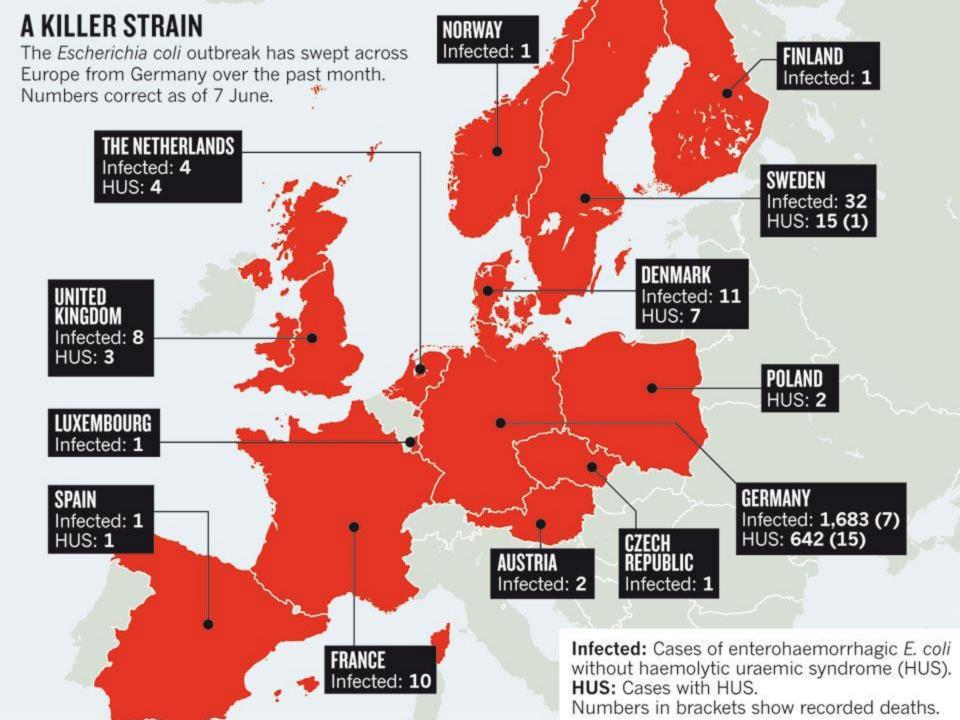
ALGORITHMS AND DATA STRUCTURES CAPSTONE

Pavel Pevzner

Department of Computer Science and Engineering University of California at San Diego

Outline

- 2011 European E. coli outbreak
- Assembling phage genome
- DNA arrays
- Assembling genomes from k-mers
- De Bruijn graphs
- Bridges of Königsberg and universal strings
- Euler theorem
- Splitting the genome into contigs
- From reads to read-pairs
- Genome assembly faces real sequencing data











2015: German government who wrongly fingered Spanish cucumbers ordered to pay





How has *E. coli* become pathogenic?

- May 2011: a girl from Hamburg got bloody diarrhea after eating sprouts.
- Doctors suspected a common pathogenic *E. coli* strain but the blood sample did not pass the tests for known *E. coli* strains.

What is the genome of *E. coli* X?





easy towards assembly program

- phi X174 virus
 - 5,386-nucleotides
 - 11 genes

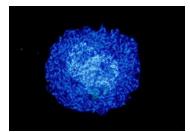


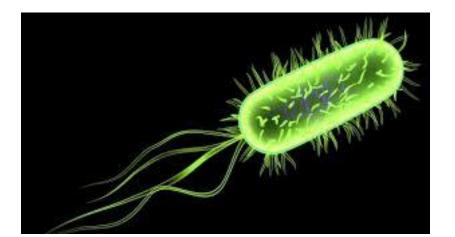
- − ≈110 thousand nucleotides
- ≈140 genes



- ≈5 million nucleotides
- ≈5 thousand genes





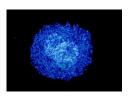


phi X174 phage



- Phages are bacterial viruses. They cannot replicate on their own and must infect bacteria to do so.
- Many phages are shaped like lunar landers, a design that helps them land on the cell wall of a bacterium and transmit their DNA into the bacterial genome
- Phage phi X174 is the first sequenced genome, completed by Fred Sanger in 1977
- You will follow in the footsteps of
 Fred Sanger to assemble the phi X174 genome.

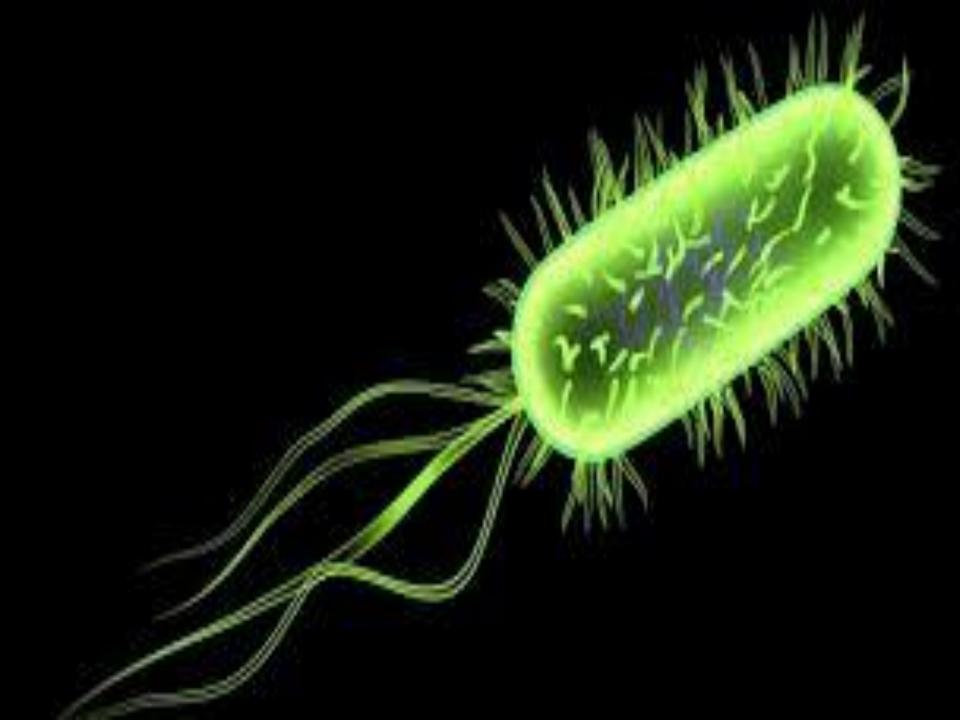
The smallest bacterial genome



- Nasuia deltocephalinicola lives inside leafhoppers.
- Its sheltered life has allowed it to reduce its genome to only ≈110,000 nucleotides and ≈140 genes.



- It lacks some genes necessary for survival, but products of these genes are supplied by its insect host.
- N. deltocephalinicola is losing its bacterial identity and turning into a part of the insect's genome
- You will follow in the footsteps of biologists who sequenced this genome in 2013



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Assembling phage genome from error-free reads

- 1000 simulated error-free reads randomly drawn from 5,386-nucleotide long circular phi X174 genome.
- Each read is 100 nucleotides long.
- All genomes in this capstone will of nucleotide-long insertion.
- Your goal will be to figure out the se

This is not a computational problem!

c cag

Genome Sequencing Problem.

Reconstruct a genome from reads.

- Input. A collection of strings Reads.
- Output. A string Genome reconstructed from Reads.

Assembling phage genome from error-free reads

- 1000 simulated error-free reads randomly drawn from 5,386-nucleotide long circular phi X174 genome.
- Each read is 100 nucleotides long.
- All genomes in our will contain a tag, a 10-nucleotidelong insertion
- Your goal will be to figure out the sequences of the tag

What does it mean to "assemble" a genome? Formulate a rigorous algorithmic problem that adequately models genome assembly.

Assembling phage genome from error-free reads

- 1000 simulated error-free reads randomly drawn from 5,386-nucleotide long circular phi X174 genome.
- Each read is 100 nucleotides long.
- All genomes in our will contain a tag, a 10-nucleotidelong insertion
- Your goal will be to figure out the sequences of the tag

Programming challenge: Assemble mutated phi X174 genome from simulated error-free reads and find out the inserted tag.

Assembling phage genome from error-prone reads

- 1000 simulated error-prone reads randomly drawn from 5,386-nucleotide long circular phi X174 genome.
- Each read is 100 nucleotides long.
- Each read has errors (substitutions of nucleotides) with probability 0.01 at each position.

What does it mean to "assemble" a genome from error-prone reads? Formulate a rigorous algorithmic problem that adequately models genome assembly from error-prone reads.

Assembling phage genome from error-prone reads

- 1000 simulated error-prone reads randomly drawn from 5,386-nucleotide long circular phi X174 genome.
- Each read is 100 nucleotides long.
- Each read has errors (substitutions of nucleotides) with probability 0.01 at each position.

Programming challenge: Assemble mutated phi X174 genome from simulated error-prone reads and find out the inserted tag.



