# Ryacas – an R interface to the yacas computer algebra system

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## 1 Introduction

Ryacas is an R package that makes the yacas computer algebra system available from within R. The name yacas is short for "Yet Another Computer Algebra System". Ryacas is available on the The

Comprehensive R Archive Network (CRAN) at http://cran.r-project.org/. In addition, Ryacas is also described on https://code.google.com/p/ryacas/.

The yacas program is developed by Ayal Pinkhuis (who is also the maintainer of yacas) and others, and is available at http://yacas.sourceforge.net for various platforms. This site also contains a comprehensive documentation and the documentation contains many examples.

The examples given here are largely taken from the yacas documentation (especially from the introductory chapter) but are organised differently.

Ressources:

http://yacas.sourceforge.net/ref.book.pdf

## 2 A small session – polynomials

The **Ryacas** package works by sending "commands" to yacas which makes the calculations and returns the result to R. This can be done in various ways.

### 2.1 Calling with yacas commands as text strings

A simple way of calling yacas from R is by submitting yacas commands as a text string to the yacas() function.

```
> out <- yacas( "Expand( (1 + u)^4 )" ); out
expression(u^4 + 4 * u^3 + 6 * u^2 + 4 * u + 1)
> class( out )
[1] "yacas"
```

Notice that the output from yacas() looks like an R expression, but it is not; see below.

The result can be displayed nicer in different forms. For example

### 2.2 The output from yacas

```
The output from yacas() looks like an R expression, bit it is not; it is a "yacas" object.
```

```
> class( out )
[1] "yacas"
> names( out )
```

```
[1] "text" "OMForm"
> lapply( out, head )
$text
expression(u^4 + 4 * u^3 + 6 * u^2 + 4 * u + 1)

$OMForm
[1] "<OMOBJ>\r\n <OMA>\r\n <OMS cd=\"arith1\" name=\"plus\"/>\r\n <OMA>\r\n <OMS cd=\"
The text slot in the output is an R expression that can be evaluated numerically as
> eval( out$text, list(u=1) )
[1] 16
in Ryacas, there is an Eval() function that operation that operates on yacas objects so one can also do
> Eval( out, list(u=1) )
[1] 16
```

#### 2.3 Calling yacas with R expressions

```
In Ryacas, R expressions can be sent to yacas so one can do
```

```
> e <- expression( Expand( (1 + u)^4 ) )
> yacas( e )
expression(u^4 + 4 * u^3 + 6 * u^2 + 4 * u + 1)
```

This works fine as long as a valid R expression can be created – but consider expanding a polynomial in two variables

```
> e <- expression( Expand((1+x-y)^2, x) )  ## This works 
> ## e <- expression( Expand((1+x-y)^2, \{x,y\}) ) ## This fails
```

The latter case fails because of the yacas syntax with curly braces which R's expression() function can not interpret. So in the latter case we have to pass the argument as a text string:

```
> yacas("Expand((1+x-y)^2, \{x,y\})")
expression(x^2 + (-2 * y + 2) * x + (y^2 - 2 * y + 1))
```

Another example of an operation that can not be handled using R expressions is differentiation. Doing

```
yacas(expression(D(x)Sin(x)))
produces an error. For such cases we can do
> yacas("D(x)Sin(x)")
expression(cos(x))
```

#### 2.4 Using Sym objects

An elegant way of working with yacas is by using Sym objects. A Sym object is a character string that has class Sym. The function Sym(x) coerces an object x to a Sym object by first coercing it to character and then changing its class to Sym. For clarity we use two different symbols here:

```
> x_ <- Sym("x")
> x_
expression(x)
```

```
> dput( x_ )
structure("x", class = c("Sym", "character"))
One can combine Sym objects with other Sym objects as well as to other R objects using +, - and other similar R operators. For example
> x_ + 4
expression(x + 4)
> Eval(x_+4, list(x=1))
[1] 5
One can apply sin, cos, tan, deriv, Integrate and other provided functions to Sym objects. For example:
> x <- Sym("x")
> Integrate(sin(x), x)
expression(-cos(x))
> deriv(cos(x), x)
expression(-sin(x))
```

