

Mathematics 221Q – Linear Algebra

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Textbook: H. Anton and C. Rorres, *Elementary Linear Algebra, Applications Version*, 11th edition.

Course Content: Mathematics 221 is an introduction to linear algebra. This course will cover systems of linear equations, vectors, matrices, determinants, vector spaces, linear transformations, eigenvalues and eigenvectors, inner product spaces, and applications.

Course Objectives: At the end of the course, the student should achieve the following goals: to know the basic definitions and theorems in the field of linear algebra as described in the course content; to solve basic problems in linear algebra using matrices and matrix operations; to prove the fundamental relationships (theorems) between the concepts in the course.

Ways of Inquiry: Linear Algebra is a highly structured mathematical system based on definitions, axioms, theorems and proofs, all of which are of fundamental importance to mathematics and mathematical activities. It has many applications to other branches of mathematics, natural sciences and social sciences. It is appropriate to adopt the “Ways of Inquiry” in this introductory course. Through the course, students interested in mathematics will gain understandings on *what mathematics is*, *how mathematics is done* and *what applications it has*.

Attendance/Class Participation: Students are expected to attend all classes and are responsible for all material covered in class as well as any changes made in the schedule regarding homework and tests.

The course is student-centered and inquiry-driven. Before class, students are expected to study the textbook (and other resources if appropriate) guided by the Reading Guides, take notes and understand most of the basic definitions, examples and concepts. A pre-lecture quiz will evaluate whether a student is well-prepared for class. Students are also expected to compile a list of questions and thoughts relevant to the material to be discussed and bring it to class. In class, following a brief presentation of the topics, most of the class time will be spent on discussing important theorems and proofs, confusing concepts and answering questions. Students are expected to be actively participating in the discussions. Being absent, late, inattentive or inactive will lead to deductions to one’s class participation grade. Attendance and consistent preparation for class will determine the success or failure the student realizes in this course.

Quizzes: At the beginning of almost every class, a short quiz will be given (see above). A student must be present to take the quiz. No make-up quizzes will be given.

Assignments: There will be about 10 assignments. A typical assignment will have 3 parts: part 1 contains basic exercises from the textbook; part 2 contains more conceptual questions (including proofs and some open-ended problems) from the textbook; and part 3 is a “Ways of Inquiry” journal. In the journal a student may write about the most difficult concept, theorem or proof he/she has encountered recently and why it is so; or thoughts on questions raised in class or by oneself, or the most impressive thing one has discovered, realized or understood. A student may also reflect on their understanding of mathematics in general, its relationship with other disciplines, and the activities of learning and doing mathematics.

Collaboration is allowed on the first two parts, however each student should make sure that ultimately he or she can *solve problems unaided by notes, the textbook or other people*. A complete solutions manual is on reserve in the library.

Tests: There will be three closed-book, comprehensive tests on the following Thursdays: **February 15, March 29, April 19, 7:45 AM – 9:30 AM**. Students are expected to take tests at the scheduled times. For legitimate reasons and conflicts students may take the tests **prior to** the scheduled time. Any emergencies will be handled on an individual basis and must be documented. **No make-up test will be given after the testing time.**

Final exam: There will be a closed-book, comprehensive final exam at the time scheduled by the registrar.

Grading:

Assignment	100
Quizzes	100
Class participation	80
Tests	320
Final Exam	200
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Total points	800

A rough guide to assign letter grades:

A: roughly more than 90 %; B: 80 – 89 %; C: 70 – 79 %; D: 60 – 69%; F: below 60 %

Grades of A-, B+, B-, C+, C-, D+ may be assigned.

Written Work: Thoughts are expressed by sentences. Your written work must be in complete sentences. Use mathematical symbols wherever appropriate. Pay attention to how the problems are worked out in the textbook. Your work should be neat and legible. It is common practice to rewrite solutions once they are found.

Calculators: In general, calculators will **not** be allowed unless the opposite is announced.

Support Services: Students should utilize the following resources:

- **Office Hours:** Office hours will be posted on Canvas. Students should use this time to ask specific questions related to this course and/or homework problems.
- **Canvas Site:** There will be a Canvas course site. Documents and announcements related to the course will be posted there. This includes topics covered that day, homework assignments, suggestions on studying the textbook, topics to preview before next lecture and other announcements. Students should check the site at least once a day.
- **Help Sessions:** Out-of-class review sessions may be scheduled as needed.
- **Study Groups:** Study groups organized by students are highly recommended. The meetings should be scheduled weekly and should be part of a regular weekly routine.

