Mathematics 110A
Fall, 2009
Instructor's Version

Textbook and materials:
Stewart, J, *Essential Calculus,* Thompson Brooks/Cole (2007).
Other content will be provided on the Blackboard site at http://classes.emory.edu.

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Course content: Mathematics 110A is the first part of a two-semester course, Mathematics 110A/B, that provides students with an integrative approach to Calculus I that includes the necessary precalculus topics.

Content of Mathematics 110A: Review of algebra, functions, trigonometric functions, logarithms and exponents. Calculus topics include limits, continuity, definition of the derivative, differentiation, extrema, Intermediate Value Theorem, Mean Value Theorem, graphing polynomial and rational functions, optimization problems.

Content of Mathematics 110B: Review of inverse trigonometric functions and differentiation, and graphing. New topics include implicit differentiation, logarithmic differentiation, related rates, graphing vertical tangents, logarithmic and exponential graphs, sums and sigma notation, induction, antiderivatives, Fundamental Theorem of Calculus, definite integral, area, volume, separable differential equations, substitution method of integration.

Course goals: After completion of the sequence Mathematics 110A/B, the student should be able to do the following: to find the limit, derivative, antiderivative, and definite derivative of a function; to understand the basic theoretical underpinnings of these processes; to understand the relationships between these processes and rates of change; to understand the relationships between these processes and the graph of a function; and to apply these processes in solving problems on rates, extrema, area, volume, and approximation. An overall goal is to provide the student with a solid foundation for Mathematics 112.

Classes: The student is responsible for what is covered in class. In addition to the regular class meetings, there will be optional SI sessions and help sessions. There will also be several tests scheduled on Tuesday or Thursday mornings. (See proposed calendar below).

Homework: A homework is assigned almost every day of class at the end of class. These exercises usually will not be collected but are for the benefit of the student. Solving problems and practicing their solution is only good way to learn mathematics. Students may ask questions about the homework, and quizzes based on the homework may be given. The instructor may ask to see a student's homework.

Although the homework exercises are not graded, it is important for the success of the student that they be completed as soon after covering the material as possible. Calculators ought to be used when appropriate, but the student should keep in mind that they are not permitted on the tests. Collaboration is encouraged, but each student should be sure that he or she ultimately can **solve problems unaided by notes, the textbook, a calculator, or other people**.

Quizzes: (100 points.) All quizzes may be announced and are usually in-class. The student must be present in class to take each quiz. Up to one quarter of the quizzes will be dropped. Each quiz will count the same amount, the average per cent being used to calculate the number of points. For example, a 94% quiz average at the end of the course will result in 94 points out of the 100. Normally an excused absence during which a student misses a quiz may not be made up; it will be dropped.

Gateway exams: (100 points.) In order to pass this course the student must pass two examinations, one algebra and graphs and one on trigonometry. All of them begin at 8:00 a.m. on the dates below:

	Algebra	**Trigonometry**
8:00 a.m.	**Tuesday, September 15**	**Tuesday, October 20**
8:00 a.m.	**Thursday, October 1**	**Tuesday, October 27**
8:00 a.m.	**Tuesday, November 10**	**Tuesday, December 1**

The student will be allowed three opportunities to pass each. Each test (of each kind) will be different but very similar to the original test.

Tests: (400 points.) In general, calculators will not be allowed on tests. Tests will be given on the following days:

In class, Wednesday, October 7 — Test 1, Part A (80 points)
8:00 a.m., Thursday, October 8 — Test 1, Part B (120 points)
8:00 a.m., Tuesday, November 3 — Test 2 (100 points)
8:00 a.m., Tuesday, November 24 — Test 3 (100 points)

Test 1 takes place over two consecutive days. There will be a practice test for Test 1 at 8:00 a.m., Tuesday, September 22. It will cover only what has been covered so far.

Final examination: (200 points.) A cumulative final exam will be given at the time scheduled by the Registrar.

