Growth of reattachment streaks in hypersonic compression ramp flow

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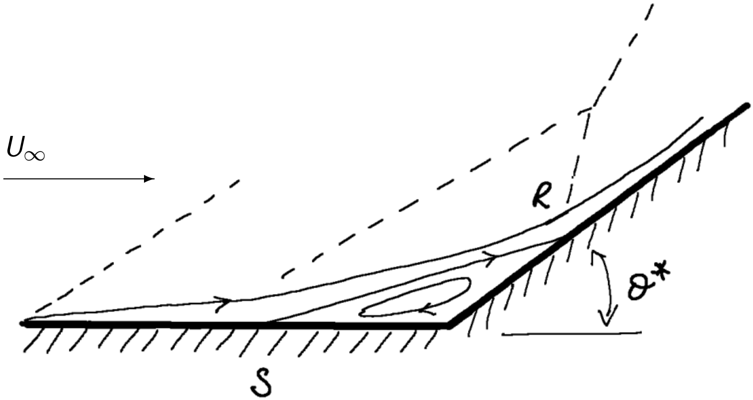
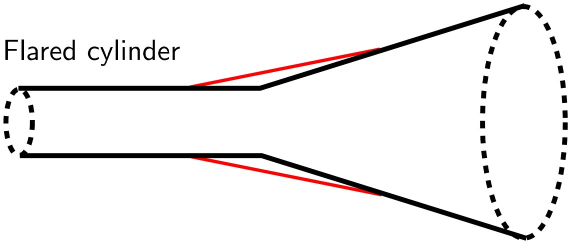
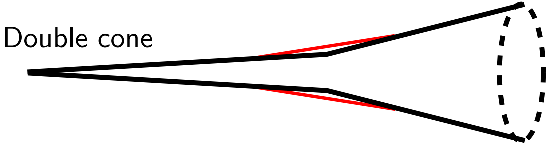
Imperial, May 9th 2025

Compression ramp flow



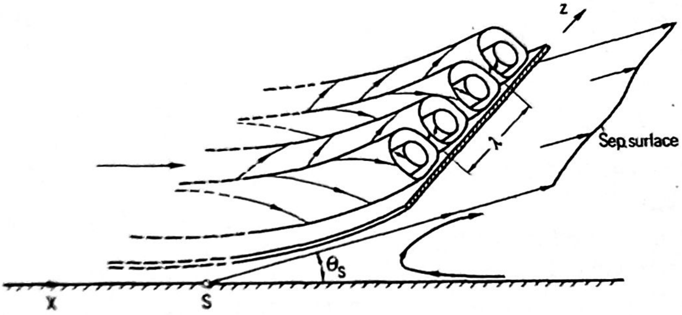
Variations on the same theme





Centrifugal effects at separation

Spanwise-periodic disturbances at separation.



From: Inger, ZAMP 1977, 28.

See also: Denier & Bassom, Stud. Appl. Math. 1996, 96.

DNS: three-dimensional examples

- Navarro-Martinez & Tutty Comp. in Fluids 2005, 34.
  3D base flow with finite spanwise extent.
  Upstream forcing.
  Downstream streaks/vortices at/post reattachment.
  Centrifugal: local estimates of Görtler number.
- Dwivedi et al. JFM 2019, 880.
  Baroclinic effects lead to streaks/vortices.
- Hao et al. JFM 2021, 919.
  2D baseflow, ARPACK GSA.
  Effects of wall cooling.
  Eigenmodes confined to separation bubble.
  Production localized to the primary vortex core.
  Suggest the mode is linked to secondary separation.

DNS stability features

- Two responses (i) an 'intrinsic' (global) instability (ii) spatial growth through the corner.

Global mode

  - develops at a critical ramp angle.
  - is localised to the separation bubble in the corner.
  - suggested it is related to secondary separation.
  - production localised to the 'eye' of the separation bubble.

Spatial

  - peak energy growth increases with ramp angle.
  - growth occurs between S & R.
  - upstream disturbances persist as downstream streaks post-R.
  - disturbance is deflected above the reverse flow region.
  - linked to both baroclinic and centrifugal mechanisms.

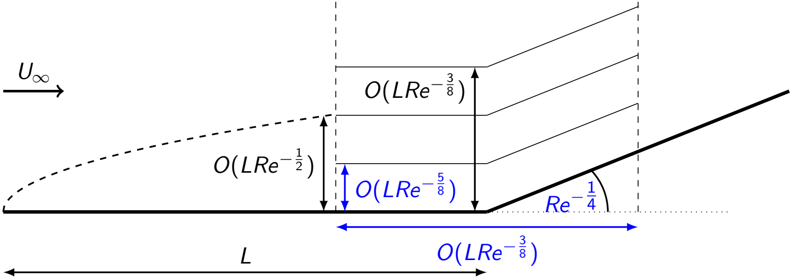
- In both cases some (not all) spanwise lengthscales are comparable to the transverse scale in the corner.

