

5118006-03 Data Structures

Tree

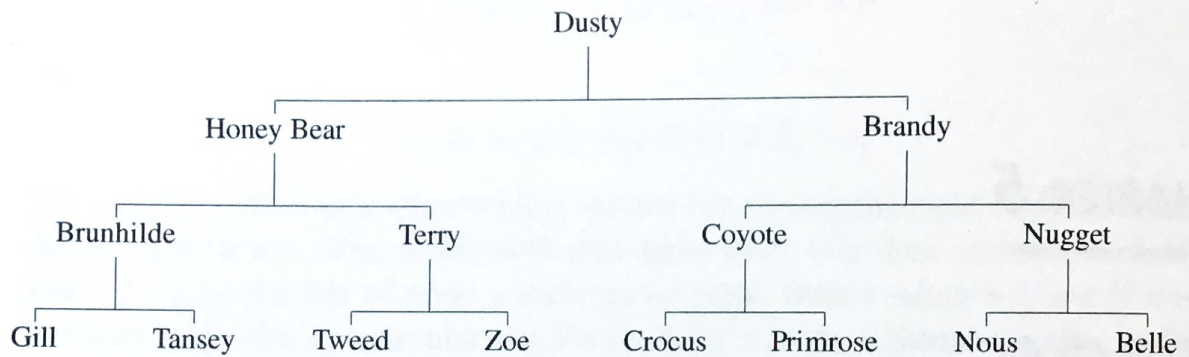
3 May 2024

Shin Hong

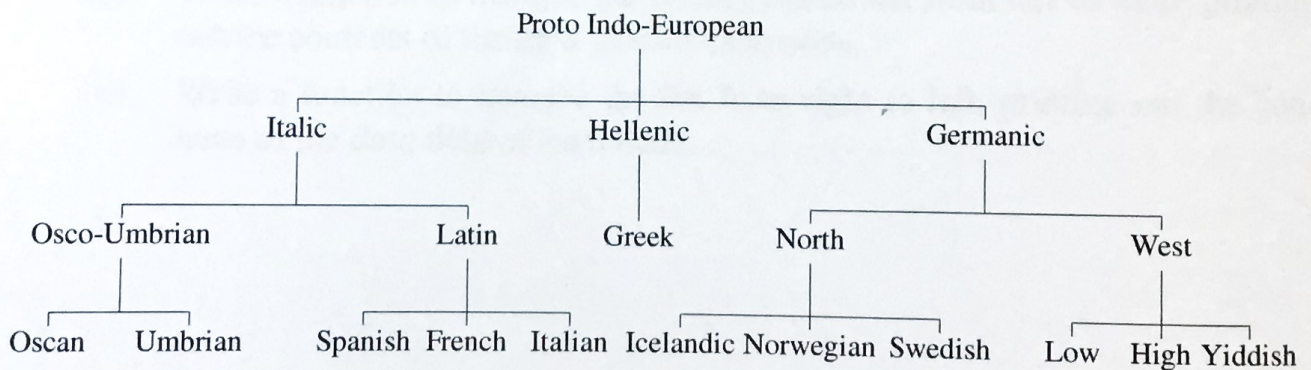
Tree

- A tree is a finite set of nodes such that:
 - each node represents a data object,
 - there exists a specifically designated node called the root, and
 - the remaining nodes are partitioned into disjoint sets T_1, T_2, \dots, T_n , where each of these sets is a tree (subtree)

