

**DSC 101Q “Physics-Based Mathematics”
Fall, 2018**

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Hours: MW 3:00–4:00p; TuTh 2:00–5:00p (Math Center). Appointments at other times welcome.

Course Content: This DSC 101Q seminar models mathematical problems in terms of a physical systems. The physical behavior of the systems are used to derive the mathematical answers to the problem. Problems from geometry, algebra, even calculus, and beyond to higher mathematics, differential equations, differential geometry, symplectic geometry, and complex analysis will be treated. The art of finding answers is called heuristics. Why the answer is valid, what assumptions are needed to ensure there is a solution (or no solution or many solutions) and so forth are the domain of mathematics. This is a course in heuristics. The initial goal is to cover a chapter a week, except chapter 3, which will take two weeks.

Text: Mark Levi, *The Mathematical Mechanic*, Princeton University Press 2009.

Course Goals: After this course, the student should have developed the following capacities:

- To model various mathematical problems as physical systems.
- To explain how to use the physical model to derive the mathematical answer to the problems.
- To explain the inverted nature of the relationship between physics and mathematics in this course material.
- To explain differences what is mathematics and what is physics insofar as they are illuminated by the material in the textbook, and to answer whether this subject matter belongs to a discipline. (See APPENDICES below.)

During this course, the class will build an effective learning community. Each student will have developed personally and have helped others develop.

Commentaries: Commentaries are **due each week on Monday at noon**, excepting weeks in which there is a test and the first week. Each is to be 400-600 words in length excluding quotations. Each is to comment on some idea or ideas in the course covered in the previous week or so. Each may provide alternate explanations of things, probe issues which cause the student confusion, amplify or clarify the successes or failures of the text, or anything which shows serious thought about some part of the course. A comment is more than an off-hand reaction like “It was interesting,” or “Here is a list of things we did: . . .” A comment is to be critical or questioning, wondering or insightful, coherent and focused. Outside sources are permitted but not encouraged; keep in mind that each commentary will be evaluated on the quality of the student’s own reflections. If outside sources are used, be sure to cite them appropriately and to avoid plagiarism as defined in the Honor Code. Unsatisfactory commentaries must be rewritten to receive credit.

Class Participation: The overarching goal of class meetings is to build an effective learning community. “Effective” means that the individual student learns effectively. “Effective” means that the class community supports the growth of each individual and teaches each one to support everyone else. Each student should have the opportunity to stretch beyond their perceived limitations. This means we should respect and have compassion for an individual’s personal goals, and, reciprocally, respect and have compassion for the intentions of another who challenges us.

The work done in the class meetings is an integral part of the course work for each student. Absences and tardiness must be counted as work not done; contact your instructor should you be unable to attend class. Further each person is to be ready to participate in each class.

The class will be divided into four teams of four students. Class activities will include small team projects (usually either solving a problem or explaining a solution given in the text), whole class discussion, and the instructor explaining mathematical or physical principles when needed. Class team activities will usually be based on individual work done outside of class; collaboration is not required on such homework but it is encouraged.

The class participation grade will have two major component categories, the class community grade and the individual grade. These components will be discussed openly in class from time to time. The improvement of each individual is a shared responsibility of the community.

Quizzes: Quizzes will have an individual and team components. The individual part is usually a “reading quiz.” It serves as the basis for the team question(s) that will immediately follow it.

Tests: Two tests will be offered. They will be in class on October 11 and November 20. Any conflicts or problems will be handled on an individual basis. If a student has an excuse deemed legitimate by the instructor, arrangements will be made to take the test **prior to** the scheduled time.

Final Examination: There will be a comprehensive final examination at the time scheduled by the registrar.

Grades: Grades will be based roughly on the following distribution of work:

| | |
|-------------------------------|-----|
| Tests | 20% |
| Class Participation & Quizzes | 35% |
| Commentaries | 20% |
| Final Examination | 20% |

The plus/minus system will be used. If the class becomes particularly small, then class participation will count more (up to 40%) and commentaries will count less.

Honor Code: The Honor Code of Oxford College applies to all work submitted for credit in this course. By placing your name on such work, you pledge that the work has been done in accordance with the given instructions and that you have witnessed no Honor Code violations in the conduct of the assignment.

