**Math 128B, Spring 2021.**

# Homework 9, due April 17.

**Prob 1.** Use fixed-point iteration to find all solutions to the following nonlinear system, accurate to within $10^{-5}$, using the $\ell_\infty$-norm. Justify that you have indeed found all solutions.

$$\begin{aligned} x_1^2 + x_2^2 - x_1 &= 0 \\ x_1^2 - x_2^2 - x_2 &= 0. \end{aligned}$$

**Prob 2.** Let $A$ be an $n \times n$ matrix and let $F : \mathbb{R}^n \to \mathbb{R}^n$ be defined by $F(x) := Ax$. Show that $F$ is continuously differentiable on $\mathbb{R}^n$. What is its Jacobian at an arbitrary point $x$?

**Prob 3.** Use Newton's method with $x^{(0)} = 0$ to compute $x^{(1)}$ and $x^{(2)}$ for the following nonlinear system:

$$\begin{aligned} x_1^2 + x_2 - 37 &= 0 \\ x_1 - x_2^2 - 5 &= 0 \\ x_1 + x_2 + x_3 - 3 &= 0. \end{aligned}$$

**Prob 4.** The nonlinear system

$$\begin{aligned} 4x_1 - x_2 + x_3 &= x_1 x_4 \\ -x_1 + 3x_2 - 2x_3 &= x_2 x_4 \\ x_1 - 2x_2 + 3x_3 &= x_3 x_4 \\ x_1^2 + x_2^2 + x_3^2 &= 1 \end{aligned}$$

has six solutions. (a) Show that if $(x_1, x_2, x_3, x_4)^T$ is a solution, then so is $(-x_1, -x_2, -x_3, x_4)^T$.
(b) Use Broyden's method three times to approximate each solution. Iterate until $\|x^{(k)} - x^{(k-1)}\|_\infty < 10^{-5}$.