Predicting Ebola Virus Spread in Sierra Leone by Applying Differential Equations in Matlab

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December 15, 2014

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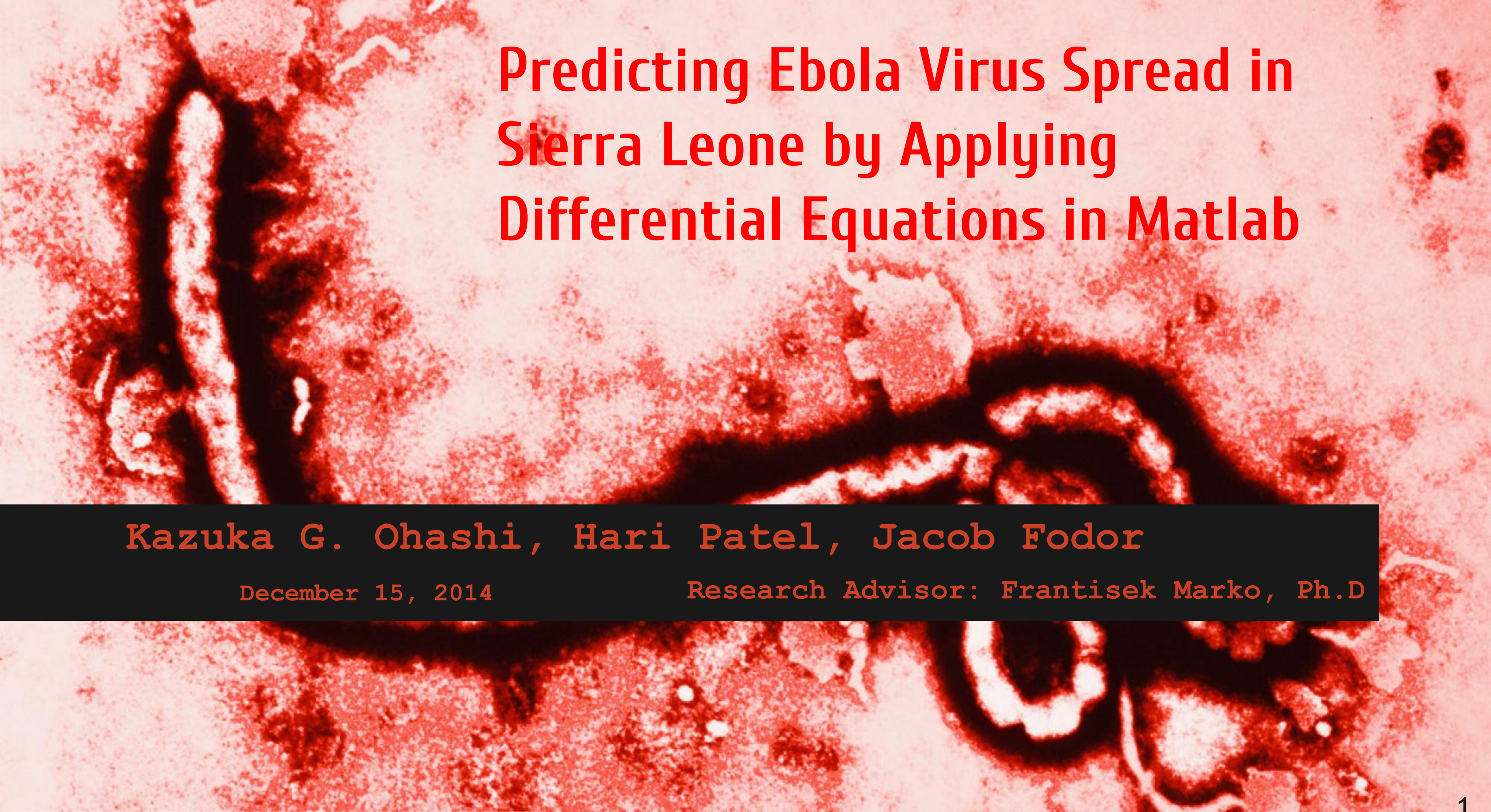
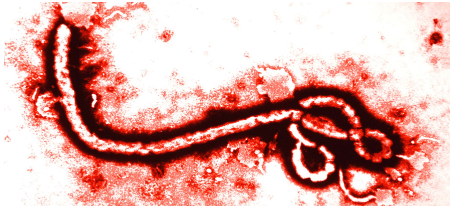