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Evolution of DNA sequencing technologies

- 1977: Sanger assembled the phi X174 genome from 500nucleotide long reads. Scaling it to the human genome would cost hundreds of billion dollars and would likely fail due to unresolved algorithmic challenges.
- 1984: US government started to plan the Human Genome Project.
- 1988: Radoje Drmanac, Andrey Mirzabekov, and Edwin Southern proposed DNA chips.







DNA arrays vs. Sanger sequencing

 Sanger technology: generate some long reads randomly sampled from the genome (read length ≈500)



 DNA chip: generate all short k-mers from a genome (k=10)



k-mer composition

Composition₃(TAATGCCATGGGATGTT)=

TAA AAT ATG TGC GCC CCA CAT ATG TGG GGG GGA GAT ATG TGT GTT

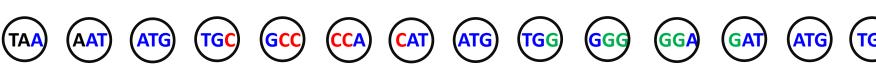
k-mer composition

Composition₃(TAATGCCATGGGATGTT)=



k-mer composition

Composition₃(TAATGCCATGGGATGTT)=



Can you construct this genome path without knowing the genome TAATGCCATGGGATGTT, only from its composition?

Connecting overlapping k-mers

Composition₃(TAATGCCATGGGATGTT)=

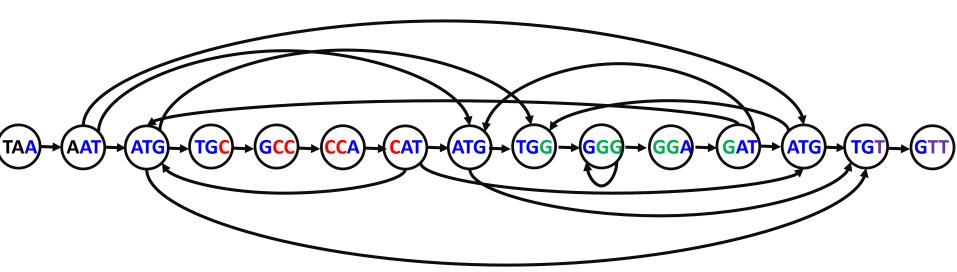


Can you construct this genome path without knowing the genome TAATGCCATGGGATGTT, only from its composition?

Connect k-mer₁ with k-mer₂ if suffix(k-mer₁)=prefix(k-mer₂). e.g. TAA \rightarrow AAT

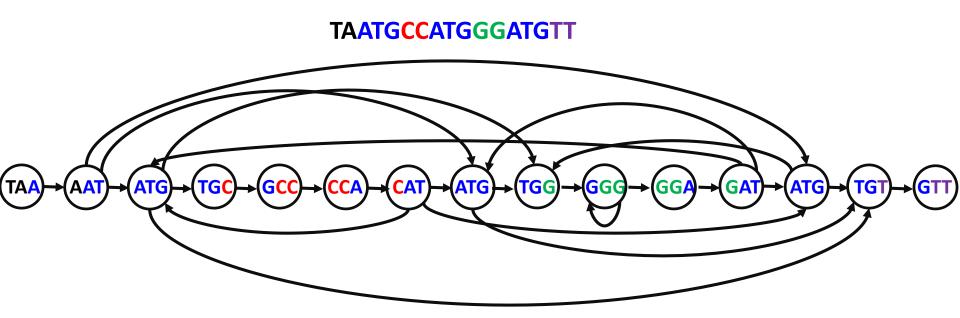
Overlap graph





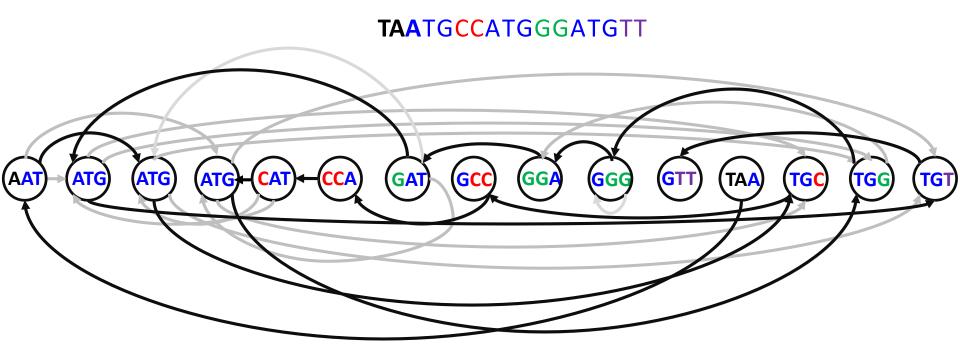
Connect k-mer₁ with k-mer₂ if suffix(k-mer₁)=prefix(k-mer₂). e.g. TAA \rightarrow AAT

Where is the genome path?



Searching for genome path

A Hamiltonian path that visits each vertex exactly once



What are we trying to find in this graph?

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Assembling phi X174 genome from k-mers

Problem: Assemble mutated phi X174 genome from all its *k*-mers and find the inserted tag.

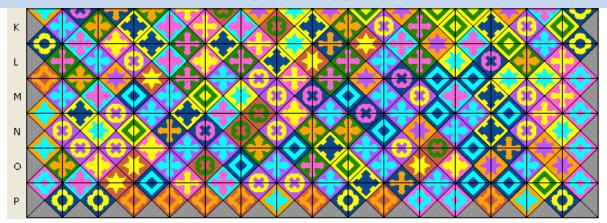


EXERCISE BREAK: What is the minimum value of *k* for which phi X174 genome can be uniquely reconstructed from its *k*-mer composition?

Eternity II puzzle (\$2,000,000 prize)



Puzzle Assembly Problem. Assemble a smaller version of the Eternity II puzzle that requires placing 25 square pieces into a 5-by-5 grid.

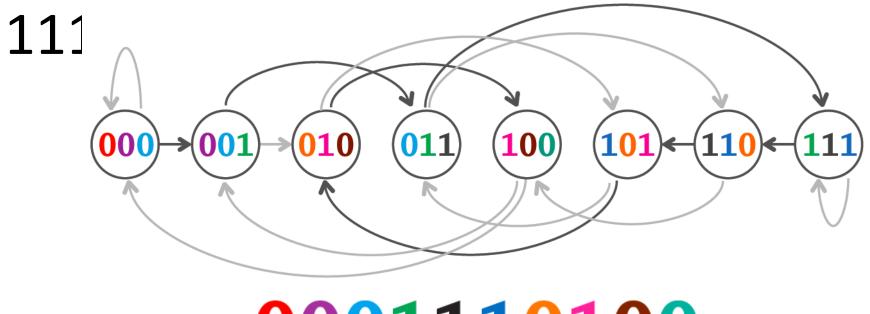


Universal strings



Universal String Problem (de Bruijn, 1946). Find a string containing each binary *k*-mer exactly once.

000 001 010 011 100 101 110



0001110100

Constructing a different graph

De Bruijn's idea: Construct a graph in which every *k*-mer corresponds to an **edge** rather than a vertex and where each *k*-universal string corresponds to an **Eulerian** path.

Eulerian Path Problem:

Construct an Eulerian path in a directed graph.

- Input: A directed graph.
- Output: A path visiting every edge in the graph exactly once (if such a path exists).



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Before: labeling vertices by k-mers

TAATGCCATGGGATGTT



3-mers as vertices

Now: labeling edges by k-mers

TAATGCCATGGGATGTT



3-mers as vertices

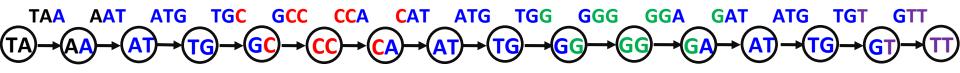


3-mers as edges

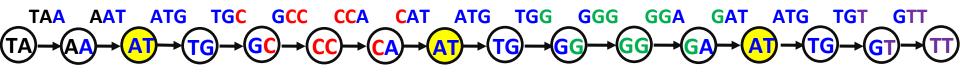
How do we label the starting and ending vertices of an edge?