Aims/contents

- Take an asymptotic approach, $\mathrm{Re} \gg 1$.
- Triple-deck formulation for $\theta^* \sim \mathrm{Re}^{-1/4}$: supersonic.
- $\mathrm{Ma} \gg 1$ problem with wall cooling: hypersonic.
  - no real gas effects.
- Solve for 2D base flows in the corner.
- Formulate a Görtler problem for lower deck disturbances.
- Solutions for the global and developing 3D disturbances.

The supersonic problem

- Take an asymptotic approach, $\mathrm{Re} = \rho_\infty U_\infty L/\mu_* \gg 1$.
- $M_\infty = U_\infty\sqrt{\gamma p_\infty/\rho_\infty} = O(1)$.



Lower deck

$$
\begin{aligned}
x &= L(1 + \mathrm{Re}^{-\frac{3}{8}})X\,,\\
y &= L\mathrm{Re}^{-\frac{5}{8}}Y\,,\\
(u, v) &= U_\infty(\mathrm{Re}_\infty^{-\frac{1}{8}}[\lambda Y + U_B(X, Y)], \quad \mathrm{Re}_\infty^{-\frac{3}{8}} V_B(X, Y)) + \cdots\,,\\
p &= p_\infty(1 + \gamma M_\infty \mathrm{Re}_\infty^{-\frac{2}{8}} P_B(X)) + \cdots\,,\\
\rho &= \rho_w + \cdots\,,\\
\mu &= \mu_w + \cdots\,.
\end{aligned}
$$

- λ is the upstream shear at the 'corner'.
- γ is the ratio of specific heats.
- We can rescale to remove λ, γ and M_∞ from the problem.

The canonical problem

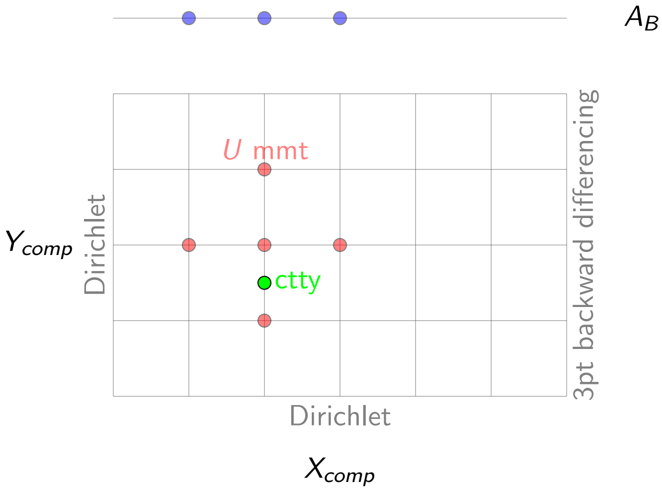
$$(Y + U_B)\frac{\partial U_B}{\partial X} + V_B\left(1 + \frac{\partial U_B}{\partial Y}\right) = -P'_B(X) + \frac{\partial^2 U_B}{\partial Y^2}\,,$$
$$\frac{\partial U_B}{\partial X} + \frac{\partial V_B}{\partial Y} = 0\,,$$

with

$$U_B \to 0 \quad \text{as} \quad X \to -\infty$$
$$U_B = V_B = 0 \quad \text{on} \quad Y = 0\,,$$
$$U_B \to A_B(X) + \alpha F(X) \quad \text{as} \quad Y \to \infty\,,$$
$$P_B = -A'_B(X)\,.$$

After moving to a new Y measured relative to the ramp surface.

Baseflow computational scheme



Newton iteration for all d.o.f

- At a given ramp angle α we are left to solve:

$$J_{NxN}(\underline{q})\,\underline{\tilde{q}} = \underline{B}(\underline{q})$$

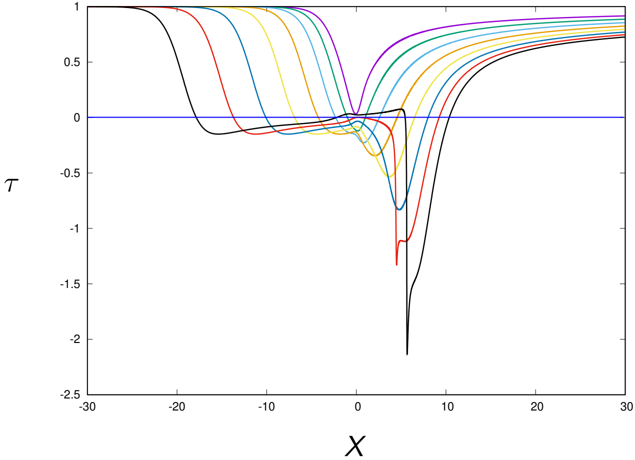
  for a correction $\underline{\tilde{q}}$. Then $\underline{q} := \underline{q} + \underline{\tilde{q}}$.
- J is sparse, with