## 2.5 Recall the most recent line – the % operator

The operator % automatically recalls the result from the previous line.

```
> yacas("(1+x)^3")
expression((x + 1)^3)
> yacas("%")
expression((x + 1)^3)
> yacas("z:= %")
expression((x + 1)^3)
> (1+x)^3
expression((x + 1)^3)
> zs <- Sym("%")
> zs
expression((x + 1)^3)
```

#### 2.6 Setting and clearing a variable

The function Set() and the operator := can both be used to assign values to global variables in yacas:

```
> yacas("n := 10")
expression(10)
> yacas("n := n + n")
expression(20)
> yacas("Set(z, Cos(a))")
expression(TRUE)
> yacas("z+z")
```

```
expression(2 * cos(a))
Notice that these are variables in yacas, not in R. Variables in yacas can be removed Clear()
> yacas("n")
expression(20)
> yacas("Clear(n, z)")
expression(TRUE)
> yacas("n")
expression(n)
Variables in yacas can also be set with Set() from R:
> Set(m, 10)
expression(10)
Now m exists as a variable in yacas (and we can make computations on this variable as above). However
we have no handle on this variable in R. Such a handle is obtained with with Sym objects:
> m <- Sym("m")
Now the R variable m refers to the yacas variable m and we can make calculations directly from R, e.g:
> Set(m, 123)
expression(123)
> m^2
expression(15129)
```

### 3 Yacas calculations

## 3.1 Symbolic and numerical calculations, precision

```
> yacas(expression( (1/2) / (4/5) ))
expression(5/8)
> Sym("1/2") / Sym("4/5") ## WRONG RESULT
expression(1/40)
> Sym("1/14 + 5/21 * (30 - 1 + 1/2)")
expression(149/21)
> Sym("55/10")
expression(11/2)
Notice that rational numbers will stay rational as long as numerators and denominators are integers.
Evaluations are generally exact:
> yacas("Exp(0)")
expression(1)
> yacas("Exp(1)")
expression(exp(1))
> yacas("Sin(Pi/4)")
```

```
expression(root(1/2, 2))
> yacas("355/113")
expression(355/113)
> exp(Sym(0))
expression(1)
> exp(Sym(1))
expression(exp(1))
> sin(Pi/4)
expression(root(1/2, 2))
> Sym("355/113")
expression(355/113)
To obtain a numerical evaluation (approximation), the N() function can be used: The N() function has
an optional second argument, the required precision:
> x <- Sym("55/10")
> N(x)
expression(5.5)
> Eval( N( x ) )
[1] 5.5
> yacas("355/113")
expression(355/113)
> N( yacas("355/113"), 8 )
expression(3.14159292)
> N( "355/113", 8 )
expression(3.14159292)
>
The command Precision(n) can be used to specify that all floating point numbers should have a fixed
precision of n digits:
> yacas("Precision(5)")
expression(Precision(5))
> yacas("N(355/113)")
expression(3.1415929203)
> ## Alternative
> ## Precision(5)
> ## N("355/113")
Combining symbolic and numerical expressions:
> x <- Sym("x")
> N(\sin(1)^2 + \cos(x)^2)
expression(cos(x)^2 + 0.708073418273571)
```

#### 3.2 Differentiation

```
> yacas("D(x) Sin(x)")
expression(cos(x))
> yacas("D(x, 2) Sin(x)")
expression(-sin(x))
> x <- Sym("x")
> deriv(sin(x), x)
expression(cos(x))
```

## 3.3 Integration

```
> yacas("Integrate(x,a,b)Sin(x)")
expression(cos(a) - cos(b))
> a <- Sym("a"); b <- Sym("b")
> Integrate(sin(x), x, a, b)
expression(cos(a) - cos(b))
```

## 3.4 Expanding polynomials

```
> yacas("Expand( (1 + x)^3)" )
expression(x^3 + 3 * x^2 + 3 * x + 1)
> yacas("Factor( x^2 - 1 )" )
expression((x + 1) * (x - 1))
> xs <- Sym("xs")
> Expand((1+xs)^3)
expression(xs^3 + 3 * xs^2 + 3 * xs + 1)
> Factor(xs^2-1)
expression((xs + 1) * (xs - 1))
```

