Math 128B, Spring 2021.

Homework 4, due February 20.

Prob 1. Create a MATLAB function that inputs a matrix A, vectors b and $x^{(0)}$ and a tolerance tol and finds an approximate solution to $Ax = b$ using the conjugate gradient method (without preconditioning). The algorithm should terminate after n steps and should output an error message in case the desired precision was not reached. Run the algorithm on a linear system of your choice.

Prob 2. Same as in Prob 1, with an additional (input) matrix C used for preconditioning. Run an example for a linear system and several matrices C. What are good and bad choices of C? Discuss.

Prob 3. Use the Gershgorin Circle theorem to determine bounds for the eigenvalues and the spectral radius of the following matrices:

$$(a)\ \begin{bmatrix} 4 & 0 & 1 & 3 \\ 0 & 4 & 2 & 1 \\ 1 & 2 & -2 & 0 \\ 3 & 1 & 0 & 4 \end{bmatrix}, \qquad (b)\ \begin{bmatrix} 1 & 0 & -1 & 1 \\ 2 & 2 & -1 & 1 \\ 0 & 1 & 3 & -2 \\ 1 & 0 & 1 & 4 \end{bmatrix}$$

Prob 4. Suppose that A and B are nonsingular $n \times n$ matrices. Show that AB is similar to BA.