

A high-magnification electron micrograph showing several Ebola virus particles. These are long, thin, and filamentous, with some appearing as distinct, slightly curved lines against a grainy, light-colored background. The particles vary in length and orientation, with some showing internal structural details.

Predicting Ebola Virus Spread in Sierra Leone by Applying Differential Equations in Matlab

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December 15, 2014

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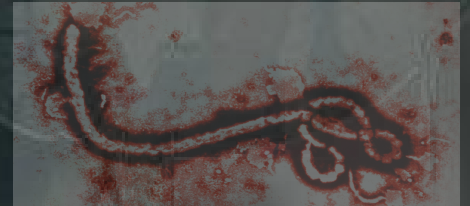
Overview

- **The Ebola Virus**
- **Introduction to differential equations in calculating the spread of the disease**
 - **Variables**
 - **Assumptions**
 - **Equations**
- **Calculating the spread of disease in modern day**
 - **Sierra Leone**
- **Conclusion**



The Ebola Virus

- Symptoms in two days to three weeks
- Fever, muscle pain
- Vomiting, diarrhea, liver and kidneys
- Internal and external bleeding
- Death in about 25 days
- Cure is yet to be found



Contamination

- Contact with blood or body fluid
- Contact with contaminated item
- Airborne spread is not documented
- Researchers believe that fruit bats are the origin of the disease

Ebola virus particle,
in 1.55 million
times its actual size

These figures
are current
estimates for
EBV cases in
three
countries.



With close contact with
infected animals. In
infected chimpanzees,
apines found in or
contact with an animal
human to human,
skin or mucous
secretions (sweat, urine,

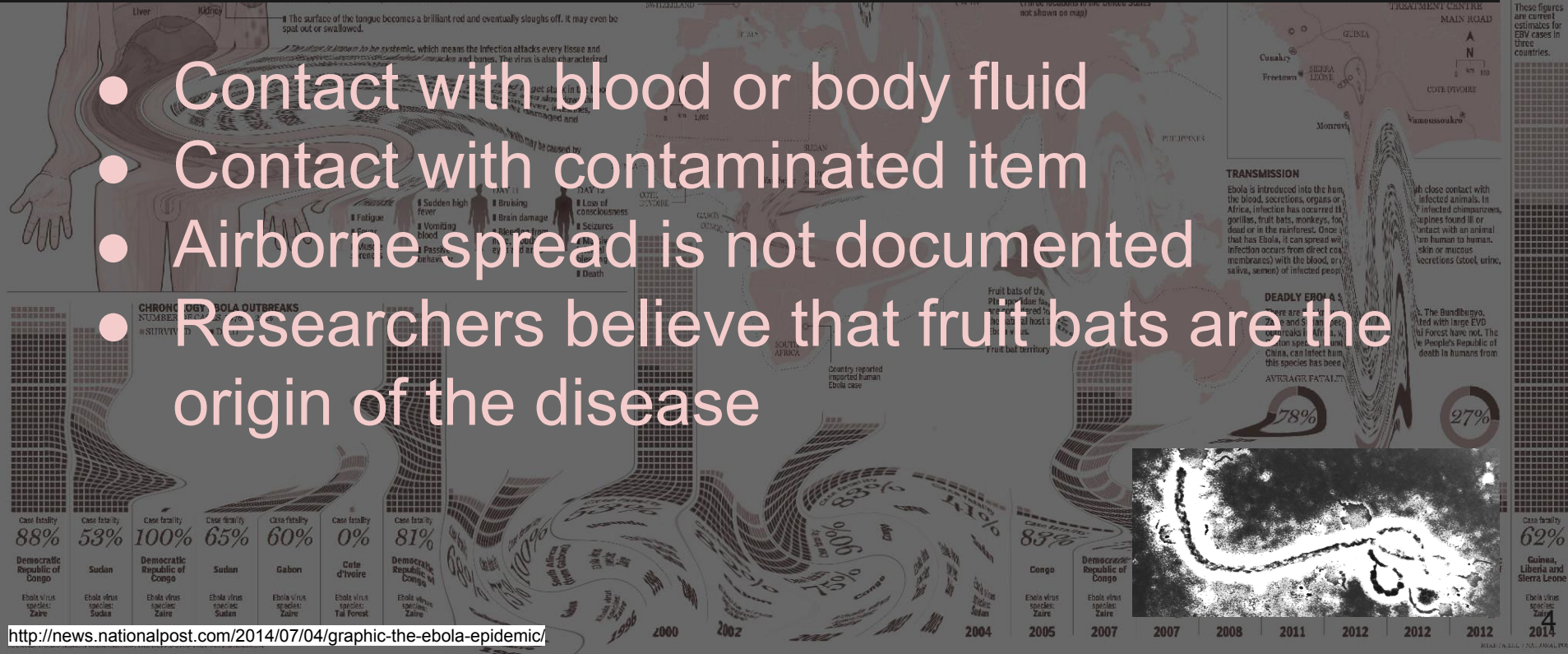
The Bundibugyo,
Forest with large EVD
in People's Republic of
death in humans from