$$N = N_x(2N_Y + 1)\,.$$

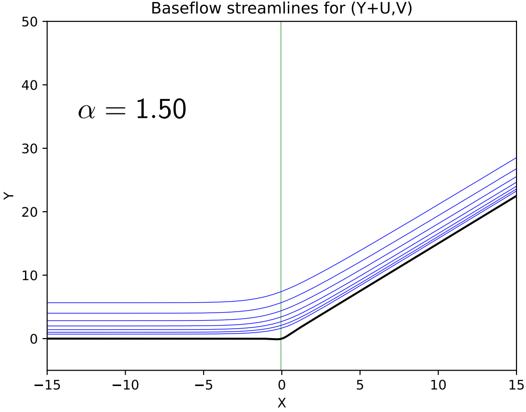
- Typical values:
  $N_x = 4001$ and $N_y = 201$.
  $X \in [-50, 50]$, $Y \in [0, 50]$.
- Sparse direct solvers: MUMPs/SuperLU.
- A single parameter problem: ramp angle α.

Supersonic baseflow results: scaled shear τ

$$\alpha = 1, 1.5, 2, \cdots, 4.5, 5$$



Supersonic baseflow results: streamlines



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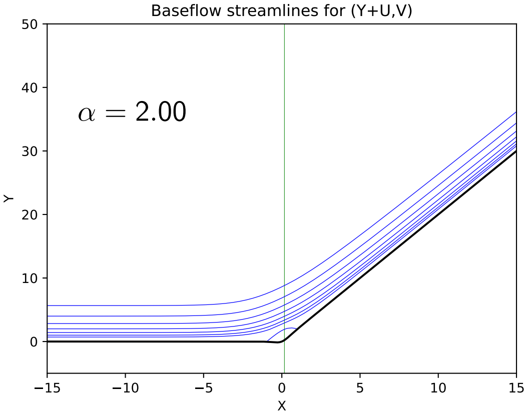

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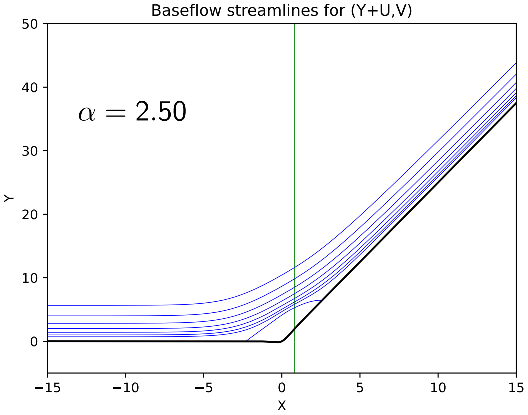
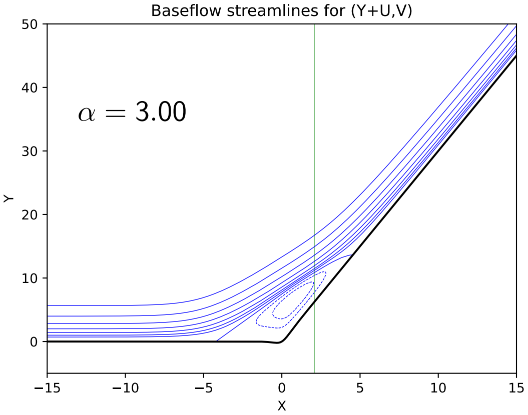
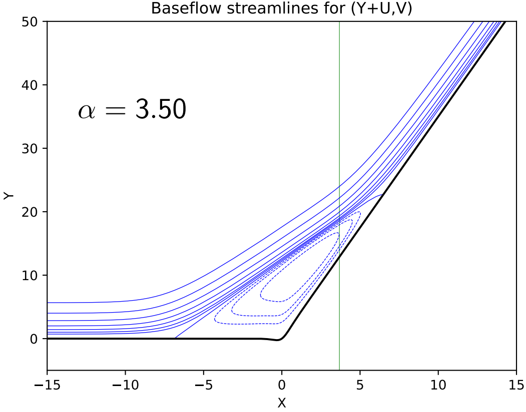


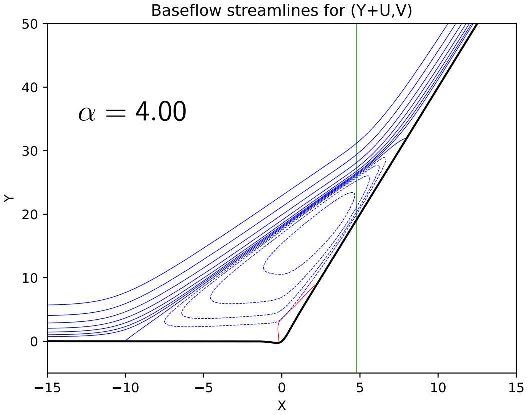
Supersonic baseflow results: streamlines