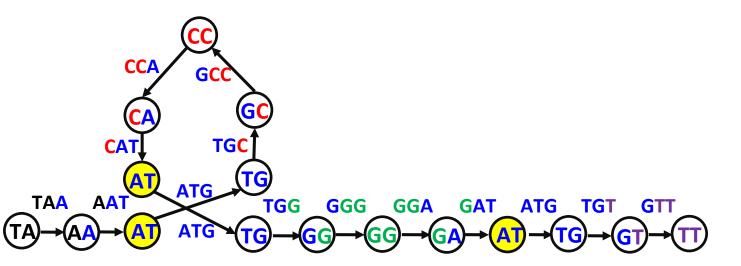
prefix of \overrightarrow{TAA} \overrightarrow{TAA} suffix of \overrightarrow{TAA}

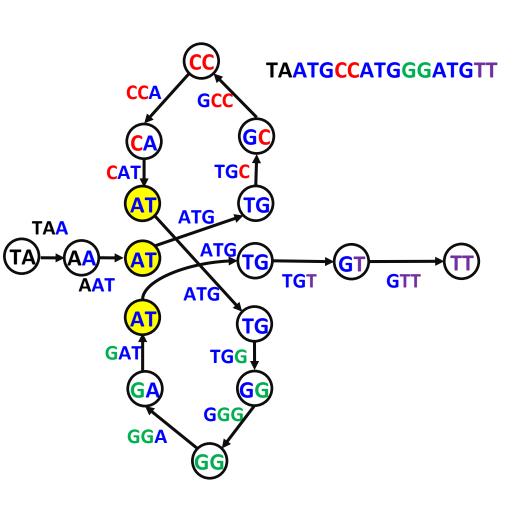
Labeling vertices by (k-1)-mers

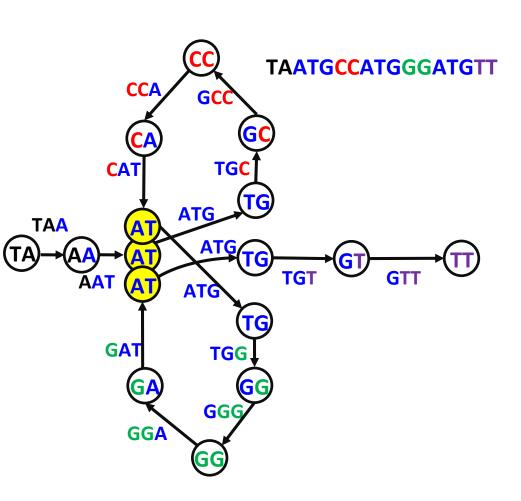


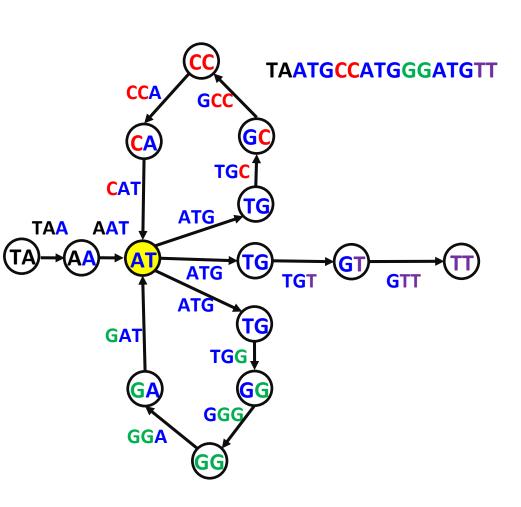
3-mers as edges and 2-mers as vertices

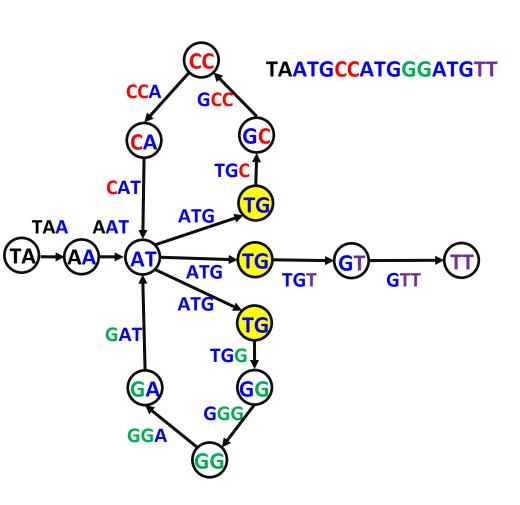


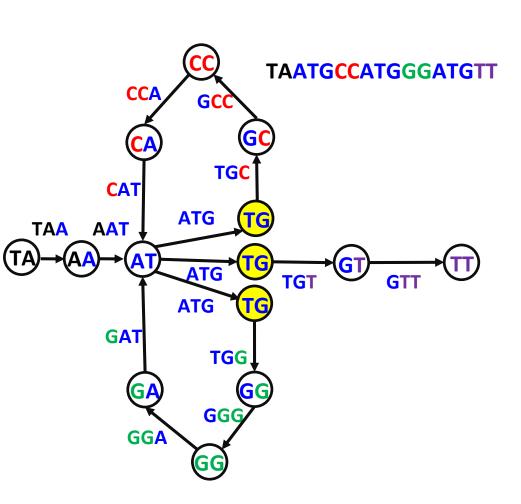


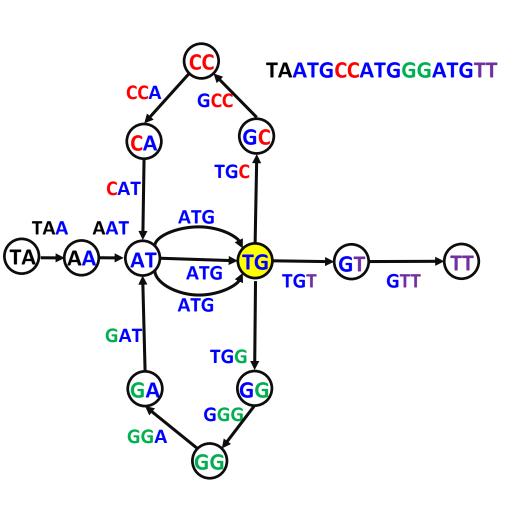


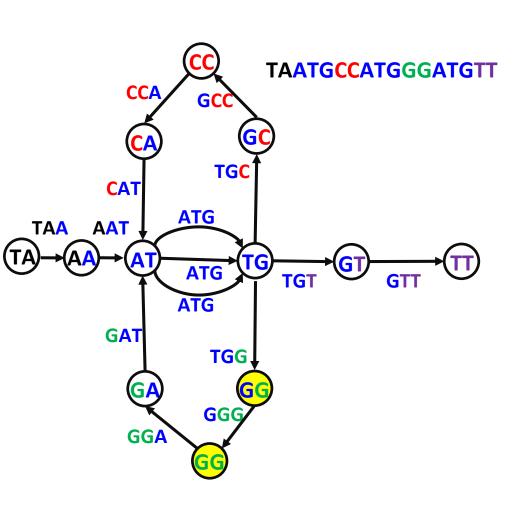


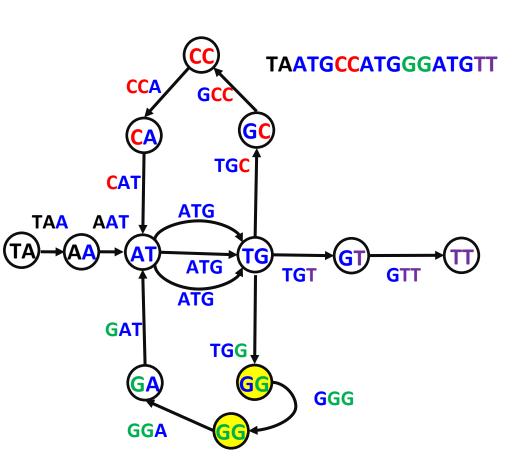












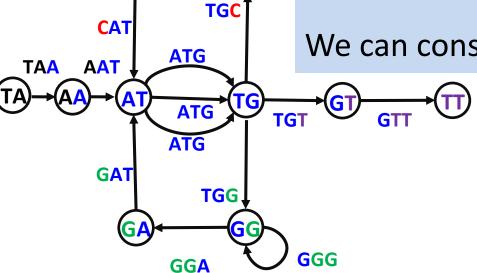
De Bruijn graph of TAATGCCATGGGATGTT

We don't need to know the string

TAATGCCATGGGATGTT

to construct the de Bruijn graph.

We can construct it from its 3-mers.



De Bruijn graph of k-mers

DeBRUIJN(*Patterns*): de Bruijn graph constructed for a set of *k*-mers *Patterns*:

- vertices: all unique (k-1)-mers occurring as a prefix or suffix of k-mers in Patterns.
- edges: each k-mer in Patterns corresponds to a directed edge that connects its prefix vertex to its suffix vertex.

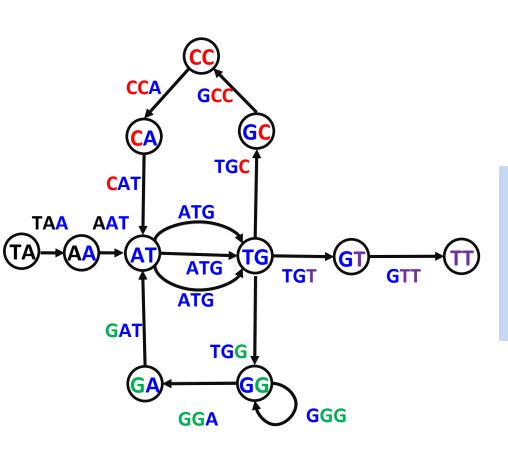
GGA

Constructing de Bruijn graph from k-mers.

- **Input:** A set of *k*-mers *Patterns*.
- Output: graph DeBRUIJN(Patterns).

Where is the *Genome* hiding in this graph?

