

# CS492: Probabilistic Programming Introduction

Hongseok Yang  
KAIST

This Review ... discusses some of the state-of-the-art advances in the field, namely, **probabilistic programming**, Bayesian optimization, data compression and automatic model discovery.

Zoubin Ghahramani  
2015 Nature Review

What is probabilistic  
programming?

# (Bayesian) probabilistic modelling of data

- I. Develop a new probabilistic (generative) model.
2. Design an inference algorithm for the model.
3. Using the algo., fit the model to the data.

# (Bayesian) probabilistic modelling of data in a prob. prog. language

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as a program

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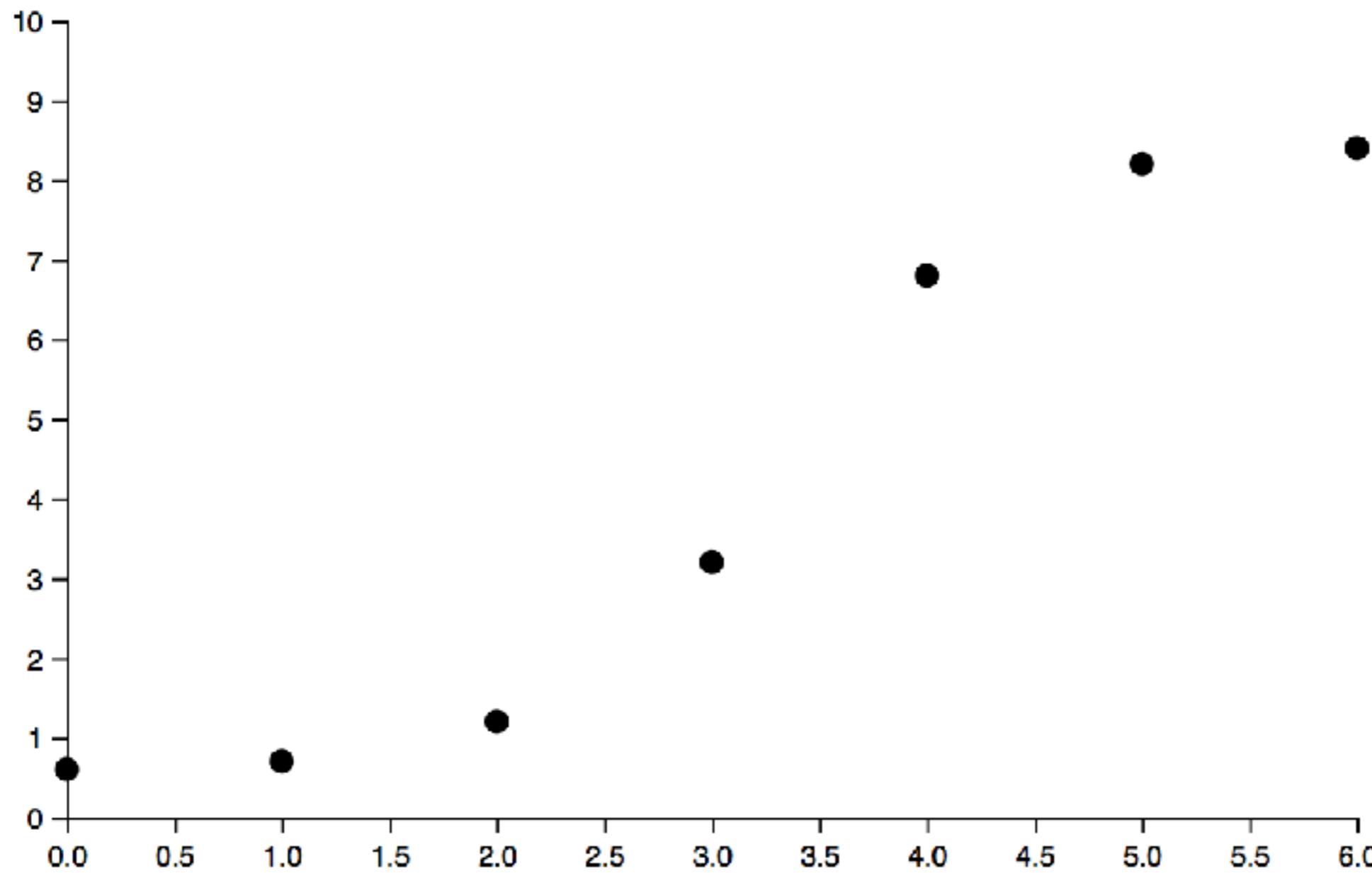
# (Bayesian) probabilistic modelling of data in a prob. prog. language

as a program

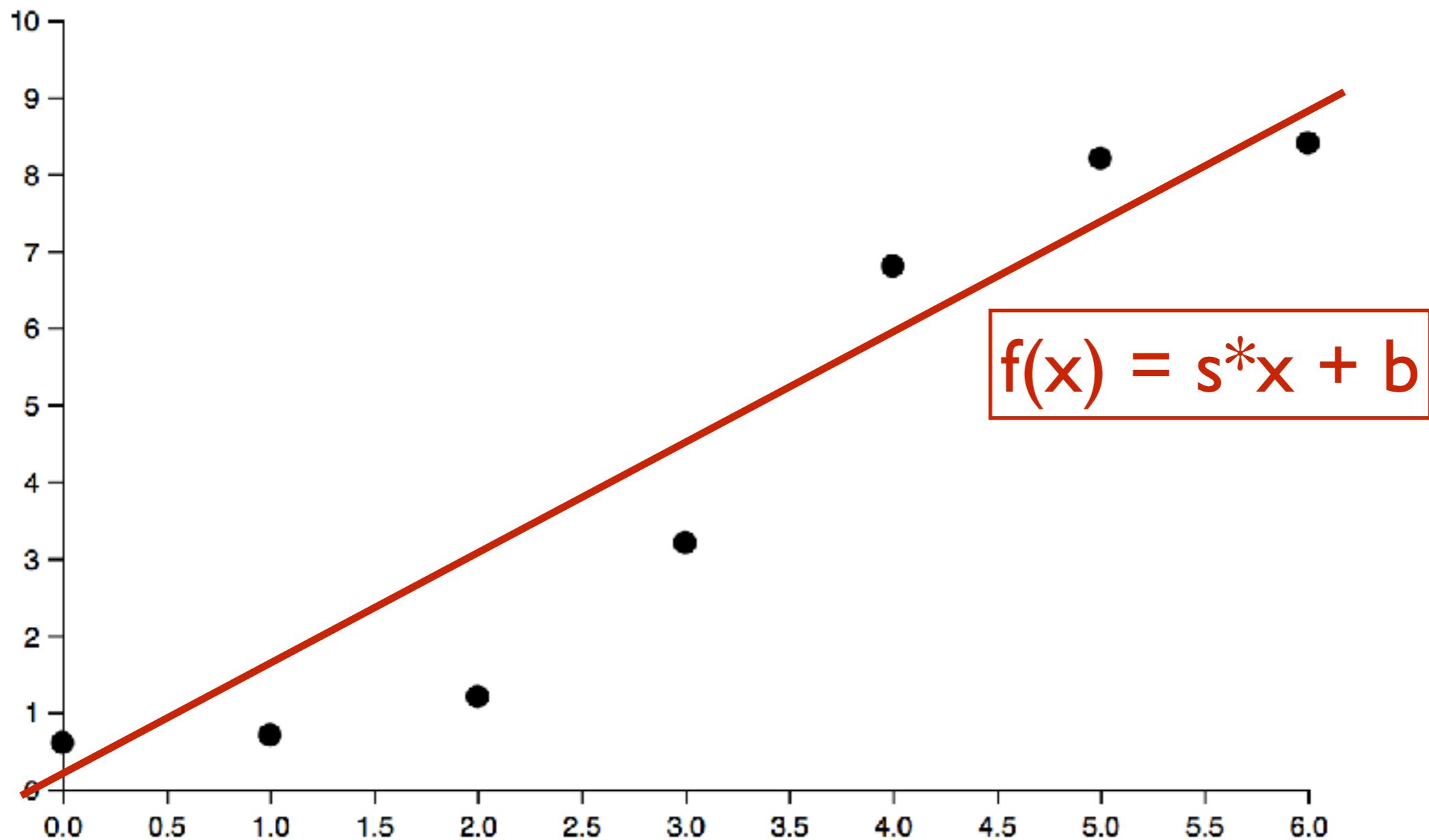
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- ~~2. Design an inference algorithm for the model.~~
3. Using ~~the algo.~~, fit the model to the data.

a generic inference algo.  
of the language

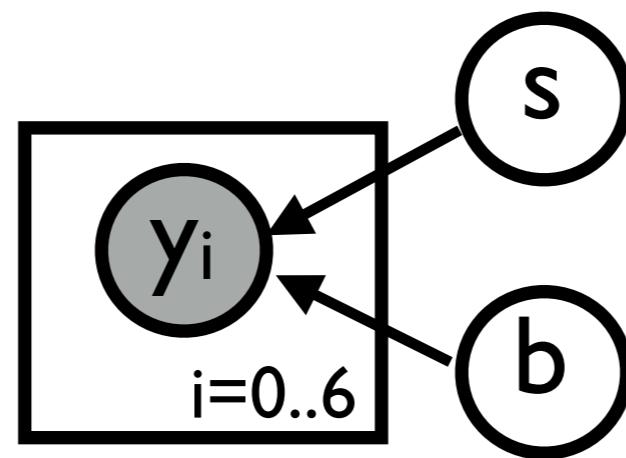
# Line fitting



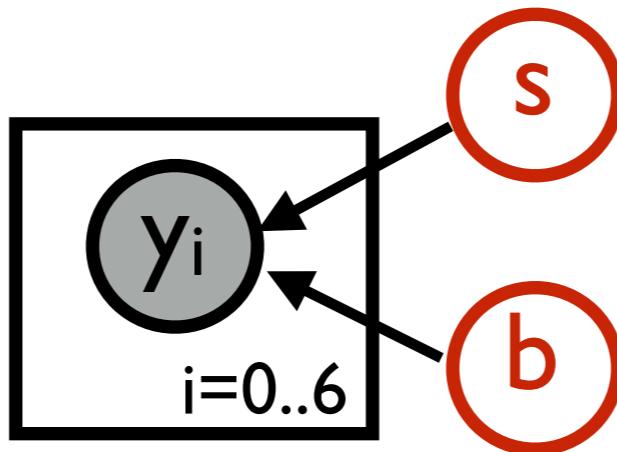
# Line fitting



# Bayesian generative model

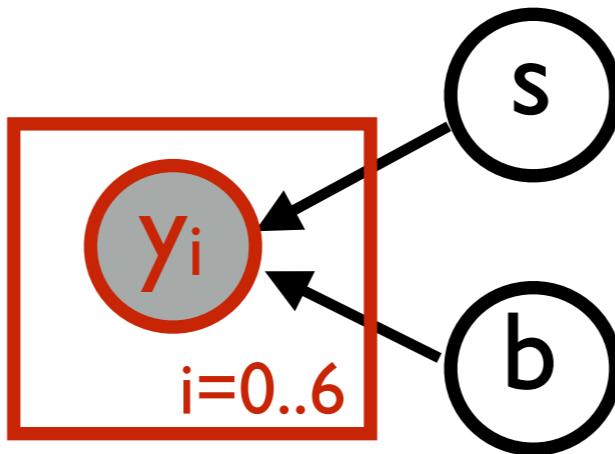


# Bayesian generative model



$s \sim \text{normal}(0, 2)$   
 $b \sim \text{normal}(0, 6)$

# Bayesian generative model



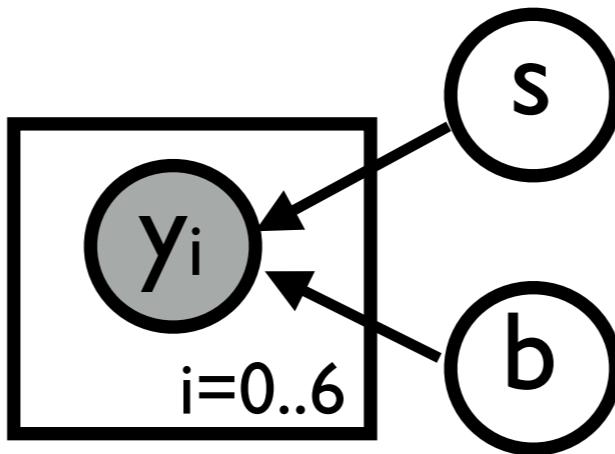
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$f(x) = s*x + b$

