Math 128B, Spring 2021.

Homework 1, due January 30.

Prob 1. Verify that the function

$$\|x\|_3 := \left(\sum_{j=1}^{n} |x_j|^3 \right)^{1/3}$$

is a norm on $\mathbb{C}^n$. As we know, it must be equivalent to the 1-norm $\|\cdot\|_1$. Find, with proof, at least one pair of constants $(0 <)c < C$ such that

$$c\|x\|_1 \leq \|x\|_3 \leq C\|x\|_1 \qquad \forall x \in \mathbb{C}^n.$$

Prob 2. Using MATLAB, plot the unit sphere for the norms $\|\cdot\|_1$ and $\|\cdot\|_3$ in the space $\mathbb{R}^3$. First plot them separately then, if you are feeling adventurous, together, in different colors.

Prob 3. For each of the following matrices, find its ∞-norm and its 2-norm, using MATLAB or by hand:

$$(a)\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \qquad (b)\begin{bmatrix} -2 & 3 \\ 3 & -2 \end{bmatrix}, \qquad (c)\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 3 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}.$$

Prob 4. Show that if a matrix A is symmetric, then its 2-norm is equal to its spectral radius.