LQR control of a Nonlinear Quadcopter System

Project Group F

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Abstract

The purpose of the project is to simulate the control and stable flight of an unmanned four rotor flying vehicle known as a quadcopter. The problem is set into three goals. The first is to have the quadcopter hover in a stable configuration. The second goal is to introduce a drone in flight and to have the quadcopter intercept that flight without knowing it's path. The third goal is to simulate the capture of the drone with a randomized disturbance force and maintain stable flight while returning to base.

Approaching the General Problem

To simulate the quadcopter's flight, we have to simulate its physical flight dynamics and place them into a state equation. Table 1 introduces the variables, or states, we used for basic flight dynamics, as well as constants used in computations:

$\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}$ *earth position in the x, y, z plane*	$\mathbf{c_1}, \mathbf{c_2}, \mathbf{c_3}$ *relative position of the x, y, z plane*
$\boldsymbol{\phi}, \boldsymbol{\theta}, \boldsymbol{\psi}$ *angles in the x, y, and z plane*	$\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ *relative position of the x, y, z plane*
$\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ *velocities in the x, y, z plane*	$\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}$ *thrust generated by quadcopter rotors*
$\boldsymbol{\omega_1}, \boldsymbol{\omega_2}, \boldsymbol{\omega_3}$ *accelerations in the x, y, z plane*	$\mathbf{g}$ *9.81 meters per second squared*
$\mathbf{m}$*mass, 0.5 kilograms*	$\mathbf{l}$ *length: 0.2 meters for each rotor arm*
$\mathbf{I_1}.\mathbf{I_2}, \mathbf{I_3}$*1.24, 1.24& 2.48 kilograms per square meter of inertia*	$\boldsymbol{\sigma}$*0.01, proportionality constant for thrust*
$\boldsymbol{\mu}$ *newtons, maximum thrust of each rotor*	

Table 1: Basic Flight Dynamic Variables and Constants

The first four variables, and states representing their derivatives, were placed into a vector named z. The purpose of the z vector is to be able to hold states which, when fed into a solver for ordinary differential equations, would generate solutions that would feed back into the controller and modify the behavior of the plant towards its desired goal. In order to do this, we have to feed back corrected values. If a controller can be placed in an invariant, state space form simply as:

$\dot{x} = Ax + Bu$

Where x are the states, and u are the inputs of the system, then we can design a control, a -Kx to affect closed loop change. The formula is then:

$\dot{x} = (A - BK)x$

We used the Linear-Quadratic Regulator (LQR) approach as an optimal control technique as a manner of generating K values dynamically. LQR operates on both the A matrix (states) and B matrix (inputs), but are given two matrices, a Q and and R, whose purpose is to weigh cost so the algorithm may calculate between two factors, usually distance and energy spent. Since we are not given a fuel tank for the quadcopter, we have left the R matrix as a diagonal matrix of ones. We have also not yet weighed the Q matrix, leaving it too as a matrix of diagonal ones, so that we may experiment in the future what is the best way to prioritize states in the system.

Quadcopter Hovering

Quadcopter Interception of Target

Quadcopter Return to Base with a Disturbance Force

References

[1] Faraz Ahmad, Pushpendra Kumar, Anamika Bhandari, Pravin P. Patil. Simulation of the Quadcopter Dynamics with LQR based Control. Materials Today: Proceedings,Volume 24, Part 2, 2020, Pages 326-332, ISSN 2214-7853. https://doi.org/10.1016/j.matpr.2020.04.282.

[2] Jinho Kim, S. Andrew Gadsden, Stephen A . Wilkerson. "A Comprehensive Survey of Control Strategies for Autonomus Quadrotors". arXiv:2005.09858v1. 20 May 2020.

[3] Madani, T, and A Benallegue. “Backstepping Control for a Quadrotor Helicopter.” 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems. IEEE, 2006. 3255–3260. Web.

[4] Jing Qiao, Zhixiang Liu, Youmin Zhang. ”Payload Dropping Control of an Unmanned Quadrotor Helicopter Based on Backstepping Controller”. MATEC Web Conf., 277 (2019) 01004. DOI: https://doi.org/10.1051/matecconf/201927701004.

[5] Jia, Zhenyue and Yu, Jianqiao and Ai, Xiaolin. ”Integral Backstepping Control for Quadrotor Helicopters”. Association for Computing Machinery. 2017. DOI:https://doi-org./10.1145/3057039.3057052

[6] Daewon Lee, H JIN Kim, Shankar Sastry. Feedback Linearization vs Adaptive Sliding Mode Control for a Quadrotor Helicopter. International Journal of Control, Automation, and Systems (2009). DOI 10.1007/s 12555-009-0311-8.