$y_i \sim \text{normal}(f(i), 0.5)$   
where  $i = 0 .. 6$

# Bayesian generative model



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 $b \sim \text{normal}(0, 6)$   
 $f(x) = s*x + b$   
 $y_i \sim \text{normal}(f(i), 0.5)$   
where  $i = 0 .. 6$

Q: posterior of  $(s, b)$  given  $y_0=0.6, \dots, y_6=8.4$ ?

# Posterior of s and b given y<sub>i</sub>'s

$$p(s, b | y_0, \dots, y_6) = \frac{p(y_0, \dots, y_6 | s, b) \times p(s, b)}{p(y_0, \dots, y_6)}$$

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# Anglican program

```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]
```

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```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]

  (observe (normal (f 0) .5) .6)
  (observe (normal (f 1) .5) .7)
  (observe (normal (f 2) .5) 1.2)
  (observe (normal (f 3) .5) 3.2)
  (observe (normal (f 4) .5) 6.8)
  (observe (normal (f 5) .5) 8.2)
  (observe (normal (f 6) .5) 8.4))
```

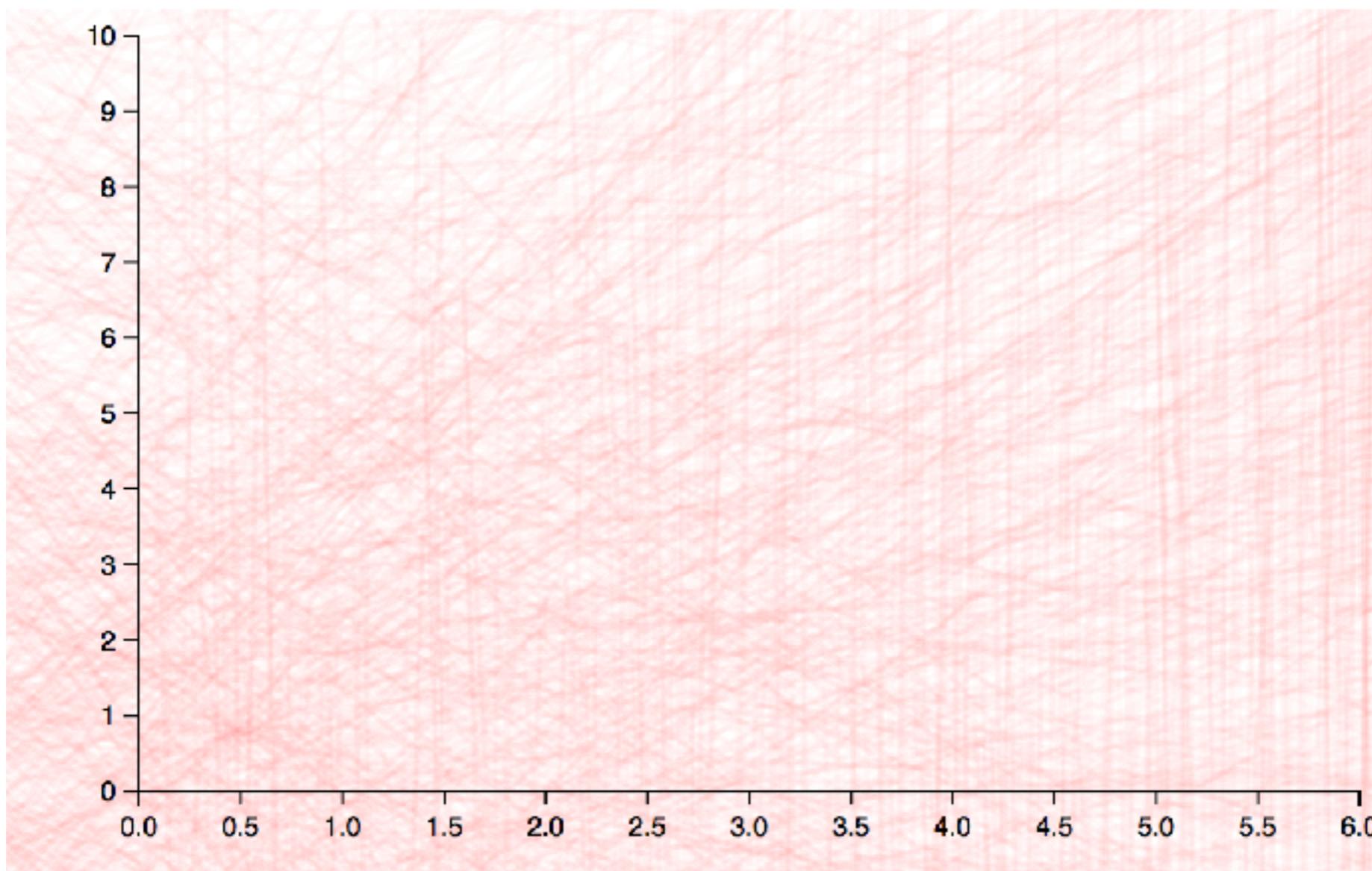
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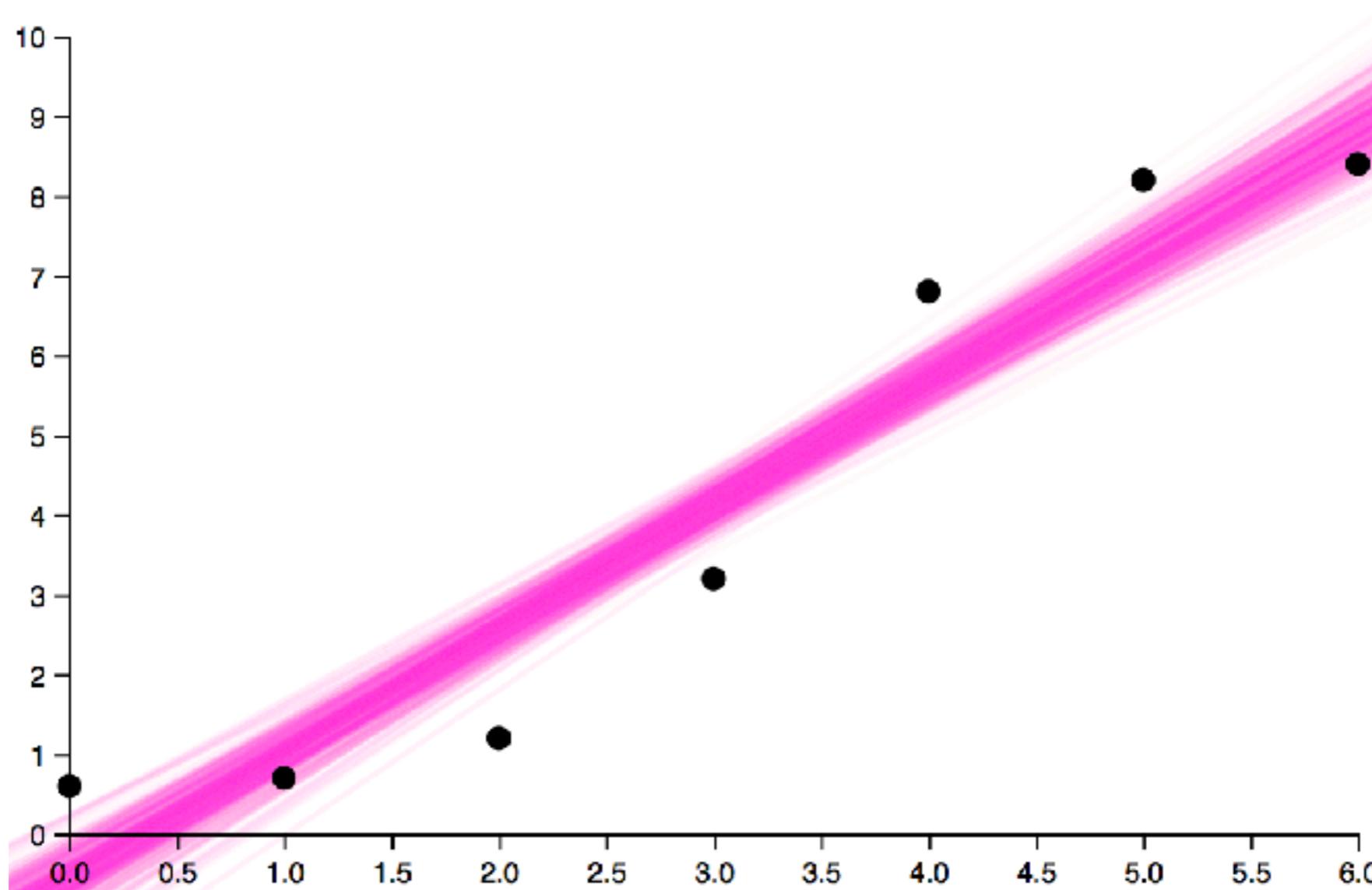
  (observe (normal (f 0) .5) .6)
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  [s b])
```