TAATGCCATGGGATGTT

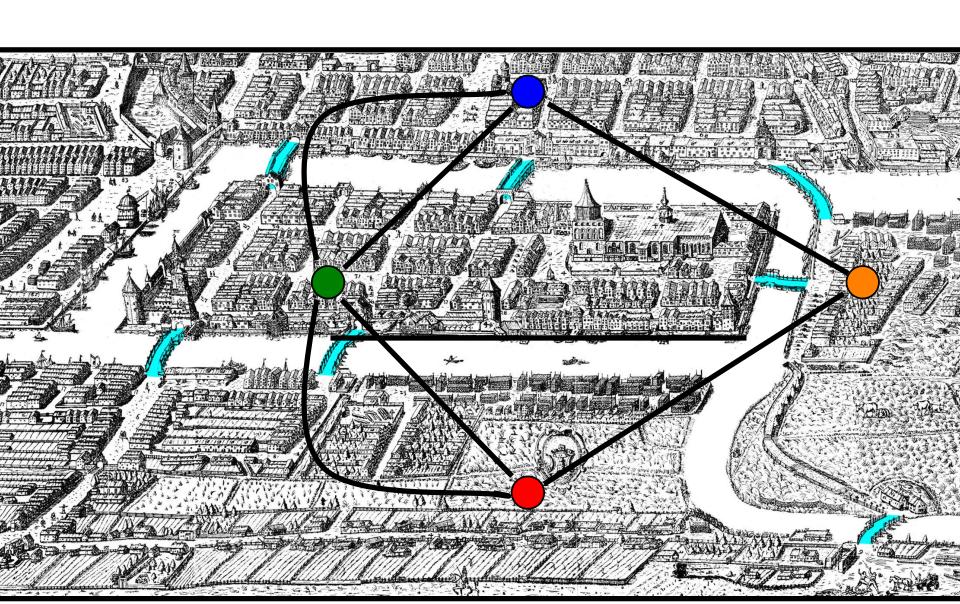


An Eulerian path that visits each edge exactly once.

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Bridges of Königsberg



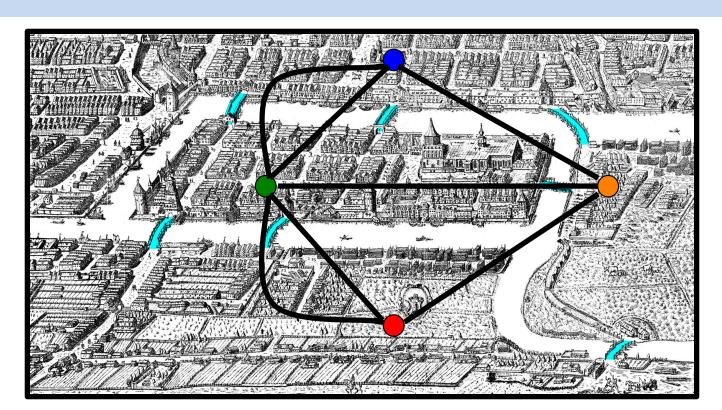
Eulerian Path Problem

Eulerian Path Problem. Find an Eulerian path in a graph.

Input. A graph.



• Output. A path visiting every edge in the graph exactly once.



Eulerian versus Hamiltonian cycles

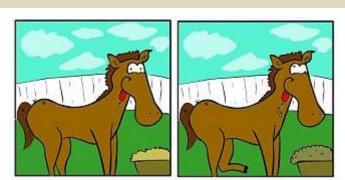
Eulerian Cycle Problem. Find an Eulerian cycle in a graph.

- Input. A graph.
- Output. A cycle visiting every edge in the graph exactly once.

Hamiltonian Cycle Problem. Find a Hamiltonian cycle in a graph.

- Input. A graph.
- Output. A cycle visiting every vertex in the graph exactly once.

Find a difference!



Eulerian versus Hamiltonian cycles

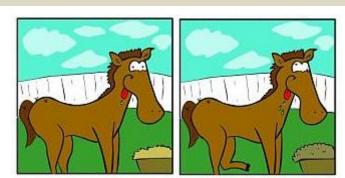
Eulerian Cycle Problem. Find an Eulerian cycle in a graph.

- Input. A graph.
- Output. A cycle visiting every edge in the graph exactly once.

Hamiltonian Cycle Problem. Find a Hamiltonian cycle in a graph.

- Input. A graph.
- Output. A cycle visiting every vertex in the graph exactly once.

Find a difference!



From Hamilton



to Euler



to de Bruijn



Universal Circular String Problem (De Bruijn, 1946). Find a circular string containing each binary *k*-mer exactly once.





to Euler

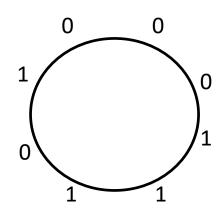


to de Bruijn



Universal Circular String Problem (De Bruijn, 1946). Find a circular string containing each binary *k*-mer exactly once.

000 001 010 011 100 101 110 111







to Euler

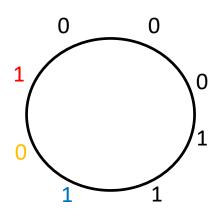


to de Bruijn



Universal Circular String Problem (de Bruijn, 1946). Find a circular string containing each binary **k**-mer exactly once.

000 001 010 011 100 101 110 111



From Hamilton



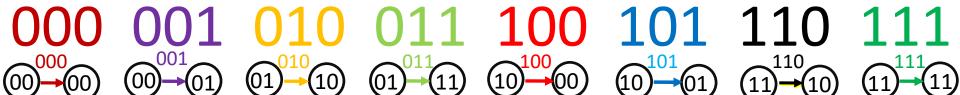
to Euler

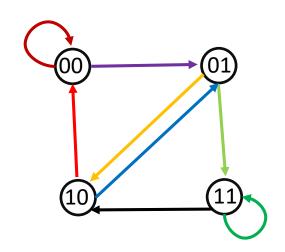






Universal Circular String Problem (de Bruijn, 1946). Find a circular string containing each binary k-mer exactly once.





From Hamilton

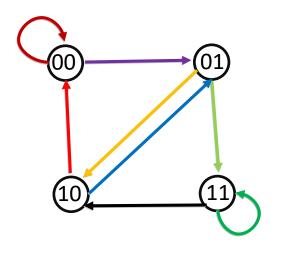


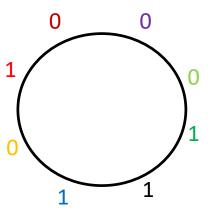
to Euler



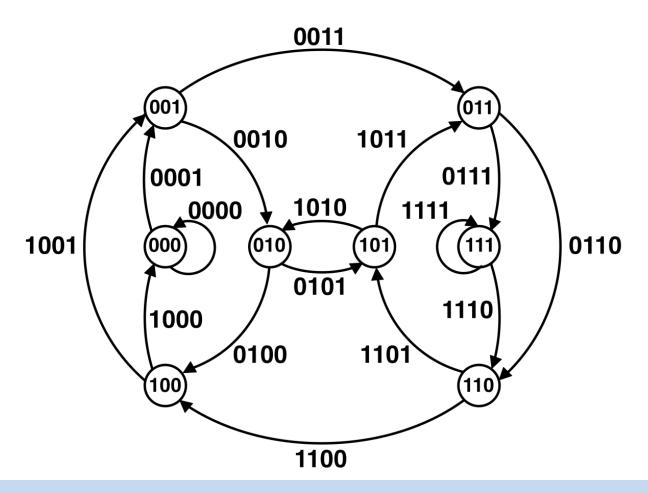
to de Bruijn







De Bruijn graph for 4-universal strings

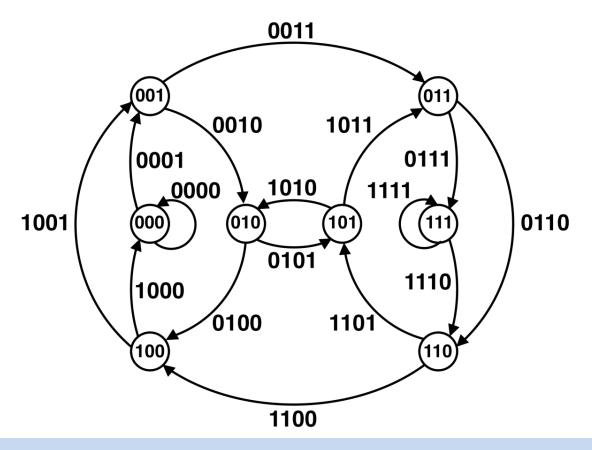


Does it have an Eulerian cycle? If yes, how can we find it?

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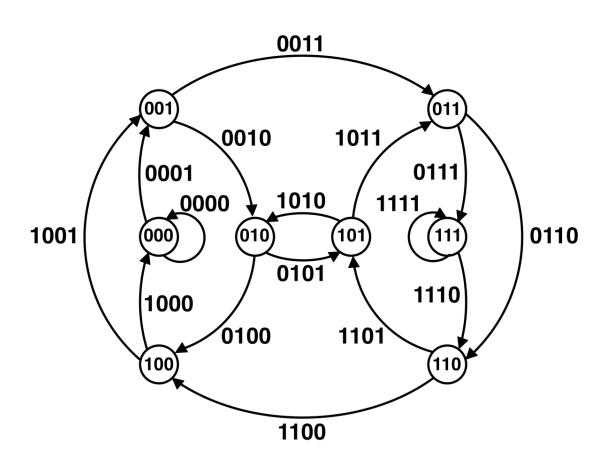
Is the graph for 4-universal strings balanced?