http://www.huffingtonpost.com/2014/09/30/ebola-us-cdc_n_5909394.html

Overview

- The Ebola Virus

- Introduction to differential equations in calculating the spread of the disease
  - Variables
  - Assumptions
  - Equations
- Calculating the spread of disease in modern day
  - Sierra Leone
- Conclusion



The Ebola Virus

- Symptoms in two days to three weeks
- Fever, muscle pain
- Vomiting, diarrhea, liver and kidneys
- Internal and external bleeding
- Death in about 25 days
- Cure is yet to be found

http://www.bbc.com/news/world-africa-28869799

Contamination

- Contact with blood or body fluid
- Contact with contaminated item
- Airborne spread is not documented
- Researchers believe that fruit bats are the origin of the disease

http://news.nationalpost.com/2014/07/04/graphic-the-ebola-epidemic/

Introduction to Applying DE

***S(t)*:** Individuals in the population who are not yet contaminated, but are in risk at time *t* - Susceptibles

***I(t)*:** Individuals who are infected and are capable of spreading the disease at time *t* - Infected

***R(t)*:** Individuals who are removed from the Infected at time *t* - Removed

***N*:** Total population, N = S + I + R

***β*:** Per Capita Contact Rate. The parameter controlling how often susceptible-infected contract results in a new infection.

***μ*:** Per Capita Removal Rate. The parameter controlling the rate of Infected becoming Removed.

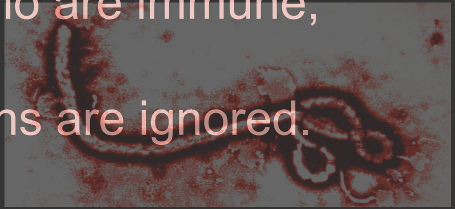
Rn*:** Reproduction number, **(β / μ).** Tells whether an epidemic will occur. (Rn*** = 2.54)

http://motherboard.vice.com/read/this-math-model-is-predicting-the-ebola-outbreak-with-incredible-accuracy

Applying Differential Equations

- Kermack - McKendrick (SIR) Model
  - Published in 1927 to predict epidemics

- Assumptions
  - Each individual has equal probability of being contaminated with the virus.
  - Only the infected individuals can spread the disease to the Susceptibles
  - The Removed population consists of individuals who are immune, cured and have died from the disease.
  - Births, immigration and any external cause of deaths are ignored.



http://www.bbc.com/news/worhttp://ooooak.deviantart.com/art/i-love-math-257509605ld-africa-28869799

SIR Model

$$\frac{d}{dt}S = -\beta \times S \times \frac{I}{N}$$

$$\frac{d}{dt}I = \beta \times S \times \frac{I}{N} - \mu I$$

$$\frac{d}{dt}R = \mu I$$

This Math Model Is Predicting the Ebola Outbreak with Incredible Accuracy

October 14, 2014 // 03:10 PM EST

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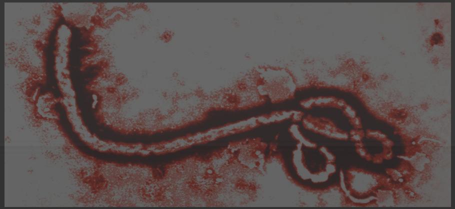
Written by
MICHAEL BYRNE
EDITOR