# Samples from prior



# Samples from posterior

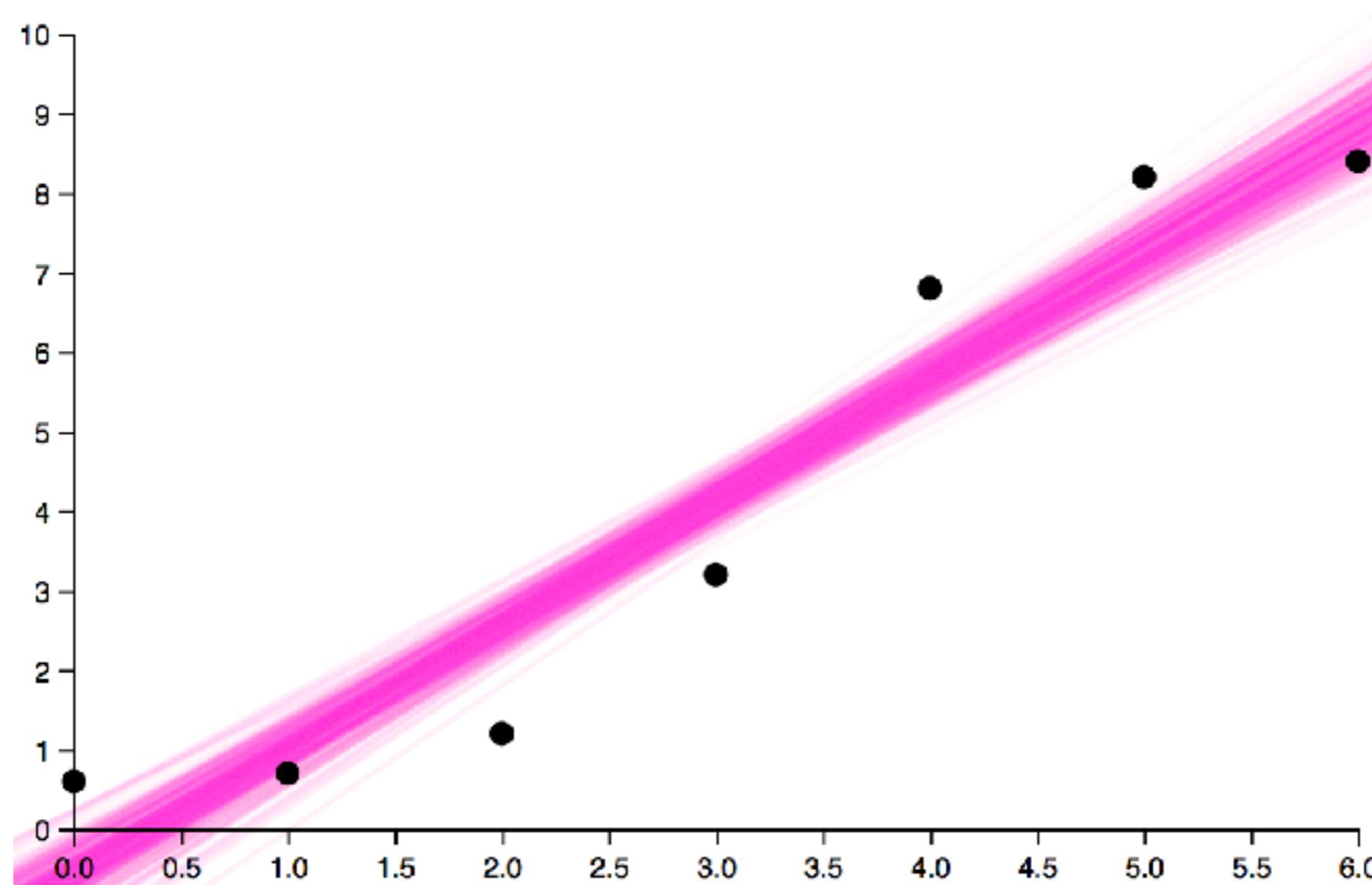


Why should one care  
about prob. programming?

“Because probabilistic programming is a good way to build an AI.” (My ML colleague)

Prob. programming languages enable one to build and explore highly complex models.

# Underfit?



```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]

  (observe (normal (f 0) .5) .6)
  (observe (normal (f 1) .5) .7)
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  [s b])
```

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(let [s (sample (normal 0 2))
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```

[s b])

**Functions as first-class citizen.**

```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]
```

```
(observe (normal (f 0) .5) .6)
(observe (normal (f 1) .5) .7)
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(observe (normal (f 6) .5) 8.4)
```

~~[s b]~~  
f)

Functions as first-class citizen.

```
(let [F (fn []
  (let [s (sample (normal 0 2))
        b (sample (normal 0 6))]
    (fn [x] (+ (* s x) b))))]
  f (F))
(observe (normal (f 0) .5) .6)
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(observe (normal (f 2) .5) 1.2)
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~~Es ist~~  
f)

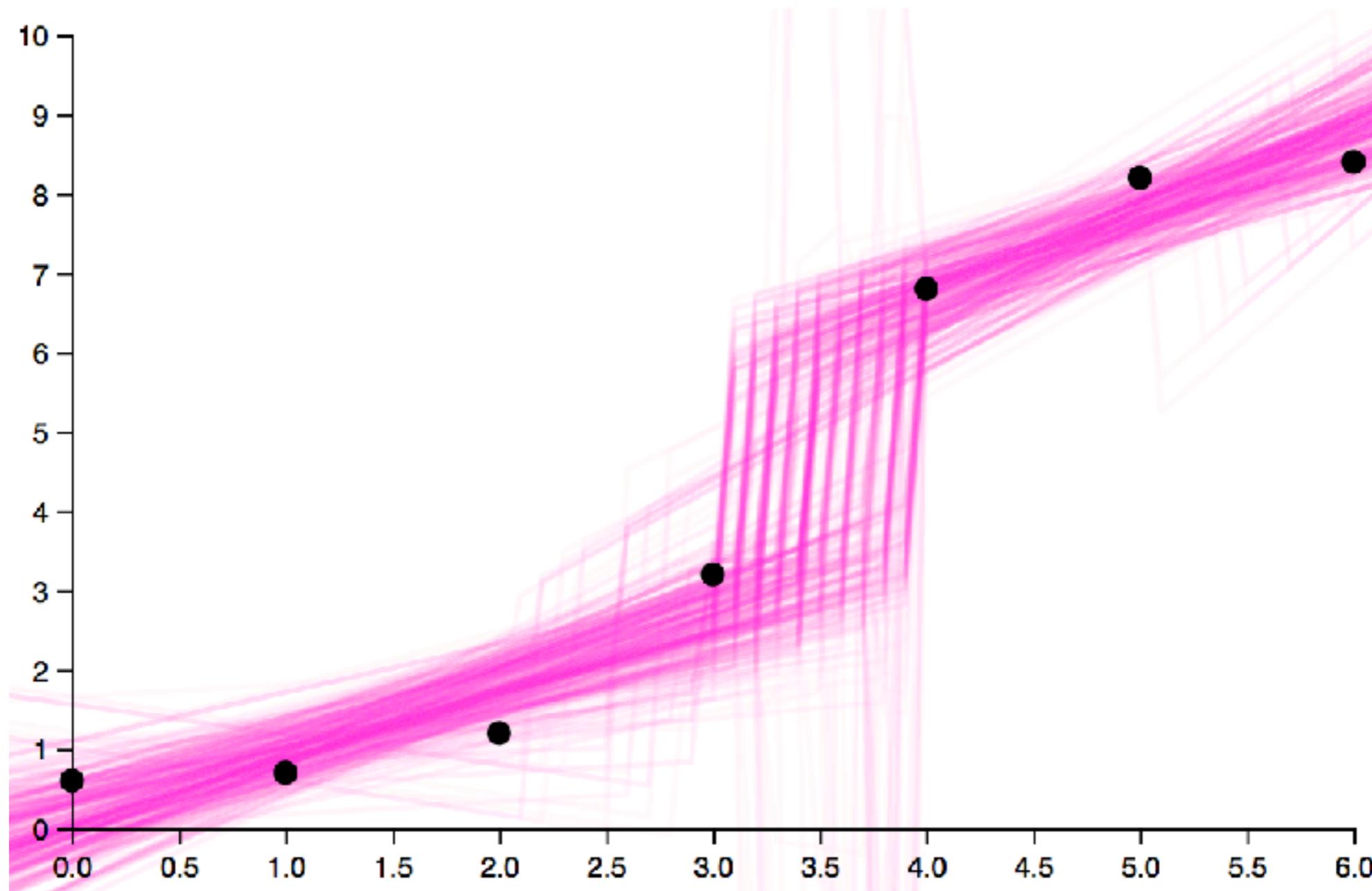
Functions as first-class citizen.

```
(let [F (fn []
  (let [s (sample (normal 0 2))
        b (sample (normal 0 6))])
    (fn [x] (+ (* s x) b))))  
f (add-change-points F 0 6)]  
(observe (normal (f 0) .5) .6)  
(observe (normal (f 1) .5) .7)  
(observe (normal (f 2) .5) 1.2)  
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```

~~Exhibit~~  
f)

Functions as first-class citizen.

# Samples from posterior



```

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    (fn [x] (+ (* s x) b)))))

  f (add-change-points F 0 6)]
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```

~~Exercise~~  
f)

[Q] Change the model so that it finds constant functions with change points.

