UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Eksamen i:	MEK 3300/4300 — Viskøs strømning og turbulens.
Eksamensdag:	Torsdag 18. desember 2003.
Tid for eksamen:	14.30 – 17.30
Oppgavesettet er på 5 sider.	
Vedlegg:	Ingen.
Tillatte hjelpemidler:	Matematisk formelsamling (K. Rottmann).

Kontroller at oppgavesettet er komplett
før du begynner å besvare spørsmålene.

Oppgave 1.

Rommet mellom to koaksiale sylindre med radius a (ytre) og b (indre) (se figur 1.1) er fylt med en viskøs væske med konstant viskositet og konstant tetthet. Det går en væskestrøm

$$\boldsymbol{u}(r,\theta) = \boldsymbol{i}_r u(r,\theta) + \boldsymbol{i}_\theta v(r,\theta) \tag{1.1}$$

i rommet mellom sylindrene. Ved den ytre sylinderen $r = a$ er det gitt at

$$\int_0^\pi \boldsymbol{u}(a,\theta)\cdot \boldsymbol{i}_r a\, d\theta = Q_0 \tag{1.2}$$

For $\pi \le \theta \le 2\pi$ og $r = a$ er

$$\boldsymbol{u}(a,\theta) = \boldsymbol{i}_r U_0 \Big[\sin\theta + \tfrac{1}{5}\sin 3\theta\Big] \tag{1.3}$$

a) Finn U_0 når $\boldsymbol{u}(b,\theta) = 0$.

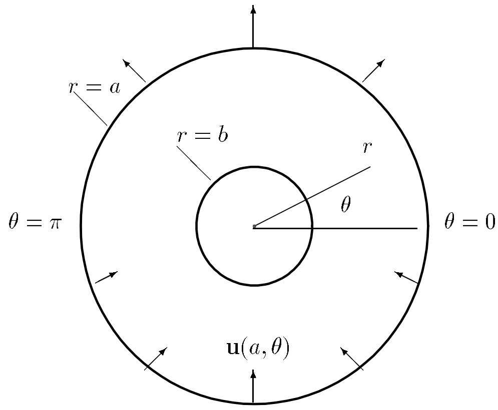
Det er også gitt at

$$\int_0^{2\pi} \boldsymbol{i}_r \cdot \big[\rho \boldsymbol{u}\,\boldsymbol{u} - \underset{\sim}{\mathcal{P}}\big]_{r=a} a\, d\theta = \boldsymbol{M} \tag{1.4}$$

for denne tidsuavhengige strømningen. $\underset{\sim}{\mathcal{P}}$ er spenningstensoren.

(Fortsettes side 2.)

b) Finn kraften som virker på den indre sylinderen pr. meter lengde uttrykt ved $\boldsymbol{M}$ når $\boldsymbol{u}(b, \theta) = 0$.



Figur 1.1 viser problemets geometri

Oppgave 2.

Rommet mellom to parallelle plan $y = \pm h$ (se figur 2.1) er fylt med homogent fluid med konstant tetthet ρ og konstant kinematisk viskositet ν. For $t < 0$ er

$$\boldsymbol{u}(x, y, z, t) = 0 \tag{2.1}$$

$$p(x, y, z, t) = p_0 \qquad \text{(konstant)} \tag{2.2}$$

der $\boldsymbol{u}(x, y, z, t)$ er hastighetsfeltet og $p(x, y, z, t)$ trykkfeltet. For $t \geq 0$ er

$$\nabla p = -c\,\boldsymbol{i} \tag{2.3}$$

der c er en positiv konstant og $\boldsymbol{i}$ enhetsvektor i x-retningen. Den gitte trykkgradienten (2.3) driver hastighetsfeltet

$$\boldsymbol{u}(x, y, z, t) = u(y, t)\boldsymbol{i} \tag{2.4}$$

Randbetingelsen er

$$u(y = \pm h, t) = 0 \tag{2.5}$$

Initialbetingelsen er

$$u(y, t = 0) = 0 \tag{2.6}$$

Det antaes at

$$\lim_{t \to \infty} [u(y, t)] = U_0(y) \tag{2.7}$$

a) Finn hastigheten $U_0(y)$.

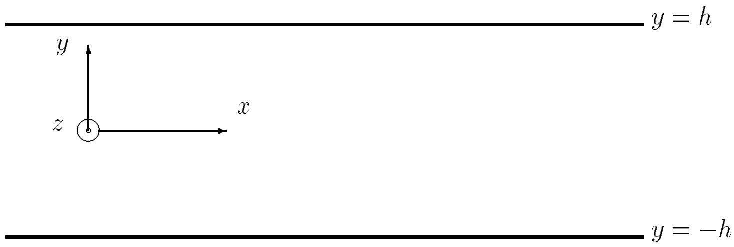
(Fortsettes side 3.)

b) Finn egenfunksjonene assosiert med det homogene problemet (svarer til $c = 0$) og bruk disse til å fremstille den generelle løsningen

$$u(y,t) = u_H(y,t) + U_o(y) \tag{2.8}$$

der $u_H(y,t)$ er løsningen av det homogene problemet. Det kan også antaes som kjent at for $-1 \leq \eta \leq 1$ er

$$1 - \eta^2 = \frac{32}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}\cos\left[\frac{(2n+1)\pi}{2}\eta\right] \tag{2.9}$$



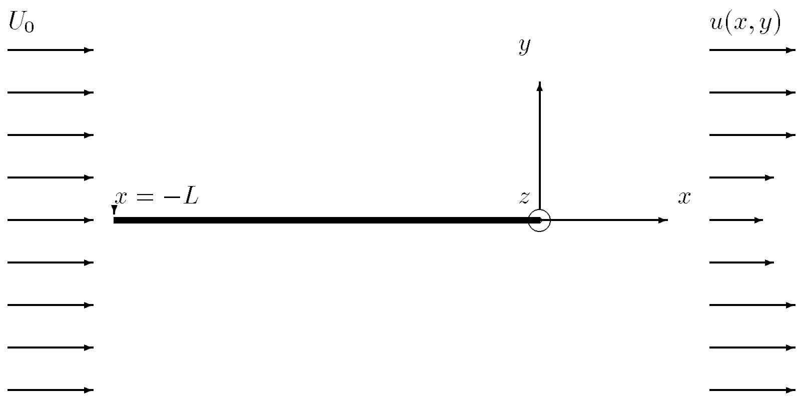
Figur 2.1 viser problemets geometri

Oppgave 3.

Hastigheten i x-retningen i kjølvannet nedstrøms en flat plate er $u(x,y)$, og

$$u(x,y) = U_0 - u_1(x,y) \tag{3.1}$$

hvor U_0 er den konstante og uniforme hastigheten inn mot plata (se figur). Væsken er viskøs med konstant kinematisk viskositet ν og konstant tetthet ρ.



Figur 3.1 viser skjematisk skisse av hastighetsprofiler og den flate plata.

(Fortsettes side 4.)

a) Forklar at for $u_1(x, y)$ gjelder approksimativt

$$U_0 \frac{\partial u_1}{\partial x} = \nu \frac{\partial^2 u_1}{\partial y^2} \tag{3.2}$$

i en slik kjølvannsstrømning.

Det antaes at løsningen av $u_1(x, y)$ er av formen

$$u_1(x, y) = Ax^m F(\eta) \tag{3.3}$$

der $\eta = Bx^m$. Forøvrig gjelder at

$$\int_{-\infty}^{+\infty} u_1(x, y) dy = Q \tag{3.4}$$

hvor Q er en konstant uavhengig av x.

b) Bestem similaritetseksponentene m og n.

Oppgave 4.

For statistisk stasjonær turbulent strømning er Reynolds momentumlikning (i-te komponent) gitt som

$$\rho U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \mu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j}(\rho \overline{u_i u_j}) \tag{4.1}$$

der ρ er væskens konstante tetthet, μ væskens konstante dynamiske viskositet, $U_i(\boldsymbol{x})$ er i-te komponent av tidsmidlet hastighet, $P(\boldsymbol{x})$ tidsmidlet trykkfelt, mens $u_i(\boldsymbol{x}, t)$ er i-te komponent av fluktuasjonsfeltet.

a) Forklar hva

$$\rho \overline{u_i u_j} = \lim_{T \to \infty} \left[\frac{1}{T} \int_t^{t+T} \rho u_i u_j \, dt \right] \tag{4.2}$$

representerer fysisk.

I en fullt utviklet, statistisk stasjonær turbulent strømning mellom to plan ($x_2 = 0$, $x_2 = h$) er

$$\left. \begin{aligned} &(U_1, U_2, U_3) = (U_1(x_2), 0, 0) \\ &\frac{\partial P}{\partial x_1} = -c \quad \text{(konstant)} \end{aligned} \right\} \tag{4.3}$$

b) Finn veggspenningen τ_w uttrykt ved c og h.

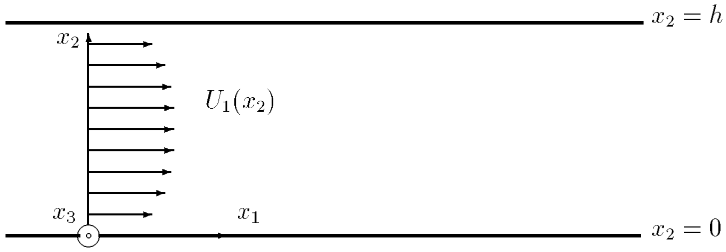
(Fortsettes side 5.)

c) Vis at

$$\frac{dU_{1+}}{dx_{2+}} - \frac{\overline{u_1 u_2}}{u_*^2} + 2\frac{x_{2+}}{R_*} - 1 = 0$$

der

$$U_{1+} = \frac{U_1}{u_*}, \quad x_{2+} = \frac{x_2 u_*}{\nu}, \quad R_* = \frac{u_* h}{\nu}$$
$$u_* = \sqrt{\frac{\tau_w}{\rho}}, \quad \nu = \frac{\mu}{\rho}$$



Figur 4.1 viser skjematisk middelstrømsprofil for turbulent strømning mellom to plan.

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Eksamen i:	MEK 3300/4300 — Viskøs strømning og turbulens.
Eksamensdag:	Mandag 13. desember 2004.
Tid for eksamen:	14.30 – 17.30.
Oppgavesettet er på 4 sider.	
Vedlegg:	Ingen.
Tillatte hjelpemidler:	Rottmann: Matematische Formelsamlung, godkjent kalkulator.

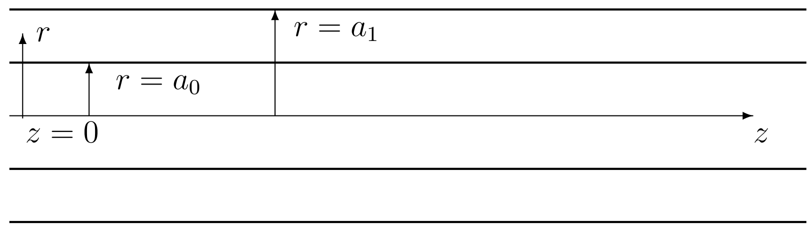
Kontroller at oppgavesettet er komplett
før du begynner å besvare spørsmålene.

Oppgave 1.

Rommet mellom to koaksiale sylindre med sirkulært tverrsnitt er fylt med en væske som har konstant tetthet ρ og konstant kinematisk viskositet ν. Den indre sylinderen har radius a_0 og er i ro. Den ytre sylinderen har radius a_1 og beveger seg i aksialretningen med hastigheten W_1 (se figur 1). Sylinderveggene er ugjennomtrengelige for væsken. Tyngdens virkning skal neglisjeres. Det er ingen annen bevegelse i væsken enn den som induseres på grunn av den ytre sylinderens bevegelse. Væskebevegelsen skal beskrives i et sylinderkoordinat system $(r.\theta, z)$ der væskens hastighet $\mathbf{u}$ dekomponeres i

$$\mathbf{u} = u\mathbf{i}_r + v\mathbf{i}_\theta + w\mathbf{i}_z \qquad (1)$$

(Fortsettes side 2.)



