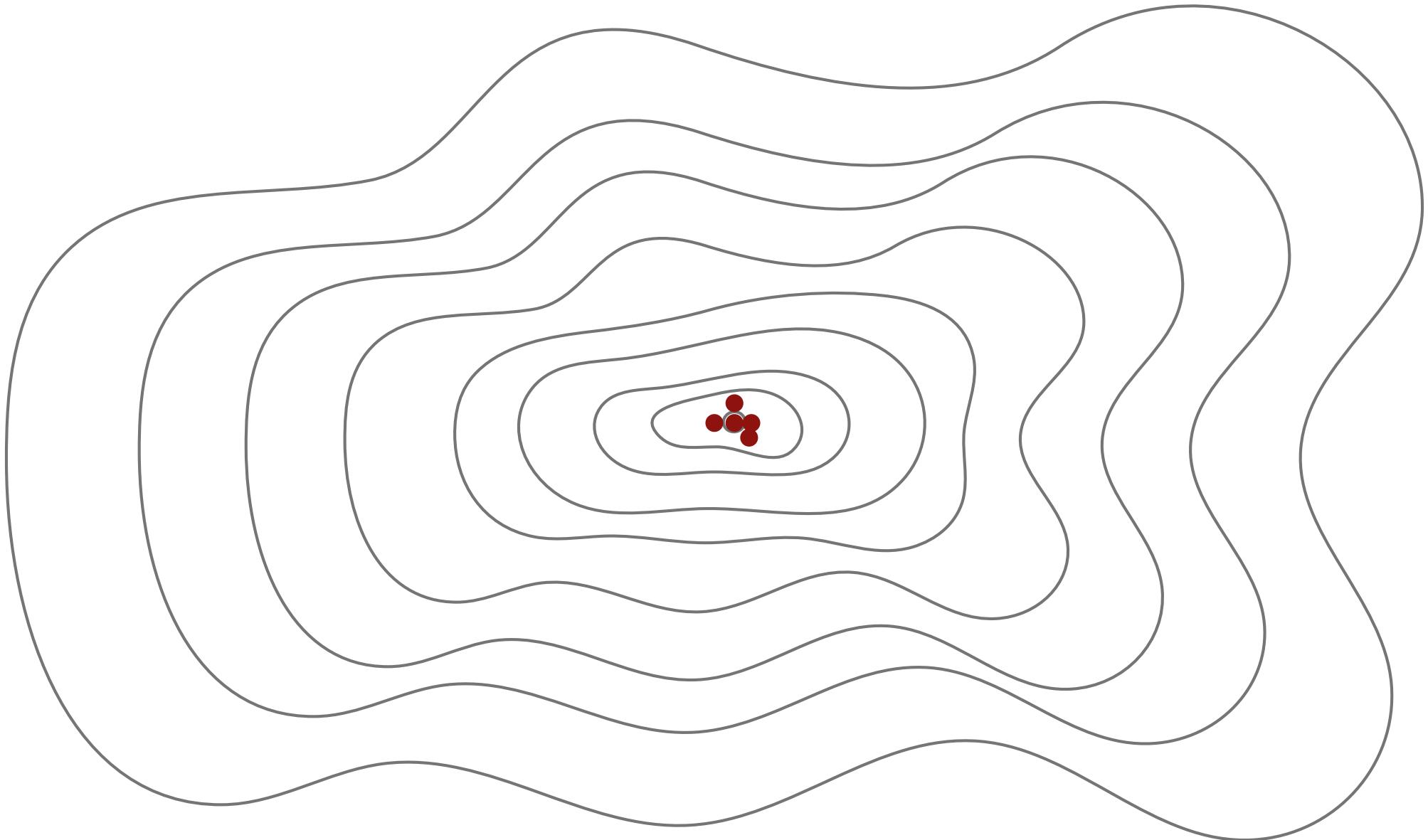
**PRESENT YOUR RESULTS
TO THE CLASS**



CODING TIPS

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LET'S TALK ABOUT VARIABILITY LOSS



THINK ABOUT

INTENSIFICATION

VS

DIVERSIFICATION

INITIAL POPULATION

- In the generation of the initial population, the main criterion to deal with is diversification.
- If the initial population is not well diversified, a premature convergence can occur for any P-metaheuristic.
- The determination of the initial population is often disregarded in the design of a P-metaheuristic.
- This step plays a crucial role in the effectiveness of the algorithm and its efficiency.



INITIAL POPULATION

TABLE 3.2 Analysis of the Different Initialization Strategies

| Strategy | Diversity | Computational Cost | Quality of Initial Solutions |
|----------------------------|-----------|--------------------|------------------------------|
| Pseudo-random | ++ | +++ | + |
| Quasi-random | +++ | +++ | + |
| Sequential diversification | ++++ | ++ | + |
| Parallel diversification | ++++ | +++ | + |
| Heuristic | + | + | +++ |

More generally, the generation of the initial population in a way to optimize a given diversification criterion can be defined as an optimization problem. This problem may be as (or more) difficult as the original problem to solve.

STOP CRITERIA

- Static procedure: In a static procedure, the end of the search may be known a priori. For instance, one can use a fixed number of iterations (generations), a limit on CPU resources, limited time or a maximum number of objective function evaluations.
- Adaptive procedure: In an adaptive procedure, the end of the search cannot be known a priori. One can use a fixed number of iterations (generations) without improvement, when an optimum or a satisfactory solution is reached, one measure of diversity is too low, etc.