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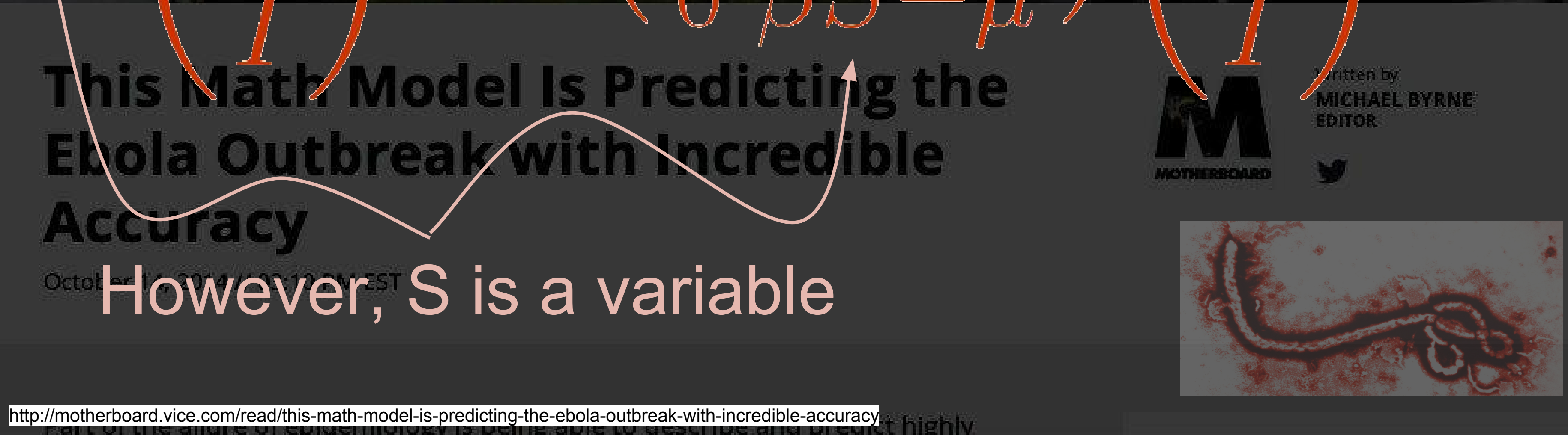
Computation

$$\begin{pmatrix} S \\ I \end{pmatrix}' = \begin{pmatrix} 0 & -\beta S \\ 0 & \beta S - \mu \end{pmatrix} \begin{pmatrix} S \\ I \end{pmatrix}$$



However, S is a variable

http://motherboard.vice.com/read/this-math-model-is-predicting-the-ebola-outbreak-with-incredible-accuracy

Modeling of SIR using MATLAB

- Appropriate MATLAB program approximates the differential equations.
- Command ‘ode45’ is used for solving differential equations using Runge - Kutta method.

http://homepages.cwi.nl/~willem/mathlinks.html

Program Script & Function

% Function *SIR.m*	% Script *p1.m*

```
function dy=SIR(t,y,beta,mu)
    dy=zeros(3,1);

if(isnumeric(beta)&&isnumeric(mu))

dy(1)=-((beta.*(y(1)).*y(2))./((y(1)+y(2)+y(3))));

dy(2)=((beta*y(1)*y(2))./((y(1)+y(2)+y(3))))-(mu.*y(2));
        dy(3)=mu.*y(2);
    end
end
```

```
clear; clc; close all;
s = input('Susceptible: ');
i = input('Infected: ');
r = input('Recovered: ');
beta = input('beta: ');
mu = input('mu: ');
t1 = input('Starting from what day? ');
t2 = input('How many total days? ');
X0 = [s,i,r];
N = sum(X0);
[T, Y] = ode45(@(t, y) SIR(t, y, beta, mu), [t1 t2], X0);
plot(T,Y(:,1),'-b','linewidth',2);
hold on;
plot(T,Y(:,2),'-m','linewidth',2);
plot(T,Y(:,3),'-r','linewidth',2);
xlabel('Time(days)');
ylabel('Susceptible,Infected,Removed');
legend('Susceptible','Infected','Removed');
hold off
disp(' --------------------- ');
disp(' --------------------- ');
No = ['Population: ', num2str(N)];
disp(No);
G = ['Susceptible: ', num2str(s)];
disp(G);
H = ['Infected: ', num2str(i)];
disp(H);
I = ['Recovered: ', num2str(r)];
disp(I);
Q = ['Total Days: ', num2str(t2)];
disp(Q);
U = ['Per Capita Contact Rate:      ', num2str(beta)];
disp(U);
E = ['Per Capita Recovered Rate:    ', num2str(mu)];
disp(E);
M = ['Total days: ', num2str(t2)];
disp(M);
C = ['Days on the Graph: ',num2str(delta_time)];
disp(C);
disp('Graph attached on left');
```