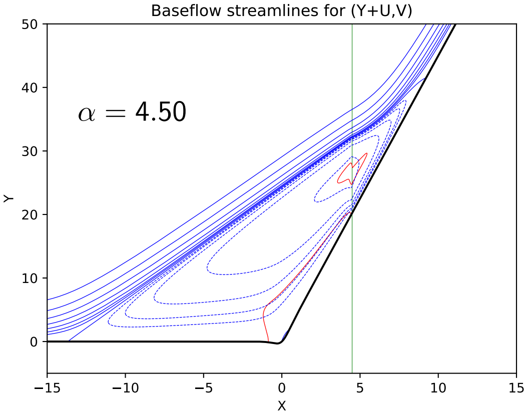
Supersonic baseflow results: streamlines



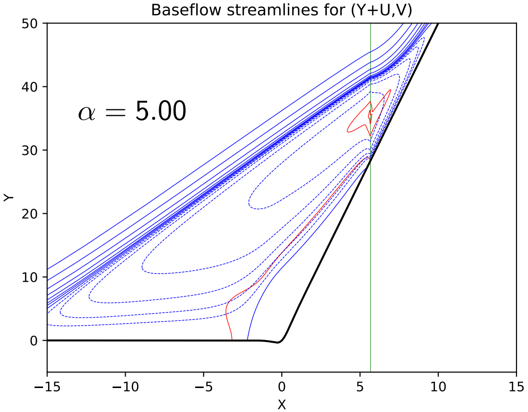
Supersonic baseflow results: streamlines



Supersonic baseflow results: streamlines



Supersonic baseflow results: streamlines



See Broadley, Hewitt & Gajjar JFM 2023, 968.

The wall-cooled hypersonic version

- Both $M_\infty, \mathrm{Re} \gg 1$.
- We will assume that

$$\chi = M_\infty^2 \mathrm{Re}^{-1/2} \ll 1\,.$$

- The impinging BL now has thickness $O(M_\infty Re^{-1/2})$ and a low density of $O(M_\infty^{-2})$.
- Enthaply and viscosity:

$$h = U_\infty^2 \hat{h}\,,$$
$$\mu = \mu_* \hat{h}^n\,, \quad (n > 0).$$

- Cooling at the wall with $\hat{h}_w = h(wall)/U_\infty^2$:

$$\hat{h}_w \ll 1.$$

Transverse scales

Upstream:

- Bulk BL is $O(M_\infty \mathrm{Re}^{-1/2})$ and density is low $O(M_\infty^{-2})$.
- Flow has to adjust to the cooled wall as $\hat{h}_w \ll 1$.
- Thin upstream near-wall region $O(M_\infty \mathrm{Re}^{-1/2} \hat{h}_w^{n+1})$.
- Density is increased to $O(M_\infty^{-2}/\hat{h}_w)$.

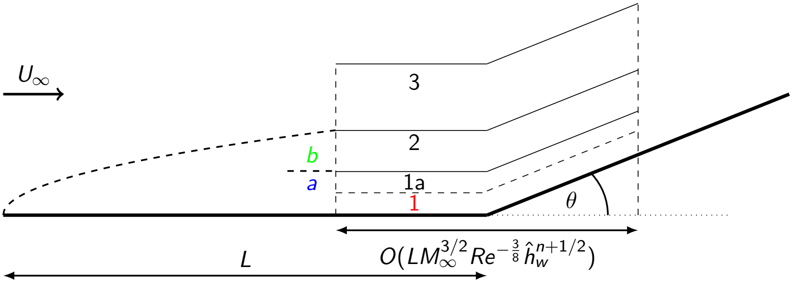
Lower deck:

- The lower deck is: $O(M_\infty^{3/2} \hat{h}_w^{n+1/2} \mathrm{Re}^{-5/8})$.

Lower deck remains thin compared to the upstream near wall region until

- $\hat{h}_w \sim M_\infty \mathrm{Re}^{-1/4} = \chi^{1/2} \ll 1$.
- Breaking this re-introduces compressibility into lower deck.

Asymptotic regions



b/a: the upstream BL has a near-wall layer a.

1: the lower deck thickness is now $M_\infty^{3/2}\mathrm{Re}^{-5/8}\hat{h}_w^{n+1/2}$

We assume a remains thicker than 1.

Baseflow equations

We obtain largely the same lower-deck problem, but the interaction condition is now:

$$P_B(X) = -A_B'(X) + S\mathcal{L}\, P_B'(X)\,,$$

where

$$S = M_\infty^{1/2}\mathrm{Re}^{-1/8}/\hat{h}_w^{(n+1/2)} = \chi^{1/4}/\hat{h}_w^{n+1/2} = O(1).$$

$\mathcal{L}$ is an integral of the upstream profile impinging on the corner - we expect $\mathcal{L} < 0$.

See: Kerimbekov, Ruban & Walker, JFM 1994, 274.