A graph is balanced if indegree = outdegree for each node

Euler's Theorem

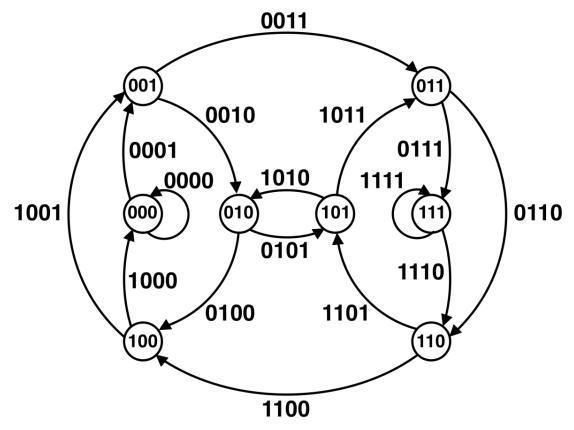
Every Eulerian graph is balanced



Euler's Theorem

- Every Eulerian graph is balanced
- Every balanced* graph is Eulerian

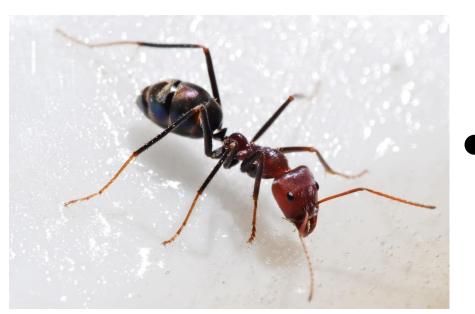


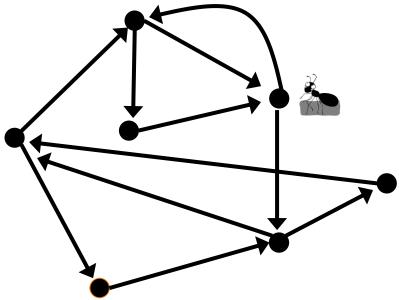


Recruiting an ant

Let an ant randomly walk through the graph.

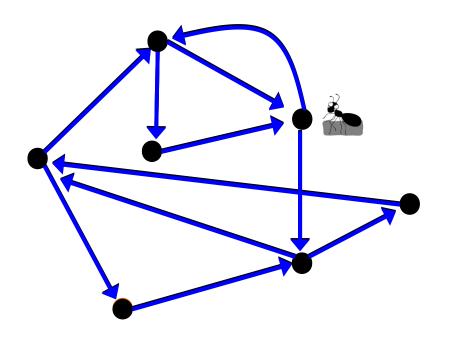
The ant cannot use the same edge twice!



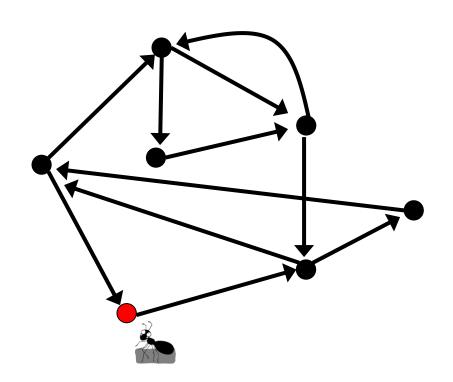


If ant was a genius...

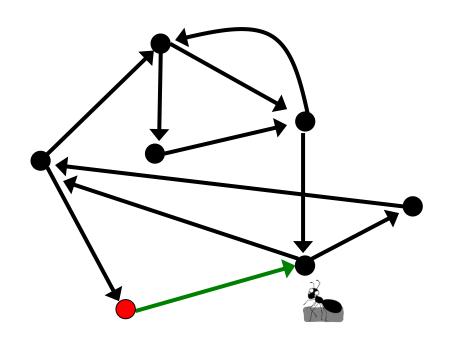
"Yay! Now can I go home please?"



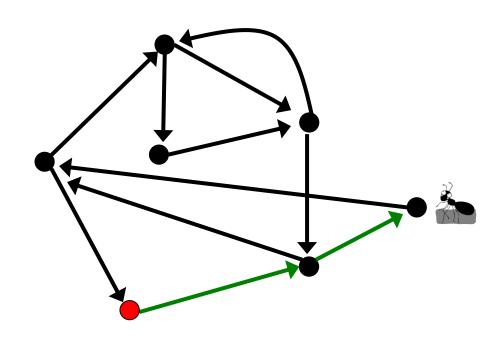
A less intelligent ant...



Walking...

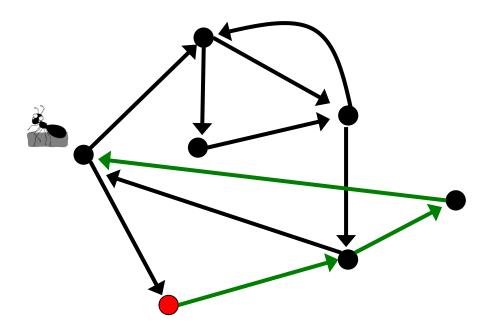


Walking... and walking...

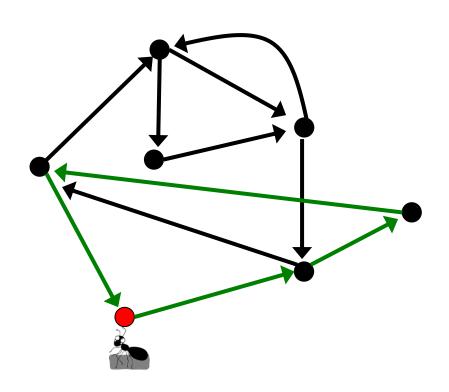


Walking... and walking... and walking...

Can it get stuck? In what vertex?

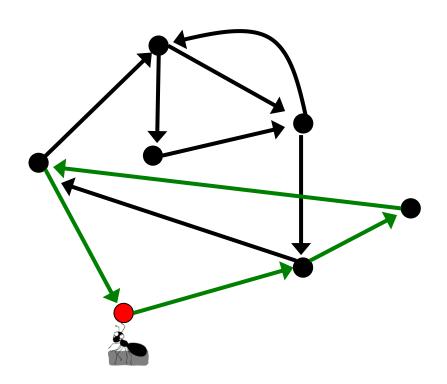


The ant can only get stuck at the starting vertex

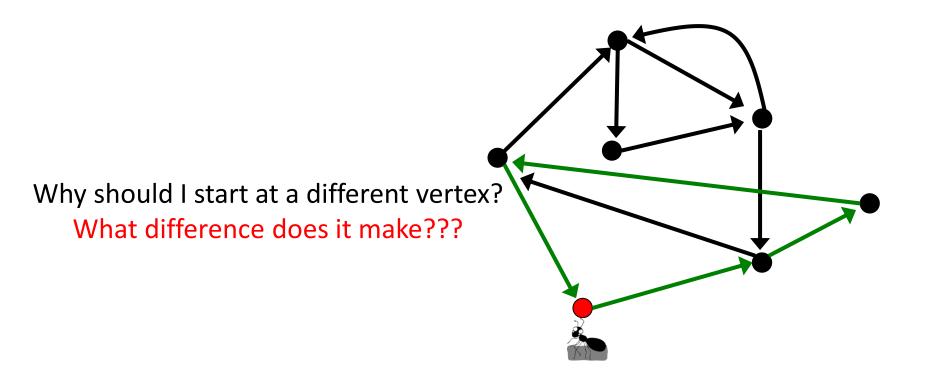


The ant has completed a cycle BUT it is not Eulerian...

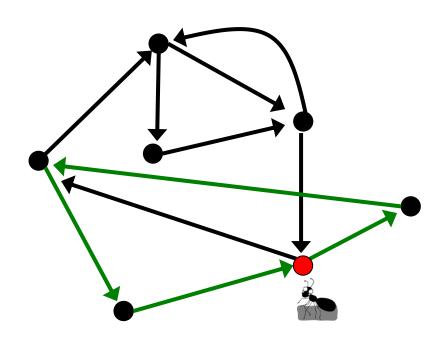
Can we enlarge this cycle?



