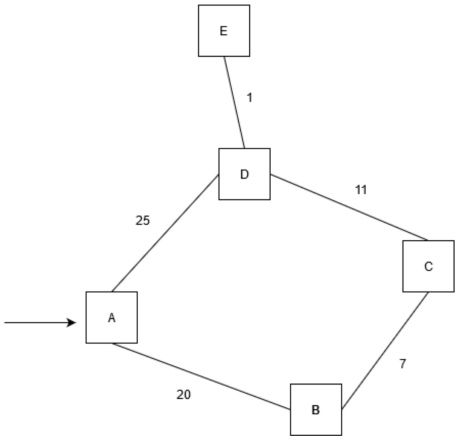
ADS 2021: Week 8 Exercises

Exercises for week 8 of Algorithms and Data Structures at ITU. The exercises are from *Algorithms, 4th Edition* by Robert Sedgewick and Kevin Wayne unless otherwise specified. Color-coding of difficulty level and alterations to the exercises (if any) are made by the teachers of the ADS course at ITU.

1 - Green Hand-run Prim's, Kruskal's and Dijkstra's algorithm on the small example graph below, and write down the order in which the edges of the graph are explored. (For Prim and Dijkstra, the graph exploration should start at vertex A.)



4.3.1 - Green Prove that you can rescale the weights by adding a positive constant to all of them or by multiplying them all by a positive constant without affecting the MST.

4.3.7 - Green Describe how you would find a maximum spanning tree of an edge-weighted graph.

4.3.12 - Green Suppose that a graph has distinct edge weights. Does its shortest edge have to belong to the MST? Can its longest edge belong to the MST? Does a min-weight edge on every cycle have to belong to the MST? Argue for your answer to each question or give a counterexample.

4.4.1 - Green True or false: Adding a constant to every edge weight does not change the solution to the single-source shortest-paths problem.

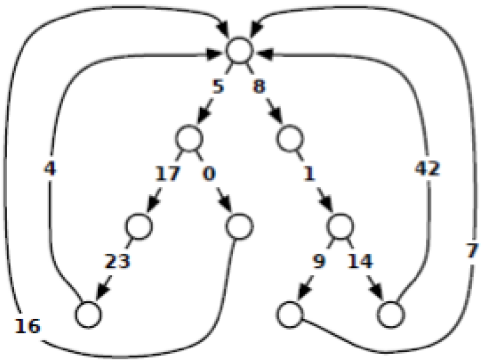
4.3.3 - Yellow Show that if a graph's edges all have distinct weights, the MST is unique.

4.3.4 - Yellow Consider the assertion that an edge-weighted graph has a unique MST only if its edge weights are distinct. Give a proof or a counterexample.

4.3.14 - Yellow Given an MST for an edge-weighted graph G, suppose that an edge in G that does not disconnect G is deleted. Describe how to find an MST of the new graph in time proportional to E.

4.4.31 - Yellow Given a weighted line graph (undirected connected graph, all vertices of degree 2, except two endpoints which have degree 1), describe how you would devise an algorithm that preprocesses the graph in linear time and can return the distance of the shortest path between any two vertices in constant time.

Exam from May 31st 2012, question 4 A looped tree is a weighted, directed graph built from a binary tree by adding an edge from every leaf back to the root. Every edge has a non-negative weight.



A looped tree.

4.A - Green Convince yourself that the shortest path from the leftmost leaf to the rightmost leaf in the above example graph has length 27.

4.B - Yellow Given such a graph and two vertices, u and v, we want to find the shortest path from u to v. Clearly Dijkstra's algorithm ([SW, Algorithm 4.9]) solves this problem. What is the running time in terms of the number of nodes, N?

4.C - Red Design a faster algorithm for this problem. If you want, you can make use of existing algorithms, models, or data structures from the book. Estimate the running time of your construction.