```

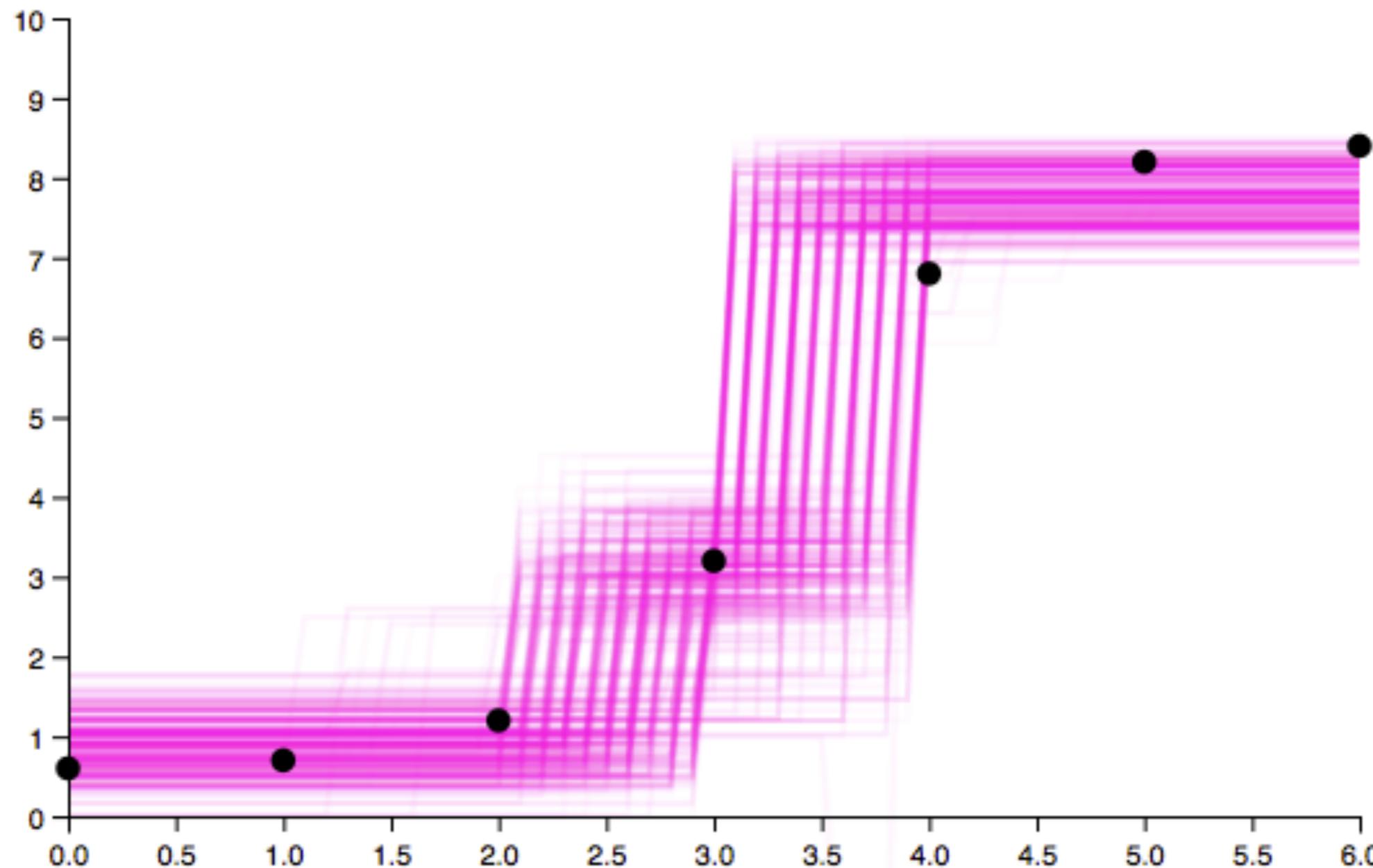
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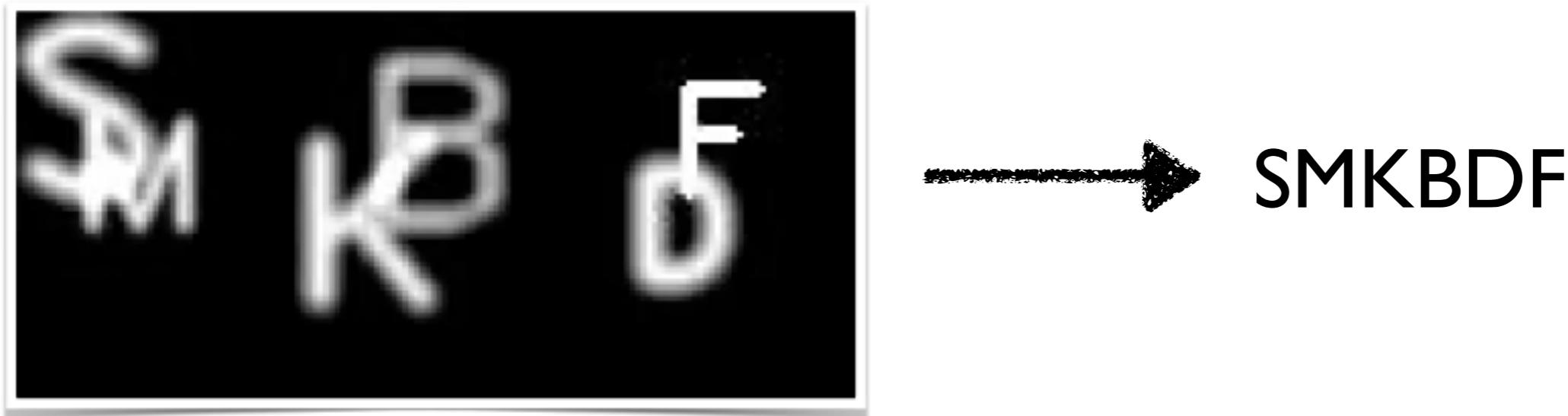
~~Exercise~~  
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[Q] Change the model so that it finds constant functions with change points.

# Samples from posterior

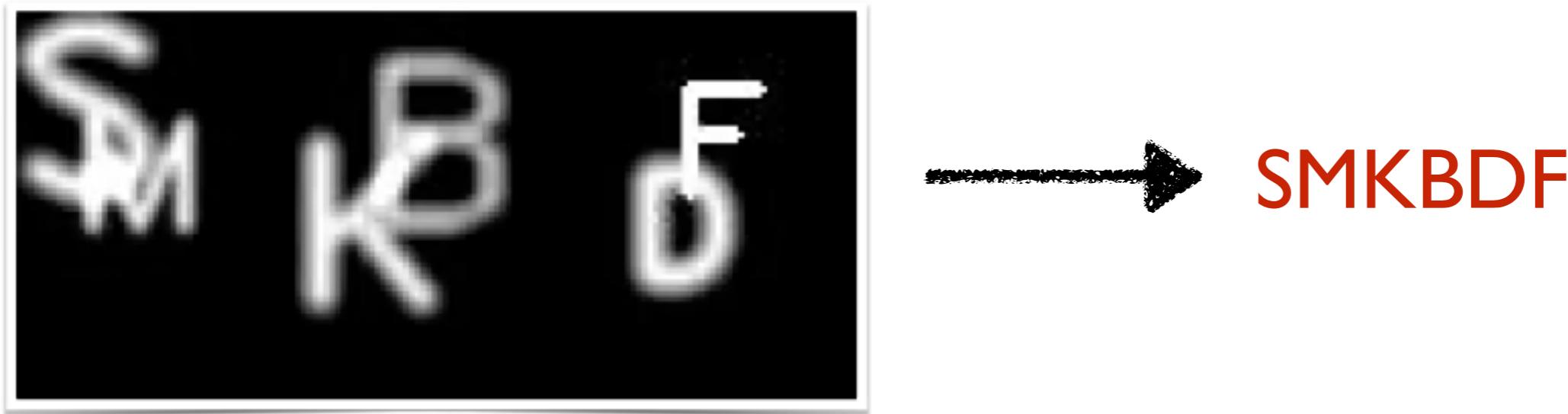


# Success story I: Captcha breaking



Le, Baydin, Wood [2016]