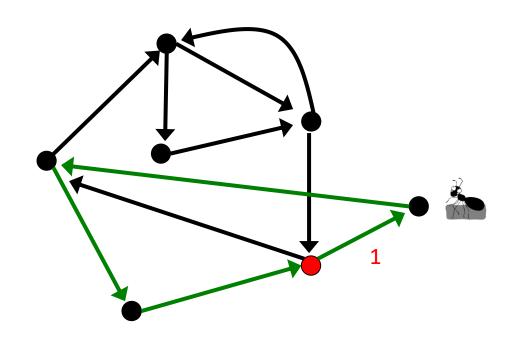
Let's start at a different vertex in the green cycle (with still unexplored edges)



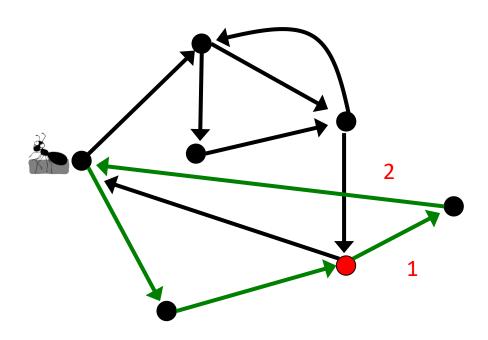
New walking instructions



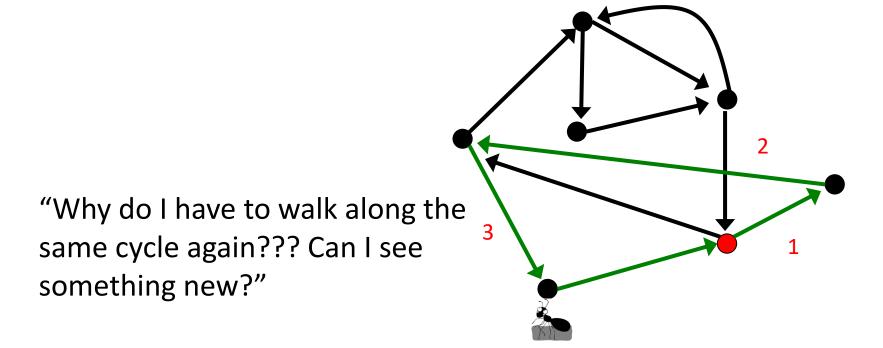
Traversing previously constructed cycle



Traversing previously constructed cycle



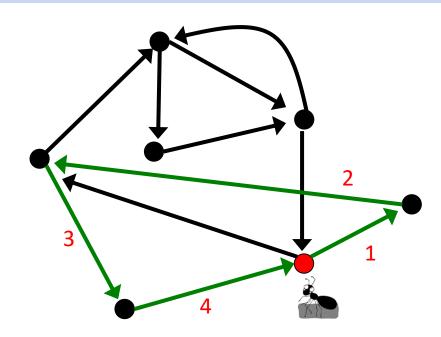
Traversing previously constructed cycle

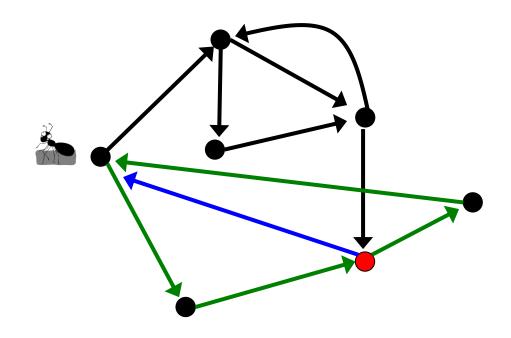


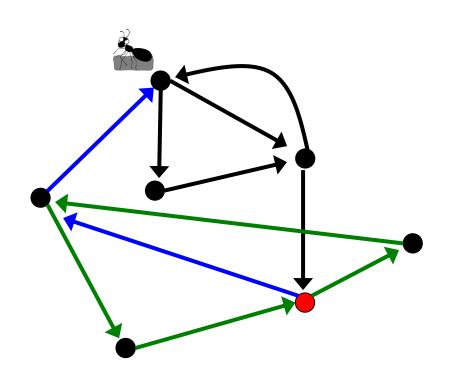
I returned back BUT... I can continue walking!

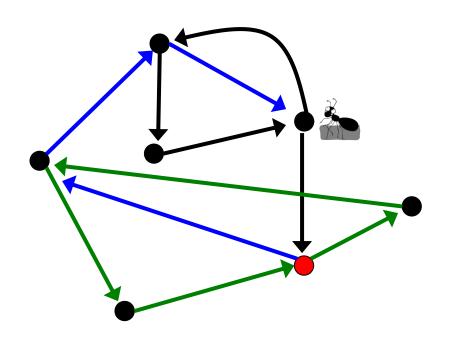
Starting at a vertex that has an unused edge, traverse the already constructed (green cycle) and return back to the starting vertex.

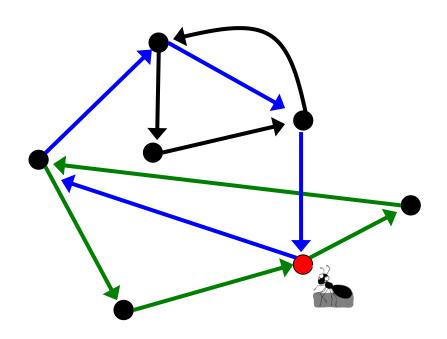
After completing the cycle, start random exploration of still untraversed edges in the graph.







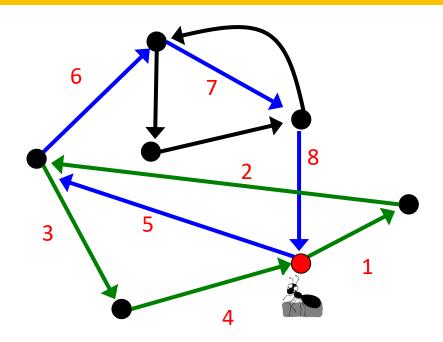




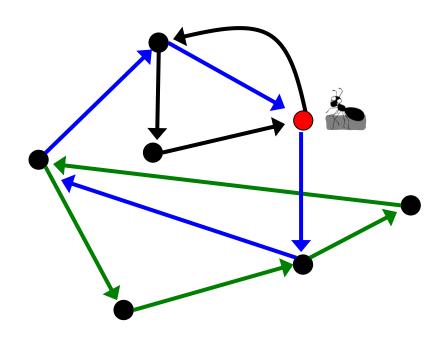
Stuck again!

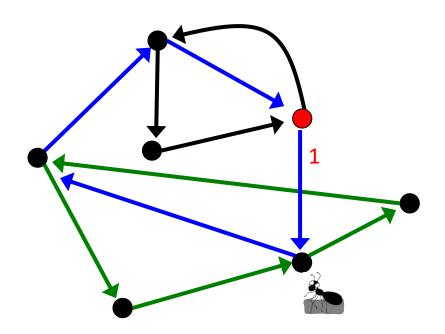
No Eulerian cycle yet... can we enlarge the green-blue cycle?

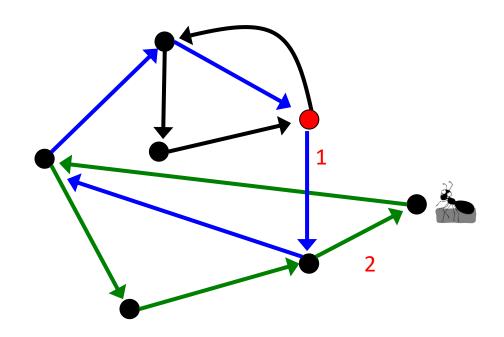
The ant should walk along the constructed green-blue cycle starting at yet another vertex. Which one?



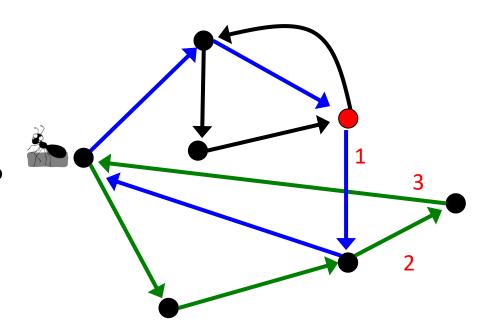
Starting at a new vertex, again...

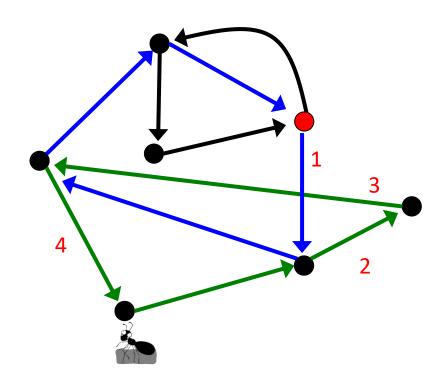




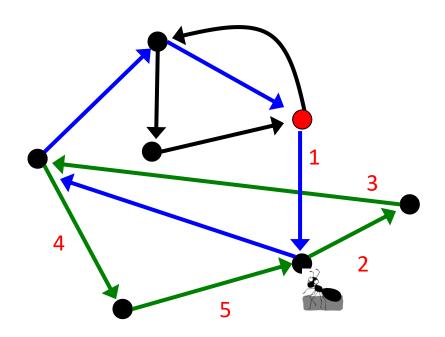


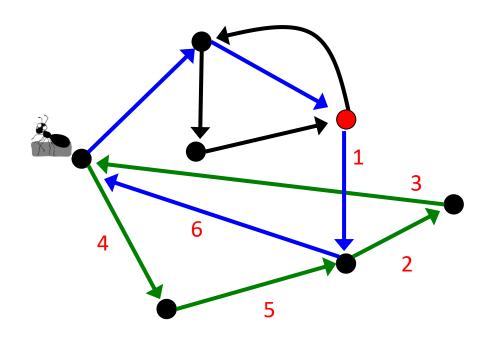
I hate to traverse the same cycle! What difference does it make where I start my walk???