Grading: Evaluation will be based on the following written work:

Gateway Exams	100 points
Tests (4 @ 100 pts)	400 points
Quizzes	100 points
Final	200 points
Total	800 points

The plus/minus system will be used with the following rough guide to letter grades:

A	720 points and up	D	480–559 points
B	640–719 points	F	below 480
C	560–639 points		

Excuses: Excuses deemed legitimate by the instructor will be handled according to the individual circumstances and college policies.

The student is expected to take all tests and exams at the scheduled times. For legitimate excuses arrangements will be made to take a test **prior to** the testing time. There will be no make-up tests given after the testing time.

Written style: Thoughts are expressed by sentences: just so in mathematics. Pay attention to your textbook: it is written in sentences. **Your written work must be in complete sentences.** Note "$1+1=2$" is a complete sentence (it has a subject "1+1", verb "=" and predicate "2"). Use mathematical symbols wherever appropriate. Your work also needs to be neat and orderly to be intelligible. See the essay, "Clean Writing in Mathematics," from *Calculus: A Liberal Art,* by W.M. Priestley and the "Calculus Style Guide." Practice good style in all your work, including uncollected homework.

Test preparation: Here are some criteria for "studying hard" and "being prepared" for a test, perhaps in order of importance.

- **Be present.** One can hardly claim to be prepared to take a test if one does not show up.
- **Be well rested.** A tired brain is not reliable, especially if a test demands *thinking.* Studying all night is not preparing for the test. *Plan* to be prepared.
- **No questions when the test starts.** You know you are ready when you are comfortable bringing only your pencils to the test.
- **Be ready to take the test the day before it's given.** That is, the night before is literally review and running through the material, just to hone your memory and confirm your understanding of the material.

Accomplishing these things is not easy. The trick is to not get behind and keep working at it. Tests are performances, similar to those by athletes, musicians, and dancers. Prepare for them in similar ways. Begin practicing for them weeks in advance.

When you are ready for and receptive to advice, your instructor, your adviser, and the counseling center are willing to help you.

Practice and insight: Gauss said the purpose of calculation is insight. Insight is an understanding into why things work the way they do. This should be the goal of working out problems. Know **why** each step is correct and **why** each step was the right step to take. This is more than knowing **that** each step is correct. This is like the three *wheres* of grammar: *whence,* where you came from; *where,* where you are; and *whither,* where you're going. You need to achieve the intended insight from doing your homework and other studying.

Hard work and the difficulty of the calculus: In general the student will need to study six to eight good hours per week exclusive of the time spent on review for tests.

"Difficult" is rarely an appropriate description of a problem. Either you can solve the problem or you cannot solve it. When you can solve it, it is not difficult but "easy." When you cannot solve it, it is also not difficult but "impossible." What is difficult is **learning** — learning **how to solve** the problem, learning **how to figure out how to solve** the problem, learning **how to learn**. These are increasingly higher levels of learning. Learning calculus is hard. It takes work: reading, asking the right questions, and, most of all, solving problems.

Mathematical knowledge: A few remarks about the proper attitude the mind should have toward the course content might help the student to study better. The basic elements of mathematics are the axioms, definitions and theorems (including all propositions and formulas). The basic method of learning, aside from studying these elements, is **to solve problems**.

- Axioms: These with the definitions "create" the subject. Calculus is based on many years of algebra and analysis developed in school. By now, the axioms are so familiar and long used that they have been forgotten or perhaps even hidden. Roughly they are those of the real number system as a continuum and those of analytic geometry.
- Definitions: Each creates an idea. They are exact and slight changes can have disastrous consequences. To know a definition is more than an exact memory of its statement.
  - Know the statement.
  - Know examples:
    - The basic examples illustrating the definitions.
    - For each condition in the definition, an example that does not satisfy that condition while satisfying any other conditions.
- Theorems: These show how the ideas relate. Again, to know a theorem is more than an exact memory of its statement. Most theorems consist of a hypothesis or hypotheses and a conclusion.
  - Know the statement.
  - Know examples:
    - An example illustrating each of three possibilities for the hypothesis and conclusion:

	Hypothesis	Conclusion
1	True	True
2	False	True
3	False	False

    - If the hypothesis is composed of multiple conditions, an example for each condition showing why it is necessary.
  - Know the proof (except when not required). Even when the proof is not required, the student should have in mind some idea of why the theorem is true; for without an account of why something is true, there is not really an understanding.
- Problems: Examples, either ones presented or ones solved, are the primary resource of the student. What does it mean to understand a solution to a problem? A solution is a sequence of steps that begins with educing what is given in the statement of the problem and leads logically to what was to be found or shown. There are three things to be known about these steps:
  - To know the correct steps.
  - To know why each step is valid (its justification, where it comes from).
  - To know why each step was taken (its purpose, what it leads to). These are the three aspects of understanding a solution. They involve relating the mathematical elements above. Moreover, finding a solution involves another important mathematical skill, guessing.

- Guessing: Skill in guessing and other forms of inductive reasoning consitute an important part of mathematical knowledge. Guesses come from analogy with similar problems, patterns in examples calculated, and recognizing alternatives. For further hints and practical advice about "heuristic," the art of discovery, see G. Polya, *How to Solve It,* Princeton (2004). These remarks are meant to give a hint to the student about how to learn mathematics. The important roles of examples should be evident. The most important examples will be the problems the student has figured out for herself or himself.

SI/Help Sessions/Tutoring: The SI program is a program of optional, organized study sessions. The sessions are not meant to be tutoring sessions. The supplemental instructor (SI) is a student who has taken the course (or a similar course) before, has a good understanding of the material (but probably not as complete as the instructor!), and knows how to be a successful student.

Help sessions will be scheduled as there is demand for them. Attendance is optional.

The schedule for tutoring in the Math Center will be announced when available. See

http://mathcenter.oxford.emory.edu

Honor Code: The Honor Code of Oxford College applies to all work submitted for credit in this course. By submitting such work, you pledge that the work has been done in accordance with the given instructions and that you have not condoned any Honor Code violations in the conduct of the assignment.

You may always ask your instructor any question about an assignment. He will answer at his discretion.