Modeling Sierra Leone (1)

- Date: 14 December, 2014
- Population: 6,348,350
- Susceptible: 6,339,597
- Infected: 6,702
- Removed: 2,051
- Removal Rate: (***μ***)
    - It takes about 24 ~ 31 days upon being contaminated to death or cure
- Contact Rate: (***β***)
    - ***Rn*** = 2.54.

$$Rn = \frac{\beta}{\mu}$$

$$\mu = \frac{1}{31}, \frac{1}{24}$$

$$\beta = 0.08, 0.11$$

http://www.who.int/csr/disease/ebola/maps/en/

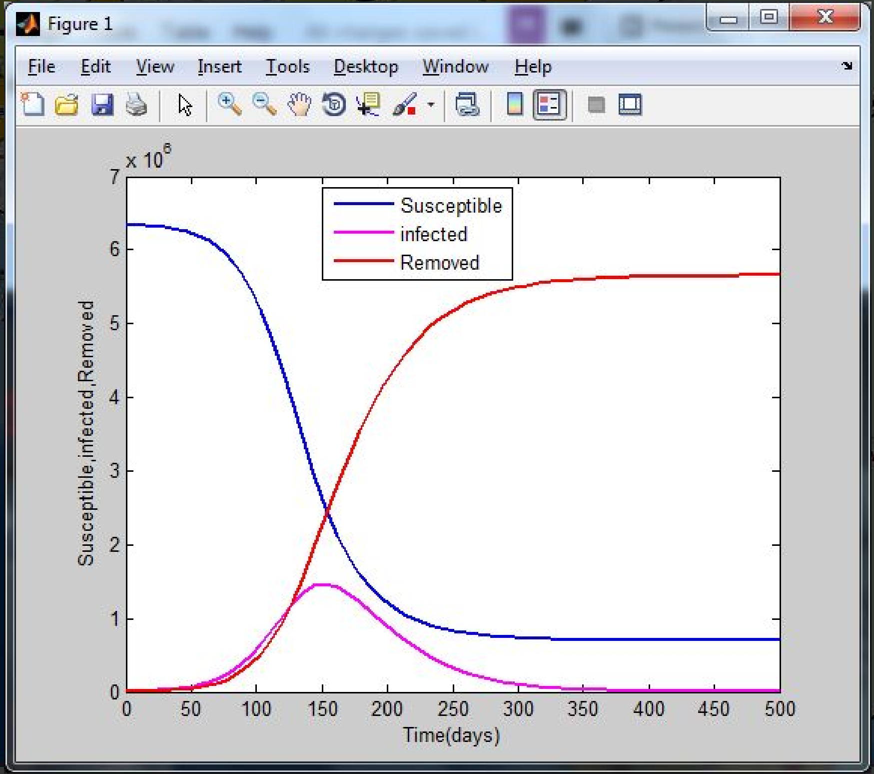
Modeling Sierra Leone (2)