DEADLY EBOLA
There are
Zaire and Sierra Leone
outbreaks in
from space and
China, can infect
this species has been

AVG. FATALITY
78%

27%

Case fatality
62%
Guinea,
Liberia and
Sierra Leone
Ebola virus
spread
Zaire
4
2014



Introduction to Applying DE

$S(t)$: Individuals in the population who are not yet contaminated, but are in risk at time t - Susceptibles

$I(t)$: Individuals who are infected and are capable of spreading the disease at time t - Infected

$R(t)$: Individuals who are removed from the Infected at time t - Removed

N : Total population, $N = S + I + R$

β : Per Capita Contact Rate. The parameter controlling how often susceptible-infected contract results in a new infection.

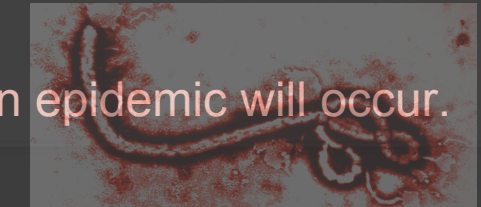
μ : Per Capita Removal Rate. The parameter controlling the rate of Infected becoming Removed.

Rn : Reproduction number, (β / μ) . Tells whether an epidemic will occur.
($Rn = 2.54$)

This Math Model Is Predicting the Ebola Outbreak with Incredible Accuracy

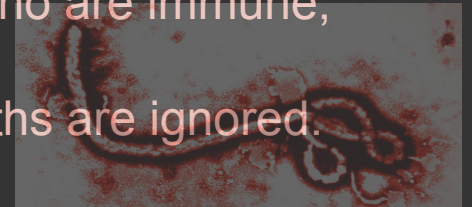
October 14, 2014 // 03:10 PM EST

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Applying Differential Equations

- Kermack - McKendrick (SIR) Model
 - Published in 1927 to predict epidemics
- Assumptions
 - Each individual has equal probability of being contaminated with the virus.
 - Only the infected individuals can spread the disease to the Susceptibles
 - The Removed population consists of individuals who are immune, cured and have died from the disease.
 - Births, immigration and any external cause of deaths are ignored.



SIR Model

$$\frac{d}{dt}S = -\beta \times S \times \frac{I}{N}$$

$$\frac{d}{dt}I = \beta \times S \times \frac{I}{N} - \mu I$$

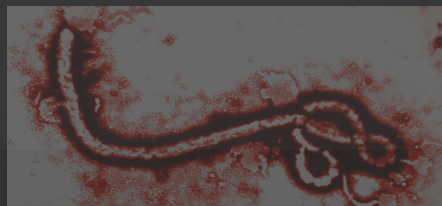
$$\frac{d}{dt}R = \mu I$$

This Math Model Is Predicting the Ebola Outbreak with Incredible Accuracy

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Computation

$$\begin{pmatrix} S \\ I \end{pmatrix}' = \begin{pmatrix} 0 & -\beta S \\ 0 & \beta S - \mu \end{pmatrix} \begin{pmatrix} S \\ I \end{pmatrix}$$

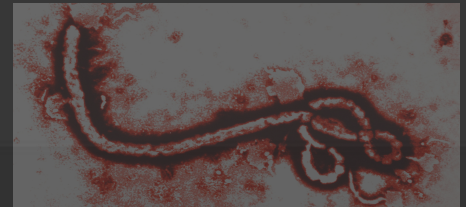
This Math Model Is Predicting the Ebola Outbreak with Incredible Accuracy

October 11, 2014 / 12:12 PM EST

However, S is a variable

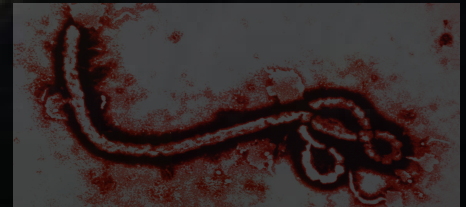


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Modeling of SIR using MATLAB

- Appropriate MATLAB program approximates the differential equations.
- Command 'ode45' is used for solving differential equations using Runge - Kutta method.