Examples



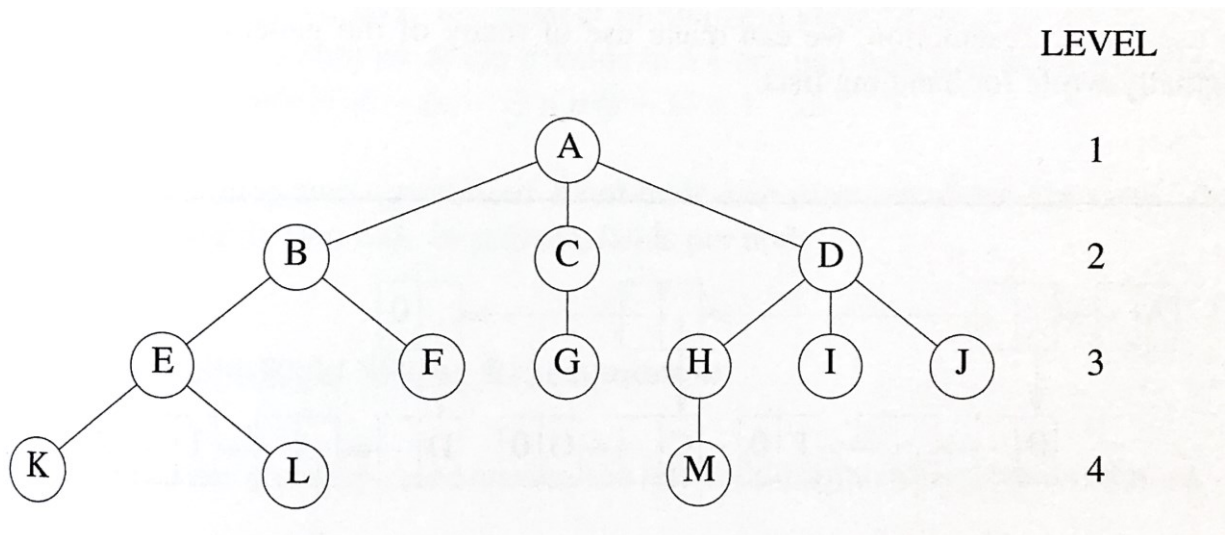
(a) Pedigree



(b) Lineal

Terminologies

- Node: the item of information
- Degree of a node: the number of subtrees
 - degree of a tree
- Leaf (terminal) node: node with degree zero
 - non-terminal nodes
- Children, Parent, Siblings, Ancestors
- Level of a node: the number of nodes between the root and the node, plus one
 - the level of the root is 1
- Height (depth) of a tree: the maximum level

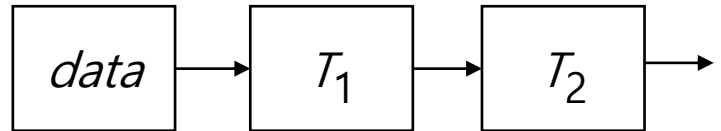


Tree Representation (1)

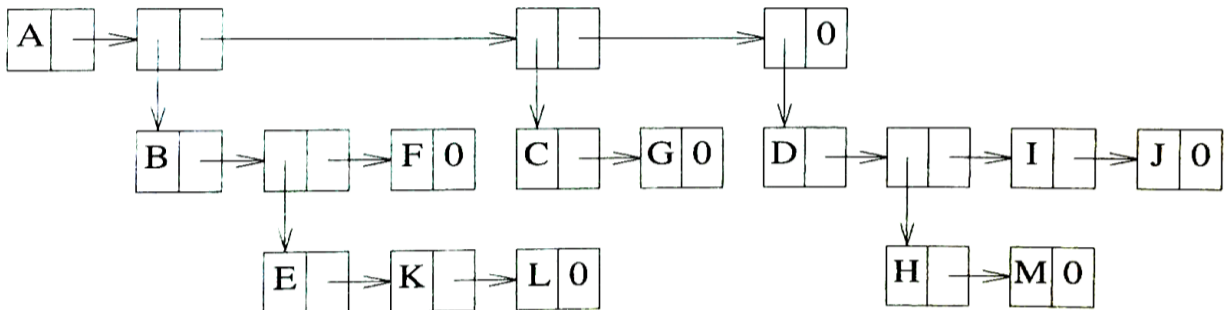
- List of subtrees

- Data*, or (*Data* (T_1, T_2, \dots, T_N))

- E.g.,



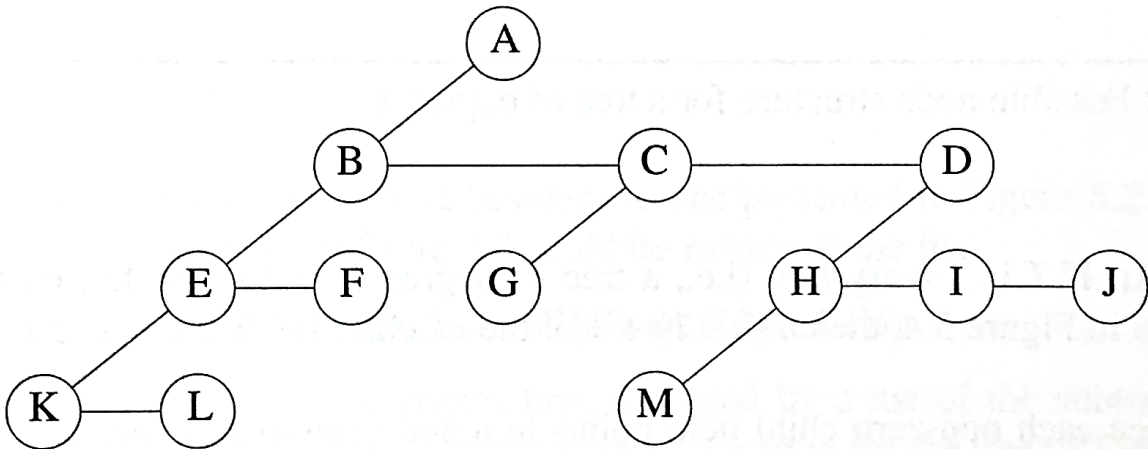
$(A(B(E(K,L),F),C(G),D(H(M),I,J)))$



Tree Representation (2)