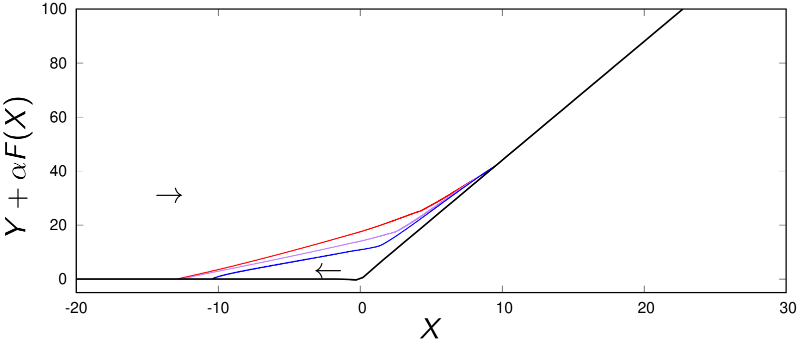
Cooling regimes

$$P_B(X) = -A'_B(X) + S\mathcal{L}\,P'_B(X)\,,$$

- $S \ll 1$: same supersonic compression ramp problem.
- $S = O(1)$: modified interaction law (moderate cooling).
- Main deck displacement dominates as S increases (strong cooling).
- $S \gg 1$: when $\hat{h}_w \sim M_\infty \mathrm{Re}^{-1/4} = \chi^{1/2} \ll 1$ compressibility effects enter the lower deck.

Baseflow results: reverse flow region

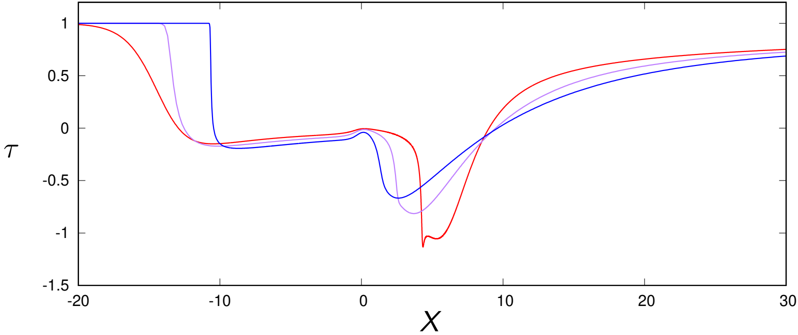
$\alpha = 4.4$, $S\mathcal{L} = 0(\mathit{red}), -2.5(\mathit{purple}), -5(\mathit{blue})$.



Recall that: $x = L(1 + M_\infty^{3/2}\mathrm{Re}^{-3/8}\hat{h}_w^{n+1/2}X)$

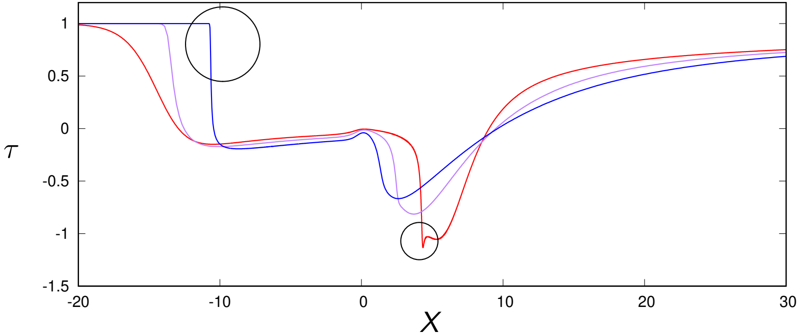
Baseflow results: τ

$\alpha = 4.4$, $S\mathcal{L} = 0(red), -2.5(purple), -5(blue)$.



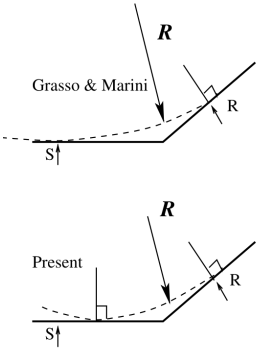
Baseflow results: τ

$\alpha = 4.4$, $S\mathcal{L} = 0(\mathit{red}), -2.5(\mathit{purple}), -5(\mathit{blue})$.



See Cassel, Ruban & Walker JFM 1996, 321.

Centrifugal instability



From: Navarro-Martinez & Tutty Comp. & Fluids, 2005.

- DNS results estimate flow curvature.
- A Görtler number is estimated.
- Attempts to compare spatial growth through the corner.
- Instead we will try to solve the Görtler problem in the lower deck.

Linear centrifugal instability

Approach of Hall & Bennett JFM 1986, 171 (incompressible).