Figur 1 viser skjematisk aksialt lengdesnitt av to koaksiale sirkulære sylindre omtalt i teksten.

a) Forklar at trykket p i væsken er konstant og finn væskens hastighet.

Vi antar så at sylinderveggene er porøse og at en gjennomstrømning i veggene induserer radialhastigheten

$$u = \frac{U_0 a_0}{r} \tag{2}$$

Den ytre sylinderen beveger seg fortsatt med hastigheten W_1 i aksialretningen.

b) Finn trykket p i væsken når

$$p(r = a_0, \theta, z) = p_0 \ = \text{konstant} \tag{3}$$

c) Finn $w(r)$ når $u = \frac{U_0 a_0}{r}$ og $w(r = a_1, \theta, z) = W_1$

Kontinuitets-likning og **momentum**-likninger i sylinderkoordinater (r, θ, z) med tilhørende hastighetskomponenter (u, v, w) er gitt som:

Kontinuitets-likningen

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0 \tag{4}$$

r-momentum

$$\frac{\partial u}{\partial t} + (\mathbf{u} \cdot \nabla)u - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right) \tag{5}$$

θ-momentum

$$\frac{\partial v}{\partial t} + (\mathbf{u} \cdot \nabla)v + \frac{uv}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \nu\left(\nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2}\frac{\partial u}{\partial \theta}\right) \tag{6}$$

(Fortsettes side 3.)

z-momentum

$$\frac{\partial w}{\partial t} + (\mathbf{u} \cdot \nabla) w = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu \nabla^2 w \tag{7}$$

der

$$\mathbf{u} \cdot \nabla = u\frac{\partial}{\partial r} + \frac{v}{r}\frac{\partial}{\partial \theta} + w\frac{\partial}{\partial z} \tag{8}$$

og

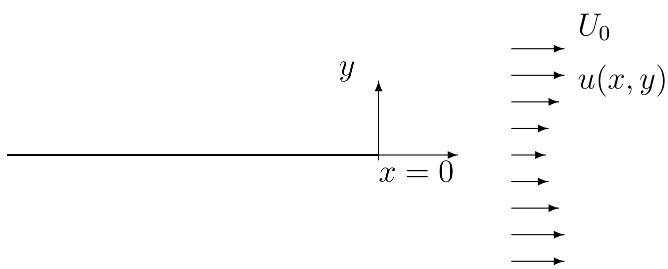
$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \tag{9}$$

Oppgave 2.

Hastighetsfordelingen $u(x,y)$ nedstrøms en flat plate i strøm er indikert i figur 2 og kan uttykkes

$$u(x,y) = U_0 - u_1(x,y) \tag{10}$$

der $u(x, y \to \pm\infty) \to U_0$ (*konstant*).



Figur 2 viser skjematisk en flat plate i et strømningsfelt. Plata ligger i planet $y = 0$ og strekker seg fra $x = -L$ til $x = 0$. Kjølvannsprofilet $u(x,y)$ er også indikert.

a) Vis at grensesjiktlikningene anvendt på denne kjølvannsstrømningen approksimativt gir

$$U_0 \frac{\partial u_1}{\partial x} = \nu \frac{\partial^2 u_1}{\partial y^2} \tag{11}$$

forutsatt $\mid u_1 \mid \ll U_0$.

b) Vis at i kjølvannet gjelder approksimativt

$$\int_{-\infty}^{+\infty} u_1(x,y) dy = konstant \ (uavhengig\ av\ x) \tag{12}$$

forutsatt $\mid u_1 \mid \ll U_0$.

(Fortsettes side 4.)

Anta at

$$u_1(x,y) = AU_0 x^m f(\eta) \tag{13}$$

$$\eta = Bx^n \tag{14}$$

c) Finn similaritetseksponentene m og n.

d) Finn $f(\eta)$. (Bestemmelse av integrasjonskonstantene kreves ikke.)

Oppgave 3.

I et statistisk stasjonært turbulent strømningsfelt er hastighets- og trykkfeltet betegnet med $\tilde{u}_i(\mathbf{x}, t)$ og $\tilde{p}(\mathbf{x}, t)$, henholdsvis ($\mathbf{x}$ er posisjon og t er tiden).

a) Innfør Reynolds dekomposisjon av feltene og utled Reynolds tidsmidlede momentum-likning og definer Reynolds-spenningene.

b) Skriv ned Boussinesq-modellen for Reynolds-spenningene og gi en kort forklaring på hva de enkelte leddene representerer fysisk.

c) Finn funksjonen $f(\epsilon, K)$ for relasjonen

$$\nu_t = f(\epsilon, K) \tag{15}$$

der ν_t ($\frac{m^2}{s}$) er eddyviskositeten , ϵ ($\frac{m^2}{s^3}$) er dissipasjonsraten pr. masseenhet i turbulensen og K ($\frac{m^2}{s^2}$) er turbulent kinetisk energi pr. masseenhet.

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Eksamen i:	MEK 3300/4300 — Viskøs strømning og turbulens.
Eksamensdag:	Torsdag 15. desember 2005.
Tid for eksamen:	14.30 – 17.30.
Oppgavesettet er på 4 sider.	
Vedlegg:	Ingen.
Tillatte hjelpemidler:	Rottmann: Matematische Formelsamlung, godkjent kalkulator.

Kontroller at oppgavesettet er komplett
før du begynner å besvare spørsmålene.

Oppgave 1.

Gjennom et sirkulært rør med radius $a(z)$ som varier med aksial posisjon z og gitt som

$$a(z) = a_0 + a_1 \sin(kz) \tag{1}$$

strømmer en viskøs væske med konstant kinematisk viskositet ν og konstant tetthet ρ. ($a_0 \gg a_1$, $ka_1 \ll 1$.)

Strømningen er laminær og skal beskrives i sylinderkoordinater (r, θ, z) med tilhørende hastighetskomponenter (u, v, w). Volumstrømmen Q_0 gjennom røret er konstant (uavhengig av z og tiden t). Tyngdens virkning skal neglisjeres. Strømningen er aksesymmetrisk og $v = 0$. Hastighetskomponenten i z-retningen $w(r, z)$ kan approksimativt skrives som

$$w(r,z) = W_0(z)\left(1 - \left(\frac{r}{a(z)}\right)^2\right) \tag{2}$$

(Fortsettes side 2.)

a) Finn $W_0(z)$ uttrykt ved Q_0 og $a(z)$.

Av kontinuitetsgrunner må det oppstå en radiell hastighetskomponent $u(r,z)$.

b) Finn $u(r,z)$.

c) Finn trykkfallet $\frac{\partial p}{\partial z}$ som må til for å drive $w(r,z)$. (Hint: Det er tilstrekkelig at $\frac{\partial p}{\partial z}$ beregnes fra den lineariserte differensiallikningen som her kommer til anvendelse.)

Kontinuitets-likning og **momentum**-likninger i sylinderkoordinater (r,θ,z) med tilhørende hastighetskomponenter (u,v,w) er gitt som:

Kontinuitets-likningen

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{\partial w}{\partial z}=0 \tag{3}$$

r-momentum

$$\frac{\partial u}{\partial t}+(\mathbf{u}\cdot\nabla)u-\frac{v^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\nabla^2 u-\frac{u}{r^2}-\frac{2}{r^2}\frac{\partial v}{\partial \theta}\right) \tag{4}$$

θ-momentum

$$\frac{\partial v}{\partial t}+(\mathbf{u}\cdot\nabla)v+\frac{uv}{r}=-\frac{1}{\rho r}\frac{\partial p}{\partial \theta}+\nu\left(\nabla^2 v-\frac{v}{r^2}+\frac{2}{r^2}\frac{\partial u}{\partial \theta}\right) \tag{5}$$

z-momentum

$$\frac{\partial w}{\partial t}+(\mathbf{u}\cdot\nabla)w=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\nabla^2 w \tag{6}$$

der

$$\mathbf{u}\cdot\nabla=u\frac{\partial}{\partial r}+\frac{v}{r}\frac{\partial}{\partial \theta}+w\frac{\partial}{\partial z} \tag{7}$$

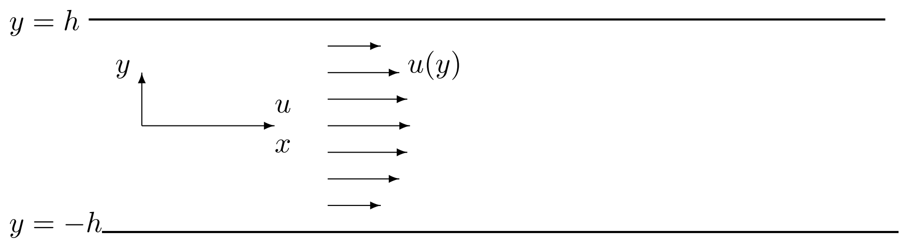
og

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial z^2} \tag{8}$$

Oppgave 2.

Rommet mellom to plan $y=h$ og $y=-h$ (se figur 2) er fylt med en væske som har konstant tetthet ρ, konstant viskositet ν og konstant termisk diffusivitet κ. Tyngdens virkning skal neglisjeres. Et kartesisk (x,y)-koordinatsystem som væskens hastighet (u,v) skal refereres til, er indikert i figur 2.

(Fortsettes side 3.)



Figur 2 viser skjematisk væskerommet mellom to plan $y = h$ og $y = -h$ som er omtalt i teksten. x-komponenten av hastigheten er også indikert

Væsken mellom planene strømmer i x-retningen og en **skal regne** med konstant hastighet U_0 over tversnittet. Det vil si

$$u(y) = U_0 \tag{9}$$

For $x \leq 0$ er temperaturen T_0 i væsken konstant. Ved $x = 0$ er det et sprang i veggtemperaturen slik at for $x > 0$ er veggtemperaturen

$$T(x > 0, y = \pm h) = T_w = T_0 - \Delta T \tag{10}$$

der ΔT er en konstant. Grensebetingelsen ved $x = 0$ kan dermed skrives

$$T(x = 0, y) = T_w + \Delta T \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \cos\left[\frac{(2n+1)\pi}{2} \frac{y}{h}\right] \tag{11}$$

For $x > 0$ kan temperaturutviklingen i væsken approksimativt beskrives med likningen

$$U_0 \frac{\partial T(x, y)}{\partial x} = \kappa \frac{\partial^2 T}{\partial y^2} \tag{12}$$

a) Finn temperaturfordelingen i væsken for $x > 0$. (Hint: Innfør $T(x, y) = T_w + \Delta T\, \Theta(x, y)$ og finn $\Theta(x, y)$)

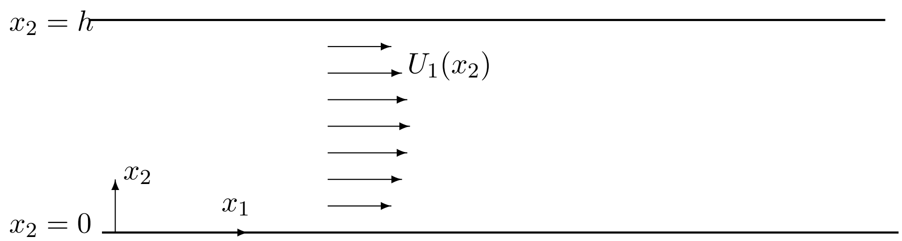
b) Foklar hvilke fysiske effekter/fenomener som er neglisjert i den approksimative likningen, likning (12), for temperaturutviklingen i væsken i forhold til den eksakte likningen for samme.

Oppgave 3.

Et statistisk stasjonært turbulent hastighetsfelt mellom to plan $x_2 = 0$ og $x_2 = h$ (se figur 3) er betegnet med $\tilde{u}_i(\mathbf{x}, \mathbf{t})$ og det tilhørende trykkfelt

(Fortsettes side 4.)

$\tilde{p}(\mathbf{x}, \mathbf{t})$. Tyngdens virkning skal neglisjeres.



Figur 3 viser skjematisk væskerommet mellom to plan $x_2 = 0$ og $x_2 = h$ som er omtalt i teksten. x_1-komponenten av hastigheten er også indikert.