- Two-legged linked list
 - left for child
 - right for sibling

<i>data</i>	
<i>left</i>	<i>right</i>

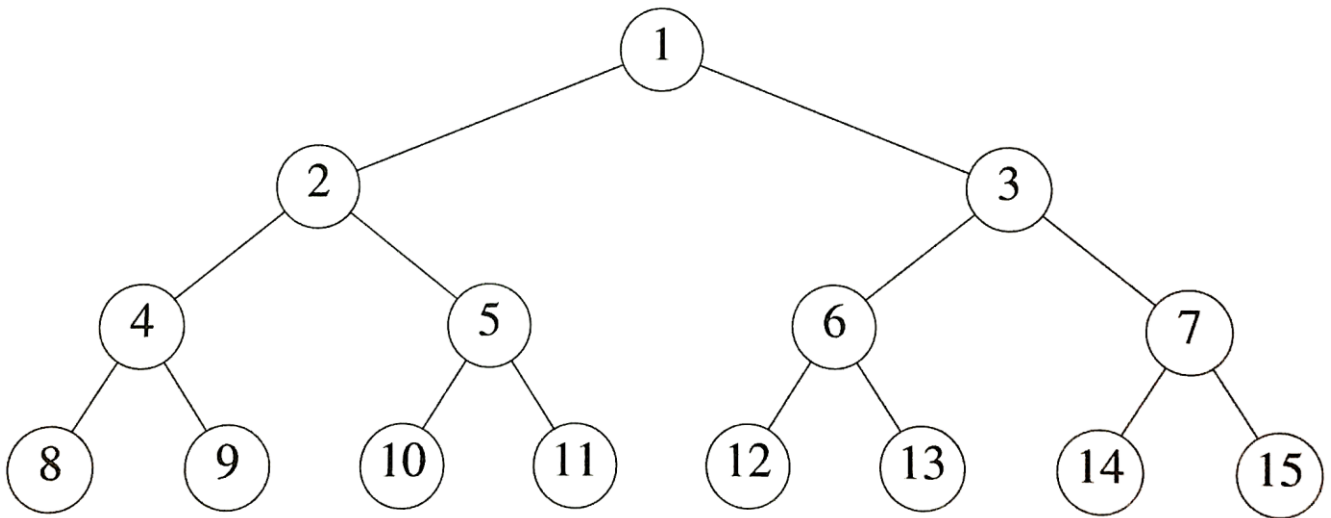


Binary Tree

- A binary tree is a finite set of nodes that is either empty or consists of a root and two disjoint binary trees
 - A binary tree is a tree with a degree 2
 - each node may have a left child and a right child
- Abstract Data Type
 - Objects: a finite set of nodes consisting of left BinTree and right BinTree, or empty
 - Operations
 - `is_empty(bt)`
 - `make_bintree(left, right)`
 - `get_data(bt)`
 - `get_left_child(bt)`
 - `get_right_child(bt)`

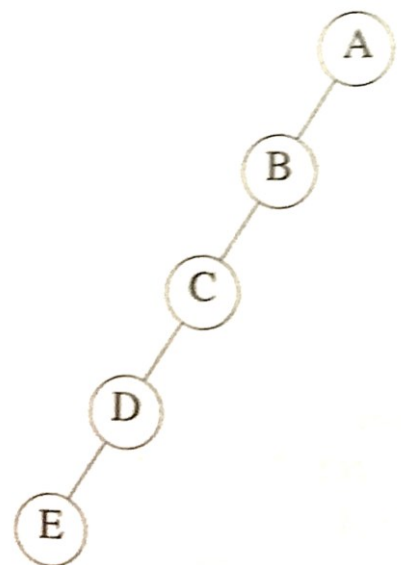
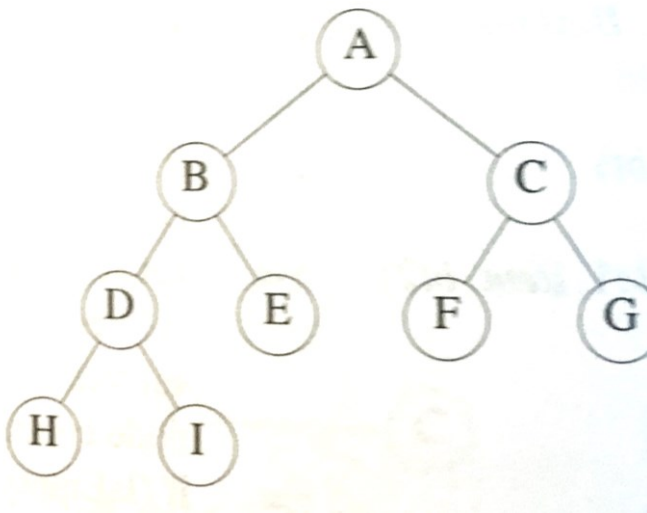
Terminologies

- A full binary tree of depth k is a binary tree of depth k having $2^k - 1$ nodes



Terminologies (2/2)

- A binary tree with n nodes and depth k is **complete** if and only if its nodes correspond to the nodes numbered from 1 to n in the full binary tree of depth k
- The height of a complete binary tree with n nodes is $\lceil \log_2(n + 1) \rceil$
- A tree is called skewed if nodes are skewed at left or right subtrees



Properties of Binary Tree

- The max number of nodes on level i of a binary tree is 2^{i-1} for $0 < i$
- The max number of nodes in a binary tree of depth k is $2^k - 1$ for $0 < k$
- For a non-empty binary tree, if n_0 is the number of leaf nodes and n_2 is the number of nodes of degree 2, then $n_0 = n_2 + 1$