Proposed Calendar

Date	Topic	Section
Wed 26 Aug	Optimization problems: Introduction to functions, domains, limits, asymptotes Notes: Cheapest fence, box; minimum area; cone circumscribed about a sphere,... ; include problems from §§4.1, 4.5 — set up equations, exploring the functional relation and identify parts that increase and decrease as the quantities vary	§1.1
Fri 28 Aug	Velocity problems: Introduction to rates and limits Other limit problems Notes: average velocity $s(t)=t^2$, (1) over [0,4] and subdividing interval & estimating change in position from average velocity and so on, and (2) @ diminishing intervals about, say, $t=2$; cone filling with water, blowing up a balloon; limits at end of domain; include §2.7 — again just to set up relation and exploring what happens as things change	§1.2, §1.3
Mon 31 Aug	Multiplication and its combinatorics; Binomial Theorem Notes: Making change; arranging stamps; expanding powers	
Wed 2 Sep	Power functions; difference of powers; graphs Notes: For appropriate $n \gtrless 1$: $x^n \gtrless x$ for positive $x \gtrless 1$; $x^n \gtrless mx$ and tangency to axes; concavity of $y=x^n$ (through algebra); opt. x^n+y if $x+y=a$, $a \gtrless 1$ for $n=2,3,1/2$.	
Fri 4 Sep	Quadratic functions; tangent to parabolas; graphs Notes: Completing the □; vertex; quadratic formula; x or y = quadratic and solving for y or x resp.; (line)^2; finding tangent to, say, $y=x^2-7x+6$ at $x=2$ through $y=(x-2)^2-3(x-2)-4$, substituting $x=(x-2)+2$, and taking linear part; can solve the least cost rectangular pen problem from the first day by inverting the function (it's a quadratic equation)	
Mon 7 Sep	*Labor Day*	
Wed 9 Sep	Absolute value and piecewise-defined functions; graphs; inequalities Notes: Got a good problem for this?	
Fri 11 Sep	Combining functions; transformations of graphs Notes: Good problem?	§1.2
Mon 14 Sep	Trigonometric functions: definitions and basic graphs	NTF A, C
Tue 15 Sep	**Gateway Exam (Algebra — First Opportunity) at 8:00 a.m.**	
Wed 16 Sep	Limits, finite and infinite	§§1.3, 1.5
Fri 18 Sep	Combining functions and limits Notes: Squeeze Theorem: $\sin x/x$ (@ ∞), $x\sin(1/x)$, $x^2\sin(1/x)$, etc.	§1.4
Mon 21 Sep	Trigonometric functions: graphs and limits Notes: Transformations of sine and cosine; limits at points of continuity and asymptotes	NTF C
Tue 22 Sep	**Practice Test 1 at 8:00 a.m.**	
Wed 23 Sep	Continuity and review of limits	§1.6
Fri 25 Sep	Intermediate value theorem and continuity	§1.6
Mon 28 Sep	Derivative	§2.1
Wed 30 Sep	Derivative	§2.2
Thu 1 Oct	**Gateway Exam (Algebra — Second Opportunity) at 8:00 a.m.**	
Fri 2 Oct	Some derivative rules	§2.3
Mon 5 Oct	Review	
Wed 7 Oct	**Test 1, Part A**	
Thu 8 Oct	**Test 1, Part B at 8:00 a.m.**	
Fri 9 Oct	Trigonometric functions: identities	NTF B
Mon 12 Oct	*Fall Break*	

Wed 14 Oct	Trigonometric functions: equations	NTF D
Fri 16 Oct	Trigonometric functions: problems Notes: Triangle problems, opt. problems in 4.5 (e.g., 29, 32, 43–50, pp. 234–236).	
Mon 19 Oct	Limits and derivatives of trigonometric functions Notes: $\sin x/x$ @ x=0	§2.3
Tue 20 Oct	**Gateway Exam (Trigonometry — First Opportunity) at 8:00 a.m.**	
Wed 21 Oct	Differentiation	§2.4
Fri 23 Oct	Chain Rule	§2.5
Mon 26 Oct	Differentials	§2.8
Tue 27 Oct	**Gateway Exam (Trigonometry — Second Opportunity) at 8:00 a.m.**	
Wed 28 Oct	Maximum and minimum values	§3.1
Fri 30 Oct	Mean value theorem	§4.1
Mon 2 Nov	Review	§4.2
Tue 3 Nov	**Test 2**	
Wed 4 Nov	Derivatives and the shapes of graphs	§4.3
Fri 6 Nov	Graphing	Handout
Mon 9 Nov	Graphing	Handout
Tue 10 Nov	**Gateway Exam (Algebra — LAST Opportunity) at 8:00 a.m.**	
Wed 11 Nov	Exponential and logarithmic functions	§3.1; NTF F
Fri 13 Nov	Inverse functions	§3.2
Mon 16 Nov	Exponential and logarithmic equations	NTF G
Wed 18 Nov	Derivatives of logarithmic and exponential functions	§3.3
Fri 20 Nov	Exponential growth and decay	§3.4
Mon 23 Nov	Review	
Tue 24 Nov	**Test 3 at 8:00 a.m.**	
Wed 25 Nov	*Thankgiving*	
Fri 27 Nov	*Thankgiving*	
Mon 30 Nov	Optimization problems	§4.5
Tue 1 Dec	**Gateway Exam (Trigonometry — Last Opportunity) at 8:00 a.m.**	
Wed 2 Dec	Graphing transcendental functions	§§4.3, 4.4
Fri 4 Dec	Review limits	
Mon 7 Dec	Review differentiation	