APPENDICES

Etymologies

seminar, from Latin *seminarium*, a seed bed, nursery (for trees); hence a place to plant thoughts and cultivate knowledge, to grow both ideas and individuals.

-ics, from Greek, *-ικά*, neuter plural of *-ικός*, the things (*τὰ*) of a certain kind (*...-ικά*); like the adjectival ending like *-ical* in English; by extension denoting science or body of knowledge, as in:

- * *physics*, from Greek *τὰ φυσικά*, natural things; *φύσις*, nature: hence the science of nature.
- * *mathematics*, from Greek *τὰ μαθηματικά*, things that can be learned; *μαθημα*, that which is learned, lesson, knowledge; hence mathematics (interesting, no?).
- * *heuristics*, formed from Greek *τὰ εὐριστικά*, things that can be found, recent word ca. 1946; cognate with *Eureka!*, I have found (it)!; similar in mathematical writings to the Latin *invenire*, to come across, find, discover, invent.; hence the art of finding solutions, more poetically, the art of inventiveness.

discipline, from Latin *disciplina*, teaching, branch of study; comes into Anglo-Norman, first meaning massacre, carnage and later teaching, thence to English first as punishment and then as teaching; perhaps the source of the I’ll-teach-you-a-thing-or-two! attitude sometimes still present in Western educational systems.

define, from Latin *definio*, to fix the limits, draw a boundary around; compare with *finite*.

compete, from Latin *con-* + *petere*, to seek or strive together; see parable below; if striving together, you develop your ability and you help another develop their ability, you are improving your community and you are improving the world.

A parable: Competition

“I was in a race, striving to catch to the person a half step in front of me. I ran hard, harder than I ever had, but I lost. Yet both my rival and I ran faster than we ever had before. Were there any true losers here? Was not my improvement more important than my loss? Was not the improvement of my friend also cause for rejoicing?”

Contributions. A community renews itself through independent, individual actions and collective actions, which actions promote some common goal or purpose. Things that contribute to the development and learning of the community:

- Asking a (pertinent) question. Questions arising out of contemplation of a problem, a text, or a discussion of these usually reflect or refract some truth. With enough angles, the truth might be discovered. Another’s angle may bump you out of your fixed position and let you see something new.
- Building on the contribution of another.
- Sharing an idea that help someone make progress.
- Explaining the text, background information, or a solution. Perhaps not as helpful as leading someone to figure it out for themselves, but sometimes it is the best way to help the group make progress.
- Showing gratitude for the contribution of another. This can be a simple acknowledgement or something more substantial such as building on another person’s idea or showing how it helped.

A physics metaphor: The protons and neutrons of nuclei of atoms are bound together by the strong interaction and is mediated by the exchange of the appropriately-named particle, the gluon. As the number of community members grows, it gets harder to keep up the number interactions needed to bind the community into a whole. This is a somewhat simplistic explanation of why atoms with a high atomic number are unstable.

What is Mathematics?

Let us begin quite practically with the curriculum at hand. These paragraphs will not attempt a full answer, but they will try to place this course in context and raise some questions and idea related to what is mathematics.

The bread and butter of the mathematics department is first-year calculus and arguably statistics (117Q). Statistics has often been treated as the step-child of mathematics, here at Emory and historically throughout the world—but we’ll leave that aside. With the growth in AAP in recent years, one might add second-year calculus (211, 212), if by “bread and butter” we include any course often recommended as a good first college mathematics course to significant segment of the incoming class. But to keep the discussion brief, let’s just stick to first-year calculus.

In particular, how does the first-year calculus course relate to mathematics and the activity of mathematicians, both current and historical? Given the noise made about the importance of mathematics, the question should be of some interest. I have looked at various calculus textbooks from somewhat before 1900 on, taking particular note of American textbooks and what they reflect on the development of the American system of education. Also in the late nineteenth century, changes evolved in both mathematics and college education that make the somewhat arbitrary choice of 1900 a reasonable rough cut-off. Throughout this time from 1900 on, the calculus course has been invented, reformed, and reinvented almost continually.

My impression is that the American calculus course was invented and reinvented, not to prepare future mathematicians or “math majors,” but to equip future mathematical scientists and engineers with the concepts and computational tools that had become so important in the natural sciences, engineering, and economics, as well as in some important parts of mathematics. Already in 1914, we have *Calculus Made Easy* by Sylvanus P. Thompson. If one goes back fifty years or so, one finds this focus on practical applications explicitly mentioned as a feature in prefaces of some of the textbooks. It is this practical, applied approach that strikes one.

