

# Math 120Q — Fall, 2015

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Hours: TuTh 3:00–5:00; MW 3:45p–4:45p; by appt.

**Course Content:** Mathematics 120 is an introduction to pure mathematics through the study of geometry. This term we will use as a theme the role of infinity in the development of mathematical thought.

**Texts:** The ones preceded by an asterisk (\*) should be purchased in the bookstore by **August 26!**

\*Plato, *Meno*., Focus Publishing (trans. by Anastaplo and Berns).

Aristotle, *Physics*, excerpt.

Euclid, *Elements*, Book I.

Galileo, *Dialogues Concerning Two New Sciences*, excerpt.

Saccheri, *Euclid Freed of All Blemish*.

Lobachevsky, “Geometrical Researches on the Theory of Parallels.”

Hilbert, *Foundations of Geometry*, excerpt.

Einstein, *Geometry and Experience*.

Escher, *Circle Limit* series.

Hausdorff, “Remark on the content of point-sets.”

**Course Goals:** After this course, the student should have developed the following capacities: to reason about geometry, to read with greater attention to detail, to understand the elements of geometry, to discuss differences in its ancient and modern conceptions. This course is for the purposes of liberal education. Thus its goals are the highest possible in a mathematics course. (Xylander, who produced the first German translation of Euclid in 1562 in which many of the proofs were omitted, tells us that his book was meant for the simple amateur who is of course content to know the facts without knowing how to prove them. But you are not mere amateurs. You are liberal arts students.)

**A Way of Inquiry:** This is a general education, Ways of Inquiry (INQ) course. The focus of a “Ways of Inquiry” course is the education of a particular person, namely you. You will develop your intellectual abilities, to ask questions, to seek answers, and to judge opinions. You will be better able to see, with foresight, and insight, and hindsight. Truly good foresight is the result of learning many ways of inquiry, and this course is part of that education.

You have chosen a class in a particular discipline, mathematics. “Discipline” comes from a word meaning teaching, but you could think of a discipline as a way of looking at things, of thinking about things. Mathematics has a particular way of thinking about things. It is exciting in itself and useful in addressing all sorts of problems in other disciplines. It is only natural that it is this way. Every other discipline has arisen and evolved in the same natural way.

In general education, however, the focus is not on disciplines but on you. We emphasize inquiry to teach you the importance of good questions. Good questions may be basic questions, radical questions, sometimes simply “Why?” or “Why not?” Each discipline offers a perspective on the world’s problems. From its viewpoint, some aspects of a problem will be more obvious and some will be obscured. Converting a real problem to mathematics involves stripping away and ignoring many things. It is sometimes amazing that the mathematical solution has any bearing on the original problem. Why does it work? What are we leaving out? Sometimes what mathematics leaves out is essential, but the questions it includes are always important.

By asking fundamental questions humans have taught themselves to think anew, people have improved their communities, and individuals have learned new ways to engage the world. Asking good, fundamental questions is crucial to learning. How can you live a free person if you cannot teach yourself? This is the emphasis of a Ways of Inquiry course. With such a liberating education you can make a difference in your life, your affairs, and even the world.

**Continuing Writing Requirement:** This course may be used to fulfill the Continuing Writing Requirement. In order to do so, the student must receive a satisfactory grade on the Commentary component. See Grades below.

**Coursework:** Learning activities in the course can be divided into three major categories, reading, speaking (in class discussion and presentations) and writing. The writing is meant to be the product of integrative learning exercises carried out by the student, who reflects on the readings and discussion in an attempt to arrive at a clearer understanding of the material, questions the material raise, and connections to other parts of the course or other disciplines. It is the principal assignment in which the student assumes the responsibility for being an intellectual individual seeking to understand ideas. It is an essential part of the course and a partial complement to the seeking of understanding through group discussion that occurs in class. Tests, too, provide a different sort of complement to class discussion.

**Tests:** Two tests will be offered. The first is currently planned for the third or fourth week in September in class. The second test will be announced a week in advance. Students are expected to take tests at the scheduled times. Any conflicts or problems will be handled on an individual basis. If a student has an excuse deemed legitimate by the instructor, arrangements will be made to take the test **prior to** the scheduled time.

**Commentaries:** Commentaries are **due each week on Friday**, excepting weeks in which there is a test and the first week. Each is to be two pages or more, typed, double-spaced, of a 12-point font, of about 600-700 words in length with the word count included at the top of the first page. Each is to comment on the ideas in the course covered in the previous week or so. Each may provide alternate explanations of things, probe issues which cause the student confusion, amplify or clarify the successes or failures of the text, or anything which shows serious thought about some part of the course. A comment is more than an off-hand reaction like “It was interesting.” A comment is to be critical or questioning, wondering or insightful, coherent and focused. Outside sources are permitted but not encouraged; keep in mind that each commentary will be evaluated on the quality of the student’s own reflections. If outside sources are used, be sure to cite them appropriately and to avoid plagiarism as defined in the Honor Code. Unsatisfactory commentaries must be rewritten to receive credit.

**Class Participation:** The work done in the class meetings is part of the course work for each student. Absences and tardiness must be counted as work not done. Further each person is to be ready to participate in each class conversation. Students will be expected to present proofs at the blackboard unaided by notes or the text. Responsibility will rotate in random order throughout the whole class.

