ADS 2021: Week 7 Exercises

Exercises for week 7 of Algorithms and Data Structures at ITU. The exercises are from *Algorithms*, 4th Edition by Robert Sedgewick and Kevin Wayne unless otherwise specified. Color-coding of difficulty level and alterations to the exercises (if any) are made by the teachers of the ADS course at ITU.

- **4.1.1 Green** What is the maximum number of edges in a graph with V vertices and no parallel edges? What is the minimum number of edges in a graph with V vertices, none of which are isolated?
- **4.2.1 Green** What is the maximum number of edges in a digraph with V vertices and no parallel edges? What is the minimum number of edges in a digraph with V vertices, none of which are isolated?
- **4.1.28 Green** Two graphs are isomorphic if there is a way to rename the vertices of one to make it identical to the other. Draw all the nonisomorphic graphs with two, three and four vertices.
- **4.2.8 Green** Draw all the nonisomorphic DAGs with two, three and four vertices.
- **4.1.12 Yellow** What does the BFS tree tell us about the distance from v to w in an (undirected) graph when neither is at the root?
- **4.1.16 Yellow** Describe how to find:
 - a) The eccentricity of a vertex (the length of the shortest path from that vertex to the furthest vertex from it)
 - b) The diameter of a graph (the maximum eccentricity of any vertex in the graph)
 - c) The radius of a graph (the smallest eccentricity of any vertex in the graph)
 - d) A center of a graph (a vertex whose eccentricity is the radius of the graph)
- 4.1.21 Yellow Show a trace of the Bipartite traversal (p. 547) of the graph described by tinyGex2.txt (p. 558). (If you insist, you can do it as detailed as in the figures in the book, but that takes a lot of time.)

Then, describe an alternative implementation that uses BFS instead of DFS. Show a trace of that traversal as well. Are the resulting colourings the same? Are the running times the same? Are there other differences?

- **4.1.32 Red** Describe a linear-time algorithm to count the parallel edges in a graph.
- **4.1.36 Red** A bridge in a graph is an edge that, if removed, would separate a connected graph into two disjoint subgraphs. A graph that has no bridges is said to be edge connected. Design and describe two algorithms for determining whether a given graph is edge connected, one using a DFS-based data type and one using a BFS-based data type. Both should run in $O(E \cdot (V + E))$.