HONOR CODE: THE HONOR CODE OF OXFORD COLLEGE APPLIES TO ALL WORK SUBMITTED FOR CREDIT. ALL SUCH WORK WILL BE PLEDGED TO BE YOURS AND YOURS ALONE. THIS IS THE CASE WHEN YOU PLACE YOUR NAME ON WORK SUBMITTED.

Math221 Linear Algebra

What might make this course different from most other mathematics courses you have taken so far...

Linear Algebra is one of the first courses where you will get a glimpse of *what mathematics is, how mathematics is done and what applications it has*.

Asking the “Right” Questions—What (fundamental) questions will you learn to ask when dealing with this discipline?

Linear Algebra is a highly structured mathematical system based on definitions, axioms, theorems and proofs, all of which are of fundamental importance to mathematics and mathematical activities. It has many applications to other branches of mathematics, natural sciences and social sciences.

Throughout the course, using the “Right” Tools, you will learn to ask the following questions:

About Definitions:

- Why is defining this object necessary?
- How does the definition emerge?
- Why is it defined this way?
- Are there other equivalent ways of defining it?
- Why is the choice of the words, are they precise?
- What happens if certain changes are made to the definition?
- What are some examples (from previously studied mathematical notions) of this object defined?
- How is this object related to other concepts?
- Can you find any real-world applications which can be modeled by the object defined?

About Axioms:

For most of you, it is the first time that you will see structures defined by a set of carefully chosen axioms. While these are important concepts of crucial significance to mathematics (e.g. General Vector Space, Inner Product Space, etc.), the idea of the axioms and using axioms to define objects might be fresh and foreign and usually difficult for you to comprehend and grasp. Learning to ask the following questions will help.

- What is an axiom? What does it mean to designate a statement as an axiom?
- Why do we use axioms to define a structure?
- When defining a specific structure, why is this set of axioms chosen? (How did people come up with them? Why these and not others? What if some axioms were eliminated, or changed? Is this the best choice?)
- Give some examples of objects that satisfy all the axioms, some of the axioms and not all.
- Can some of the axioms be deleted or added so as to describe more general or restricted objects?

About Theorems:

Theorems are central to mathematics. While typically you have encountered theorems in your previous experiences (such as Geometry and Calculus), in Linear Algebra, it is probably the first time you will be asked to fully understand every aspect and detail of a theorem. The following set of questions offers a starting point to guide the students. As you mature as a thinker of mathematics, you will ask more and more questions about a theorem.

About the conditions (hypotheses) and conclusions:

- What are the conditions of the theorem?
- What are the conclusions of the theorem? Can the conclusions be stated in an equivalent but different way?

- Are the conditions necessary, sufficient or both?
- Can one change/delete some of the conditions? How does that affect the conclusions? Do we obtain a meaningful conclusion? Do we obtain a stronger, or weaker or the same conclusion?
- Are there other (similar) conditions leading to the same conclusion?

About some related statements:

- What is the converse of the theorem?
- Does the converse always, or sometimes or never hold?
- If the converse is always true, can you prove it? If it is sometimes true, can you determine when it is true? If it never holds, why? Can you provide examples when it does not hold?
- What is the contrapositive? (Note: the contrapositive of a statement holds whenever the statement holds).
- When is it easier to apply the theorem in its contrapositive form?

About the significance of the statement:

Being a textbook for undergraduates, the book adopted for Linear Algebra calls almost every statement proven a “theorem.” In real mathematical activities however, mathematicians make a clear and necessary distinction between propositions and theorems, lemmas and theorems. Being able to evaluate the significance of a statement and distinguish between major theorems and minor facts is a sign of gaining deeper understanding and mathematical maturity.

- Is this theorem important?
- Why or why not is the theorem important? What is its significance? How is it related to other facts and concepts?
- Can you generalize the theorem? What are some meaningful generalizations?
- What are some related questions that worth investigating? Can you think of some ways to approach them?

About Proofs:

In Linear Algebra, you will be asked to study all the proofs of the theorems encountered, memorize and be able to reproduce them. You will also be asked to prove statements in homework and tests.

When studying an existing proof:

- What is the main idea in the proof?
- How does one think of the ideas?
- How are the conditions used?
- How is the argument written and why is it written this way?