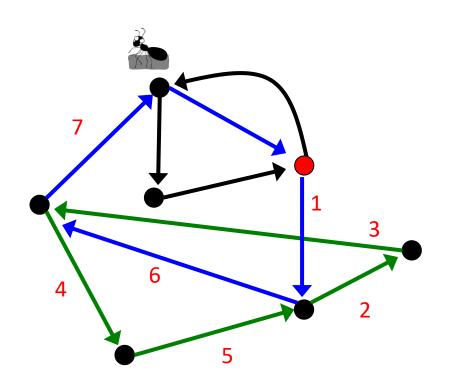




"These instructions are stupid..."

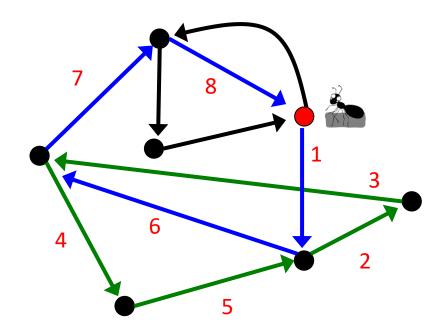




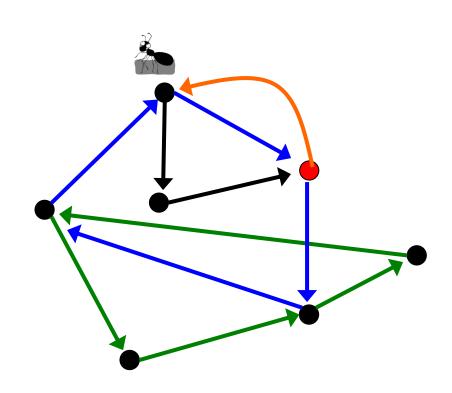


I returned back BUT... I can continue walking!

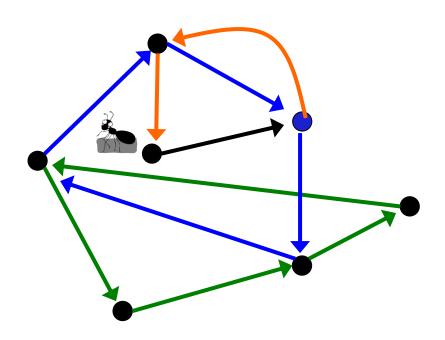
"Hmm, maybe these instructions were not that stupid..."



Enlarging the green-blue cycle



Enlarging the green-blue cycle



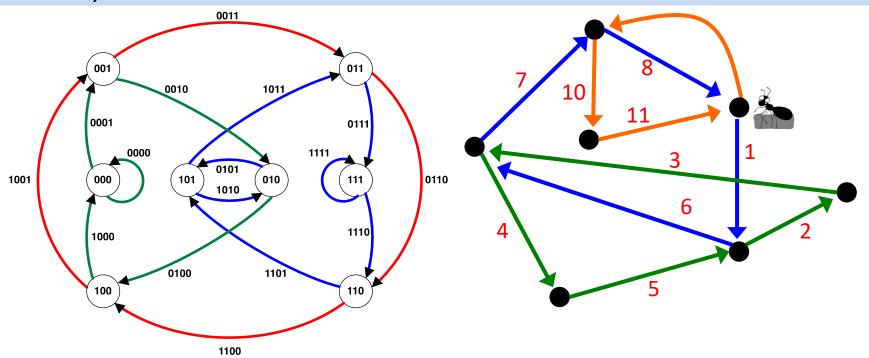
I proved Euler's Theorem! Can I go home please?

EulerianCycle(BalancedGraph)

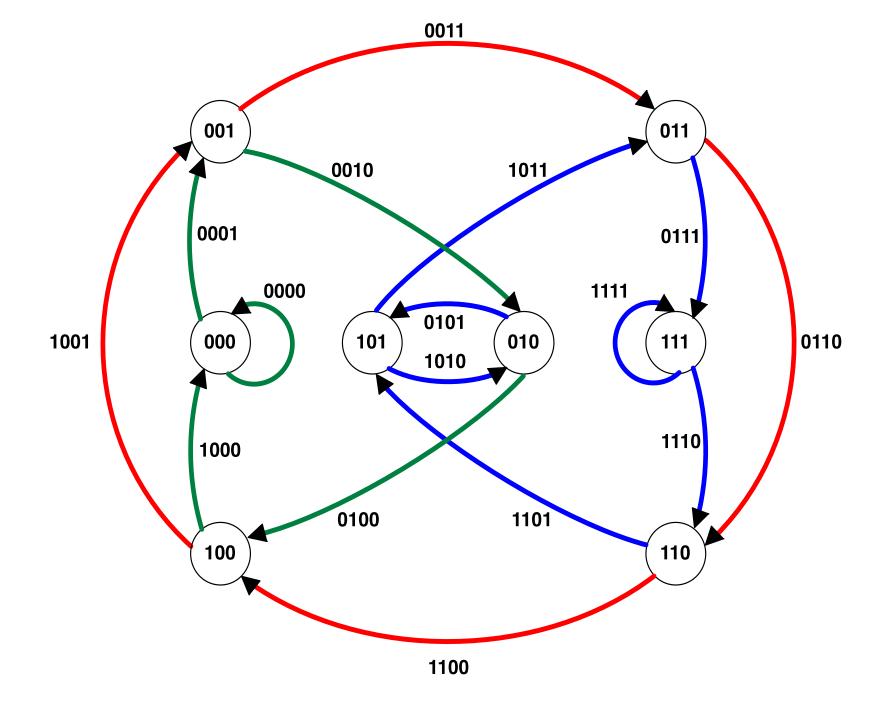
form a *Cycle* by randomly walking in *BalancedGraph* (dont visit the same edge twice!) **while** *Cycle* is not Eulerian

select a node *newStart* in *Cycle* with still unexplored outgoing edges form a *Cycle'* by traversing *Cycle* from *newStart* and randomly walking afterwards $Cycle \leftarrow Cycle'$

return Cycle



Problem: Implement linear-time algorithm for constructing Eulerian cycles

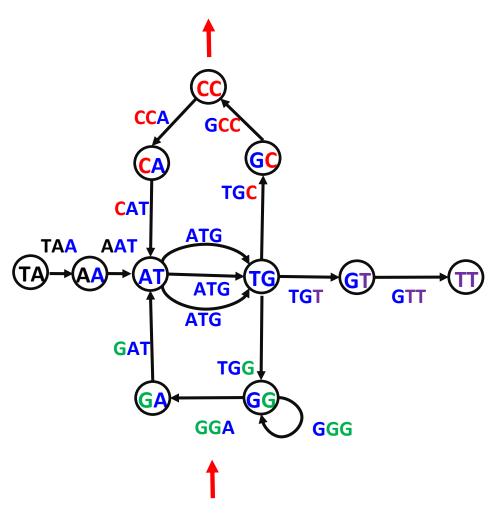


Outline

- 2011 European *E. coli* outbreak
- Assembling phage genome
- DNA arrays
- Assembling genomes from k-mers
- De Bruijn graphs
- Bridges of Königsberg and universal strings
- Euler theorem
- Splitting the genome into contigs
- From reads to read-pairs
- Genome assembly faces real sequencing data