Class conversation, including conversation in class needs to follow certain guidelines, if it is to be productive. Each person must feel free to contribute. This requires each person to be open to and willing to explore others’ opinions. This is not to say that every person’s opinion is equally valid. But every opinion, seriously proposed, equally merits investigation until we all can see in what ways it is valid and in what ways it is not. Let us keep in mind that there is not necessarily only one correct opinion, and that opinions are not necessarily strictly correct or incorrect. Usually there are ways in which an opinion is correct and ways in which it is not. We should respond to what others say and refer to whom we are responding. To remind us to be civil and polite, we should use formal address, “Ms” or “Mr” plus the name. A sign of a good conversation is that it makes us want to reread the text.

**Questions:** From time to time, each student will be asked to submit a written question on a 3" by 5" card. The question is to be over the reading assigned for that day. The student’s name shall be written in the top right corner of the card.

**Quizzes:** All quizzes are announced; some may be take-home. The student must be present in class to receive a quiz unless a **prior** arrangement has been made with the instructor. They will figure in the class participation component of the grade.

**Final Examination:** There will be a comprehensive final examination at the time scheduled by the registrar.

**Grades:** Grades will be based roughly on the following distribution of work:

Tests	20%
Class Participation	35%
Commentaries*	25%
Final Examination	20%

The plus/minus system will be used.

\*A student who does not pass the Commentaries portion of the class cannot receive a grade above a C– and will not earn credit for the Continuing Writing Requirement.

**Homework: Rereading.** I want to point out to you at the outset the importance for you, both in this course and particularly in your future life, of the development of skill in reading. Reading is more than sounding out and recognizing words. It is more than putting the words in a sentence together to get the meaning. The deepest reading is ever mindful that the text was written by a human for a human audience and with human purposes in mind. A sensitive reader will discover these purposes, and see what evidence there is in the text that the author has these purposes and why. All this requires rereading.

Mathematical writing usually lacks the poetical subtleties of irony and metaphor, but it does have its own difficulties. Words, phrases, definitions, and logic are used precisely, and the ideas fit together precisely. This requires the reader to pay close attention to detail and have a good memory. You must practice these things. The reader needs to follow the *logic* of the argument. One must pay attention to the scope of the hypotheses, that is, when the text assumes some statement and when the text stops assuming it. The subtle differences in the words “would/will” and “could/can” are important.

**A resource:** Carl Boyer, *History of Mathematics*. For information about Euclid, non-euclidean geometry, Lobachevksy. Quite readable.

**All Questions Answered** Knuth: In every class that I taught at Stanford, the last day was devoted to “all questions answered.” The students didn’t have to come to class if they didn’t want to, but if they did, they could ask any question on any subject except religion or politics or the final exam. I got the idea from Richard Feynman, who did the same thing in his classes at Caltech, and it was always interesting to see what the students really wanted to know. [Donald Knuth, “All Questions Answered,” *Notices of the AMS*, 49(3), March 2002, p. 318]

**Honor Code:** The Honor Code of Oxford College applies to all work submitted for credit in this course. By placing your name on such work, you pledge that the work has been done in accordance with the given instructions and that you have witnessed no Honor Code violations in the conduct of the assignment.

## Introduction Letter to the student

My Very Dear Student,

Welcome and thank you for enrolling in this course, a great favorite of mine. It is different from most mathematics courses. The word pure, traditionally when modifying the word mathematics, is opposed to the word applied. What I like to say is that we will not study anything that is applicable to real life — no formulas, no methods. This is a somewhat fuzzy division that goes back to the ancient liberal arts, the quadrivium, comprising geometry, arithmetic, astronomy, and music (*harmonike*). Astronomy and music were thought of as geometry and arithmetic in motion. The distinction continues today in certain subjects being thought of as arising from applications to science and engineering (applied mathematics) and those that do not, often from mathematics itself or mathematical ways of thinking about the world (pure mathematics).

I have said that this course will avoid anything that applies to real life. Yet you might be wondering if that could really be so, since perhaps skills seem useful to you and what is useful seems good. Among the abstracted skills that one might imagine constitute an education — writing, critical thinking, quantitative skills, and so on — the *technical* skill of **reading** is the one I most particularly wish to show you and whose growth in you I hope to cultivate. And I am glad to advise you that, if education is your goal and you wish to make plans for it in terms of such abstract skill, that becoming a good reader ought to figure highly in your plans. Literature is a good training ground, but so is science, and mathematics, and sociology, and so on. Listening is reading's correlative and an estimable skill. Both are means of being attentive to the worlds and what is outside you. Their adjuncts are writing and speaking. It turns that often to be a good writer or speaker one has to be a good listener. One obvious reason is that to be a good writer or speaker, you have to say something, that is, to have something to say; and to be a great writer or speaker, you have to have something to say that others want to hear. To do that, it helps to have paid attention to what is outside of you. Thus reading and listening can be important aids to becoming a good writer or speaker.

**Logic**, too, is sometimes viewed as a technical skill. By the mathematical nature of the course, we will be reflecting on reasoning in the texts and in our discussions. Some technical terms will be introduced, and they will be helpful; however, we will not be treating logic in the technical way of Philosophy 110 or Mathematics 250 (which is an *unmathematical* way, except when one uses logic to study logic itself).

While such skills seem useful, almost by definition, the primary learning in the course will be mathematical. I hope my meaning will become clearer as we progress through the course, just as I hope all the uses of “mathematical” will become clearer.