Linear perturbation $|\epsilon| \ll 1$:

$$\tilde{\mathbf{u}} = \epsilon M_\infty^{1/2} \hat{h}_w^{1/2} \left(\begin{array}{c} \mathrm{Re}^{-1/8} \tilde{U}(X, Y, Z, T) \\ \mathrm{Re}^{-3/8} \tilde{V}(X, Y, Z, T) \\ \mathrm{Re}^{-3/8} \tilde{W}(X, Y, Z, T) \end{array} \right) ,$$

$$\tilde{p} = \epsilon \gamma M_\infty^{-1} \mathrm{Re}^{-3/4} \hat{h}_w^{n+1/2} \tilde{Q}(X, Y, Z, T) ,$$

$$z = M_\infty^{3/2} \hat{h}_w^{n+1/2} \mathrm{Re}^{-5/8} Z \quad \text{(lower deck scale)},$$
$$t = M_\infty \hat{h}_w^n \mathrm{Re}^{-1/4} T .$$

Linear perurbation equations in the lower deck

$$\tilde{U}_X + \tilde{V}_Y + \tilde{W}_Z = 0,$$
$$\tilde{U}_T + \overline{U}_B \tilde{U}_X + \tilde{U}\overline{U}_{BX} + V_B \tilde{U}_Y + V\overline{U}_{BY} = \nabla_2^2 \tilde{U},$$
$$\tilde{V}_T + \overline{U}_B \tilde{V}_X + \tilde{U} V_{BX} + \tilde{V} V_{BY} + V_B \tilde{V}_Y + 2\alpha F_{XX} \overline{U}_B \tilde{U} = -\tilde{Q}_Y + \nabla_2^2 \tilde{V},$$
$$\tilde{W}_T + \overline{U}_B \tilde{W}_X + V_B \tilde{W}_Y = -\tilde{Q}_Z + \nabla_2^2 \tilde{W}.$$

$$\overline{U}_B = \lambda Y + U_B(X, Y)$$
$$\tilde{U} = \hat{U}(X, Y, T) e^{i\beta Z} \quad \text{etc}$$

$$\hat{U} = \hat{W} = 0, \ \hat{V} = \hat{V}_{blowing}(X, T) \quad \text{at} \quad Y = 0,$$
$$\hat{U}, \hat{V}, \hat{W} \to 0 \quad \text{as} \quad Y \to \infty,$$

Upstream forcing

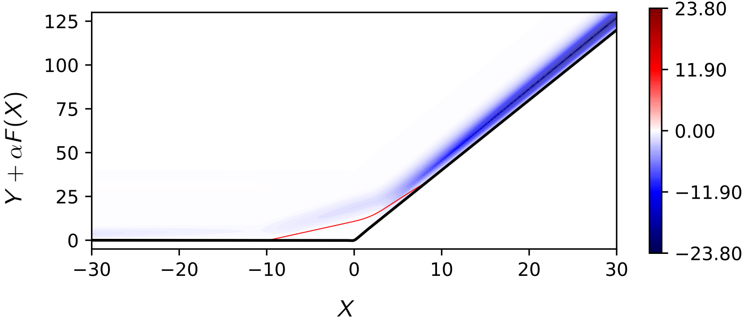
- Ad-hoc forcing applied upstream:

$$\hat{V}_{blowing} = \exp^{-(X-X_0)^2/4}$$

- This could obviously be something more 'realistic'.
- Look for steady solutions to the disturbance equations.
- No attempt to optimize the input forcing for peak growth.

