Mathematics 221Q – Linear Algebra **Spring, 2013**

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Textbook: H. Anton and C. Rorres, *Elementary Linear Algebra, Applications Version*, 10th edition.

Course Content: Mathematics 221 is an introduction to linear algebra. This course will cover systems of linear equations, vectors, matrices, determinants, vector spaces, linear transformations, eigenvalues and eigenvectors, inner product spaces, and applications.

Course Objectives: At the end of the course, the student should achieve the following goals: to know the basic definitions and theorems in the field of linear algebra as described in the course content; to solve basic problems in linear algebra using matrices and matrix operations; to prove the fundamental relationships (theorems) between the concepts in the course.

Ways of Inquiry: Linear Algebra is a highly structured mathematical system based on definitions, axioms, theorems and proofs, all of which are of fundamental importance to mathematics and mathematical activities. It has many applications to other branches of mathematics, natural sciences and social sciences. It is appropriate to adopt the "Ways of Inquiry" in this introductory course. Through the course, students interested in mathematics will gain understandings on *what mathematics is*, *how mathematics is done* and *what applications it has*.

The course is entirely student-centered and inquiry-driven. Instead of lectures, students will learn the material through studying the textbook, class facilitation/participation/discussions and completing homework.

Class facilitation: Most of the topics covered will be done through in-class discussions led by students. For each topic, a group of two students will be designated as facilitators. Before a topic is discussed, a reading guide will be posted on Blackboard. Every student is expected to study the textbook (and other resources if appropriate) guided by the questions, come up with a list of answers, thoughts and more questions to be brought to class. This will be the basis for class discussion. In class, the facilitators will give a brief presentation and then be in charge of discussions.

Each group is expected to perform as facilitators about three times in the semester. Before the scheduled facilitation the group is required to meet with the instructor briefly to go over the material. The members of the group are expected to contribute equally.

Attendance: Students are expected to attend all classes and are responsible for all material covered in class as well as any changes made in the schedule regarding homework and tests. Class attendance and consistent preparation for class will determine the success or failure the student realizes in this course. <u>For each unexcused absence 5 points will be deducted from the participation grade.</u> A student may not miss a facilitation class unless there is a legitimate (documented) reason. In this case the student will discuss with the professor about how that might affect the grades. <u>Being late for more than 10 minutes will be counted as half of an absence.</u>

Homework: Homework will be collected almost weekly. Collaboration is allowed, but each student should be sure that he or she ultimately can s**olve problems unaided by notes, the textbook, or other people**. Solutions will be posted on Blackboard.

Ways of Inquiry Journal: Students are required to keep a journal and make at least one entry every week. The entry can be your reflection on anything about the course, especially related to the "ways of inquiry." Most of the entries should be about mathematics and learning of mathematics. For example, it can be your thoughts on the questions raised in class discussions; or the most impressive thing you have discovered/realized/understood; or the most puzzling concept/theorem/proof you have encountered recently and why it is so; or a question not raised in the book but asked by the students and why it is a meaningful and interesting question. Journals will be collected and graded regularly.

Quizzes: At the beginning of the discussion of a new topic, a short quiz will be given. A student must be present to take the quiz. No make-up quizzes will be given.

Tests: There will be 2 tests on the following **Thursday evenings: February 21 and April 11, 6 – 8 pm.** The tests will be closed book. You are expected to take tests at the scheduled times. Any emergencies will be handled on an individual basis and must be documented. **<u>No make-up test will be given after the testing time and date</u>**.

Final exam: There will be a closed-book, comprehensive final exam at the time scheduled by the registrar.

Grading:

Class facilitation and participation	200 (150 pts for facilitation and 50 pts for participation)
Ways of Inquiry Journal	50
Homework	50
Quizzes	100
Tests (2 @ 150 points each)	300
Final Exam	200
Total points	900

<u>A rough guide for letter grades:</u>
A: roughly more than 90 %; B: 80 – 89 %; C: 70 – 79 %; D: 60 – 69%; F: below 60 %
Grades of A-, B+, B-, C+, C-, D+ may be assigned.

Written Work: Thoughts are expressed by sentences. <u>Your written work must be in complete sentences.</u> Use mathematical symbols wherever appropriate. Pay attention to how the problems are worked out in the textbook. Your work should be neat and legible. It is common practice to rewrite solutions once they are found.

Calculators: In general, calculators will **<u>not</u>** be allowed unless the opposite is announced.

Office Hours/Outside Help: Office hours will be announced. Students should use this time to come by and ask specific questions related to this course and/or homework problems. In addition, students may email your instructors privately. The Math Center is open Mondays through Thursdays, 3 – 6 pm.