Ved Reynolds dekomposisjon kan feltene skrives

$$\tilde{u_i}(\mathbf{x}, t) = U_1(x_2) + u_i(\mathbf{x}, t) \tag{13}$$

$$\tilde{p}(\mathbf{x}, t) = P(x_1, x_2) + p(\mathbf{x}, t) \tag{14}$$

Det er gitt at

$$\frac{\partial P(x_1, x_2)}{\partial x_1} = -\beta \tag{15}$$

der $\beta > 0$ og konstant.

a) Finn veggspenningen τ_w uttrykt ved β og h.

b) Vis at

$$[\overline{u_1 u_2}]_{y_+ \to 0} \to c y_+^3 \tag{16}$$

der $y_+ = x_2 u_* / \nu$, $u_* = \sqrt{(\tau_w/\rho)}$, og c er en ukjent konstant. ($\overline{(\cdot)}$ betyr tidsmidling.)

Anta som kjent at for $30 < y_+ < 1000$ er

$$u_+ = \frac{1}{\kappa} \ln y_+ + B \tag{17}$$

en approksimativ løsning for $u_+ = U_1/u_*$ der κ og B er modell-parametre.

c) Bruk relasjonene (16) og (17), samt at $u_+(y_+)_{y_+ \to 0} \to y_+$, til å konstruere et approksimativ, uniformt gyldig uttrykk for $y_+ = f(u_+)$ i området $0 < y_+ < 1000$. (Hint: Spaldings metode).

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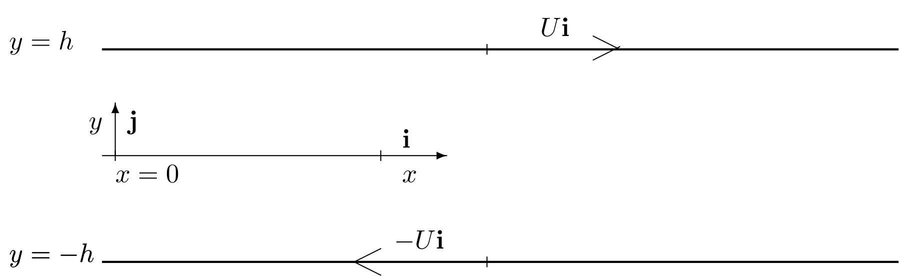
Det matematisk-naturvitenskapelige fakultet

Eksamen i:	MEK 3300/4300/9300 — Viskøs strømning og turbulens.
Eksamensdag:	Tirsdag 19. desember 2006.
Tid for eksamen:	14.30 – 17.30.
Oppgavesettet er på 3 sider.	
Vedlegg:	Ingen.
Tillatte hjelpemidler:	Rottmann: Matematische Formelsamlung, godkjent kalkulator.

Kontroller at oppgavesettet er komplett
før du begynner å besvare spørsmålene.

Oppgave 1.

Rommet mellom to plan $y = h$ og $y = -h$ (se figur 1) er fylt med et homogent fluid med konstant tetthet ρ og konstant kinematisk viskositet ν.



Figur 1 viser skjematisk rommet mellom to plan $y = h$ $y = -h$ omtalt i teksten. Det kartesiske (x, y)-koordinatsystemet referert til i teksten er også vist i figuren.

Det er etablert et fullt utviklet, tidsuavhengig hastighetsfelt ved at planet $y = h$ har hatt hastigheten $U\mathbf{i}$ og planet $y = -h$ har hatt hastigheten $-U\mathbf{i}$ (se figur 1) i meget lang tid. U er konstant. Trykket er konstant.

a) Finn det etablerte hastighetsfeltet.

(Fortsettes side 2.)

Ved tiden $t = 0$ stanses begge planene instantant og forblir i ro for $t > 0$.

b) Skriv ned initialbetingelsene og randbetingelsene for den bevegelsen fluidet får for $t > 0$.

c) Finn hastighetsfeltet i fluidet for $t > 0$.

Hint: Funksjonen $f(\eta)$ gitt ved

$$f(\eta) = \begin{cases} \eta & \text{for } -1 < \eta < 1 \\ 0 & \text{for } \eta = \pm 1 \\ f(\eta + 2) & \text{for alle } \eta \end{cases}$$

kan representeres ved en Fourier-rekke som

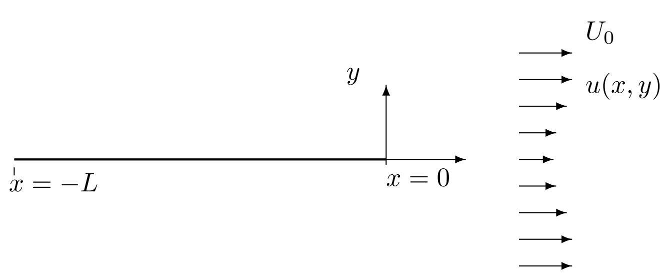
$$f(\eta) = \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \sin(k\pi\eta) \tag{1}$$

Oppgave 2.

Hastighetsfordelingen $u(x, y)$ nedstrøms en flat plate i strøm er indikert i figur 2 og kan uttykkes

$$u(x, y) = U_0 - u_1(x, y) \tag{2}$$

der $u(x, y \to \pm\infty) \to U_0$ (*konstant*).



Figur 2 viser skjematisk en flat plate i et strømningsfelt. Plata ligger i planet $y = 0$ og strekker seg fra $x = -L$ til $x = 0$. Kjølvannsprofilet $u(x, y)$ er også indikert.

a) Verifiser at grensesjiktlikningene anvendt på denne kjølvannsstrømningen approksimativt gir

$$U_0 \frac{\partial u_1}{\partial x} = \nu \frac{\partial^2 u_1}{\partial y^2} \tag{3}$$

forutsatt $| u_1 | \ll U_0$.

(Fortsettes side 3.)

b) Verifiser at i kjølvannet gjelder approksimativt

$$\int_{-\infty}^{+\infty} u_1(x,y)dy = \text{konstant (uavhengig av } x) \tag{4}$$

forutsatt $\mid u_1 \mid \ll U_0$.

Anta at

$$u_1(x,y) = AU_0 x^m g(\eta) \tag{5}$$

$$\eta = Bx^n y \tag{6}$$

c) Finn similaritetseksponentene m og n.

Oppgave 3.

I et statistisk stasjonært turbulent strømningsfelt er hastighetskomponentene og trykkfeltet betegnet med $\tilde{u}_i(\mathbf{x},\mathbf{t})$ og $\tilde{p}(\mathbf{x},\mathbf{t})$, henholdsvis.

a) Innfør Reynolds dekomposisjon av feltene, utled Reynolds tidsmidlede momentum-likning og definer Reynolds-spenningene.

b) Skriv ned Boussinesq-modellen for Reynolds-spenningene og gi en kort forklaring på hva de enkelte leddene representerer fysisk.

c) Finn en dimensjonsriktig relasjon for

$$\nu_t = f(\epsilon, K) \tag{7}$$

der ν_t ($\frac{m^2}{s}$) er eddyviskositeten, ϵ ($\frac{m^2}{s^3}$) er dissipasjonsraten pr. masseenhet i turbulensen og K ($\frac{m^2}{s^2}$) er turbulent kinetisk energi pr. masseenhet.

SLUTT

UNIVERSITETET I OSLO

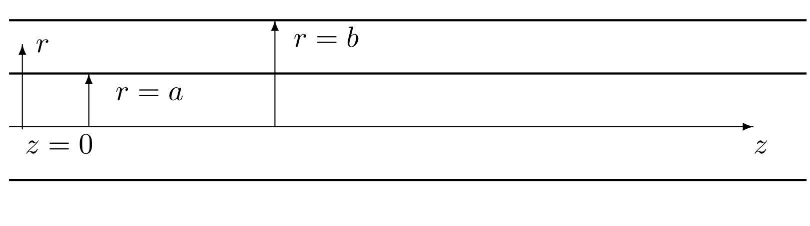
Det matematisk-naturvitenskapelige fakultet

Eksamen i:	MEK 3300/4300 — Viskøs strømning og turbulens.
Eksamensdag:	Fredag 13. juni 2008.
Tid for eksamen:	14.30 – 17.30.
Oppgavesettet er på 4 sider.	
Vedlegg:	Vedlegget "Kontinuitetslikning og momentumlikninger i sylinderkoordinater" er inkludert i oppgavesettet.
Tillatte hjelpemidler:	Rottmann: Matematische Formelsamlung, godkjent kalkulator.

Kontroller at oppgavesettet er komplett
før du begynner å besvare spørsmålene.

Oppgave 1.

Rommet mellom to koaksiale sylindre med sirkulært tverrsnitt er fylt med en væske som har konstant tetthet ρ og konstant kinematisk viskositet ν. Det strømningsproblemet som defineres nedenfor skal beskrives i et sylinderkoordinat system (r, θ, z) som indikert i figur 1.



Figur 1 viser skjematisk aksialt lengdesnitt av to koaksiale sirkulære sylindre omtalt i teksten.

Væskens hastighet $\mathbf{u}$ vil generelt kunne dekomponeres som

$$\mathbf{u} = u\mathbf{i}_r + v\mathbf{i}_\theta + w\mathbf{i}_z \tag{1}$$

(Fortsettes side 2.)

der $\mathbf{i}_r, \mathbf{i}_\theta$ og $\mathbf{i}_z$ er enhetsvektorer i $r-$, $\theta-$ og $z-$retning, henholdsvis. Den indre sylinderen har radius a og er i ro. Den ytre sylinderen har radius b og beveger seg med hastigheten $V_b \sin(\omega t)\,\mathbf{i}_\theta$ (se figur 1) slik at en kan skrive

$$\mathbf{u}(r = b, \theta, z; t) = V_b \sin(\omega t)\,\mathbf{i}_\theta \tag{2}$$

V_b er konstant og tyngdens virkning skal neglisjeres. Det er ingen annen bevegelse i væsken i dette tilfellet enn den som induseres på grunn av den ytre sylinderens bevegelse. Væskebevegelsen er laminær og stabil.

a) Vis at den induserte væskebevegelsen under de gitte betingelser kan beskrives med likningen

$$\beta\frac{\partial V}{\partial \tau} = \frac{\partial^2 V}{\partial R^2} + \frac{1}{R}\frac{\partial V}{\partial R} - \frac{V}{R^2} \tag{3}$$

der $\beta = \frac{\omega a^2}{\nu}$, $\tau = \omega t$, $R = \frac{r}{a}$ og $V = \frac{v}{V_b}$.

Vi søker en approksimativ løsning av likning (3) ved en perturbasjonsutvikling av formen

$$V(R, \tau; \beta) = V_0(R, \tau) + \alpha_1(\beta)V_1(R, \tau) + \cdots \tag{4}$$

forutsatt $\beta << 1$ og $1 >> \alpha_1(\beta) >> \cdots$.

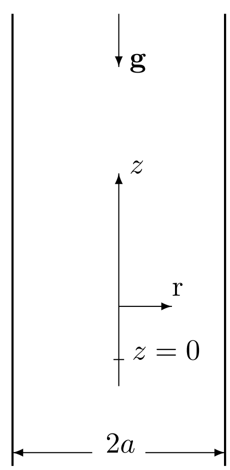
b) Finn $V_0(R, \tau)$ under de gitte betingelser.

c) Finn trykkfordelingen $p(r, t)$ i væsken når det er gitt at

$$p(r = a, t) = p_0 \tag{5}$$

hvor p_0 er en konstant.

Oppgave 2.



Figur 2 viser skjematisk aksialsnitt av det sirkulære røret som er omtalt i oppgaveteksten. z-aksen og r-koordinaten er også indikert.

