

# Manual of FXPACK

## 1 Outline

This is a package of subroutines to compute FFT. Since the radix of the transform is 2, 3, and 5, the data-length to be transformed must be factorized by these prime numbers.

In each group of subroutines shown below, the initialization subroutine, the name of which ends with the letter I, must be called once before other subroutines in the group are called.

For computational efficiency, the transform subroutines treat multiple data streams simultaneously. If a two-dimensional array,  $X(I, J)$ ,  $I=1, 2, \dots, M$ ,  $J=1, 2, \dots, N$ , is given, Fourier transform of  $X(I, 1), X(I, 2), \dots, X(I, N)$  is computed for each  $I$ . That is, Fourier transform of length- $N$  data stream is computed  $M$  times. If only one data stream is given, you should set as  $M=1$ .

If you did make ISPACK with setting either  $SSE=avx$  or  $SSE=fma$  on Intel x86 CPUs and using  $M=4$ , the array that contains the data streams to be transformed must be aligned with 32byte boundary. If you did make ISPACK with setting  $SSE=avx512$  on Intel x86 CPUs and using  $M=8$ , the array that contains the data streams to be transformed must be aligned with 64byte boundary. In those cases, you should allocate a memory area with `MXALLC`, for example.

## 2 List of subroutines

Discrete complex Fourier transform

<code>FXZINI(N, IT, T)</code>	Initialization
<code>FXZTFA(M, N, X, IT, T)</code>	Forward transform
<code>FXZTBA(M, N, X, IT, T)</code>	Backward transform

Discrete real Fourier transform

<code>FXRINI(N, IT, T)</code>	Initialization
<code>FXRTFA(M, N, X, IT, T)</code>	Forward transform
<code>FXRTBA(M, N, X, IT, T)</code>	Backward transform

### 3 Usage of each subroutine

#### 3.1 FXZINI/FXZTFA/FXZTBA

##### 1. Purpose

Discrete complex (forward/backward) Fourier transforms of  $M$  sets of one-dimensional data streams of length  $N$ ,  $\{x_j\}$  or  $\{\alpha_k\}$  are computed using FFT algorithm. Note that  $N$  must be factorized as  $N = 2^a 3^b 5^c$  ( $a, b, c$ : 0 or natural number). **FXZINI** does initialization; **FXZTFA** computes forward Fourier transform; **FXZTBA** computes backward Fourier transform.

##### 2. Definition

###### • Forward Fourier transform

The input  $\{x_j\}$  is transformed as follows to give the output  $\{\alpha_k\}$ .

$$\alpha_k = \frac{1}{N} \sum_{j=0}^{N-1} x_j \exp\left(-2\pi i \frac{jk}{N}\right), \quad (k = 0, 1, \dots, N-1)$$

###### • Backward Fourier transform

The input  $\{\alpha_k\}$  is transformed as follows to give the output  $\{x_j\}$ .

$$x_j = \sum_{k=0}^{N-1} \alpha_k \exp\left(2\pi i \frac{jk}{N}\right), \quad (j = 0, 1, \dots, N-1)$$

##### 3. Synopsis

```
FXZINI(N,IT,T)
FXZTFA(M,N,X,IT,T)
FXZTBA(M,N,X,IT,T)
```

##### 4. Parameters

```
INTEGER(8) :: M,N,IT(N)
REAL(8) :: T(N*2),X(M*2*N)
```

**M** Input. The number of data streams  $M$  to be transformed simultaneously.  
**N** Input. The length of data stream  $N$ .  
**X** Input/Output. For input:  $\{x_j\}$  or  $\{\alpha_k\}$ . For output:  $\{\alpha_k\}$  or  $\{x_j\}$ .  
**IT** Output(**FXZINI**) or Input(**FXZTFA**, **FXZTBA**).  
**T** Outupt(**FXZINI**) or Input(**FXZTFA**, **FXZTBA**).