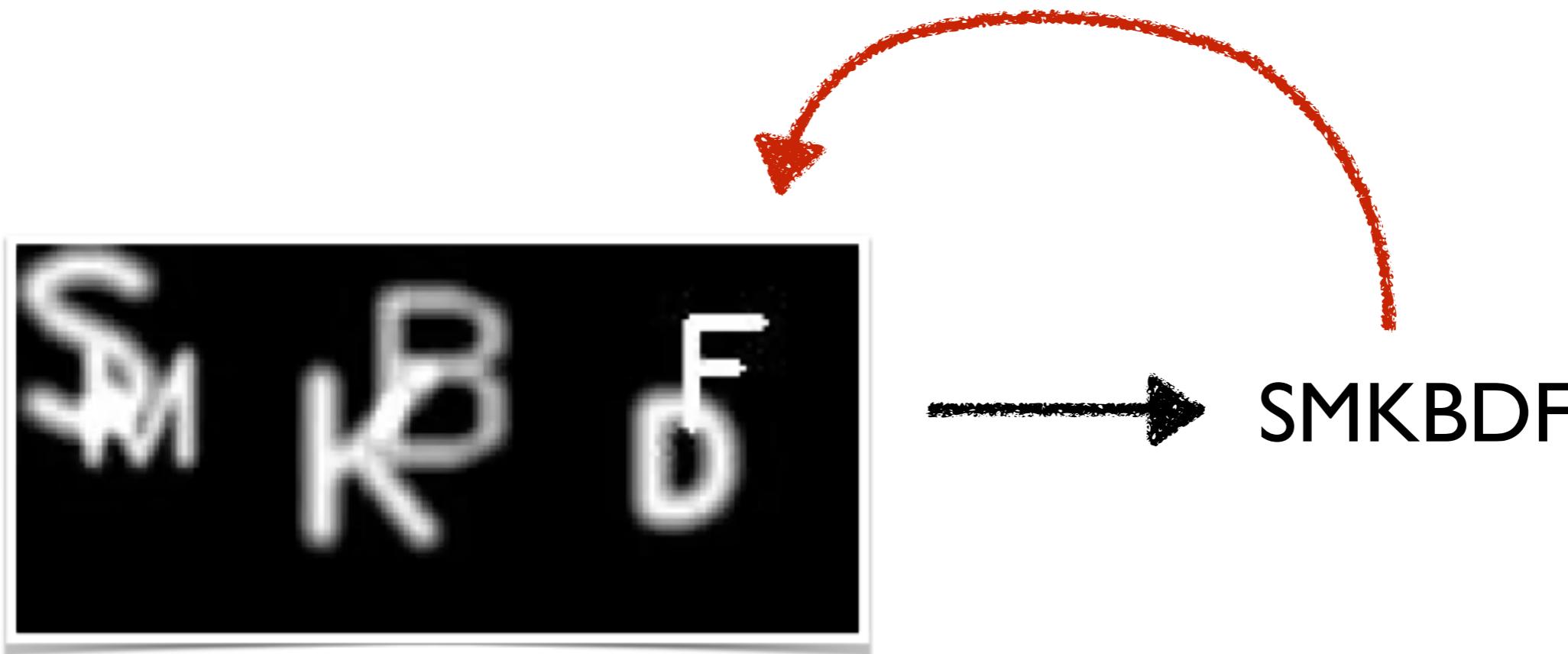
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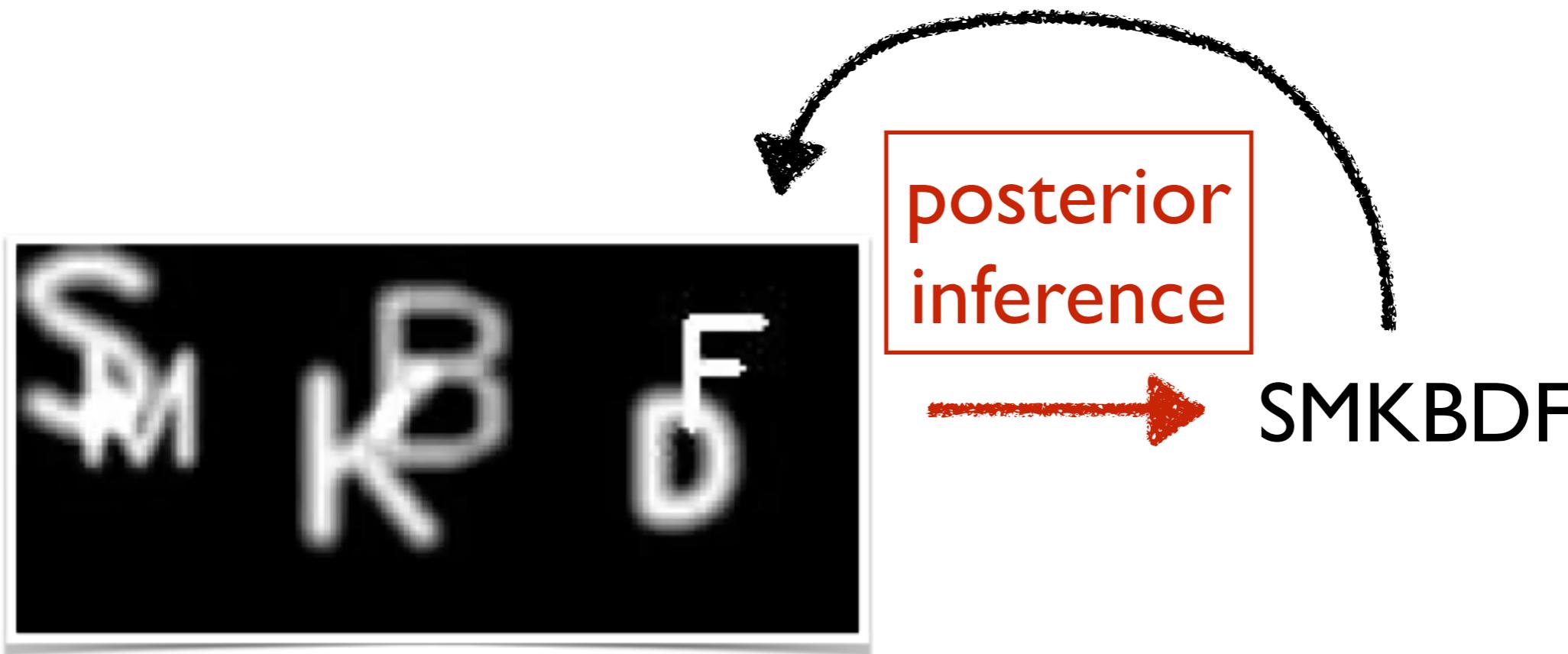
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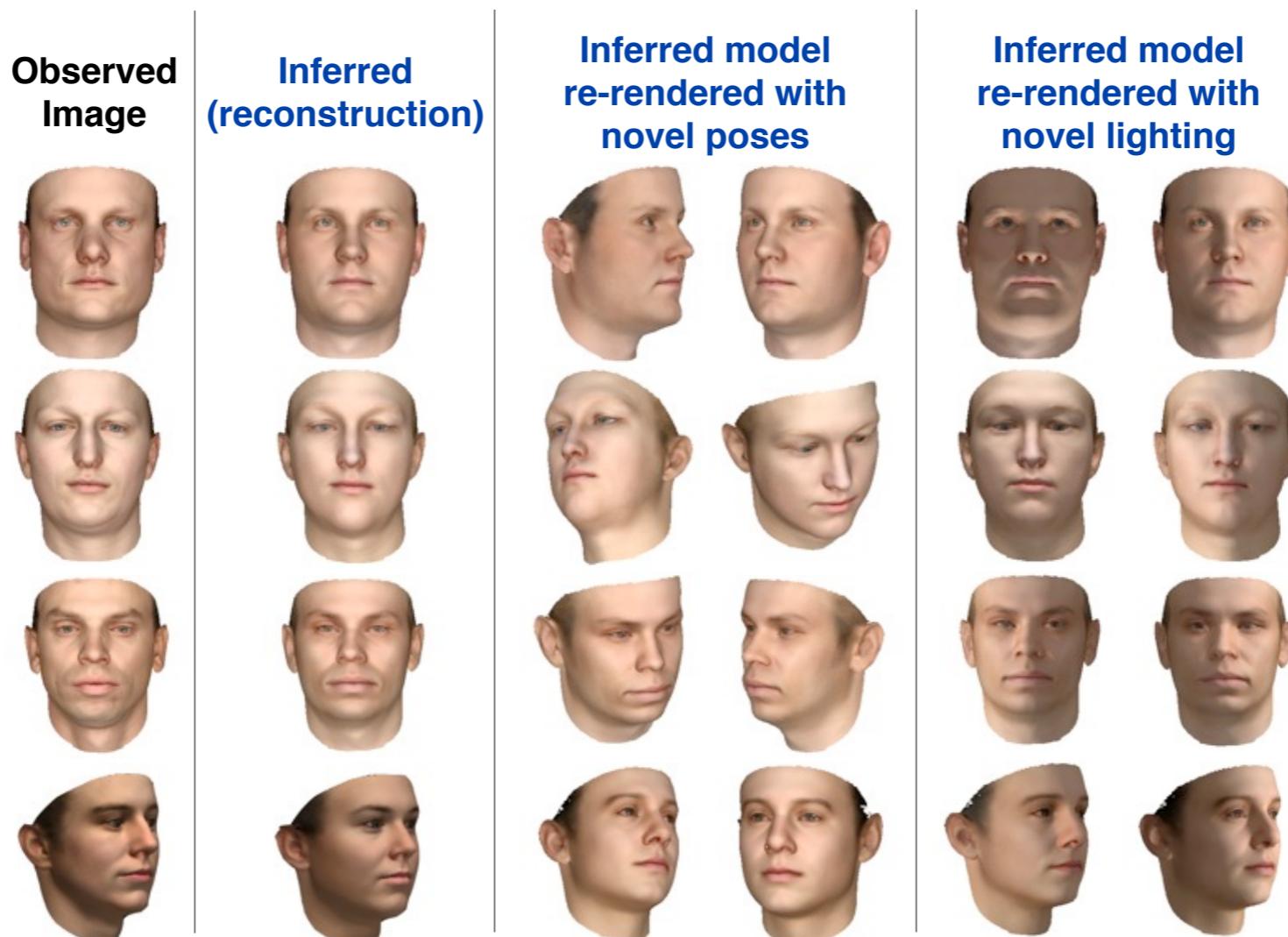
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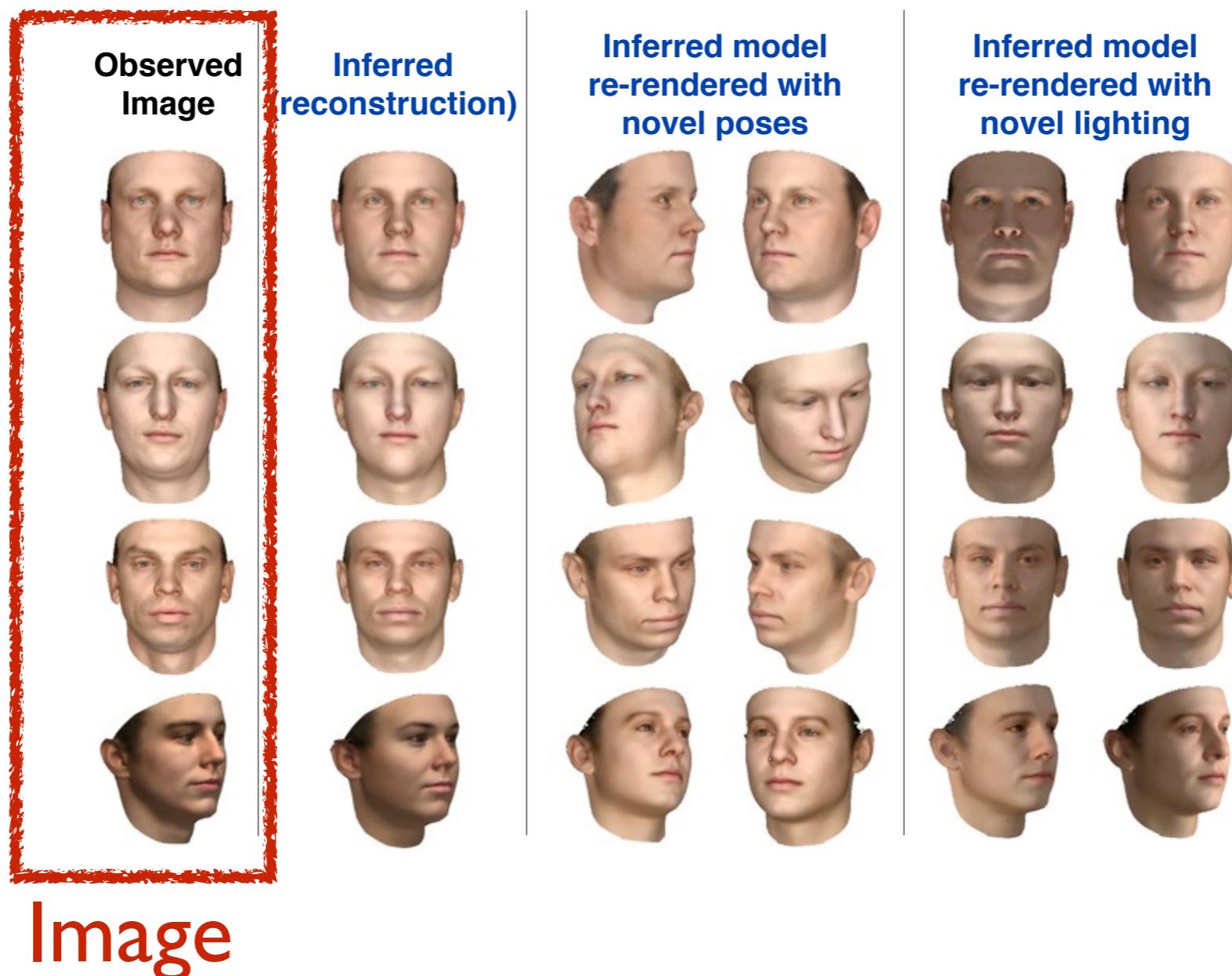
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# Success story 2: Inverse graphics



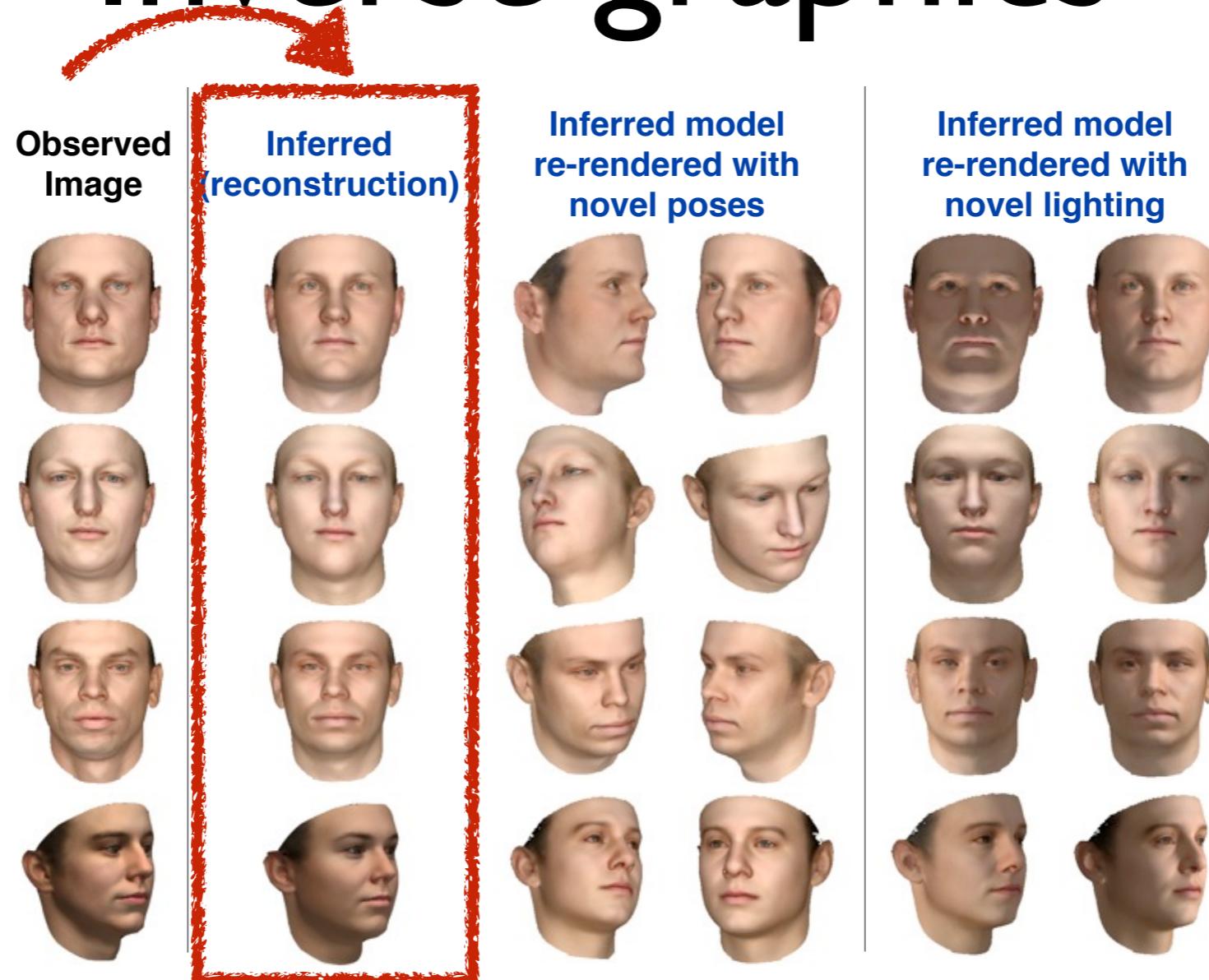
Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