Et rett vertikalt rør med sirkulært tverrsnitt og diameter $2a$ har ugjennomtrengelig vegg for $z < 0$, men fror $z \geq 0$ er veggen permeabel. Et fluid

(Fortsettes side 3.)

med konstant kinematisk viskositet ν og konstant tetthet ρ strømmer gjennom røret drevet av et trykkfelt $p(r, z)$. Tyngdefeltet er representert ved akselerasjonen

$$\mathbf{g} = -\mathbf{i}_z g \tag{6}$$

hvor $g > 0$. Strømningen skal beskrives i et sylinderkoordinatsystem (r, θ, z) som indikert i figur 2. Strømningen er tidsuavhengig og aksesymmetrisk og hastigheten $\mathbf{u}$ kan skrives

$$\mathbf{u} = \mathbf{i}_r u(r, z) + \mathbf{i}_z w(r, z) \tag{7}$$

For $z < 0$ er strømningen fullt utviklet og gitt som

$$\mathbf{u}(r, z) = \mathbf{i}_z W_0 \left[1 - \left(\frac{r}{a}\right)^2\right] \tag{8}$$

hvor W_0 er en konstant. For $z \geq 0$ gir rørveggens permeabilitet følgende lekkasjehastighet

$$\mathbf{u}(r = a, z) = \mathbf{i}_r u_0 \left(1 - \frac{z}{h}\right) \tag{9}$$

hvor u_0 er en konstant, og lekkasjen begrenser fluidkolonnens høyde til h målt fra $z = 0$. Alle overgangseffekter omkring $z = 0$ skal neglisjeres.

a) Finn h.

b) Anta at for $z \geq 0$ er

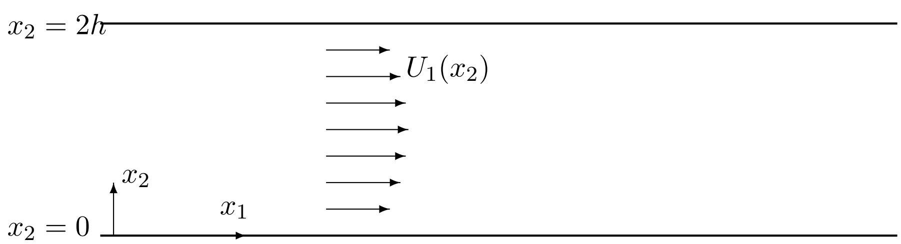
$$w(r, z) = W_1(z) \left[1 - \left(\frac{r}{a}\right)^2\right] \tag{10}$$

og bestem $W_1(z)$.

c) Finn en løsning for radialkomponenten $u(r, z)$ som kontinuitetsmessig er konsistent med den $w(r, z)$ du finner av svaret på spørsmål b).

Oppgave 3.

Et fullt utviklet statistisk stasjonært turbulent hastighetsfelt mellom to plan $x_2 = 0$ og $x_2 = 2h$ (se figur 3) er betegnet med $\tilde{u}_i(\mathbf{x}, t)$ og det tilhørende trykkfelt er $\tilde{p}(\mathbf{x}, t)$. Tyngdens virkning skal neglisjeres.



Figur 3 viser skjematisk væskerommet mellom to plan $x_2 = 0$ og $x_2 = 2h$ som er omtalt i teksten. x_1-komponenten $U_1(x_2)$ av hastigheten er også indikert.

Ved Reynolds dekomposisjon kan feltene skrives

$$\tilde{u}_i(\mathbf{x}, t) = U_i(\mathbf{x}) + u_i(\mathbf{x}, t) \tag{11}$$

$$\tilde{p}(\mathbf{x}, t) = P(x_1, x_2) + p(\mathbf{x}, t) \tag{12}$$

(Fortsettes side 4.)

der $U_i(\mathbf{x})$ og $P(x_1, x_2)$ er tidsmidlede komponenter, mens $u_i(\mathbf{x}, t)$ og $p(\mathbf{x}, t)$ er fluktuerende komponenter ($U_2 = U_3 = 0$). Det er gitt at

$$\frac{\partial P(x_1, x_2)}{\partial x_1} = -\beta \tag{13}$$

der $\beta > 0$ og konstant.

a) Finn veggspenningen τ_w uttrykt ved β og h.

b) Vis at

$$[\overline{u_1 u_2}]_{y_+ \to 0} \to c y_+^3 \tag{14}$$

der $y_+ = x_2 u_*/\nu$, $u_* = \sqrt{(\tau_w/\rho)}$, og c er en ukjent konstant. ($\overline{(\cdot)}$ betyr tidsmidling.)

c) Angi fysisk relevante og samhørende lengdeskalaer og skalaer for variasjon av $U_1(x_2)$ i vegglaget og i det ytre området.

d) Anta at det eksisterer et overlappingsområdet for løsningen av $U_1(x_2)$ i vegglaget og i det ytre området og finn ved hjelp av 'matching' et approksimativt uttrykk for $U_1(x_2)$ i overlappingsområdet.

VEDLEGG

Kontinuitets-likning og **momentum**-likninger i sylinderkoordinater (r, θ, z) med tilhørende hastighetskomponenter (u, v, w) er gitt som:

Kontinuitets-likningen

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0 \tag{15}$$

r-momentum

$$\frac{\partial u}{\partial t} + (\mathbf{u} \cdot \nabla)u - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right) \tag{16}$$

θ-momentum

$$\frac{\partial v}{\partial t} + (\mathbf{u} \cdot \nabla)v + \frac{uv}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \nu\left(\nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2}\frac{\partial u}{\partial \theta}\right) \tag{17}$$

z-momentum

$$\frac{\partial w}{\partial t} + (\mathbf{u} \cdot \nabla)w = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w \tag{18}$$

der

$$\mathbf{u} \cdot \nabla = u\frac{\partial}{\partial r} + \frac{v}{r}\frac{\partial}{\partial \theta} + w\frac{\partial}{\partial z} \tag{19}$$

og

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \tag{20}$$

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UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in MEK4300 — Viscous Flow and Turbulence.

Day of examination: Friday, June 12, 2009.

Examination hours: 14.30 – 17.30.

This problem set consists of 4 pages.

Appendices: None.

Permitted aids: Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

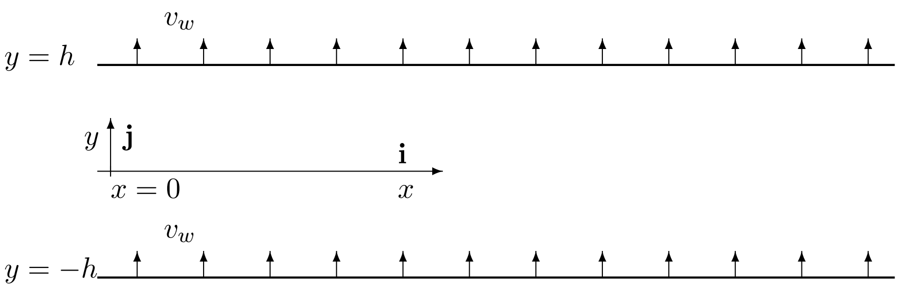


Figure 1 shows schematically the space between two planes $y = h$ and $y = -h$ referred to in the text. The leakage velocity v_w is indicated.
The cartesian coordinate system (x, y) used in the problem is also indicated.

We define the space between two planes $y = -h$ and $y = h$ as a duct where flow experiments are carried out. In the first experiment there is a laminar flow in the duct with velocity

$$\mathbf{u} = \mathbf{i}u + \mathbf{j}v \tag{1}$$

The fluid is homogeneous with constant density ρ and constant kinematic viscosity ν. The planes are porous allowing the following boundary conditions

$$v(x, y = -h) = v(x, y = h) = v_w > 0 \tag{2}$$

where v_w is a constant, while

$$u(x, y = -h) = u(x, y = h) = 0 \tag{3}$$

The flow is time independent and fully developed with

$$\nabla p = -\mathbf{i}\rho\beta, \quad \beta > 0 \tag{4}$$

where β is constant.

(Continued on page 2.)

a) Find the velocity components u og v. (The algebraic equations determining the constants of integration in the solution for u, are *not* required solved.)

The same duct is used for another flow experiment where the boundary conditions for v are changed to

$$v(x, y = -h) = -v_w \tag{5}$$

$$v(x, y = h) = v_w \tag{6}$$

while the boundary conditions for u still are $u(x, y = \pm h) = 0$.

b) Find find the volume flow rate $Q(x) = \int_{-h}^{h} u(x, y)dy$ given that $Q(x = 0) = Q_0$.

The stream function associated with (u, v) is $\psi(x, y) = g(x)f(y)$.

c) Define the velocity components u og v expressed in terms of the stream function and find $g(x)$ presupposed $f(h) = 1$ and $f(-h) = -1$.

d) Explain what the difference $\psi(x, h) - \psi(x, -h)$ represents physically.

Problem 2

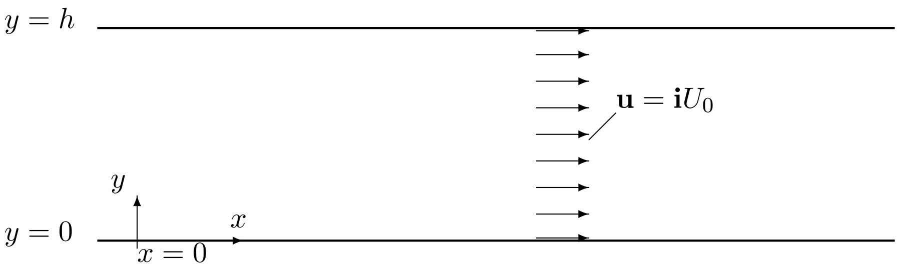


Figure 2 shows schematically the space between two planes $y = 0$ and $y = h$ referred in the text. The planes have temperature distributions as given in the text. The cartesian coordinate system (x, y) referred in the text is shown in the figure.

The space between two planes $y = 0$ and $y = h$ is filled with a *non-viscous* fluid with constant density ρ and constant thermal conductivity k. The fluid is flowing through the space with constant velocity $\mathbf{u} = \mathbf{i}U_0$. The planes have the following temperature distributions

$$T(x, y = 0) = T_0 \tag{7}$$

$$T(x < 0, y = h) = T_0 + \Delta T \tag{8}$$

$$T(x \geq 0, y = h) = T_0 \tag{9}$$

where ΔT is a constant. The temperature distribution in the fluid for $x \leq 0$ is regarded known and given as

$$T(x \leq 0, 0 < y < h) = T_0 + \Delta T \, \frac{y}{h} \tag{10}$$

The development of a temperature field is generally governed by the following equation

$$\rho c_p \frac{DT}{Dt} = k\nabla^2 T + \Phi \tag{11}$$

(Continued on page 3.)

where Φ is the dissipation function. We suppose here that $|\rho c_p U_0 \frac{\partial T}{\partial x}| >> k|\frac{\partial^2 T}{\partial x^2}|$.

a) Explain why the temperature field in the fluid for $x > 0$ in the problem considerd here, can be described approximately by the following equation

$$\rho c_p U_0 \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} \tag{12}$$

b) Find the temperature distribution in the fluid for $x > 0$.

Hint: It might be useful to know that the function $H(\eta)$ given as

$$H(\eta) = \eta \; for \; -1 < \eta < 1 \tag{13}$$

$$H(\eta) = 0 \; for \; \eta = \pm 1 \tag{14}$$

$$H(\eta \pm 2) = H(\eta), \; \forall \; \eta \tag{15}$$

can be expressed by

$$H(\eta) = \frac{2}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m} \sin(m\pi\eta) \tag{16}$$

Problem 3

The velocity and pressure fields in a strictly stationary turbulent flow field are denoted $u_i(x_j, t)$ og $p(x_j, t)$ where $i = 1, 2, 3$ and $j = 1, 2, 3$. The fluid is homogeneous with constant density ρ and constant dynamic viscosity μ. The fields may be decomposed into mean fields $U_i(x_j)$, $P(x_j)$ and fluctuating fields $u_i'(x_j, t)$, $p'(x_j, t)$ so that

$$u_i(x_j, t) = U_i(x_j) + u_i'(x_j, t) \tag{17}$$

$$p(x_j.t) = P(x_j) + p'(x_j, t) \tag{18}$$

a) Derive Reynolds avereaged Navier-Stokes equations and define Reynolds stress tensor.

b) Derive a balance equation for the kinetic energy $\frac{1}{2} U_i U_i$ associated with the mean flow and define the production of turbulent kinetic energy.