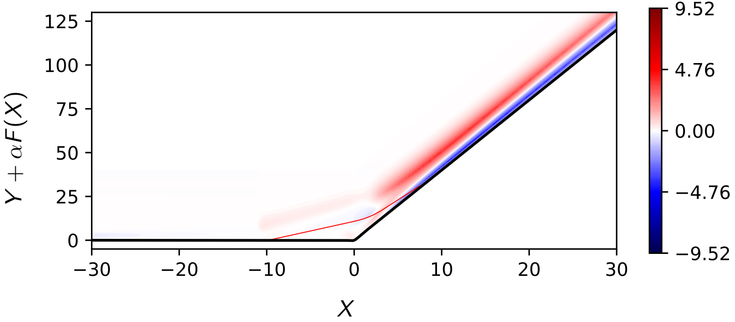
Streamwise perturbation $\hat{U}(X, Y)$

$$\alpha = 4,\ S\mathcal{L} = -2.5,\ \beta = 0.4,\ X_0 = -40.$$



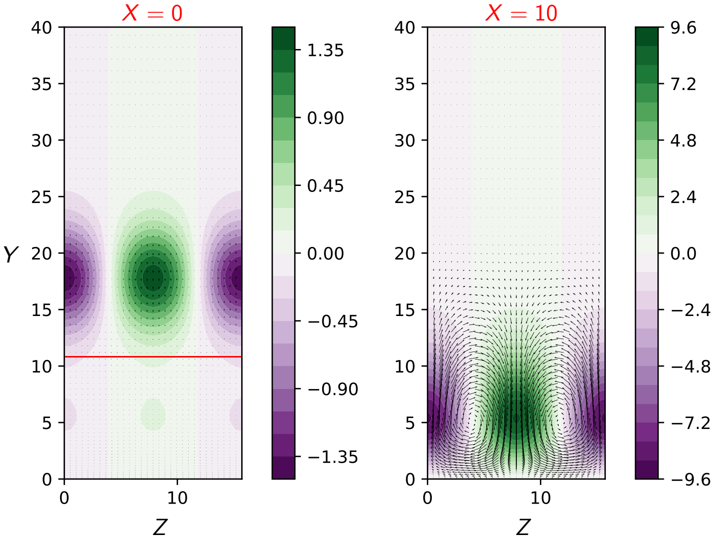
Spanwise perturbation $\hat{W}(X, Y)$

$$\alpha = 4,\ S\mathcal{L} = -2.5,\ \beta = 0.4,\ X_0 = -40.$$



Cross-sectional view

$$\alpha = 4.0,\ \beta = 0.4,\ S\mathcal{L} = -2.5.$$



Upstream disturbance, downstream growth

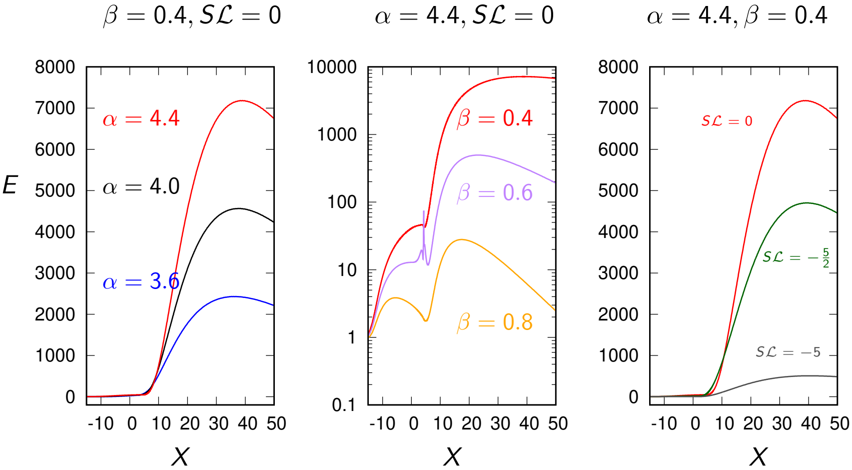
- Introduce a metric for the response

$$E(X) = \frac{1}{\Gamma} \int_0^\infty \tilde{U}^2(X, Y)\, \mathrm{d}Y\,.$$

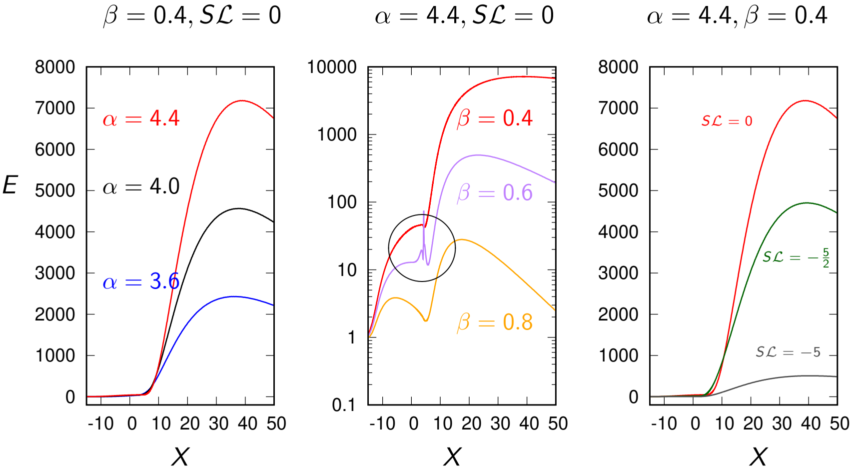
- Γ normalises the energy to $E = 1$ at the separation point.
- We also define a local spatial growth rate