From Reads to de Bruijn Graph to Genome

TAATGCCATGGGATGTT

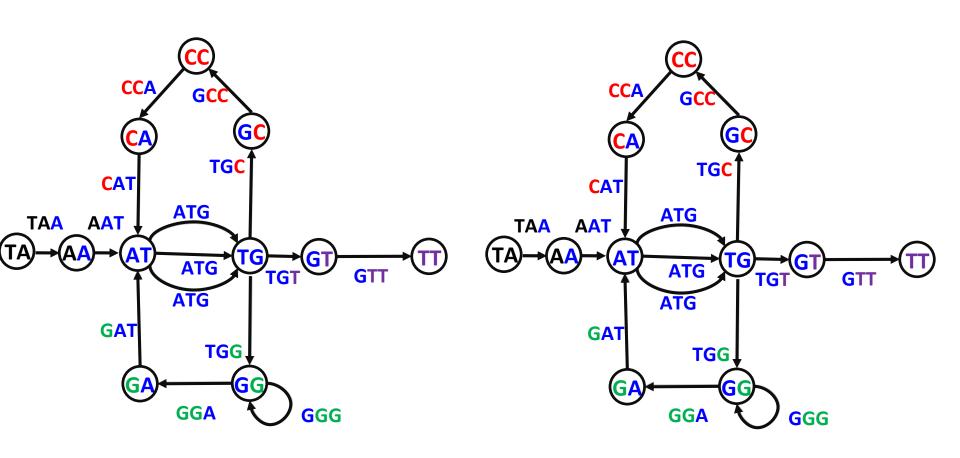


AAT ATG ATG CAT CCA GAT GCC GGA GGG GTT TAA TGC TGG TGT

Multiple Eulerian Paths

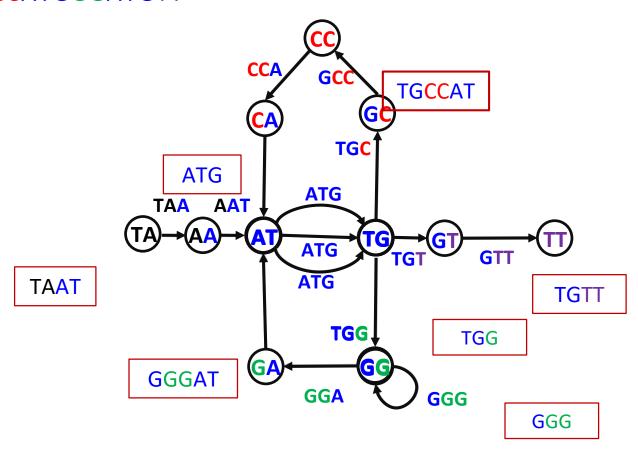
TAATGCCATGGGATGTT

TAATGGGATGCCATGTT



Breaking Genome Into Contigs

TAATGCCATGGGATGTT



Contig Generation Problem: Given a set of k-mers Patterns, generate all contigs in **DeBRUIJN** $_k$ (Patterns)

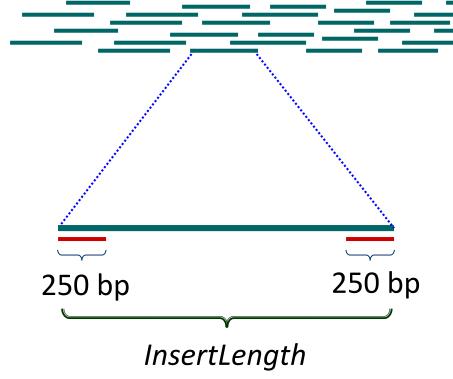
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Genome Sequencing with Read-Pairs

Multiple identical copies of genome

Randomly cut genomes into large equally sized fragments of size *InsertLength*



Generate read-pairs: two reads from the ends of each fragment (separated by a fixed distance)

From k-mers to paired k-mers

Genome

Read 1

Read 2

...A T C A G A T T A C G T T C C G A G ...

distance d=11 -----

A paired k-mer is a pair of k-mers at a fixed distance d apart in Genome. E.g. TCA and TCC are at distance d=11 apart.

Disclaimer: in reality, the distance *d* between reads is measured with errors.

What is PairedComposition(TAATGCCATGGGATGTT)? TAA GCC

paired 3-mer

```
What is PairedComposition(TAATGCCATGGGATGTT)?

TAA GCC

AAT CCA

ATG CAT

TGC ATG

GCC TGG

CCA GGG

CAT GGA

ATG GAT

TGG ATG

GGG TGT
```

GGA GTT

String Reconstruction from Read-Pairs

String Reconstruction from Read-Pairs Problem. Reconstruct a string from its paired k-mers.

- Input. A set of paired k-mers.
- Output. A string Text such that PairedComposition(Text)
 coincides with the set of paired k-mers.

How would de Bruijn assemble paired k-mers?



Hint:



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Unrealistic assumptions

 Perfect coverage of genome by reads (every k-mer from the genome is represented by a read)

Reads are error-free.

Multiplicities of k-mers are known

Distances between reads within read-pairs are exact.

In reality...

- Imperfect coverage of a genome by reads (reads do not start at each position of a genome)
- Reads are error-prone.
- Multiplicities of k-mers are unknown.
- Distances between reads within read-pairs are inexact.
- Etc., etc., etc.

1st unrealistic assumption: perfect coverage

```
atgccgtatggacaacgact
atgccgtatg
gccgtatgga
gtatggacaa
gacaacgact
```

250-nucleotide reads generated by Illumina technology capture only a small fraction of 250-mers from the genome, thus violating the key assumption of the de Bruijn graphs.

Breaking reads into shorter k-mers

```
atgccgtatggacaacgact
atgccgtatg
gccgtatgga
gtatggacaa
gtatggacaa
gacaacgact
```

```
atgccgtatggacaacgact
atgcc
tgccg
 gccgt
   ccqta
    cgtat
     gtatg
      tatgg
       atgga
        tggac
         ggaca
          gacaa
           acaac
             caacq
              aacqa
               acgac
                cgact
```

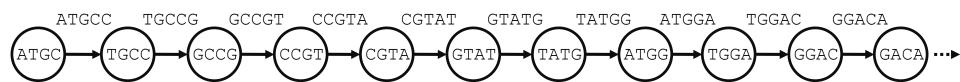
2nd unrealistic assumption: error-free reads

```
atgccgtatggacaacgact
atgccgtatg
gccgtatgga
gtatggacaa
gtatggacaa
gacaacgact
cgtaCggaca
```

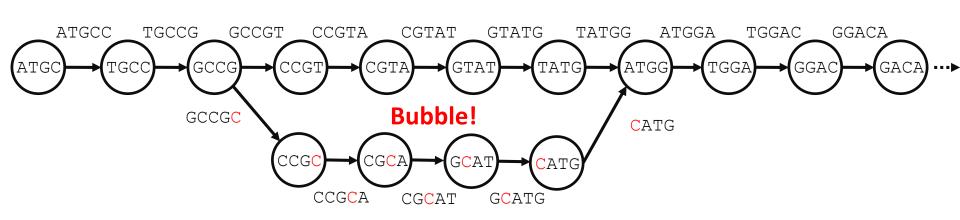
Erroneous read (change of t into C)

```
atgccgtatggacaacgact
atgcc
 tgccg
 gccgt
 ccgta
    cgtat
     gtatg
      tatqq
       atgga
        tggac
         ggaca
          qacaa
            acaac
             caacq
              aacqa
               acgac
                cgact
    cqtaC
     gtaCg
      taCgg
       aCgga
        Cggac
```

De Bruijn graph of ATGGCGTGCAATG... constructed from error-free reads

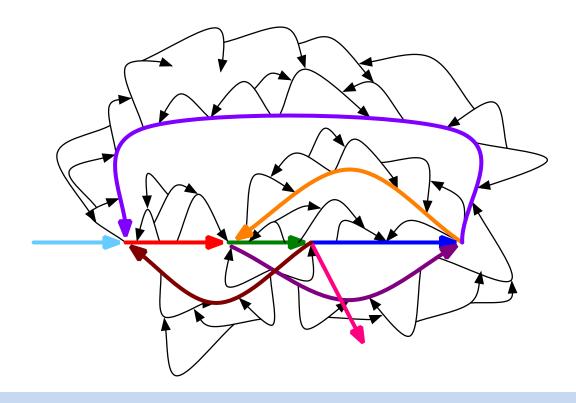


Errors in reads lead to "bubbles" in de Bruijn graphs



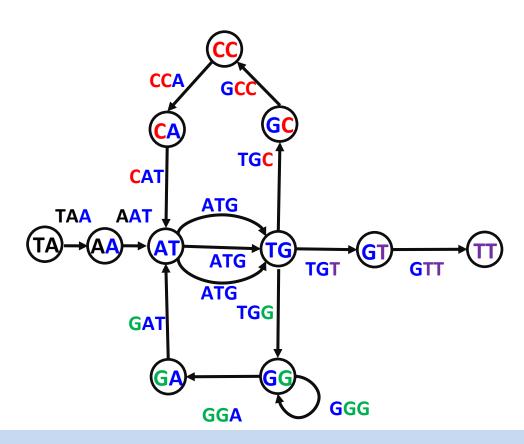
Bubble Detection Problem. Design an algorithm for detecting bubbles in a directed graph. Output the number of bubbles in the de Bruijn graph constructed from the *k*-mers occurring in 1000 error-prone reads from a mutated phi X174 genome.

Bubble explosion... where are the correct edges of the de Bruijn graph?



Reconstructing the phi X174 genome from error-prone reads using de Bruijn graphs. Given 1000 error-prone reads from a mutated phi X174 genome, construct their de Bruijn graph and simplify it by removing bubbles and other artifacts.

3rd unrealistic assumption: multiplicities of *k*-mers are known



STOP and Think: The problem of inferring multiplicities of *k*-mers can be formulated as one of the problems you have already studied in this Specialization. Which one?



Assembling *E. coli* **X genome from real reads.** Given a set of real reads from *E. coli* **X**, reconstruct the *E. coli* **X** genome. How many contigs does your reconstruction have? Use the QUAST tool to generate the assembly quality report.

Assembling *E. coli* X genome from real read-pairs. Given a set of real read-pairs from *E. coli* X, reconstruct the *E. coli* X genome. How many contigs does your reconstruction have? Use the QUAST tool to generate the assembly quality report.