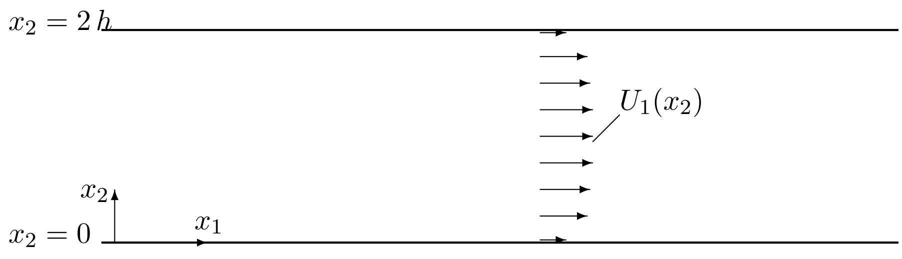


Figure 3 shows schematically the space between two planes $x_2 = 0$ and $x_2 = 2\,h$ referred to in the text. The velocity profile $U_1(x_2)$ is indicated. Cartesian coordinates (x_1, x_2) referred to in the text is also indicated.

(Continued on page 4.)

There is a fully developed turbulent fluid flow in the space between two planes $x_2 = 0$ og $x_2 = 2h$ (as indicated in figure 3) driven by the pressure drop

$$\frac{\partial P}{\partial x_1} = -\alpha \tag{19}$$

where α is a positive constant.

c) Find the shear stress distribution $\tau_{12}(x_2; \alpha; h) \equiv \mu \frac{dU_1}{dx_2} - \rho \overline{u_2' u_1'}$.

   Hint: Observe that τ_{12} can be expressed as a function of x_2 with α and h as parameters.

END

UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MEK4300/9300 — Viscous Flow and Turbulence.

Day of examination: Friday, June 11, 2010.

Examination hours: 9.00 – 12.00.

This problem set consists of 3 pages.

Appendices: None.

Permitted aids: Rottmann: Matematische Formelsamlung, approved calculator.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

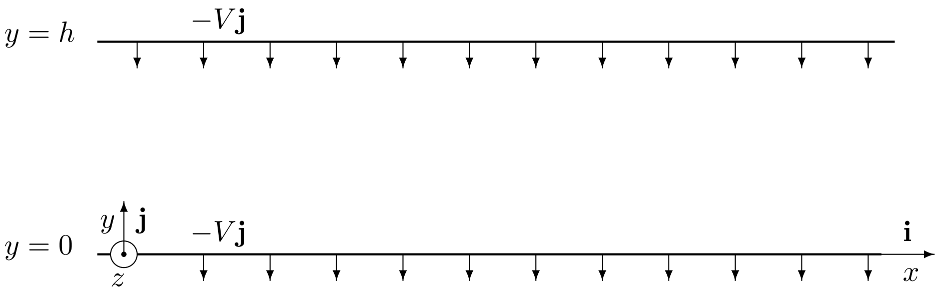


Figure 1 shows schematically the space between two planes $y = h$, $y = 0$ referred to in the text. The leakage velocity $-V\mathbf{j}$ referred to in the text is indicated. The cartesian (x, y, z)-coordinate system used in the problem, is also shown in the figure.

A homogeneous newtonian fluid with constant density ρ and constant kinematic viscosity ν is flowing through the space between two planes $y = 0$ and $y = h$ as indicated in figure 1. The velocity of the fluid, $\mathbf{u} = \mathbf{i}u + \mathbf{j}v$, is induced by the following boundary conditions

$$u(x, y = 0, z) = 0 \tag{1}$$

$$u(x, y = h, z) = U \tag{2}$$

$$v(x, y = 0, z) = v(x, y = h, z) = -V \tag{3}$$

$$w(x, y = 0, z) = w(x, y = h, z) = 0 \tag{4}$$

The flow is fully developed. There is no other motion in the fluid than that induced by the given boundary conditions, and $V > 0$. We consider a control volume bounded by the channel walls $y = 0$ and $y = h$, and the

(Continued on page 2.)

mathematically defined planes $x = 0$ and $x = a$, $z = 0$ and $z = b$. The stress tensor $\mathcal{P}$ is given on the boundary of the control volume as the following

$$\mathcal{P}(0 \le x \le a, y = 0, 0 \le z \le b) = -p_0\mathcal{E} + (\mathbf{ij} + \mathbf{ji})\tau_0 \tag{5}$$

$$\mathcal{P}(0 \le x \le a, y = h, 0 \le z \le b) = -p_0\mathcal{E} + (\mathbf{ij} + \mathbf{ji})\tau_1 \tag{6}$$

$$\mathcal{P}(x = 0, 0 \le y \le h, 0 \le z \le b) = \mathcal{P}(x = a, 0 \le y \le h, 0 \le z \le b) \tag{7}$$

$$\mathcal{P}(0 \le x \le a, 0 \le y \le h, z = 0) = \mathcal{P}(0 \le x \le a, 0 \le y \le h, z = b) \tag{8}$$

where

$$\mathcal{E} = \mathbf{ii} + \mathbf{jj} + \mathbf{kk} \tag{9}$$

and p_0, τ_0 and τ_1 are constants.

a) Find the x-component of the integral momentum balance equation using the given boundary conditions.

b) Find the velocity components u and v.

The temperature on the plane $y = 0$ is T_0 (constant). The temperature on the plane $y = h$ is

$$T(x, y = h, z, t) = T_0 + \Delta T \sin(\omega t) \tag{10}$$

where ΔT is a constant. The temperature equation is

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + \phi(y) \tag{11}$$

c) Neglect the dissipation $\phi(y)$ and verify when using

$$T(y, t) = T_0 + \Delta T \Theta(y, t) \tag{12}$$

that the solution of $\Theta(y, t)$ can be written

$$\Theta(y, t) = \Re\{[A_1 \exp(\alpha_1 y) + A_2 \exp(\alpha_2 y)] \exp(i\omega t)\} \tag{13}$$

$$\alpha_1 = \frac{-V + \sqrt{V^2 + 4i\omega\kappa}}{2\kappa} \tag{14}$$

$$\alpha_2 = \frac{-V - \sqrt{V^2 + 4i\omega\kappa}}{2\kappa} \tag{15}$$

Problem 2

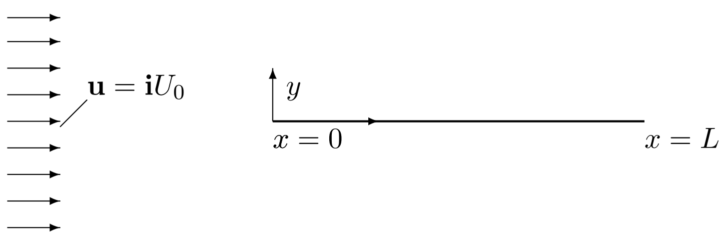


Figure 2 indicate schematically the flat plate at $y = 0$ and $0 < x < L$, referred to in the text.

(Continued on page 3.)

A fully developed viscous boundary layer flow is established along a flat plate, as indicated in figure 2. The x-component of the velocity field in the boundary layer, $u(x,y)$, is approximated by

$$u(x,y) = U_0\,(1 - \exp(-\alpha(x)y)) \tag{16}$$

U_0 is constant, and $\alpha(x) = \frac{1}{5}\sqrt{\frac{U_0}{\nu x}}$, where ν is the constant kinematic viscosity of the fluid.

a) Find y-component $v(x,y)$ of the velocity field in the boundary layer.

b) Find the displacement thickness $\delta^*(x)$ in the boundary layer.

Problem 3

In a newtonian fluid with constant density ρ and constant kinematic viscosity ν, a statistically stationary turbulent state of motion has been established. The velocity field is denoted $\mathbf{u}(\mathbf{x},t)$, and the pressure field is denoted $p(\mathbf{x},t)$. Reynolds decomposition of the fields should be introduced so that

$$\mathbf{U}(\mathbf{x}) = \lim_{T\to\infty}\left[\frac{1}{T}\int_0^T \mathbf{u}(\mathbf{x},t)dt\right] \tag{17}$$

$$P(\mathbf{x}) = \lim_{T\to\infty}\left[\frac{1}{T}\int_0^T p(\mathbf{x},t)dt\right] \tag{18}$$

and thereby $\mathbf{u}(\mathbf{x},t) = \mathbf{U}(\mathbf{x}) + \mathbf{u}'(\mathbf{x},t)$, og $p(\mathbf{x},t) = P(\mathbf{x}) + p'(\mathbf{x},t)$.

a) Find the Reynolds averaged momentum equation and the Reynolds averaged continuity equation.

b) Find the equation for the mean flow kinetic energy pr mass unit $\frac{1}{2}U_iU_i$.

c) Verify that the equation for the kinetic energy of the turbulence $\frac{1}{2}\overline{u_i'u_i'}$ can be written

$$\begin{aligned} U_j\frac{\partial}{\partial x_j}\left[\frac{1}{2}\overline{u_i'u_i'}\right] &= -\frac{\partial}{\partial x_j}\left[\overline{p'u_j'} + \frac{1}{2}\overline{u_i'u_i'u_j'} - 2\nu\overline{u_i's_{ij}'}\right] \\ &\quad - \overline{u_i'u_j'}S_{ij} - 2\nu\overline{s_{ij}'s_{ij}'} \end{aligned} \tag{19}$$

where $S_{ij} = \frac{1}{2}(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i})$ and $s_{ij}' = \frac{1}{2}(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i})$.

d) What does $\overline{u_i'u_j'}S_{ij}$ and $2\nu\overline{s_{ij}'s_{ij}'}$ represent physically.

END

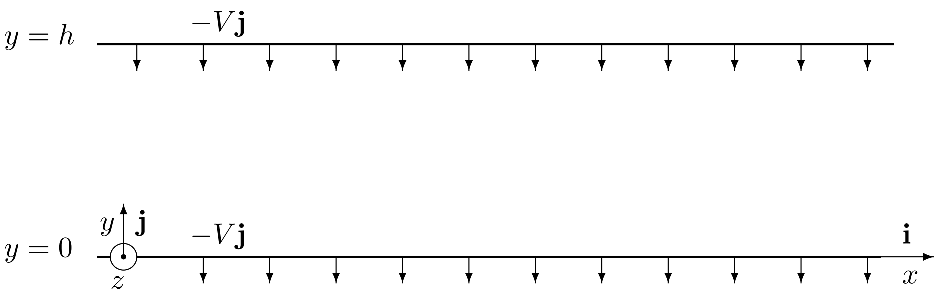
UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Eksamen i:	MEK4300/9300 — Viskøs strømning og turbulens.
Eksamensdag:	Fredag 11. juni 2010.
Tid for eksamen:	9.00 – 12.00.
Oppgavesettet er på 3 sider.	
Vedlegg:	Ingen.
Tillatte hjelpemidler:	Rottmann: Matematische Formelsamlung, godkjent kalkulator.

Kontroller at oppgavesettet er komplett før du begynner å besvare spørsmålene.

Oppgave 1



Figur 1 viser skjematisk rommet mellom to plan $y = h$, $y = 0$ omtalt i teksten. Lekkasjehastigheten $-V\mathbf{j}$ referert til i oppgaveteksten er indikert. Det kartesiske (x, y, z)-koordinatsystemet som brukes i oppgaven, er også vist i figuren.

