Math 128B, Spring 2021.

Homework 7, due March 27.

Prob 1. Find the least squares polynomial approximation of degrees 2 and 3 to the function $f(x) = e^x$ on the interval $[-1, 1]$.

Prob 2. Show that for any positive integers $i > j$, the following identity holds:

$$T_i(x)T_j(x) = \frac{1}{2}\left[T_{i+j}(x) + T_{i-j}(x)\right].$$

Prob 3. Derive the 3-term recurrence relation for the Laguerre polynomials L_n, which are orthogonal with respect to the weight function $w(x) = e^{-x}$ on the interval $(0, \infty)$. Plot the polynomials L_0, L_1, L_2 and L_3 on an interval containing all their zeros. Do you observe interlacing? Discuss.

Prob 4. Determine the Padé approximation of degree 6 with $n = 2$, $m = 4$ to the function $f(x) = \sin x$.

Prob 5. Express the following rational functions as continued fractions:

$$(a)\ \frac{4x^2+3x-7}{2x^3+x^2-x+5}, \qquad (b)\ \frac{2x^3+x^2+3x-1}{3x^3+x^2-x+1}.$$