### 3.5 Taylor expansion

```
> yacas("texp := Taylor(x,0,3) Exp(x)")
expression(x + x^2/2 + x^3/6 + 1)
> xs <- Sym("xs")
expression(xs)
> texp <- Taylor(exp(xs), xs, 0, 3)
expression(xs + xs^2/2 + xs^3/6 + 1)
Expand exp(x) in three terms around 0 and a:
> yacas("Taylor(x,0,3) Exp(x)")
expression(x + x^2/2 + x^3/6 + 1)
> yacas("Taylor(x,a,3) Exp(x)")
```

## 3.6 Inverse Taylor

The InverseTaylor() function builds the Taylor series expansion of the inverse of an expression. For example, the Taylor expansion in two terms of the inverse of  $\exp(x)$  around x = 0 (which is the Taylor expansion of  $\ln(y)$  around y = 1):

```
> yacas("InverseTaylor(x,0,2)Exp(x)")
expression(x - 1 - (x - 1)^2/2)
> yacas("Taylor(y,1,2)Ln(y)")
expression(y - 1 - (y - 1)^2/2)
> ys <- Sym("ys"); xs <- Sym("xs")
> InverseTaylor(exp(xs),xs,0,2)
expression(xs + xs^2/2 + 1)
> Taylor(log(ys),ys,1,2)
expression(ys - 1 - (ys - 1)^2/2)
```

#### 3.7 Factorial

```
> yacas("40!")
expression(8.15915283247898e+47)
> yacas("40!", retclass = "character")
815915283247897734345611269596115894272000000000
> Factorial(40)
expression(Factorial(40))
```

#### 3.8 Limits

```
> yacas( "Limit(x,0) Sin(x)/x" )
expression(1)
> yacas( "Limit(n,Infinity) (1+(1/n))^n" )
expression(exp(1))
> yacas( "Limit(h,0) (Sin(x+h)-Sin(x))/h" )
expression(cos(x))
```

```
> x <- Sym("x"); n <- Sym("n"); h <- Sym("h")
> Limit( sin(x) / x, x, 0)
expression(1)
> Limit( (1 + (1 / n))^n, n, Infinity )
expression(exp(1))
> Limit( (sin(x + h) - sin(x) ) / h, h, 0)
expression(cos(x))
```

### 3.9 Simplifying an expression

The function Simplify() attempts to reduce an expression to a simpler form.

```
> y <- Sym("y")
> yacas("(x+y)^3-(x-y)^3")
expression((x + y)^3 - (x - y)^3)
> yacas("Simplify(%)")
expression(6 * (x^2 * y) + 2 * y^3)
> (x+y)^3-(x-y)^3
expression((x + y)^3 - (x - y)^3)
> Simplify("%")
expression(6 * (x^2 * y) + 2 * y^3)
```

#### 3.10 Complex numbers and the imaginary unit

The imaginary unit i is denoted I and complex numbers can be entered as either expressions involving I or explicitly Complex(a,b) for a+ib.

```
> yacas("I")
expression(complex_cartesian(0, 1))
> yacas("I^2")
expression(-1)
> yacas("7+3*I")
expression(complex_cartesian(7, 3))
> yacas("Conjugate(%)")
expression(complex_cartesian(7, -3))
> yacas("Exp(3*I)")
expression(complex_cartesian(cos(3), sin(3)))
> I
expression(complex_cartesian(0, 1))
> I^2
expression(-1)
> 7+3*I
```

```
expression(complex_cartesian(7, 3))

> Conjugate("%")
expression(complex_cartesian(7, -3))

> exp(3*I)
expression(complex_cartesian(cos(3), sin(3)))

> z < -7+3*I
> 1/z
expression(complex_cartesian(7/58, -3/58))

> z*(1/z)
expression(1)

We get the famous identity exp(i\pi) = -1 with

> exp(I*Pi)
expression(-1)
```

### 3.11 Solving equations

#### 3.11.1 Solving equations symbolically

Solve equations symbolically with the Solve() function:

```
> yacas("Solve(x/(1+x) == a, x)")
expression(list(x == a/(1 - a)))
> yacas("Solve(x^2+x == 0, x)")
expression(list(x == 0, x == -1))
> Solve(xs/(1+xs) == as, xs)
expression(list(xs == as/(1 - as)))
> Solve(xs^2+xs == 0, xs)
expression(list(xs == 0, xs == -1))
> Solve(List(xs^2+ys^2==6, xs-ys==3), List(xs,ys))
expression(list(list(xs == root(6 - ys^2, 2), ys == ys)))
> mu <- Sym("mu") # mean
> v <- Sym("v") # variance
> Solve(List(mu==(xs/(xs+ys)), v==((xs*ys)/(((xs+ys)^2) * (xs+ys+1)))),
+ List(xs,ys))
expression(list(list(xs == mu^2 * (1 - mu)/v - mu, ys == xs/mu - xs)))
```

(Note the use of the == operator, which does not evaluate to anything, to denote an "equation" object.)