- $\beta = 0.08$ & $\mu = 0.03$

$$\frac{d}{dt}S = -\frac{\beta SI}{N}$$

$$\frac{d}{dt}I = \frac{\beta SI}{N} - \mu I$$

$$\frac{d}{dt}R = \mu I$$



http://www.who.int/csr/disease/ebola/maps/en/

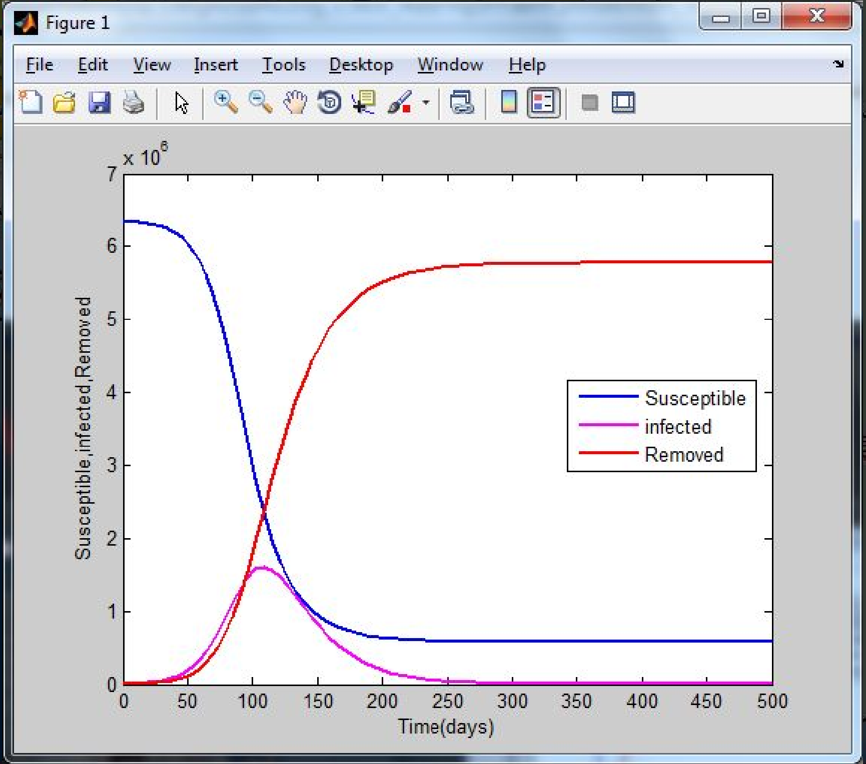
Modeling Sierra Leone (3)

- $\beta = 0.11$ & $\mu = 0.04$

$$\frac{d}{dt}S = -\frac{\beta SI}{N}$$

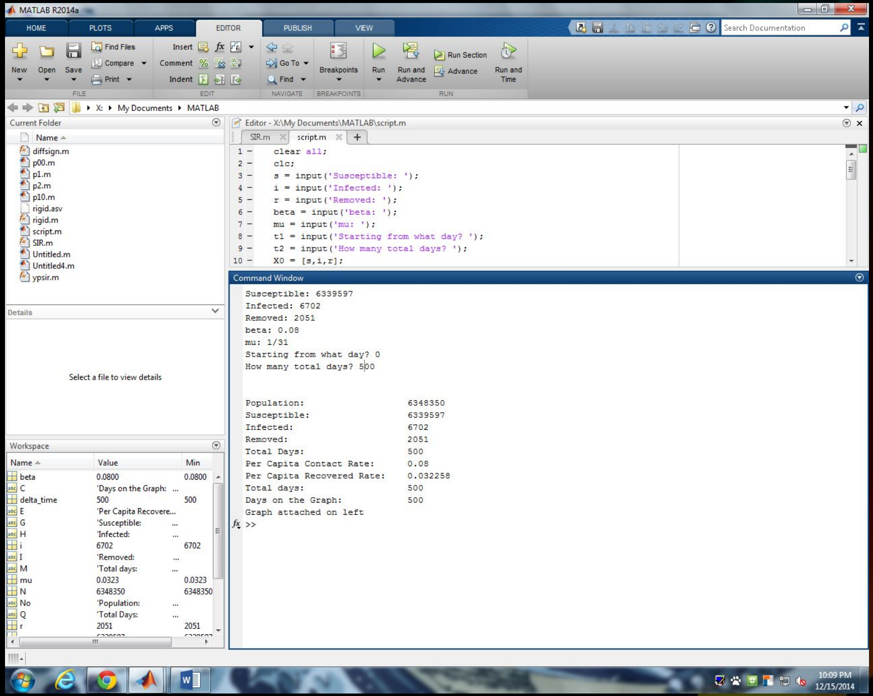
$$\frac{d}{dt}I = \frac{\beta SI}{N} - \mu I$$