Et homogent newtonsk fluid med kontant tetthet ρ og konstant kinematisk viskositet ν strømmer gjennom rommet mellom to plan $y = 0$ og $y = h$ som skissert i figur 1. Hastigheten i fluidet, $\mathbf{u} = \mathbf{i}u + \mathbf{j}v$, er indusert av følgende randbetingelser

$$u(x, y = 0, z) = 0 \tag{1}$$

$$u(x, y = h, z) = U \tag{2}$$

$$v(x, y = 0, z) = v(x, y = h, z) = -V \tag{3}$$

$$w(x, y = 0, z) = w(x, y = h, z) = 0 \tag{4}$$

Bevegelsen er fullt utviklet. Det er ingen annen bevegelse i fluidet enn den som induseres av ovenstående randbetingelser, og $V > 0$. Vi betrakter et kontrollvolum begrenset av kanalveggene $y = 0$ og $y = h$, og de matematisk definerte planene $x = 0$ og $x = a$, $z = 0$ og $z = b$. Spenningstensoren $\mathcal{P}$ er gitt på kontrollvolumets begrensningsflater som følger

$$\mathcal{P}(0 \le x \le a, y = 0, 0 \le z \le b) = -p_0\mathcal{E} + (\mathbf{ij} + \mathbf{ji})\tau_0 \tag{5}$$

(Fortsettes på side 2.)

$$\mathcal{P}(0 \le x \le a, y = h, 0 \le z \le b) = -p_0\mathcal{E} + (\mathbf{ij} + \mathbf{ji})\tau_1 \tag{6}$$

$$\mathcal{P}(x = 0, 0 \le y \le h, 0 \le z \le b) = \mathcal{P}(x = a, 0 \le y \le h, 0 \le z \le b) \tag{7}$$

$$\mathcal{P}(0 \le x \le a, 0 \le y \le h, z = 0) = \mathcal{P}(0 \le x \le a, 0 \le y \le h, z = b) \tag{8}$$

hvor

$$\mathcal{E} = \mathbf{ii} + \mathbf{jj} + \mathbf{kk} \tag{9}$$

og p_0, τ_0 og τ_1 er konstanter.

a) Finn x-komponenten av integral momentum balanse likning ved bruk av de gitte randbetingelser.

b) Finn hastighetskomponentene u og v.

Temperaturen på planet $y = 0$ er T_0 (konstant). Temperaturen på planet $y = h$ er

$$T(x, y = h, z, t) = T_0 + \Delta T \sin(\omega t) \tag{10}$$

hvor ΔT er en konstant. Temperaturlikningen er

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + \phi(y) \tag{11}$$

c) Neglisjer dissipasjonen $\phi(y)$ og verifiser at med

$$T(y, t) = T_0 + \Delta T \Theta(y, t) \tag{12}$$

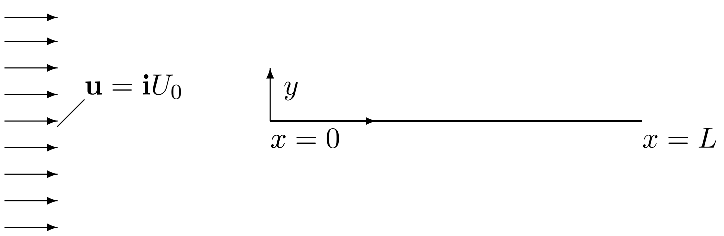
kan løsningen for $\Theta(y, t)$ skrives

$$\Theta(y, t) = \Re\{[A_1 \exp(\alpha_1 y) + A_2 \exp(\alpha_2 y)] \exp(i\omega t)\} \tag{13}$$

$$\alpha_1 = \frac{-V + \sqrt{V^2 + 4i\omega\kappa}}{2\kappa} \tag{14}$$

$$\alpha_2 = \frac{-V - \sqrt{V^2 + 4i\omega\kappa}}{2\kappa} \tag{15}$$

Opgave 2



Figur 2 viser skjematisk en flat plate i $y = 0$ og $0 < x < L$ omtalt i teksten.

(Fortsettes på side 3.)

En fullt utviklet viskøs grensesjiktsstrømning er etablert langs en flat plate. som indikert i figur 2. Hastighetsfeltet i grensesjiktet har en x-component som er approksimert med

$$u(x,y) = U_0 \left(1 - \exp(-\alpha(x)y)\right) \tag{16}$$

U_0 er konstant, og $\alpha(x) = \frac{1}{5}\sqrt{\frac{U_0}{\nu x}}$, hvor ν er fluidets konstante kinematiske viskositet.

a) Finn y-komponenten $v(x,y)$ av hastighetsfeltet i grensesjiktet.

b) Finn forskyvningstykkelsen (displacement thickness) $\delta^*(x)$ i grensesjiktet.

Opgave 3

I et newtonsk fluid med konstant tetthet ρ og konstant kinematisk viskositet ν er det etablert en statistisk stasjonær turbulent strømningstilstand. Hastighetsfeltet betegnes $\mathbf{u}(\mathbf{x},t)$ og trykkfeltet $p(\mathbf{x},t)$. Reynolds dekomposisjon av feltene skal innføres slik at

$$\mathbf{U}(\mathbf{x}) = \lim_{T\to\infty}\left[\frac{1}{T}\int_0^T \mathbf{u}(\mathbf{x},t)dt\right] \tag{17}$$

$$P(\mathbf{x}) = \lim_{T\to\infty}\left[\frac{1}{T}\int_0^T p(\mathbf{x},t)dt\right] \tag{18}$$

og dermed $\mathbf{u}(\mathbf{x},t) = \mathbf{U}(\mathbf{x}) + \mathbf{u}'(\mathbf{x},t)$, og $p(\mathbf{x},t) = P(\mathbf{x}) + p'(\mathbf{x},t)$.

a) Finn Reynolds-midlet momentum likning og Reynolds-midlet kontinuitetslikning.

b) Finn likningen for middelstrømmens kinetiske energi pr. masseenhet $\frac{1}{2}U_iU_i$.

c) Vis at likningen for turbulensens kinetiske energi pr. masseenhet $\frac{1}{2}\overline{u_i'u_i'}$ kan skrives

$$\begin{aligned} U_j\frac{\partial}{\partial x_j}\left[\frac{1}{2}\overline{u_i'u_i'}\right] &= -\frac{\partial}{\partial x_j}\left[\overline{p'u_j'} + \frac{1}{2}\overline{u_i'u_i'u_j'} - 2\nu\overline{u_i's_{ij}'}\right] \\ &\quad - \overline{u_i'u_j'}S_{ij} - 2\nu\overline{s_{ij}'s_{ij}'} \end{aligned} \tag{19}$$

hvor $S_{ij} = \frac{1}{2}(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i})$, og $s_{ij}' = \frac{1}{2}(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i})$.

d) Hva representerer $\overline{u_i'u_j'}S_{ij}$ og $2\nu\overline{s_{ij}'s_{ij}'}$ fysisk?

SLUTT

UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MEK4300/9300 — Viscous flow og turbulence

Day of examination: Wednesday 15. June 2011

Examination hours: 9.00 – 13.00

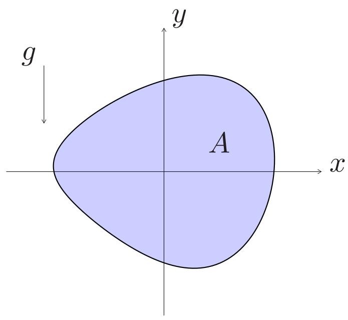
This problem set consists of 4 pages.

Appendices: None

Permitted aids: Rottmann: Matematische Formelsamlung, certified calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 Poiseuille flow (weight 36%)



A viscous flow in a duct of arbitrary cross-section (see figure) is driven by a pressure change along the duct. The cross section is parallel to the xy plane and is denoted by A, while the z axis is aligned along the duct. The fluid is incompressible, the dynamic viscosity coefficient is μ, gravity is directed in the negative y direction and the duct wall is impenetrable. Moreover, we assume that the flow is stationary and uniform in the z direction.

1a (weight 6%)

Write down the equations and the boundary conditions for this problem.

1b (weight 6%)

Assume a velocity on the form

$$\mathbf{v} = w(x, y)\mathbf{k},$$

and find an expression for the pressure. Furthermore, show that

$$\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = -\beta \tag{1}$$

and state how the constant β relates to the pressure.

(Continued on page 2.)

1c (weight 6%)

Find the wall stress τ_w expressed by derivatives of w. Show also that the dissipation in this case becomes

$$\Phi = \mu \left\{ \left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right\}.$$

1d (weight 6%)

Find w, expressed by β, when the duct perimeter is elliptical and defined as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0.$$

Hint: observe that the Laplacian of the left hand side of this expression is constant.

1e (weight 6%)

In this sub-problem we assume $a = b$. Find the total volume flux, Q, along the duct. Do also determine the wall stress τ_w expressed by the pressure gradient and other parameters of the problem.

1f (weight 6%)

We again assume a duct of general shape. Use (1) to show the energy equation on the form

$$Q \frac{\partial p}{\partial z} = - \iint_A \Phi \, dx \, dy,$$

where Q is the volume flux in the duct, and explain the physical content of this relation. Hint: you need to manipulate the integrals to obtain the right hand side.

Problem 2 Turbulence (weight 28%)

2a (weight 10%)

Derive the Reynolds averaged Navier-Stokes (RANS) equations for an incompressible, Newtonian fluid and identify the Reynolds stress tensor. You may use, without proof, that $\mathbf{v} \cdot \nabla \mathbf{v} = \nabla \cdot (\mathbf{v}\mathbf{v})$ for any divergence-free velocity field $\mathbf{v}$.

2b Turbulent duct flow (weight 10%)

We assume fully developed pressure-driven duct flow between two planes at $y = -h$ and $y = h$. The mean velocity will then be reduced to $\overline{\mathbf{v}} = \overline{u}(y)\mathbf{i}$. Find the pressure variation across the duct from the RANS equations. Show that

$$0 = -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial y},$$

where the shear stress is

$$\tau = \mu \frac{\mathrm{d}\overline{u}}{\mathrm{d}y} - \rho \overline{u'v'},$$

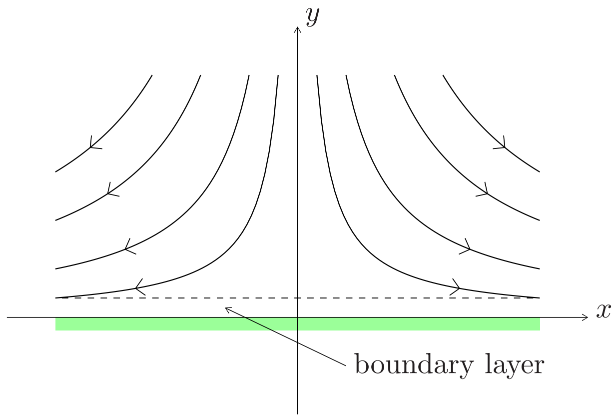
(Continued on page 3.)

ρ is the density and $\frac{\partial \overline{p}}{\partial x}$ is a constant.

2c (weight 8%)

Why must there be a viscous sublayer close to a no-slip boundary? The velocity profile in the sublayer may be approximated by a linear function. Find this expressed by the wall stress τ_w and the other relevant parameters.

Problem 3 Stagnation flow (weight 36%)



An inviscid solution for stagnation flow at a wall (see figure) is

$$\psi^* = Bx^*y^*, \quad u^* = Bx^*, \quad v^* = -By^*,$$

where ψ^* is the stream function, and u^* and v^* are the velocity components. The star indicates quantities with dimension. We seek the modification of this flow due to the no-slip condition at the wall and viscosity.

3a (weight 9%)

We denote the density by ρ and the dynamic viscosity coefficient by μ. Dimensionless variables are then introduced according to

$$\begin{aligned} x^* = Lx, \quad & y^* = Hx, \quad t^* = Bt, \\ u^* = LBu, \quad & v^* = HBv, \quad p^* = \hat{p}p, \end{aligned}$$

where L is some length scale in the x direction and $H = \sqrt{\frac{\mu}{\rho B}}$ is a scale related to the boundary layer. Determine the pressure scale, $\hat{p}$, such that the non-dimensional equations become

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \gamma\frac{\partial^2 u}{\partial x^2}, \\ \gamma\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) &= -\frac{\partial p}{\partial y} + \gamma\left(\frac{\partial^2 v}{\partial y^2} + \gamma\frac{\partial^2 v}{\partial x^2}\right), \end{aligned}$$

where $\gamma = H^2/L^2$. For any sensible choice of L the parameter γ is small. However, we seek exact solutions of the equations and will not discard terms of order γ as such.

(Continued on page 4.)