##### 5. Order of data stream

If the input/output array is declared as **X(M,2,0:N-1)**, the data is contained in it for each  $I$  as follows.

<b>X(I,1,0)</b>	<b>X(I,2,0)</b>	<b>X(I,1,1)</b>	<b>X(I,2,1)</b>	$\cdots$	<b>X(I,1,N-1)</b>	<b>X(I,2,N-1)</b>
$\text{Re}(x_0)$	$\text{Im}(x_0)$	$\text{Re}(x_1)$	$\text{Im}(x_1)$	$\cdots$	$\text{Re}(x_{N-1})$	$\text{Im}(x_{N-1})$
$\text{Re}(\alpha_0)$	$\text{Im}(\alpha_0)$	$\text{Re}(\alpha_1)$	$\text{Im}(\alpha_1)$	$\cdots$	$\text{Re}(\alpha_{N-1})$	$\text{Im}(\alpha_{N-1})$

### 3.2 FXRINI/FXRTFA/FXRTBA

#### 1. Purpose

Discrete real (forward/backward) Fourier transforms of  $M$  sets of one-dimensional data streams of length  $N$ ,  $\{x_j\}$  are computed using FFT algorithm. Note that  $N$  must be an even number and  $N/2$  must be factorized as  $N = 2^a 3^b 5^c$  ( $a, b, c$ : 0 or natural number). FXRINI does initialization; FXRTFA computes forward Fourier transform; FXRTBA computes backward Fourier transform.

#### 2. Definition

- Forward Fourier transform

The input  $\{x_j\}$  is transformed as follows to give the output  $\{a_k\}, \{b_k\}$ .

$$a_k = \frac{1}{N} \sum_{j=0}^{N-1} x_j \cos \frac{2\pi jk}{N}, \quad (k = 0, 1, \dots, N/2)$$

$$b_k = -\frac{1}{N} \sum_{j=0}^{N-1} x_j \sin \frac{2\pi jk}{N}, \quad (k = 1, 2, \dots, N/2 - 1)$$

- Backward Fourier transform

The input  $\{a_k\}, \{b_k\}$  is transformed as follows to give the output  $\{x_j\}$ .

$$x_j = a_0 + a_{N/2}(-1)^{j+2} \sum_{k=1}^{N/2-1} \left( a_k \cos \left( \frac{2\pi jk}{N} \right) - b_k \sin \left( \frac{2\pi jk}{N} \right) \right) \quad (j = 0, 1, \dots, N-1)$$

#### 3. Synopsis

```
FXRINI(N, IT, T)
FXRTFA(M, N, X, IT, T)
FXRTBA(M, N, X, IT, T)
```

#### 4. Parameters

```
INTEGER(8) :: M, N, IT(N/2)
REAL(8) :: T(N*3/2), X(M*N)
```

**M** Input. The number of data streams  $M$  to be transformed simultaneously.  
**N** Input. The length of data stream  $N$ .  
**X** Input/Output. For input:  $\{x_j\}$  or  $\{a_k\}, \{b_k\}$ . For output:  $\{a_k\}, \{b_k\}$  or  $\{x_j\}$ .  
**IT** Output(FXRINI) or Input(FXRTFA, FXRTBA).  
**T** Outupt(FXRINI) or Input(FXRTFA, FXRTBA).

#### 5. Order of data stream

If the input/output array is declared as  $X(M, 0:N-1)$ , the data is contained in it for each  $I$  as follows.

$X(I, 0)$	$X(I, 1)$	$X(I, 2)$	$X(I, 3)$	$\dots$	$X(I, N-2)$	$X(I, N-1)$
$x_0$	$x_1$	$x_2$	$x_3$	$\dots$	$x_{N-2}$	$x_{N-1}$
$a_0$	$a_{N/2}$	$a_1$	$b_1$	$\dots$	$a_{N/2-1}$	$b_{N/2-1}$