There is a Blackboard site, **Math_OX 221Q Linear Algebra – Spring 2013**. Documents and announcement related to the course will be posted there. This includes topics covered that day, homework assignments, suggestions on studying the textbook, topics to preview before next lecture and other announcements. Students should check the site at least once a day.

Out-of-class review sessions may be scheduled as needed.

Study groups organized by students are highly recommended. The meetings should be scheduled weekly and should be part of a regular weekly routine.

Student work submitted as part of this course may be reviewed by Oxford College and Emory College faculty and staff for the purposes of improving instruction and enhancing Emory education.

HONOR CODE: THE HONOR CODE OF OXFORD COLLEGE APPLIES TO ALL WORK SUBMITTED FOR CREDIT. ALL SUCH WORK WILL BE PLEDGED TO BE YOURS AND YOURS ALONE. THIS IS THE CASE WHEN YOU PLACE YOUR NAME ON WORK SUBMITTED.

Ways of Inquiry
Math221Q Linear Algebra

It seems appropriate to adopt "Ways of Inquiry" in Linear Algebra because for most of you, Linear Algebra is one of the first courses where you will get a glimpse of *what mathematics is*, *how mathematics is done* and *what applications it has*.

Asking the "Right" Questions—What (fundamental) questions will you learn to ask when dealing with this discipline?

Linear Algebra is a highly structured mathematical system based on definitions, axioms, theorems and proofs, all of which are of fundamental importance to mathematics and mathematical activities. It has many applications to other branches of mathematics, natural sciences and social sciences.

Throughout the course, using the *"Right" Tools*, you will learn to ask the following questions:

<u>About Definitions:</u>
- Why is defining this object necessary?
- How does the definition emerge?
- Why is it defined this way?
- Are there other equivalent ways of defining it?
- Why is the choice of the words, are they precise?
- What happens if certain changes are made to the definition?
- What are some examples (from previous studied mathematical notions) of this object defined?
- How is this object related to other concepts?
- Can you find any real-world applications which can be modeled by the object defined?

<u>About Axioms:</u>
For most of you, it is the first time that you will see structures defined by a set of carefully chosen axioms. While these are important concepts of crucial significance to mathematics (e.g. General Vector Space, Inner Product Space, etc.), the idea of the axioms and using axioms to define objects might be fresh and foreign and usually difficult for you to comprehend and grasp. Learning to ask the following questions will help.

- What is an axiom? What does it mean to designate a statement as an axiom?
- Why do we use axioms to define a structure?
- When defining a specific structure, why is this set of axioms chosen? (How did people come up with them? Why these and not others? What if some axioms were eliminated, or changed? Is this the best choice?)
- Give some examples of objects that satisfy all the axioms, some of the axioms and not all.
- Can some of the axioms be deleted or added so as to describe more general or restricted objects?

<u>About Theorems:</u>
Theorems are central to mathematics. While typically you have encountered theorems in your previous experiences (such as Geometry and Calculus), in Linear Algebra, it is probably the first time you will be asked to fully understand every aspect and detail of a theorem. The following set of questions offers a starting point to guide the students. As you mature as a thinker of mathematics, you will ask more and more questions about a theorem.

About the conditions (hypotheses) and conclusions:
- What are the conditions of the theorem?
- What are the conclusions of the theorem? Can the conclusions be stated in an equivalent but different way?

- Are the conditions necessary, sufficient or both?
- Can one change/delete some of the conditions? How does that affect the conclusions? Do we obtain a meaningful conclusion? Do we obtain a stronger, or weaker or the same conclusion?
- Are there other (similar) conditions leading to the same conclusion?

About some related statements:
- What is the converse of the theorem?
- Does the converse always, or sometimes or never hold?
- If the converse is always true, can you prove it? If it is sometimes true, can you determine when it is true? If it never holds, why? Can you provide examples when it does not hold?

- What is the contrapositive? (Note: the contrapositive of a statement holds whenever the statement holds).
- When is it easier to apply the theorem in its contrapositive form?

About the significance of the statement:
Being a textbook for undergraduates, the book adopted for Linear Algebra calls almost every statement proven a "theorem." In real mathematical activities however, mathematicians make a clear and necessary distinction between propositions and theorems, lemmas and theorems. Being able to evaluate the significance of a statement and distinguish between major theorems and minor facts is a sign of gaining deeper understanding and mathematical maturity.
- Is this theorem important?
- Why or why not is the theorem important? What is its significance? How is it related to other facts and concepts?

- Can you generalize the theorem? What are some meaningful generalizations?
- What are some related questions that worth investigating? Can you think of some ways to approach them?

<u>About Proofs:</u>
In Linear Algebra, you will be asked to study all the proofs of the theorems encountered, memorize and be able to reproduce them. You will also be asked to prove statements in homework and tests.