3b (weight 9%)

We assume that v is independent of x

$$v = -F(y),$$

where F is some function that must be determined. Show that this implies that the other velocity component is on the form

$$u = xF'(y),$$

and that F must fulfill the boundary conditions

$$F(0) = 0, \quad F'(0) = 0, \quad \lim_{y\to\infty} F'(y) = 1.$$

3c (weight 9%)

Show that the pressure must be on the form

$$p = -\gamma(F' + F^2) + p_0(x).$$

3d (weight 9%)

Show that F must solve the equation

$$F''' + FF'' - \left(F'\right)^2 = -1.$$

The End

UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MEK4300/9300 — Viscous flow og turbulence

Day of examination: Friday 15. June 2012

Examination hours: 9.00 – 13.00

This problem set consists of 3 pages.

Appendices: Formula sheet

Permitted aids: Rottmann: Matematische Formelsamlung, certified calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 Turbulence (weight 15%)

Derive the Reynolds averaged Navier-Stokes (RANS) equations for an incompressible, Newtonian fluid and identify the Reynolds stress tensor. You may use, without proof, that $\mathbf{v} \cdot \nabla \mathbf{v} = \nabla \cdot (\mathbf{v}\mathbf{v})$ for any divergence-free velocity field $\mathbf{v}$.

Problem 2 Gravity driven viscous flow (weight 35%)

A film of liquid is flowing down at the outside of a vertical cylinder of radius a, under the action of gravity. We assume that the thickness of the fluid film, $b - a$, is constant along (and around) the cylinder and that the flow is stationary and radially symmetric. At the surface of the fluid there is an external pressure, p_0, while the shear stress is negligible. The cylinder is at rest.

2a (weight 10%)

Formulate equations and boundary conditions, invoking the simplifying assumptions.

2b (weight 20%)

Find the pressure and the velocity distribution.

2c (weight 5%)

Calculate the drag (D) on the cylinder per height. Show that D obeys

$$D = \rho g A,$$

(Continued on page 2.)

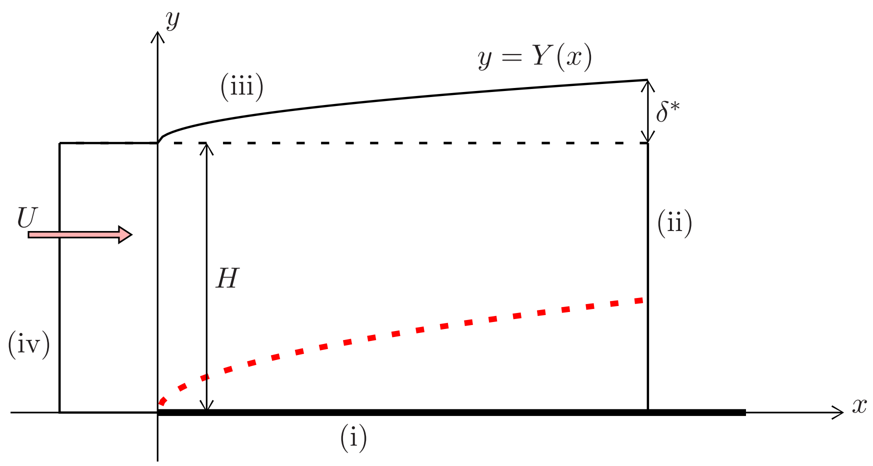


Figure 1: A control volume for mass balance analysis. The upper boundary of the volume, $y = Y(x)$, is a streamline outside the boundary layer. Also the line $y = H$ is outside the boundary layer, which is indicated by the bold dashes.

where A is the cross-sectional area of the fluid. Explain this relation physically. Correspondingly, the total dissipation per height and the volume flux, Q, through the cross-sectional area, fulfill the relation

$$\iint_A \Phi \, dx \, dy = \rho g Q,$$

where Φ is the dissipation pr volume. Do not (!) calculate Φ and Q, but explain the relation physically.

Problem 3 Boundary layer (weight 50%)

In this problem we consider the Blasius boundary layer, which develops along a semi-infinite plate, corresponding to the positive x-axis, with the leading edge at the origin. The fluid is homogeneous and incompressible and the background flow is $U\mathbf{i}$, where $\mathbf{i}$ is the unit vector in the x-direction. Moreover, the y-axis is aligned normal to the plate and the velocity components in the x and y directions are u and v, respectively.

3a (weight 10%)

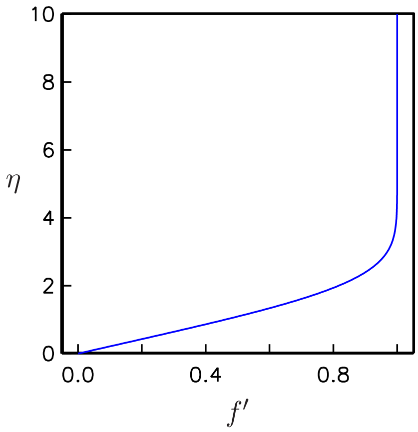
A control volume is depicted in figure 1. The sides are numbered (i), (ii), (iii) and (iv). The boundary (iii) corresponds to a streamline outside the boundary layer. Use the mass balance argument to show that the displacement thickness is given by

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

3b (weight 20%)

Give the boundary layer equations and boundary conditions for the Blasius flow. A derivation is not required, but the main differences from the full Navier-Stokes equations should be listed.

(Continued on page 3.)



η	f	f'	f''
0.000000	0.000000	0.000000	0.469600
0.500000	0.058643	0.234227	0.465030
1.000000	0.232990	0.460633	0.434379
1.500000	0.515032	0.661474	0.361805
2.000000	0.886797	0.816695	0.255669
2.500000	1.322440	0.916808	0.147475
3.725000	2.509840	0.994972	0.014234
4.975000	3.758240	0.999929	0.000283
7.475000	6.258220	1.000000	0.000000

Figure 2: The form function of the Blasius profile.

3c (weight 10%)

We assume the existence of a similarity solution

$$u = Uf'(\eta), \quad \eta = \frac{y}{\Delta(x)},$$

where the functions f and Δ are to be determined. Find a corresponding expression for v, show that we may employ $\Delta = \sqrt{2\nu x/U}$ and find an ordinary differential equation, with boundary equations, for f. The solution for f is depicted and tabulated in figure 2. However, a discussion of its computation is not required.

3d (weight 10%)

Use the preceding results to find explicit expressions for the displacement thickness, the shear stress at the plate and the drag (per width) of a section of length D from the front of the plate. Discuss briefly the validity of these expressions.

The End

UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MEK4300/9300 — Viscous flow og turbulence

Day of examination: Friday 14. June 2013

Examination hours: 9.00 – 13.00

This problem set consists of 4 pages.

Appendices: None

Permitted aids: Rottmann: Matematische Formelsamlung, certified calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 Turbulence (weight 35%)

Fluid flow is often described mathematically through the Navier-Stokes equations:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f}, \tag{1}$$

$$\nabla\cdot\boldsymbol{u} = 0, \tag{2}$$

where $\boldsymbol{u}$, p, ρ, ν and $\boldsymbol{f}$ are the velocity vector, pressure, density, kinematic viscosity and volume forces respectively.

1a (weight 5%)

Which two physical laws were used in the derivation of the Navier-Stokes equations? What other assumptions have been made?

1b (weight 10%)

Introduce Reynolds decomposition of velocity and pressure and derive from (1) og (2) the Reynolds Averaged Navier-Stokes (RANS) equations.

1c (weight 10%)

Turbulent flows are often 'defined' through a list of characteristic properties. Give at least 5 characteristic properties that define turbulence.

1d (weight 10%)

Explain in words how it is possible to derive a transport equation for $\overline{u_i u_j}$ (Note, not $\overline{u_i' u_j'}$). Explain also how $\overline{u_i' u_j'}$ can be modelled through an algebraic expression.

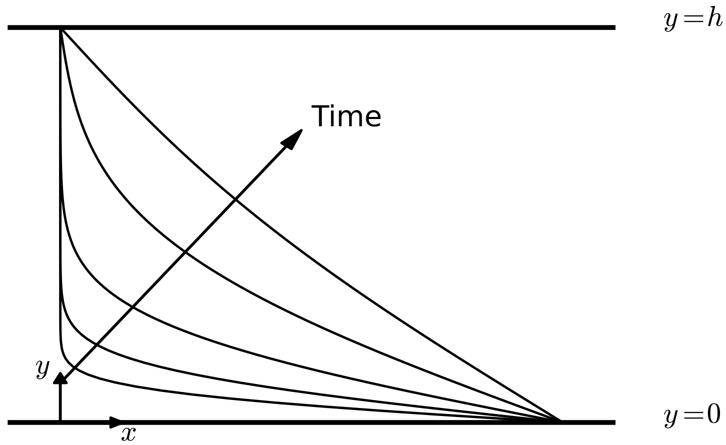


Figure 1: Sketch of the flow between two plates located at $y = 0$ og $y = h$. The thin lines illustrate how the flowfield evolves in time. The velocity is then plotted along the x-axis as a function of y.

Problem 2 Transient flow between two parallel plates (weight 25%)

Consider a uniform, straight flow of an incompressible Newtonian fluid between two parallel plates of infinite extension in x and z-directions. The lower plate is located at $y = 0$ and the upper at $y = h$. The velocity vector is $\boldsymbol{u} = (u(y,t), 0, 0)$. The fluid between the plates is initially at rest. At $t = 0$ the velocity of the lower plate is suddenly accelerated to $u(0,t) = U$, while the upper is kept still. The flow field is illustrated in Figur 1.

2a (weight 10%)

Formulate the equations, including boundary and initial conditions for this problem.

2b (weight 15%)

Find the velocity $u(y,t)$. (Hint: Introduce $v(y,t) = u(y,t) - U(1 - y/h)$ and solve the homogeneous problem for $v(y,t)$ first. Also, $\int_0^h \sin^2(n\pi y/h)dy = h/2$ and $\int_0^h (1 - y/h)\sin(n\pi y/h)dy = h/(n\pi)$, for $n = 1, 2, 3 \ldots$).

Problem 3 Laminar boundary layer (weight 40%)

We consider a laminar boundary layer that evolves over a plate lying at position $y = 0$ in the plane spanned by the x- and z-axes. The boundary layer evolves from the origin and grows in the positive x-direction. The boundary layer is generated by the outer flow $U\mathbf{i}$, where $\mathbf{i}$ is the unit normal vector in the x-direction. The velocitycomponents in x and y directions are, respectively, u and v. Two common quantities often used in defining boundary layers are

$$\delta^*(x) = \int_0^{y\to\infty} \left(1 - \frac{u}{U}\right) \mathrm{d}y,$$
$$\theta(x) = \int_0^{y\to\infty} \frac{u}{U}\left(1 - \frac{u}{U}\right) \mathrm{d}y.$$

3a (weight 10%)

What are the quantities δ^* and θ called and from which two physical laws are they derived?

3b (weight 10%)

Assume that the velocity u is known and given as

$$u(y;\delta) = U \sin\left(\frac{\pi y}{2\delta}\right), \tag{3}$$

where $\delta(x)$ is the actual boundary layer thickness (defined through $u(x,\delta) = 0.99U$). The skinfriction coefficient, C_f, is defined as

$$C_f = \frac{\tau_w(x)}{0.5\rho U^2} = 2\frac{\mathrm{d}\theta}{\mathrm{d}x}, \tag{4}$$

where $\tau_w(x) = \mu \mathrm{d}u/\mathrm{d}y$ at $y = 0$. Use this to find δ^* and θ expressed in terms of x. Feel free to simplify the results using $\mathrm{Re}_x = \rho U x/\mu$.