# Success story 2: Inverse graphics



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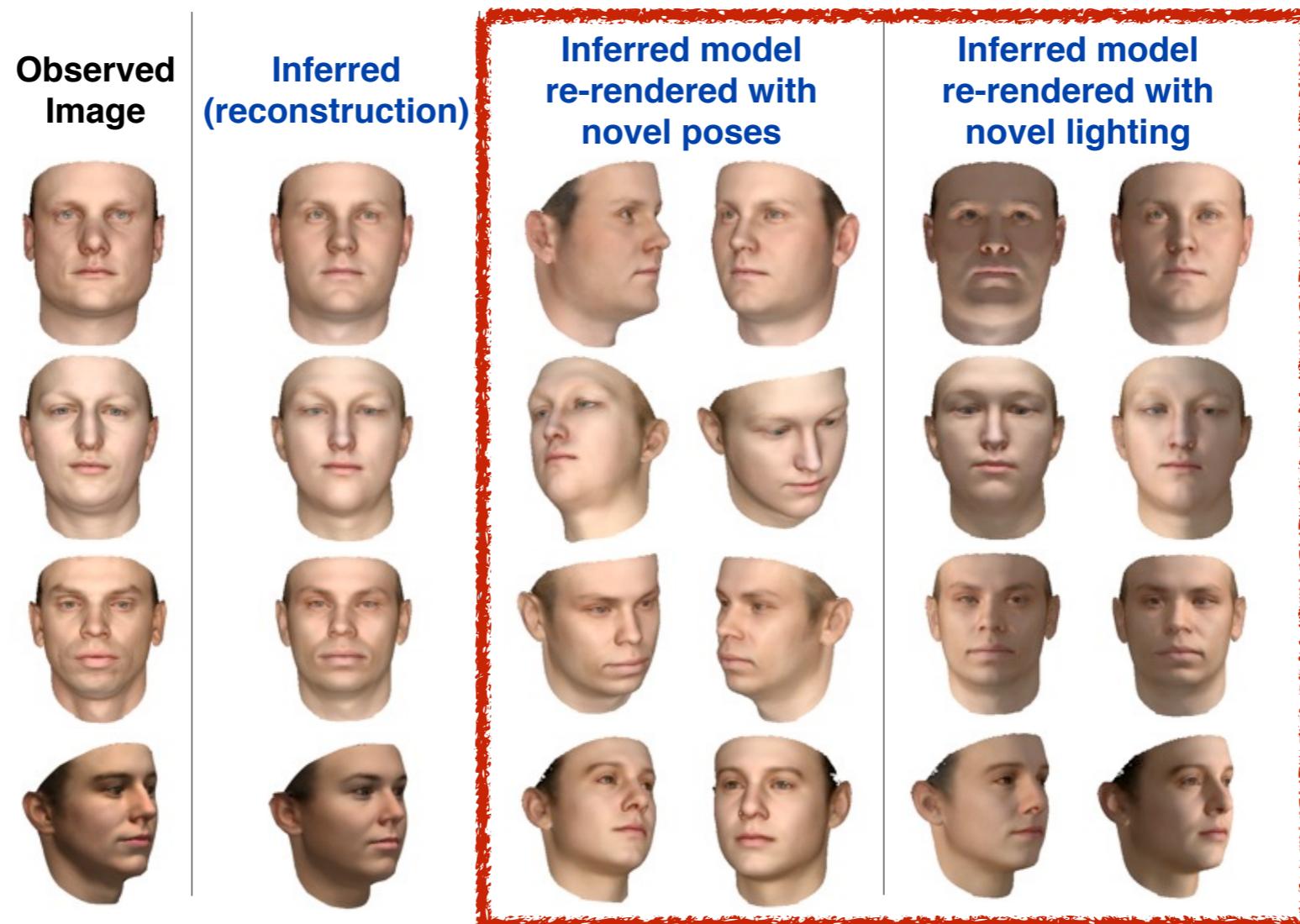
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3D model

Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

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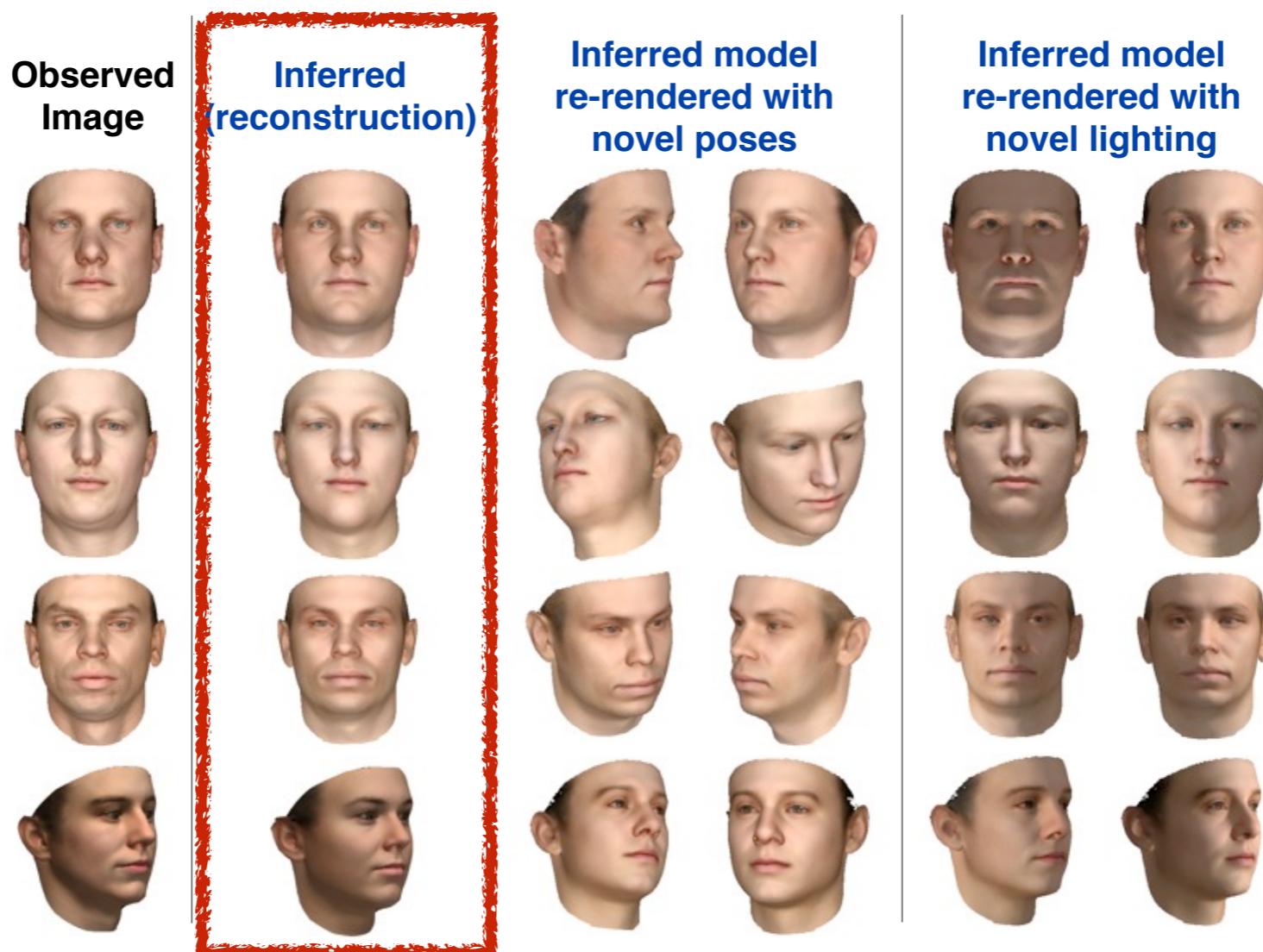


Transformed 3D models

Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

I. Sample a 3D model.

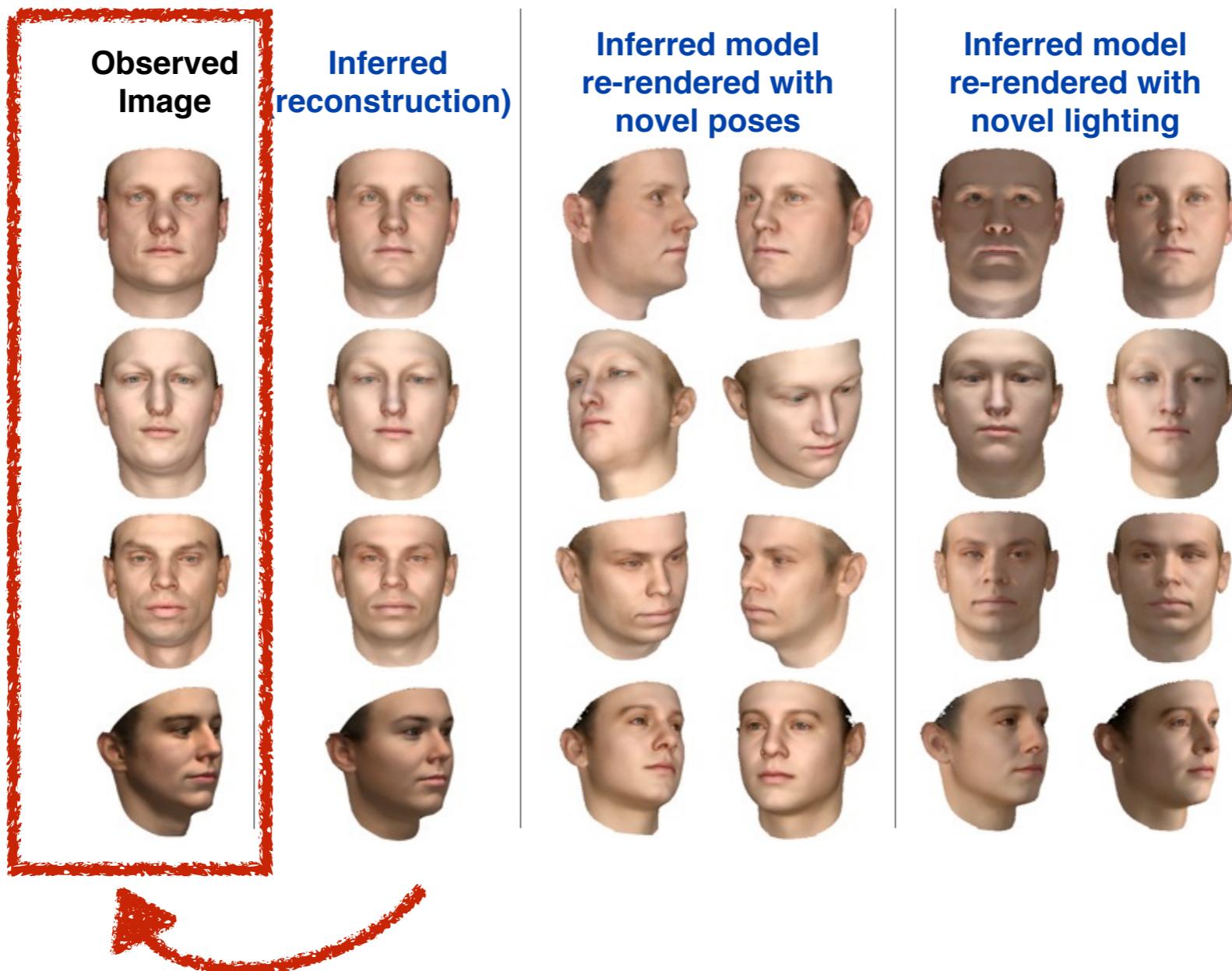
# LESS story 2: inverse graphics



Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

- I. Sample a 3D model.
2. Generate an image.