When proving a statement:

- What is the main idea?
- Can it be proven directly?
- Can it be proven indirectly? Can one argue by contradiction?
- Can one reduce it to some other statements to prove?
- Can one dissect the conclusions to cases and prove each case?
- Can one get an approach by starting backwards, from the conclusions?
- Are there different proofs? Which approach is better and why?
- Can a more general statement be proven?
- How does one structure the written proof? Are the statements logical, precise and succinct? Can one write it better and clearer?

About the Role of Linear Algebra:

Linear Algebra requires a student to think abstractly. Many key notions are obtained by extracting the common features from other objects in mathematics and models in natural and social sciences. Asking the following questions will help one to see the connections and appreciate the role of Linear Algebra in mathematics and its applications.

- What previously studied objects in other fields of mathematics and/or natural and social sciences does this concept remind you of?
- What are some known applications? How is Linear Algebra applied?
- What more knowledge/knowhow in Linear Algebra is desirable in order to further its applications?
- Can you think of applications not mentioned in the book and maybe not yet thought of by others?

Using the “Right” Tools—What methods of analysis and argument will students learn to help them investigate questions in this discipline?

You will learn to read a mathematics text critically. Guided by the questions listed above, you will study your textbooks, “chew on” definitions and investigate theorems and their proofs in depth.

You will understand what a mathematical proof is, study and practice different proof techniques.

You will practice *thinking specifically(concretely)*, for example, giving examples, counterexamples, imagining how the concepts apply to a specific situation.

You will practice *thinking generally*, for example, generalizing a theorem, a concept, or a situation.

You will practice *thinking abstractly*, for example, observing how seemingly different objects in mathematics share certain commonalities, identifying these common features and finding a way to describe them both comprehensively and precisely.

Following your interests, you will search for and investigate various applications of Linear Algebra.

Meta-Level Reflection – How will students be asked to reflect upon, question, and appreciate the ways of inquiry used in this discipline?

You are required to keep a journal and reflect on how you are doing and think about what you do at least every week. Most of the time you will include your thoughts in the free response part of your assignment. It can be your reflection on anything related to the course. For example, it can be your thoughts on the questions raised above; or the most impressive thing you have discovered/realized/understood in the past two weeks; or the most puzzling concept/theorem/proof you have encountered recently and why it is so; or a question not raised in the book but asked by the students and why it is a meaningful and interesting question.

Connections to Something Bigger—What specific “real-world” questions, interdisciplinary connections, or ethical issues will students explore in an attempt to deepen their understanding and appreciation of the class content?

Linear Algebra has many applications in other fields of mathematics, natural sciences and social sciences. The textbook contains quite a few diverse applications. For example,

- In economics, the Leontief Input-Output Models developed by the 1973 Nobel prize winner in economics, Wassily Leontief, to study the relationships between different sectors in an economy by using matrix methods;
- Dynamical systems and Markov chains, which uses matrix methods to analyze the behavior of physical systems that evolve over time, such as problems in business, ecology, demographics, sociology and most of the physical sciences;
- Internet search engines (applications of matrices);
- Games of strategy, game theory (application of matrix multiplication and basic probability concepts);
- Computer graphics (application of matrix algebra and analytic geometry);
- Computed Tomography such as X-ray imaging (application of linear systems, natural logarithms and Euclidean space);
- Fractals and Chaos (application of geometry of linear operators, eigenvalues and eigenvectors, limits and continuity);
- Cryptography (application of matrices, Gaussian elimination, matrix operations, linear independence, linear transformations);
- Genetics, investigate the propagation of an inherited trait in successive generations (application of eigenvalues and eigenvectors, diagonalization of a matrix, limits).

To succeed in this course, you are expected to do the following:

- **Before class**, guided by the Reading Guide you should study each section. You must digest the definitions, follow the examples and try to understand the theorem statements and their proofs. Furthermore, you should attempt the easier problems at the beginning of the exercise section. Write down any questions that you have.
- **In class**, listen to the presentation carefully. Actively participate by taking notes, asking and answering questions.
- **After class**, study the textbook and your class notes. If you have more questions, ask your instructor or another student. Complete the assignment as soon as you can. Ask questions if you run into difficulties.
- **Reflect on how you are doing at least once a week. Think about what you do.**
- **After getting graded assignment back**, check the solutions and correct any mistakes you might have made. Ask questions if you don't understand.
- **Forming study groups** is a very good idea and take advantage of your instructor's **office hours**.