3c (weight 10%)

The laminar boundary layer equations can be derived from the Navier-Stokes equations by proper scaling and subsequent elimination of small terms in the limit of large Reynolds numbers ($\mathrm{Re} = \rho U L/\mu$). Use the following normalizations

$$\overline{x} = \frac{x}{L} \qquad \overline{u} = \frac{u}{U} \qquad \overline{v} = \frac{v}{U}\sqrt{Re}$$
$$\overline{y} = \frac{y}{L}\sqrt{Re} \qquad \overline{t} = \frac{tU}{L} \qquad \overline{p} = \frac{p}{\rho U^2}$$

and show that the boundary layer equations can be written as

$$\frac{\partial \overline{u}}{\partial \overline{t}} + \overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} = \frac{\partial^2 \overline{u}}{\partial \overline{y}^2} - \frac{\partial \overline{p}}{\partial \overline{x}},$$
$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0,$$
$$\frac{\partial \overline{p}}{\partial \overline{y}} = 0.$$

3d (weight 10%)

We introduce, like Blasius did in 1908, a similarityvariabel $\eta(x,y)$ and a similaritysolution $\psi(x;\eta)$ defined as

$$\eta(x,y) = y\sqrt{\frac{U}{2\nu x}},$$
$$\psi(x,\eta) = \sqrt{2\nu U x} f(\eta),$$

where $f(\eta)$ is an unknown function to be determined and ψ is the streamfunction defined as

$$u = \frac{\partial \psi}{\partial y} \quad \text{og} \quad v = -\frac{\partial \psi}{\partial x}.$$

Derive, starting from the boundary layer equations, the Blasius equation

$$f''' + ff'' = 0.$$

Formulate also the boundary conditions for these equations.

The End

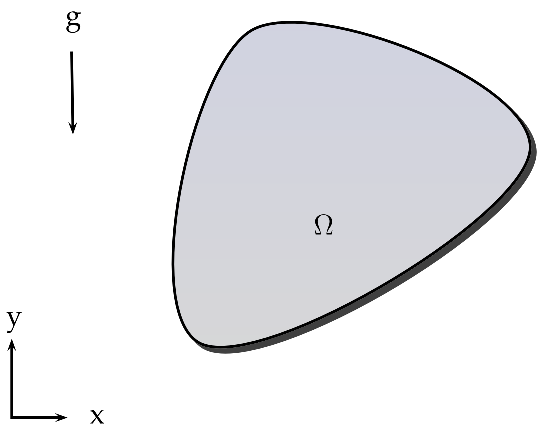
UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Eksamen i:	MEK4300 — Viskøs strømning og turbulens
Eksamensdag:	Fredag 13. juni 2014
Tid for eksamen:	14.30 – 18.30
Oppgavesettet er på 3 sider.	
Vedlegg:	Ingen
Tillatte hjelpemidler:	Rottmann: Matematische Formelsamlung, godkjent kalkulator

Kontroller at oppgavesettet er komplett før du begynner å besvare spørsmålene.

Oppgave 1 Strømning gjennom et rør med vilkårlig tverrsnitt (vekt 50%)



Vi ser på inkompressibel strømning av en Newtonsk væske ved konstant temperatur gjennom et rør med vilkårlig tverrsnitt, som vist i figuren. Røret antas å være uendelig langt og uforandret i z-retningen (normalt på arket). Strømningen i røret er drevet i positiv z-retning av en konstant trykkgradient og vi kan se bort ifra initialbetingelser. Tyngdekraften g virker i negativ y-retning som vist i figuren.

1a (vekt 5 %)

Uten videre antagelser, sett opp gjeldende likninger og grenseverdier for dette problemet. Hvilke to fysiske lover ligger til grunn for likningene? Hvilket navn går disse likningene under?

(Fortsettes på side 2.)

1b (vekt 5 %)

Anta nå at strømningen er laminær. Gjør alle mulige forenklinger og formuler likninger og grenseverdier for problemet på nytt.

1c (vekt 10 %)

Beskriv en numerisk løsning av problemet (fra 1b) i programmeringsspråket Python. Anta at du har installert FEniCS.

1d (vekt 10%)

Anta videre i oppgaven at strømningen er turbulent. Gi minst fem karakteristikker av turbulent strømning som gjør at den skiller seg fra den laminære strømningen.

1e (vekt 10 %)

Innfør Reynolds dekomponering av hastighet og trykk og utled de Reynolds midlede (RANS) likningene.

1f (vekt 10 %)

Gjør alle mulige forenklinger og sett opp gjeldene RANS likninger og grenseverdier for problemet beskrevet i denne oppgaven. Forklar hvorfor disse likningene ikke kan løses og formuler en eddy-viskositetsmodell som gjør liknings-settet løsbart.

Oppgave 2 Oppstart av kanalstrøm mellom to parallelle plater (vekt 50%)

Vi ser på en inkompressibel Newtonsk væske som ved tiden $t = 0$ ligger i ro mellom to parallelle plater av uendelig utstrekning. Platene ligger i plan utspent av x- og z-aksene, den øverste plata ved $y = 1$ og den nederste ved $y = -1$. Hastighetsvektoren er gitt ved $\boldsymbol{u} = (u(y,t), 0, 0)$.

Ved tiden $t = 0$ settes det på en konstant trykkgradient $1/\rho\nabla p = (-\beta, 0, 0)$, der β er en konstant som er større enn 0. Denne trykk-kraften setter væsken i bevegelse i retning av den positive x-aksen. Anta at Reynolds nummeret er lavt.

2a (vekt 5 %)

Hva er den matematiske beskrivelsen av en Newtonsk væske?

2b (vekt 5%)

Skriv ned de fullt forenklede likninger, initial- og grensebetingelser for dette problemet.

(Fortsettes på side 3.)

2c (vekt 10%)

Finn den endelige stasjonære hastigheten $\overline{u}(y)$ som væsken vil nå etter lang tid.

2d (vekt 15%)

Finn hastigheten $u(y,t)$. Hint: Bruk den stasjonære hastigheten og reformuler problemet slik at du løser for $v(y,t) = u(y,t) - \overline{u}(y)$. Videre kan man benytte at $\int_{-1}^{1} \cos(\lambda_k y)(1-y^2)\, dy = 4(-1)^{k-1}/\lambda_k^3$, der $\lambda_k = (2k-1)\pi/2$, for $k = 1, 2, 3 \ldots$.

2e (vekt 15%)

Implementer en numerisk løsning av problemet i Python/FEniCS. Implementer også den eksakte analytiske løsningen funnet i 2d og vis hvordan man hvert tidsskritt kan beregne et estimat på feilen i den numeriske løsningen.

Slutt

UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MEK4300/9300 — Viscous flow and turbulence

Day of examination: Wednesday 10. June 2015

Examination hours: 14.30 – 18.30

This problem set consists of 3 pages.

Appendices: None

Permitted aids: Rottmann: Matematische Formelsamlung

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 Laminar and turbulent boundary layers (weight 50%)

We consider a boundary layer forming over a flat plate located at $y = 0$ and starting at $x = 0$, as shown in Figure 1. The flow is assumed to be incompressible and the fluid is Newtonian at constant temperature. The outer flow is given as $U\boldsymbol{i}$, where U is constant and $\boldsymbol{i}$ is the unit normal in the x direction. We assume first that the flow is laminar.

1a (weight 10 %)

Use conservation of mass and show, with the help of von Kármán's control volume (see Fig. 1), that

$$\delta^* = \int_0^{y\to\infty} \left(1 - \frac{u(y)}{U}\right) dy \qquad (1)$$

The quantity δ^* is shown in Figure 1. What is δ^* called?

1b (weight 10%)

Use conservation of momentum in the x direction and show, with the help of the control volume, that

$$\theta = \frac{\text{Drag}}{\rho U^2} = \int_0^{y\to\infty} \frac{u(y)}{U}\left(1 - \frac{u(y)}{U}\right) dy \qquad (2)$$

What is θ^1 called?

1c (weight 15 %)

Assume now that the flow is turbulent. Reynolds decomposition of the velocity vector, $\boldsymbol{u} = (u, v, w)$, is given as $\boldsymbol{u}(\boldsymbol{x}, t) = \overline{\boldsymbol{u}}(\boldsymbol{x}) + \boldsymbol{u}'(\boldsymbol{x}, t)$, where

(Continued on page 2.)

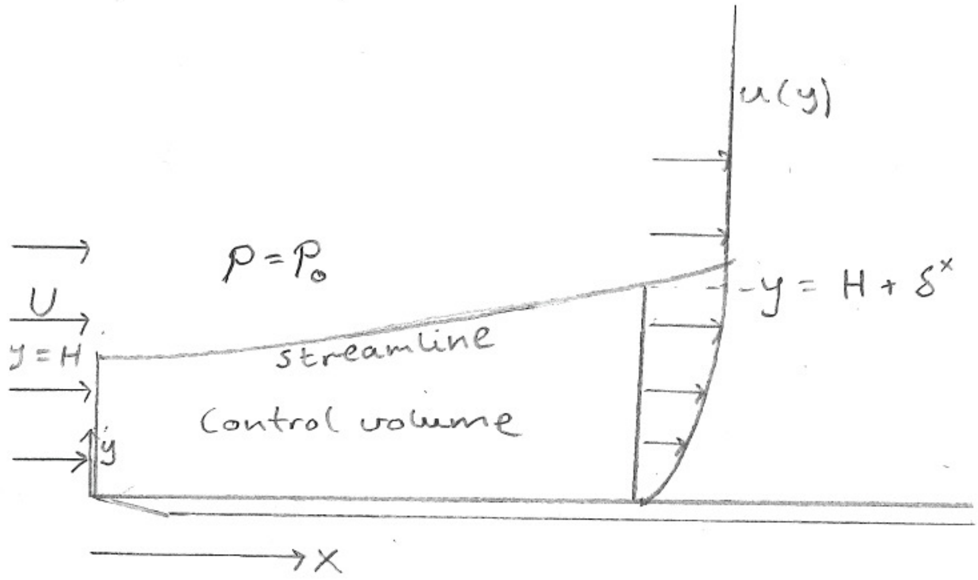


Figure 1: Development of a boundary layer over a flat plate. von Kármán's control volume is shown. The velocity U and the pressure given by P_0 are both constant.

$\overline{\boldsymbol{u}}$ and $\boldsymbol{u}'$ are, respectively, the mean velocity vector and the fluctuation about this mean. Derive the Reynolds averaged Navier Stokes equations governing this problem. Simplify as much as possible. Define also the boundary conditions for a given x (disregarding the inlet and outlet). Hint: $\overline{v} << \overline{u}$, $\frac{\partial}{\partial x} << \frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z} = 0$ (same as for laminar boundary layers).

1d (weight 15%)

The turbulent (Reynolds) shear stress is given as $-\rho\overline{u'v'}$, where ρ is density. Explain how $\rho\overline{u'v'}$ can be modelled and explain what is meant by a *complete turbulence model*. Also explain (using drawings if you will) why we should expect that $\overline{u'v'} \leq 0$ throughout the boundary layer.

Problem 2 Mixed Poiseuille-Couette flow between two parallel plates (weight 50%)

We consider a mixed Poiseuille-Couette flow of an incompressible Newtonian fluid between two parallel plates of infinite extension in the plane spanned by the x and z axes. The plates are located at $y = \pm 1$ and initially the flow between the plates is steady and the plates have constant velocities $u(1,t) = U$ and $u(-1,t) = -U$, where $t < 0$ and U is a positive constant. The velocity vector is given as $\boldsymbol{u} = (u(y,t), 0, 0)$. At time $t = 0$ a pressure gradient is suddenly applied $\frac{\partial p}{\partial x} = -\rho\beta$, where β is a constant.

2a (weight 5%)

What is meant, respectively, by Couette and Poiseuille flows?

(Continued on page 3.)

2b (weight 10%)

Formulate the fully simplified set of equations describing this problem, with initial and boundary conditions.

2c (weight 5%)

Find the steady state solution $u_s(y)$ that the flow will reach eventually.

2d (weight 15%)

Find the velocity $u(y,t)$. Hint: Use the steady solution from 2c and solve for $v(y,t) = u(y,t) - u_s(y)$. You may also use that $\int_{-1}^{1} \cos(\lambda_k y)(1-y^2)\, dy = 4(-1)^{k-1}/\lambda_k^3$, where $\lambda_k = (2k-1)\pi/2$, for $k = 1, 2, 3 \ldots$.

2e (weight 15%)

Implement a numerical solution to the problem using Python/FEniCS. Also, implement the exact analytical solution found in 2d and show how the numerical error may be estimated at every time step.

The End