When studying an existing proof:
- What is the main idea in the proof?
- How does one think of the ideas?
- How are the conditions used?
- How is the argument written and why is it written this way?

When proving a statement:
- What is the main idea?
- Can it be proven directly?
- Can it be proven indirectly? Can I argue by contradiction?
- Can I reduce it to some other statements to prove?
- Can I dissect the conclusions to cases and prove each case?
- Can I get an approach by starting backwards, from the conclusions?
- Are there different proofs? Which approach is better and why?
- Can a more general statement be proven?
- How do I structure the written proof? Are the statements logical, precise and succinct? Can I write it better and clearer?

About the Role of Linear Algebra:
Linear Algebra requires a student to think abstractly. Many key notions are obtained by extracting the common features from other objects in mathematics and models in natural and social sciences. Asking the following questions will help one to see the connections and appreciate the role of Linear Algebra in mathematics and its applications.

- What previously studied objects in other fields of mathematics and/or natural and social sciences does this concept remind you of?
- What are some known applications? How is Linear Algebra applied?
- What more knowledge/knowhow in Linear Algebra is desirable in order to further its applications?
- Can you think of applications not mentioned in the book and maybe not yet thought of by others?

Using the "Right" Tools—What methods of analysis and argument will students learn to help them investigate questions in this discipline?

You will learn to read a mathematics text critically. Guided by the questions listed above, you will study your textbooks, "chew on" definitions and investigate theorems and their proofs in depth.

You will understand what a mathematical proof is, study and practice different proof techniques.

You will practice *thinking specifically(concretely)*, for example, giving examples, counterexamples, imagining how the concepts apply to a specific situation.

You will practice *thinking generally*, for example, generalizing a theorem, a concept, or a situation.

You will practice *thinking abstractly*, for example, observing how seemingly different objects in mathematics share certain commonalities, identifying these common features and finding a way to describe them both comprehensively and precisely.

Following your interests, you will search for and investigate various applications of Linear Algebra.

Students Actively Practice Inquiry – How will students "discover" knowledge in this course? How will students actively practice the process of inquiry?

The course will be entirely student-centered and inquiry-driven.

Most of the topics will be presented by the students. Before such a topic is discussed, a group of two students will be designated as facilitators of the topic. A reading guide (mostly with questions like the list of questions above put into context for that particular topic) will be distributed. Students are expected to study the textbook (and other resources if appropriate) guided by the questions. Each student will come up with a list of answers, thoughts and more questions to bring to class. In class, the facilitators will give a brief presentation of the topics and then be in charge of discussions.

Meta-Level Reflection – How will students be asked to reflect upon, question, and appreciate the ways of inquiry used in this discipline?

You are required to keep a journal and make at least one entry every week. The entry can be your reflection on anything related to the course, especially the "ways of inquiry." For example, it can be your thoughts on the questions raised above; or the most impressive thing you have discovered/realized/understood in the past two weeks; or the most puzzling concept/theorem/proof you have encountered recently and why it is so; or a question not raised in the book but asked by the students and why it is a meaningful and interesting question.

Connections to Something Bigger—What specific "real-world" questions, interdisciplinary connections, or ethical issues will students explore in an attempt to deepen their understanding and appreciation of the class content?

Linear Algebra has many applications in other fields of mathematics, natural sciences and social sciences. For example,

- In economics, the Leontief Input-Output Models developed by the 1973 Nobel prize winner in economics, Wassily Leontief, to study the relationships between different sectors in an economy by using matrix methods.
- Dynamical systems and Markov chains, which uses matrix methods to analyze the behavior of physical systems that evolve over time, such as problems in business, ecology, demographics, sociology and most of the physical sciences.
- Internet search engines (applications of matrices)
- Games of strategy, game theory (application of matrix multiplication and basic probability concepts)
- Computer graphics (application of matrix algebra and analytic geometry)
- Computed Tomography such as X-ray imaging (application of linear systems, natural logarithms and Euclidean space)
- Fractals and Chaos (application of geometry of linear operators, eigenvalues and eigenvectors, limits and continuity)
- Cryptography (application of matrices, Gaussian elimination, matrix operations, linear independence, linear transformations)
- Genetics, investigate the propagation of an inherited trait in successive generations (application of eigenvalues and eigenvectors, diagonalization of a matrix, limits)

Appropriate Assessment for Inquiry-based Learning—What specific assignments will students be asked to do, to demonstrate their increased abilities in reading critically, writing, analyzing, or speaking with clarity?

Because of the structure of the course, every assignment will be an assessment of students' abilities in the above listed. More specifically, students' class facilitation, participation in class discussions, and journal entries will be direct ways for you to demonstrate your increased abilities in reading critically, writing, analyzing, or speaking with clarity.