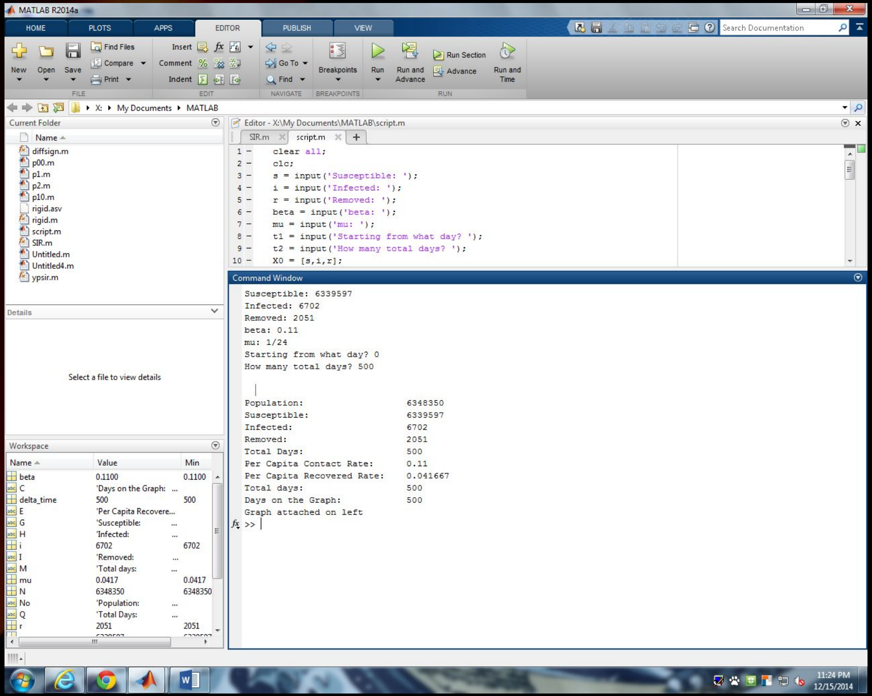
$$\frac{d}{dt}R = \mu I$$



http://www.who.int/csr/disease/ebola/maps/en/

Program Executions

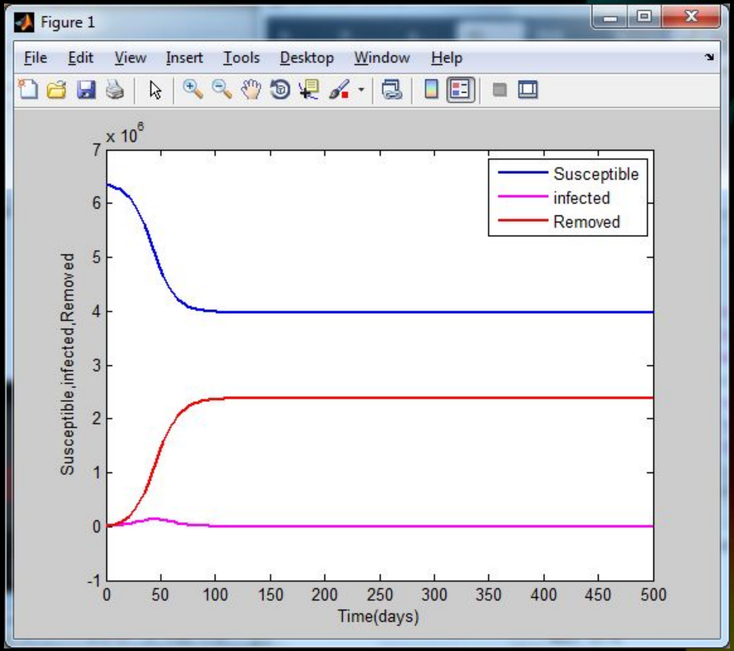




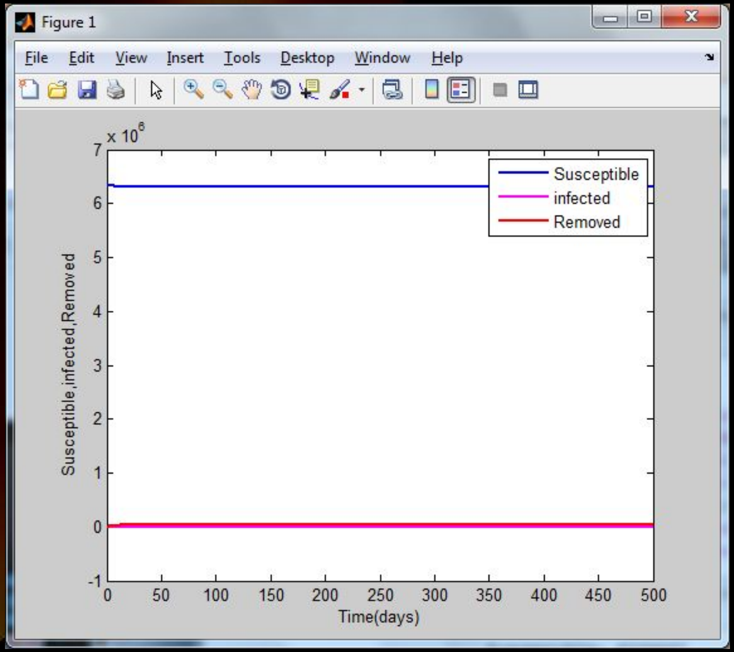
http://homepages.cwi.nl/~willem/mathlinks.html

Graphs with Different inputs

$\beta = 0.5$ & $\mu = 0.4$



$\beta = 0.5$ & $\mu = 0.6$



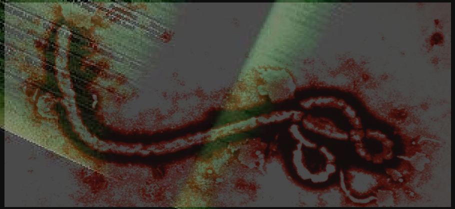
Rn ≤ 1,

No Epidemic will occur

http://homepages.cwi.nl/~willem/mathlinks.html

Further Research

- Quarantine
- Assumptions
  - Variables



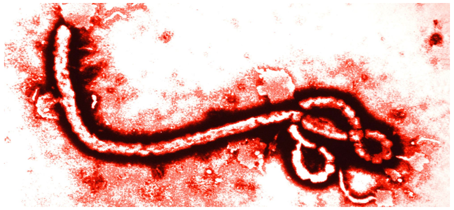
http://wallpaperswide.com/girl_studying-wallpapers.html

Conclusion

- Based on our assumptions and the use differential equations, we predict about 1.5 million people in Sierra Leone will be infected by the Ebola virus in between 100~120 days if epidemic starts on 14 December 2014.
- Model can not be used when Infected population is 0 at time, *t > 0*
  - There must exist at least one Infected individual at time *t.*
- Epidemic scenarios will vary depending on ***S***, ***I***, ***R***, and the rate constants.
- Epidemic can only occur when ***β > μ.***

$$\begin{pmatrix} S \\ I \end{pmatrix}' = \begin{pmatrix} 0 & -\beta S \\ 0 & \beta S - \mu \end{pmatrix} \begin{pmatrix} S \\ I \end{pmatrix}$$

$$\frac{d}{dt}S = -\frac{\beta S I}{N} \qquad \frac{d}{dt}I = \frac{\beta S I}{N} - \mu I \qquad \frac{d}{dt}R = \mu I$$



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Thank you!