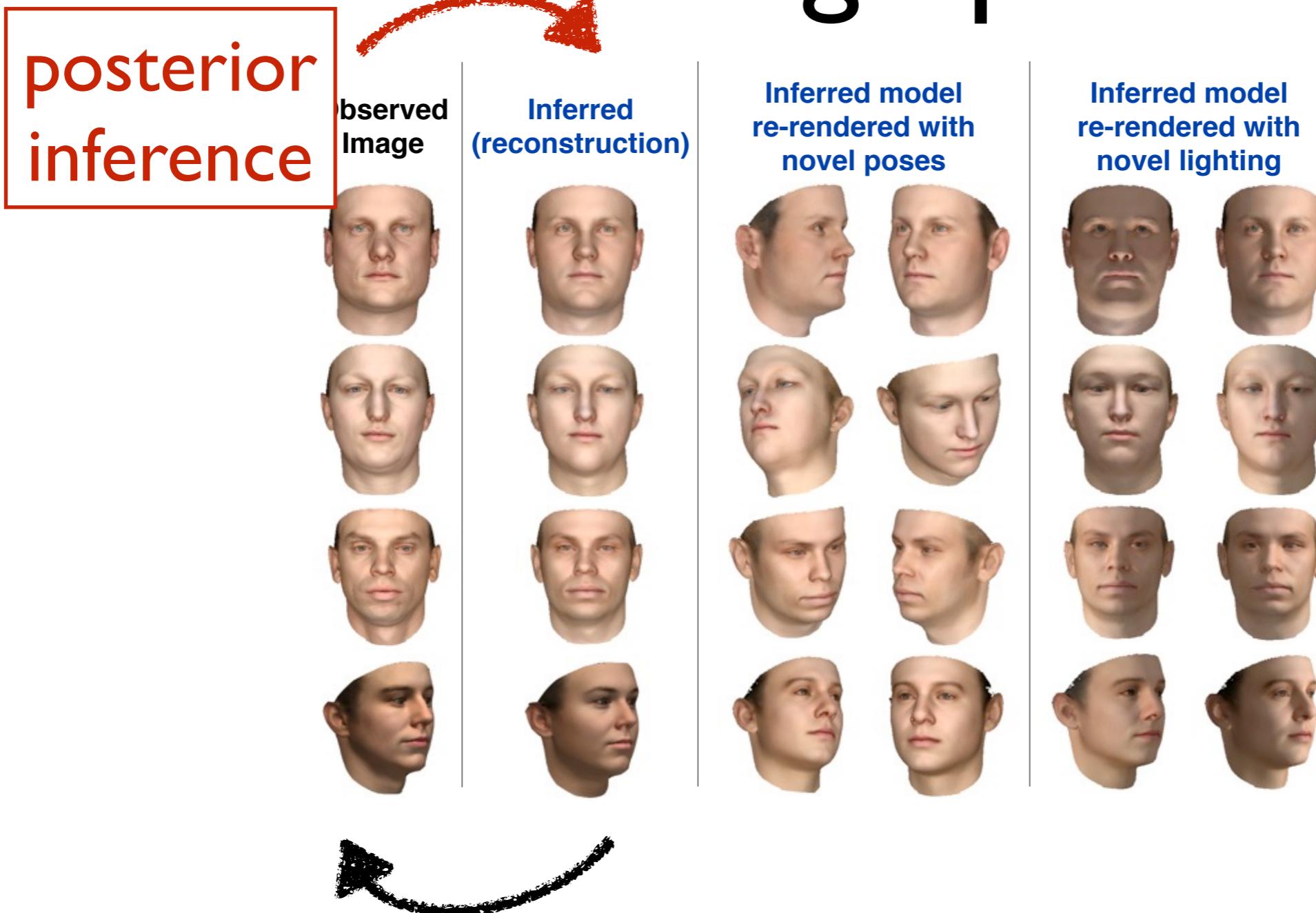
## SS story 2: inverse graphics



Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

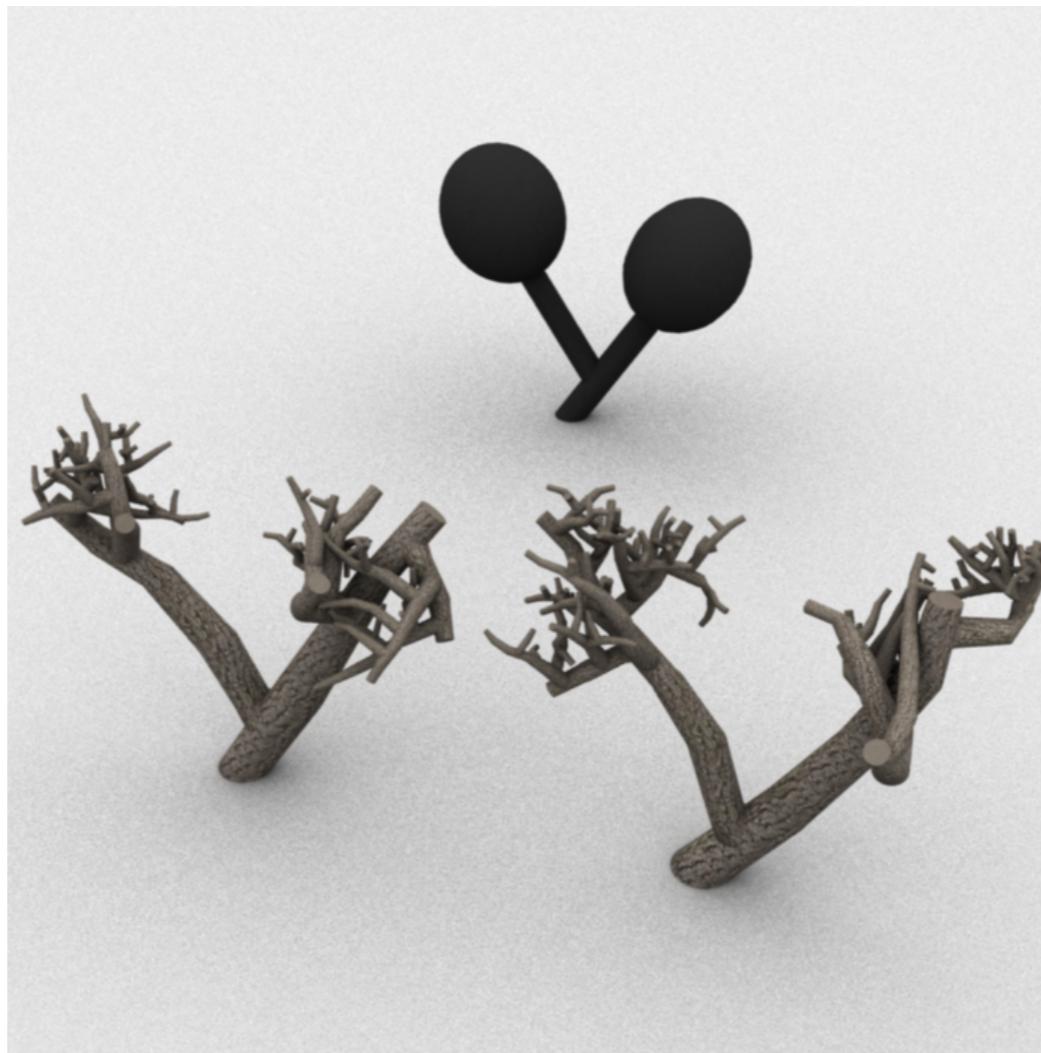
- I. Sample a 3D model.
- II. Generate an image.

## SS story 2: Face graphics



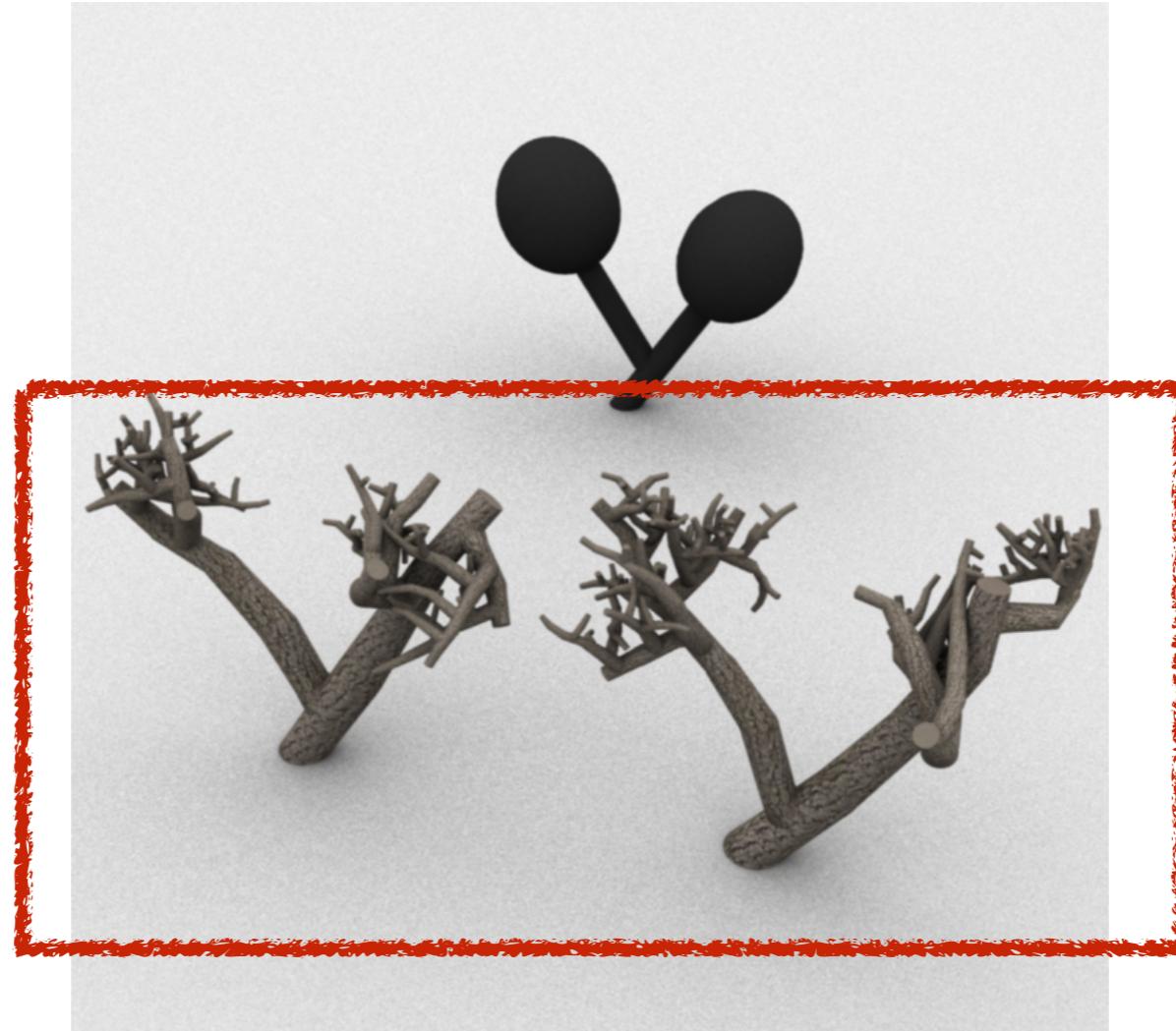
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# Success story 3: Procedural modelling