#### 3.11.2 Solving equations numerically

To solve an equation (in one variable) like sin(x) - exp(x) = 0 numerically taking 0.5 as initial guess and an accuracy of 0.0001 do:

```
> yacas("Newton(Sin(x)-Exp(x),x, 0.5, 0.0001)")
```

```
expression(-3.1830630118)
> Newton(sin(xs)-exp(xs),xs, 0.5, 0.0001)
expression(-3.1830630118)
```

## 3.12 Solving ordinary differential equations

```
> yacas("OdeSolve(y''==4*y)")
expression(C12403 * exp(2 * x) + C12407 * exp(-2 * x))
> yacas("OdeSolve(y'==8*y)")
expression(C12437 * exp(8 * x))
```

### 4 Matrices

```
> PrettyPrinter()
expression(PrettyPrinter())
> A <- yacas( "A:={{4,-2,4,2},{-2,10,-2,-7},{4,-2,8,4},{2,-7,4,7}}" )
> PrettyForm(A) ## FIXME: SHOULD THIS WORK??
list( list( 4 , -2 , 4 , 2 ) , list( -2 , 10 , -2 , -7 ) ,
list(4,-2,8,4), list(2,-7,4,7))
<OMOBJ>
 <OMS cd="logic1" name="true"/>
</OMOBJ>
> yacas("PrettyForm(A)")
| (4) (-2) (4) (2)
| ( -2 ) ( 10 ) ( -2 ) ( -7 )
| (4) (-2)(8) (4)
| (2) (-7)(4) (7)
<OMOBJ>
 <OMS cd="logic1" name="true"/>
</OMOBJ>
> R <- yacas("R := Cholesky(A)")
> yacas("PrettyForm(R)")
| (2)(-1)(2)(1)
| (0)(3)(0)(-2)
| (0)(0)(2)(1)
```

```
| (0)(0)(0)(1)
<OMOBJ>
 <OMS cd="logic1" name="true"/>
</OMOBJ>
> yacas("RtR:=Transpose(R) * R")
expression(list(list(4, -2, 4, 2), list(-2, 10, -2, -7), list(4,
   -2, 8, 4), list(2, -7, 4, 7)))
> yacas("PrettyForm(RtR)")
| (4) (-2) (4) (2)
| ( -2 ) ( 10 ) ( -2 ) ( -7 )
| (4) (-2)(8) (4)
| (2) (-7) (4) (7)
<OMOBJ>
 <OMS cd="logic1" name="true"/>
</OMOBJ>
> yacas("M:={ {u1,u1,0},{u1,0,u2},{0,u2,0} }")
expression(list(list(u1, u1, 0), list(u1, 0, u2), list(0, u2,
   0)))
> yacas("PrettyForm(M)")
(u1)(u1)(0)
(u1)(0)(u2)
| (0) (u2)(0)
<OMOBJ>
 <OMS cd="logic1" name="true"/>
</OMOBJ>
> u1 <- Sym("u1"); u2 <- Sym("u2")
> M <- List(List(u1, u1, 0), List(u1, 0, u2), List(0, u2, 0))
> PrettyForm( M )
| (u1)(u1)(0)
| (u1)(0)(u2)
(0) (u2)(0)
```

```
<OMS cd="logic1" name="true"/>
> yacas(TeXForm(M), retclass = "unquote")
4.1
             Inverse
> yacas("Mi:=Inverse( M )")
expression(list(list(u2^2/(u1 * u2^2), 0, -(u1 * u2)/(u1 * u2^2)),
            list(0, 0, u1 * u2/(u1 * u2^2)), list(-(u1 * u2)/(u1 * u2^2),
                        u1 * u2/(u1 * u2^2), u1^2/(u1 * u2^2))))
> yacas("Simplify( Mi )")
expression(list(1/u1, 0, -1/u2), list(0, 0, 1/u2), list(-1/u2, 1/u2), list(-1/u2), list(-1/
            1/u2, u1/u2^2)))
> yacas("PrettyForm(Simplify( Mi ))")
| / 1 \ ( 0 ) / -1 \
| | -- |
                                             | -- |
| \ u1 /
                                               \ u2 /
|(0)(0)/1\
                                                | -- |
                                                \ u2 /
| / -1 \ / 1 \ / u1 \
| | -- | | -- | | --- |
|\u2/\u2/| 2||
                                               \ u2 / |
\
<OMOBJ>
      <OMS cd="logic1" name="true"/>
</OMOBJ>
> Mi <- Inverse( M )</pre>
> Simplify( Mi )
expression(list(list(1/u1, 0, -1/u2), list(0, 0, 1/u2), list(-1/u2,
            1/u2, u1/u2^2)))
> PrettyForm(Simplify( Mi ))
| / 1 \ ( 0 ) / -1 \
| | -- |
                                               | -- |
| \ u1 /
                                               \ u2 /
| (0) (0) /1 \
                                                | -- |
                                                \ u2 /
| / -1 \ / 1 \ / u1 \
```