$$\sigma(X) = \frac{1}{E} \frac{\mathrm{d}E}{\mathrm{d}X}\,.$$

Downstream disturbance E growth



Downstream disturbance E growth



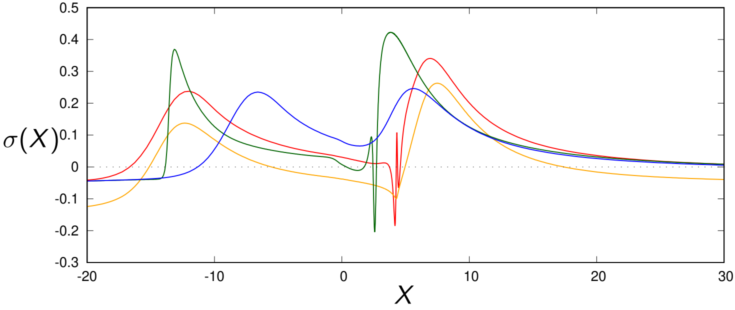
A local growth rate

$$\sigma(X) = \frac{1}{E}\frac{\mathrm{d}E}{\mathrm{d}X}\,.$$

$\beta = 0.4 : \alpha = 4.4, \alpha = 3.6,\ S\mathcal{L} = 0.$

$\beta = 0.8$: $\alpha = 4.4,\ S\mathcal{L} = 0.$

$\beta = 0.4 : \alpha = 4.4, S\mathcal{L} = -2.5.$



Bi-global eigenmodes

- Suppose instead we look for bi-global modes localised to the corner

$$\tilde{U} = \hat{U}(X, Y)e^{i\beta Z}e^{sT}$$

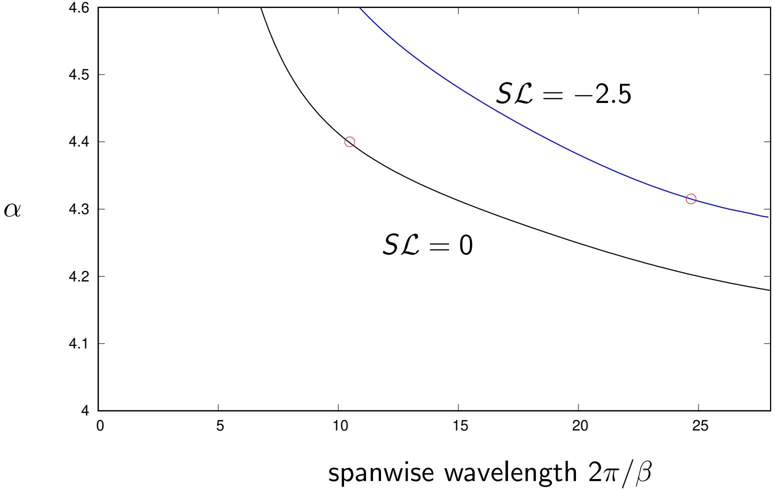
subject to upstream and downstream decay.

- Gives a sparse bi-global eigenvalue problem for the temporal growth

$$s = s(\alpha, \beta, S\mathcal{L})$$

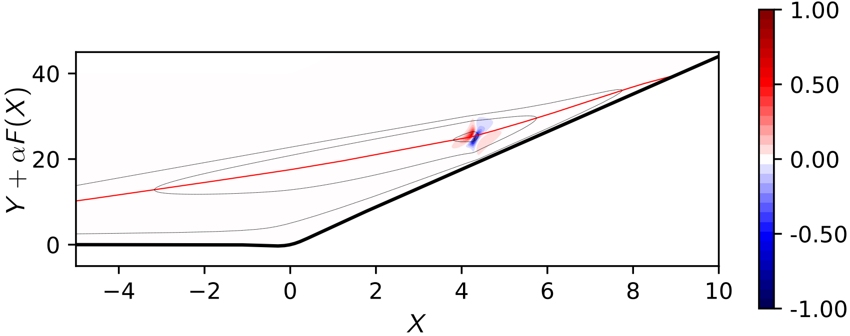
- SLEPc library to iteratively find s.
- Checked against an initial-value computation forced by transient blowing.

Bi-global neutral modes



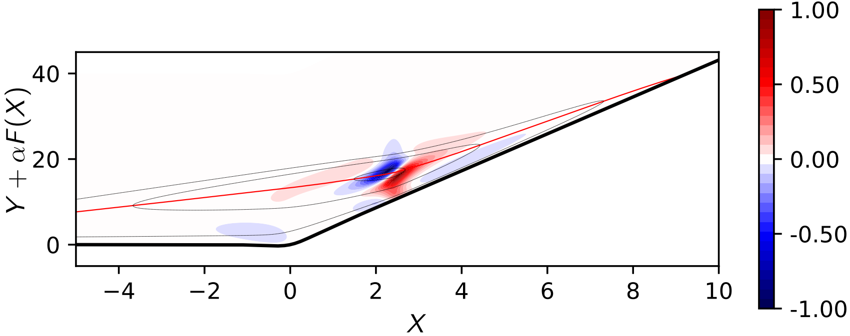
Bi-global eigenmode: spanwise velocity $\hat{W}(X, Y)$

$\alpha = 4.4,\ \beta = 0.6,\ S\mathcal{L} = 0.$



Bi-global eigenmode: spanwise velocity $\hat{W}(X, Y)$

$\alpha = 4.315,\ \beta = 0.2545,\ S\mathcal{L} = -2.5.$



Conclusions

Spatial growth of upstream disturbances

- For $\lambda_z \sim \text{Re}^{-5/8}$ upstream disturbances evolve to downstream streaks.
- Görtler equations give localised growth at S & R.
- Streaks lie over the separation bubble.
- The lower deck is incompressible, **no** baroclinic mechanism on this scale.
- Wall cooling reduces separation and streak growth*.

Global instability

- The separation bubble bifurcates to 3D at critical α.
- The eigenmode is localised to the primary eye of the recirculation (for $\lambda_z \sim \text{Re}^{-5/8}$).
- It is **not** connected to secondary separation.

To do: nonlinear effects, secondary instability, larger λ_z scales.