Mathematics 120
Spring, 2004

Instructor: Michael Rogers
Office: 115 Seney Hall
Phone: x4-8419
E-mail: LearnLink "Michael Rogers"
Hours: **MWF 9:30–12:00, Th 2:00–5:00, and by appointment.**

Course Content: Mathematics 120 is an introduction to pure mathematics through the study of geometry and number theory. This term we will use as a theme the role of infinity in the development of mathematical thought.

Texts: Aristotle, *Physics*.
Euclid, *Elements*.
E. Landau, *Foundations of Analysis.*
Lobachevsky, *The Theory of Parallels.*

Course Goals: After this course, the student should have developed the following capacities: to reason about geometry and numbers, to read with greater attention to detail, to understand the elements of two of the classical liberal arts, geometry and number theory (arithmetic), to discuss differences in the ancient and modern conceptions of these arts. This course is for the purposes of liberal education. Thus its goals are the highest possible in a mathematics course. (Xylander, who produced the first German translation of Euclid in 1562 in which many of the proofs were omitted, tells us that his book was meant for the simple amateur who is of course content to know the facts without knowing how to prove them. But you are not mere amateurs. You are liberal arts students.)

Tests: Two tests will be offered. The first is currently planned for the second week in February in class. The second test will be announced a week in advance. Students are expected to take tests at the scheduled times. Any conflicts or problems will be handled on an individual basis. If a student has an excuse deemed legitimate by the instructor, arrangements will be made to take the test **prior to** the scheduled time.

Quizzes: All quizzes are announced; some may be take-home. The student must be present in class to receive a quiz unless a **prior** arrangement has been made with the instructor.

Commentaries: Commentaries are **due each week on Friday**, excepting weeks in which there is a test and the first week. Each is to be two pages, typed, double-spaced, of a 12-point font. Each is to comment on the ideas in the course covered in the previous week or so. Each may provide alternate explanations of things, probe issues which cause the student confusion, amplify or clarify the successes or failures of the text, or anything which shows serious thought about some part of the course. A comment is more than an off-hand reaction like "It was interesting." A comment is to be critical or questioning, wondering or insightful, coherent and focused. Outside sources are permitted but not encouraged; keep in mind that each commentary will be evaluated on the quality of the student's own reflections. If outside sources are used, be sure to cite them appropriately and to avoid plagiarism as defined in the Honor Code.

Class Participation: The work done in the class meetings is part of the course work for each student. Absences and tardiness must be counted as work not done. Further each person is to be ready to participate in each class conversation. Students will be expected to present proofs at the blackboard unaided by notes or the text. Responsibility will rotate in random order throughout the whole class. A LearnLink conference, "120 Rogers", has been created within the "Oxford: Mathematics" conference for this course. Contributions to the conference count toward class participation, but they may not completely replace oral participation in class.

Class conversation, including conversation in class and in the LearnLink conference, needs to follow certain guidelines, if it is to be productive. Each person must feel free to contribute. This requires each person to be open to and willing to explore other's opinions. This is not to say that every person's opinion is equally valid. But every opinion, seriously proposed, equally merits investigation until we all can see in what ways it is valid and in what ways it is not. Let us keep in mind that there is not necessarily only one correct opinion, and that opinions are not necessarily strictly correct or incorrect. Usually there are ways in which an opinion is correct and ways in which it is not. We should respond to what others say and refer to whom we are responding. To remind us to be civil and polite, we should use formal address, "Ms" or "Mr" plus the name. A sign of a good conversation is that it makes us want to reread the text.

Original Proofs: Propositions will be handed out which the student may attempt to prove. Such proofs are to be worked out by the student alone. The student may refer to the text used in class, any notes handed out in class, and his or her own notes. No one else's notes may be used. Such proofs may be submitted for credit any time on or before **Monday, April 19, 2004**.

Final Examination: There will be a comprehensive final examination at the time scheduled by the registrar.

Grades: Grades will be based roughly on the following distribution of work:

Tests	20%
Class Participation	35%
Commentaries	20%
Original Proofs	5%
Final Examination	20%

The plus/minus system will be used. If the class becomes particularly small, then class participation will count more (up to 40%) and commentaries will count less.

Homework: Rereading. I want to point out to you at the outset the importance for you, both in this course and particularly in your future life, of the development of skill in reading. Reading is more than sounding out and recognizing words. It is more than putting the words in a sentence together to get the meaning. The deepest reading is ever mindful that the text was written by a human for a human audience and with human purposes in mind. A sensitive reader will discover these purposes, and see what evidence there is in the text that the author has these purposes and why. All this requires rereading.

Mathematical writing usually lacks the poetical subtleties of irony and metaphor, but it does have its own difficulties. Words, phrases, definitions, and logic are used precisely, and the ideas fit together precisely. This requires the reader to pay close attention to detail and have a good memory. You must practice these things. The reader needs to follow the *logic* of the argument. One must pay attention to the scope of the hypotheses, that is, when the text assumes some statement and when the text stops assuming it. The subtle differences in the words "would/will" and "could/can" are important.

Course Outline: The course consists of an introduction to geometry and analysis. Many of the advances in mathematics rest on developments in our understanding of the infinite. In particular we will examine some of the conceptions of infinity and how they are used in geometry and analsys. In geometry we read Book I of Euclid's *Elements* and one of Lobachevsky's papers on non-Euclidean geometry. In analysis, we read parts of Books VII and IX of Euclid's *Elements* and Landau's *Foundations of Analysis* (for Peano's development of arithmetic). We will also read selections from Aristotle's *Physics* and Galileo's *Two New Sciences*, which discuss the infinite in both fields. We will also study the real number system, Cantor's idea of transfinite numbers, Gödel's incompleteness theorem, fractals, space-filling curves, and such things as time allows.

Other resources:

- Carl Boyer, *History of Mathematics*. For information about Euclid, noneuclidean geometry, Lobachevksy, Peano. Quite readable.
- Morris Kline, *Mathematical Thought in the Western World*. Similar to Boyer. The mathematical level of the writing varies, but most of the material on the topics in this course is accessible.

Honor Code: The Honor Code of Oxford College applies to all work submitted for credit in this course. By placing your name on such work, you pledge that the work has been done in accordance with the given instructions and that you have witnessed no Honor Code violations in the conduct of the assignment.