The courses themselves, as represented by the textbooks, being written by mathematicians, tend to have a particular mathematical point of view about what is a concept, a definition, the meaning of “applied” or “practical,” and so forth. The point view tends to differ from the scientist’s or the engineer’s and probably differed from the student’s, given the continual cry for “real-world” applications in calculus reform over the last hundred or so years. Being at a liberal arts college also affects our attitudes at Oxford, since the liberal arts are the basic tools for finding things out. Different systems of knowledge have different systems of evidence. As mathematicians at a liberal arts college, we feel a responsibility to teach mathematics and its peculiar ways of inquiry and knowing, even in an introductory course. Since mathematics is a lowly but fundamental discipline that serves the ways of inquiry in many other fields, it is important for students to understand its strengths and limitations.

In a mathematician’s view, concepts from the physical sciences are considered separate from mathematics. If they are introduced at all, they are merely treated as words signifying certain numbers—in algebra,

we let a variable stand for an unknown number—numbers that may be used in calculations with certain formulas. In a similar way, science strips away irrelevant detail, but not as much. In physics, two masses colliding are treated not just as numbers but as masses (“Remember the units!”). It is irrelevant whether the masses are rocks or heads, even though it matters a great deal to the persons whose heads are colliding. This *selection of elements* with which the problem-solver creates a model redefines the problem (i.e., puts new limits on it) and *creates a system* that brings into focus a certain question (of all possible questions) and limits the sort of answers that may be given. For instance, if the heads of two friends collided, the first question one thought of would probably not be with what velocities did each head ricochet? (Note I tried to sneak in the element “friend,” which redefines the problem and makes a natural physical question seem even more heartless.)

Now when a mathematician gets hold of something, certain questions arise. Let us suppose that a method is to be discussed that calculates the solution to a certain kind of general problem. Does the method always work? (Probably not.) OK, under what conditions does it work? When the method fails, does it fail outright, or does it yield an approximate solution? Under what conditions does it yield a good approximation? Often the questions keep coming.

A couple of observations are in order. (1) Mathematics often fails. Thus finding sufficient conditions for success is important. (2) Something very interesting occurs as one continues to investigate such questions. Patterns arise that seem to connect unconnected things. Even in related things, when you get to the bottom of each, you see new connections, what mathematicians sometimes call “structure.” This is what the mathematics that mathematicians do is about; however, generally it is not what the activity of calculus students is about. One can enter college “liking math” without knowing what mathematics is about.

It is interesting to ponder why mathematics is like this. Why doesn’t math just stop at cool and efficient ways of calculating answers? Why are there these patterns and connections that turn out to be important to elucidate? I don’t have a clear idea about why. One thing I can say is that symmetry somehow plays a role, at least in a lot of important mathematics, either as a reason or a key to be explained. Another thing is that nature works—you can see it work. It’s so obvious and ever-present that ignoring that it works is a habit, and we might not even notice it. Given that it works *and* given that it is accurately described by mathematical models, there have to be connections between mathematical concepts.

One sees the symmetry of regular polygon or star polygons in the arrangement of petals around a flower. One also sees that each flower breaks the symmetry, and that is interesting, too, an interesting function of symmetry. A less obvious example of symmetry is that the median divides a triangle into two equal halves and is the line on which triangular plate would balance. It’s clear that a uniform rod should balance on its midpoint. If we divide a triangle into a bunch of thin rods parallel to the base of the triangle, the midpoints describe a median. If all the rods balance and the rods form the triangle, then the triangular plate must balance.

In this example, we are looking at something outside of mathematics, the plate. We are asserting that a connection between the physics of the plate and the mathematical triangle implies some mathematical relationship or property has to be true. It would be better, from the mathematician’s point of view to give a justification wholly within mathematics. Such a justification is called a *proof* in mathematics. Perhaps a proof of this example was given in your high school geometry class. This example is not particularly deep, not of the deep kind I was alluding to. But I chose it hoping it was familiar and understandable. It is also an example of the approach our textbook takes. I wanted to illustrate this approach and to raise