Ritchie, Mildenhall, Goodman,  
Hanrahan [SIGGRAPH'15]

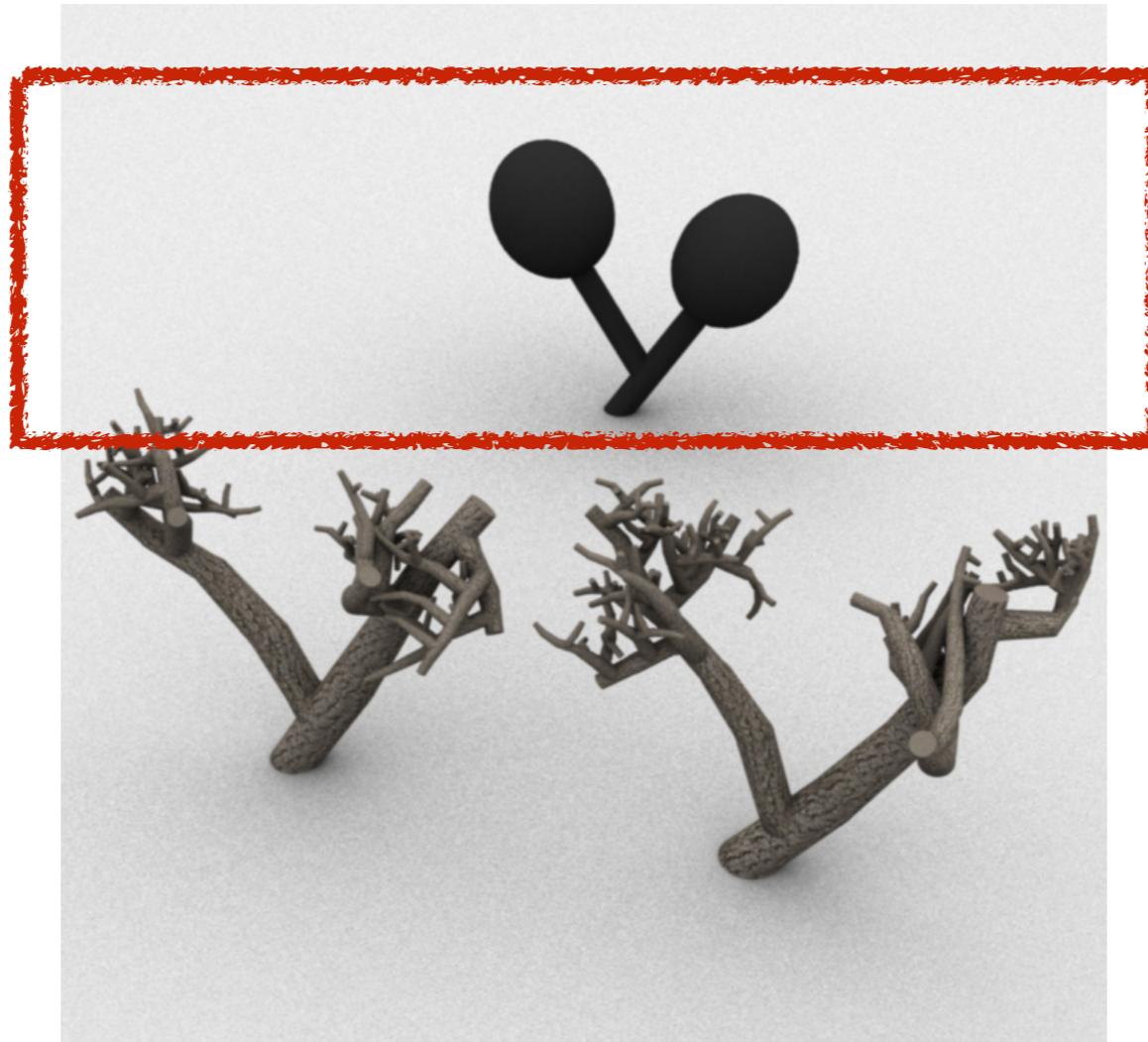
# Success story 3: Procedural modelling



I. Sample a 3D object.

Ritchie, Mildenhall, Goodman,  
Hanrahan [SIGGRAPH'15]

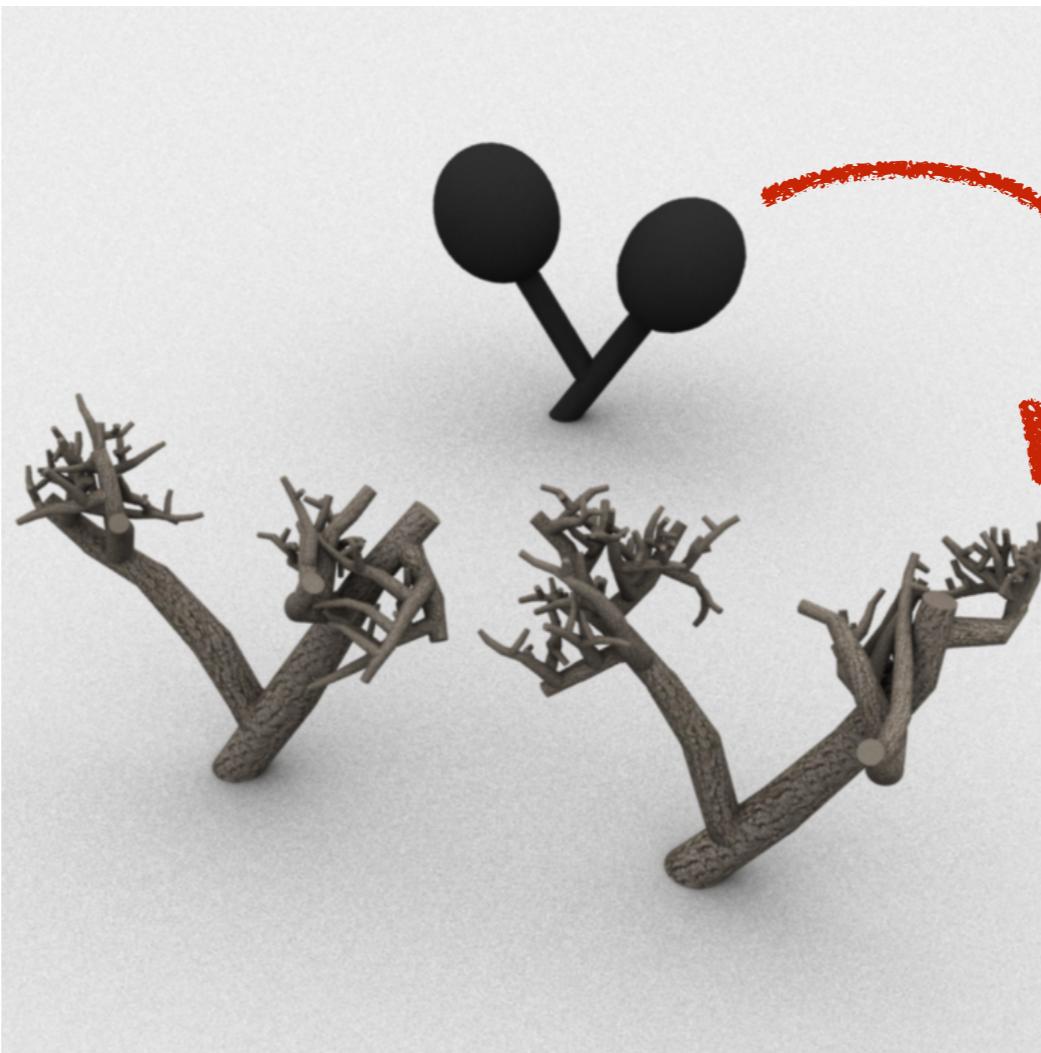
# Success story 3: Procedural modelling



1. Sample a 3D object.
2. Score the object.

Ritchie, Mildenhall, Goodman,  
Hanrahan [SIGGRAPH'15]

# Success story 3: Procedural modelling



1. Sample a 3D object.
2. Score the object.

Posterior inference generates  
objects with high scores.

# Techniques used

- Changepoints: CPS transformation and new foundation of probability theory.
- Captcha and inverse graphics: neural nets and inference amortisation.
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Prog. Languages,

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Prog. Languages, Machine Learning,

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- Captcha and inverse graphics: neural nets and inference amortisation.
- Procedural modelling: sequential Monte Carlo algorithms and stochastic future.

Prog. Languages, Machine Learning, Probability Theory

# Overview of the course

# Objective

1. Learn how to write and reason about models in a prob. prog. language (PPL).
2. Learn results from ML/PL/prob. theory that are used for building effective PPLs.
3. Contribute to probabilistic programming.

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Group project by a group of 2-3 students.

# Webpage

<https://github.com/hongseok-yang/probprog17>

All the important announcements will be made  
in this webpage.

# Evaluation

- 3-5 homework exercises (20%).
- Group project (40%).
- Final exam (40%).

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2-3 students form a group.

- Track A: New cool application of a PPL.
- Track B: Research on PPLs. Some topics suggested in the course webpage.

Tasks: i) project. ii) 3 meetings with me. iii) lecture on project and its outcome. iv) report.

# Really important announcement

1. Form a group and inform me and Byungsoo by the midnight of Sep 11 (Monday).
2. Each group will meet me twice privately.
  - (a) 7-9pm on Sep 21-22 :Topic selection.
  - (b) 7-9pm on Oct 26-27 :Progress check.
  - (c) 7-9pm on Nov 9-10 :Progress check.
3. No lectures on Sep 4 and 6.

# Somewhat important announcement

- Lecturer: Prof Hongseok Yang  
email: [hongseok.yang@kaist.ac.kr](mailto:hongseok.yang@kaist.ac.kr)
- TA: Byungsoo Kim  
email: [bskim90@kaist.ac.kr](mailto:bskim90@kaist.ac.kr)
- Install Anglican. Try the changepoint example. Get hints from the webpage.

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