

Math 128B, Spring 2021.  
**Homework 8, due April 10.**

**Prob 1.** Find the continuous least squares trigonometric polynomial  $S_n$  (following the book notation) for  $f(x) = e^x$  on the interval  $[-\pi, \pi]$ .

**Prob 2.** Determine the discrete least squares trigonometric polynomial  $S_4$  for  $f(x) = e^x$  on the interval  $[-\pi, \pi]$ , and compute the error  $E(S_4)$  of this approximation.

**Prob 3.** Determine the trigonometric interpolating polynomial of degree 4 for  $f(x) = x(\pi - x)$  on the interval  $[-\pi, \pi]$  using (a) direct calculation; (b) FFT.

**Prob 4.** (a) Show that  $c_0, \dots, c_{2m-1}$  in Algorithm 8.3 are given by

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_{2m-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{2m-1} \\ 1 & \omega^2 & \omega^4 & \cdots & \omega^{4m-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{2m-1} & \omega^{4m-2} & \cdots & \omega^{(2m-1)^2} \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_{2m-1} \end{bmatrix},$$

where  $\omega := e^{\pi i/m}$ . (b) What are the eigenvalues of this matrix?