hw9_p3_newtons

April 28, 2021

[190]:
```
# P3 Newton's Method
using LinearAlgebra
function newtons(x0, F, J, random_start, tol=1e-10, maxit=100, verbose=false)
    #==
    Implementation of Newton's method assuming direct access to J
    ==#
    if random_start == true
        # generate random start
        x0 = rand(Float64, (length(x0), 1));
    end
    N = length(x0);
    all_x = zeros(N, maxit);
    all_x[:, 1] = x0;
    num_it = 1;
    all_err = [];
    x_curr = x0;
    while (num_it <= maxit)
        i = num_it;
        x_prev = x_curr;
        # solve for y
        y = -J(x_curr)\F(x_curr);
        # update x_curr
        x_curr = x_curr + y;
        # store history
        all_x[:, i] = x_curr;
        # check convergence
        err = norm(y);
        all_err = append!(all_err, err);
        if err < tol
            println("===== converged");
            break
        end
        num_it += 1;
    end
    return all_x[:, 1:num_it-1], all_err
end
```

```
# P3 functions
function F(x)
    #== helper function, takes in x of size 3 by 1==#
    x1 = x[1];
    x2 = x[2];
    x3 = x[3];
    return [x1^2+x2-37; x1-x2^2-5; sum(x)-3];
end

function J(x)
    #== helper function, takes in x of size 3 by 1 and
    computes Jacobian of size 3 by 3 ==#
    x1, x2, x3 = x;
    N = length(x);
    Jx = zeros(N, N);
    Jx[1, 1] = 2*x1;
    Jx[1, 2] = 1;
    Jx[2, 1] = 1;
    Jx[2, 2] = -2*x2;
    Jx[3, 1] = 1;
    Jx[3, 2] = 1;
    Jx[3, 3] = 1;
    return Jx;
end
```

[190]:
```
J (generic function with 1 method)
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[191]:
```
x0 = [100;1000;13210];
all_x, all_err = newtons(x0, F, J, true, 1e-10);
```

```
===== converged
```

[192]:
```
all_x
```

[192]:
```
3×10 Matrix{Float64}:
  49.2311   24.8454   13.0171    7.77926  …   5.99749   5.99996   6.0   6.0
  28.0316   14.3698    7.46384   3.9181       1.03049   1.00043   1.0   1.0
 -74.2627  -36.2152  -17.4809   -8.69736     -4.02798  -4.0004   -4.0  -4.0
```

[193]:
```
all_err
```

[193]:
```
11-element Vector{Any}:
 93.51644466724949
 47.21145333109397
 23.207141626254703
 10.823973300783608
  4.240792760066012
  1.2817266334696407
```

0.3461529384813618
0.040870629089773
0.0005898078630249815
1.2286133417257642e-7
4.8255478180902664e-15