

Math 128B, Spring 2021.
Homework 3, due February 13.

Prob 1. Find the first two iterations of the SOR method with $\omega = 1.2$ for the following linear system:

$$\begin{aligned} 10x_1 - x_2 &= 9 \\ -x_1 + 10x_2 - 2x_3 &= 7 \\ -2x_2 + 10x_3 &= 6. \end{aligned}$$

Is the corresponding matrix T_ω convergent?

Prob 2. Let $\kappa(A)$ denote the condition number of a (square) matrix A . Show that any singular matrix B satisfies the inequality

$$\frac{1}{\kappa(A)} \leq \frac{\|A - B\|}{\|A\|}.$$

Hint: The null space of B contains a vector of norm 1.

Prob 3. Use Gaussian elimination and three-digit rounding arithmetic to find an approximate solution to

$$0.03x_1 + 58.9x_2 = 59.2$$

$$5.31x_1 - 6.1x_2 = 47.0.$$

Then use one iteration of iterative refinement. Compare both approximations to the exact solution.

Prob 4. The linear system $Ax = b$ with

$$A = \begin{bmatrix} 1 & 2 \\ 1.00001 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 3.00001 \end{bmatrix}$$

has (exact) solution $[1, 1]^T$. Use seven-digit rounding arithmetic to solve the perturbed system with

$$\tilde{A} = \begin{bmatrix} 1 & 2 \\ 1.000011 & 2 \end{bmatrix}, \quad \tilde{b} = \begin{bmatrix} 3.00001 \\ 3.00003 \end{bmatrix}$$

and compare the actual error to the Perturbation Estimate from class (formula (7.25) in our main textbook).