<OMOBJ>

```
| \ u2 / \ u2 / | 2 | |
           \ u2 / |
\
<OMOBJ>
  <OMS cd="logic1" name="true"/>
</OMOBJ>
4.2 Determinant
> yacas("Determinant( M )")
expression(-(u1 * u2^2))
> yacas("Determinant( Mi )") ## FIXME: Whats up here?
expression(-(u2^2 * (u1 * u2)^2)/(u1 * u2^2)^3)
> yacas("Simplify( Mi )")
expression(list(list(1/u1, 0, -1/u2), list(0, 0, 1/u2), list(-1/u2,
   1/u2, u1/u2^2)))
> yacas("Simplify(Determinant( Mi ))") ## FIXME: Whats up here?
expression(-1/(u2^2 * u1))
> determinant( M )
expression(-(u1 * u2^2))
> determinant( Mi )
expression(-(u2^2 * (u1 * u2)^2)/(u1 * u2^2)^3)
> Simplify( Mi )
expression(list(list(1/u1, 0, -1/u2), list(0, 0, 1/u2), list(-1/u2,
   1/u2, u1/u2^2)))
```

## 5 Printing with PrettyForm, PrettyPrint, TexForm and TeX-Form

Printing the result in nice forms:

> Simplify(determinant( Mi ))
expression(-1/(u2^2 \* u1))

#### 5.0.1 Standard form

```
The (standard) yacas form is:
> yacas("A:={{a,b},{c,d}}")
expression(list(list(a, b), list(c, d)))
> yacas("B:= (1+x)^2+k^3")
expression((x + 1)^2 + k^3)
> yacas("A")
expression(list(list(a, b), list(c, d)))
> yacas("B")
expression((x + 1)^2 + k^3)
> as <- Sym("as"); bs <- Sym("bs"); cs <- Sym("cs"); ds <- Sym("ds")
> A <- List(List(as,bs), List(cs,ds))</pre>
> ks <- Sym("ks")
> B <- (1+xs)^2+ks^3
> A
expression(list(list(as, bs), list(cs, ds)))
expression((xs + 1)^2 + ks^3)
```

#### 5.0.2 Pretty form

The Pretty form is:

```
<OMOBJ>
 <OMS cd="logic1" name="true"/>
</OMOBJ>
> yacas("PrettyForm(B)")
       2 3
(x + 1) + k
<OMOBJ>
  <OMS cd="logic1" name="true"/>
</OMOBJ>
> PrettyForm(A)
| (as)(bs) |
| (cs)(ds) |
<OMOBJ>
  <OMS cd="logic1" name="true"/>
</OMOBJ>
> PrettyForm(B)
         2
(xs + 1) + ks
<OMOBJ>
  <OMS cd="logic1" name="true"/>
</OMOBJ>
```

#### 5.0.3 TeX form

The output can be displayed in TeX form:

```
> yacas("TeXForm(B)", retclass = "character")
'$\left( x + 1\right) ^{2} + k ^{3}$'
```

This function sets up the function printer to print out the results on the command line. This can be reset to the internal printer with PrettyPrinter().

Currently implemented prettyprinters are: PrettyForm, TeXForm, Print and DefaultPrint.

```
> PrettyPrinter()
expression(PrettyPrinter())
```