MATH 128B, Spring 2021, midterm test.

All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit. Partial credit **may** be given but only for significant progress towards a solution. Cross out all work you do not wish considered. Books and notes are allowed. Electronic devices are not allowed during the test.

1. (10pts.) Determine, with proof, the limit $\lim_{n\to\infty} A^n v$ where

$$\begin{bmatrix} 2/3 & -3 & 12 \\ 0 & 1/2 & 1/2 \\ 0 & 1/3 & 1/2 \end{bmatrix}, \qquad v = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

2. (10pts.) Determine, with proof, the ℓ_2-norm and the 2-condition number of the matrix

$$\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}.$$

3. (10pts.) Using the initial vector $x^{(0)} = 0$, find two iterations of the Jacobi method for the linear system

$$\begin{aligned} -2x_1 + x_2 + \frac{1}{2}x_3 &= 4 \\ x_1 - 2x_2 - \frac{1}{2}x_3 &= 2 \\ x_2 + 2x_3 &= 0. \end{aligned}$$

4. (10pts.) Find all eigenvalues of a real $n \times n$ Householder matrix $I - 2w^t w$ where $\|w\|_2 = 1$.