Program Script & Function

% Function *SIR.m*

```
function
dy=SIR(t,y,beta,mu)
    dy=zeros(3,1);

if(isnumeric(beta)&&is
numeric(mu))

dy(1)=-((beta.*(y(1)).*y
(2))./((y(1)+y(2)+y(3))))
;

dy(2)=((beta*y(1)*y(2))
./((y(1)+y(2)+y(3))))-(m
u.*y(2));
    dy(3)=mu.*y(2);
end
end
```

% Script *p1.m*

```
clear; clc; close all;
s = input('Susceptible:
');
i = input('Infected: ');
r = input('Recovered:
');
beta = input('beta: ');
mu = input('mu: ');
t1 = input('Starting
from what day? ');
t2 = input('How many
total days? ');
X0 = [s,i,r];
N = sum(X0);
```

```
[T, Y] = ode45(@(t, y)
SIR(t, y, beta, mu), [t1
t2], X0);
plot(T, Y(:,1), '-b', 'linewi
dth', 2);
hold on;
plot(T, Y(:,2), '-m', 'linewi
dth', 2);
plot(T, Y(:,3), '-r', 'linewid
th', 2);
xlabel('Time(days)');
ylabel('Susceptible, Inf
ected, Removed');
legend('Susceptible', 'I
nfected', 'Removed');
```

```
hold off
disp(' ----- ');
disp(' ----- ');
No = ['Population:
', num2str(N)];
disp(No);
G = ['Susceptible:
', num2str(s)];
disp(G);
H = ['Infected:
', num2str(i)];
disp(H);
I = ['Recovered:
', num2str(r)];
disp(I);
Q = ['Total Days:
', num2str(t2)];
disp(Q);
```

```
U = ['Per Capita Contact
Rate: ',
num2str(beta)];
disp(U);
E = ['Per Capita
Recovered Rate: ',
num2str(mu)];
disp(E);
M = ['Total days:
', num2str(t2)];
disp(M);
C = ['Days on the Graph:
', num2str(delta_time)];
disp(C);
disp('Graph attached on
left');
```

Modeling Sierra Leone (1)

- Date:
- Population:
- Susceptible:
- Infected:
- Removed:
- Removal Rate:
- Contact Rate:

14 December, 2014

6,348,350

6,339,597

6,702

2,051

(μ)

It takes about 24 ~ 31 days upon being contaminated to death or cure

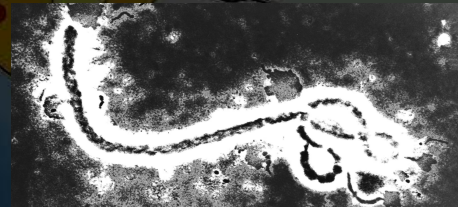
(β)

$Rn = 2.54$.

$$Rn = \frac{\beta}{\mu}$$

$$\beta = 0.08, 0.11$$

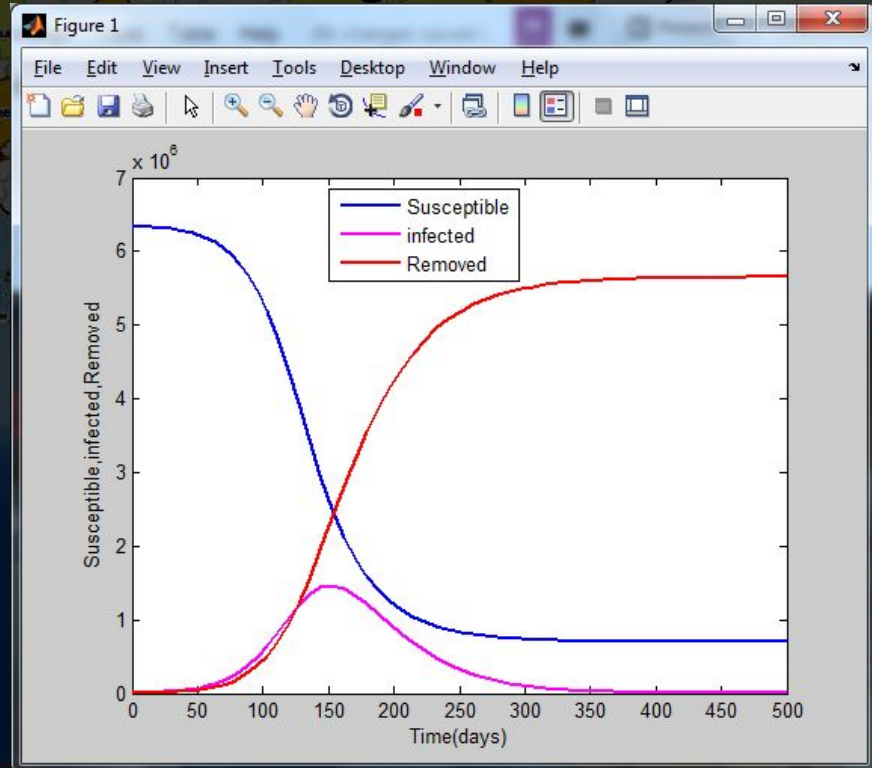
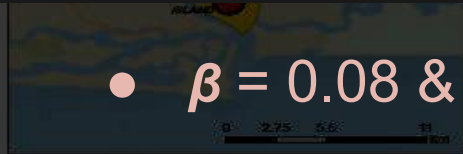
$$\mu = \frac{1}{31}, \frac{1}{24}$$



Map Scale (A3): 1:3,700,000
1 cm = 37 km
0 40 80 160 km

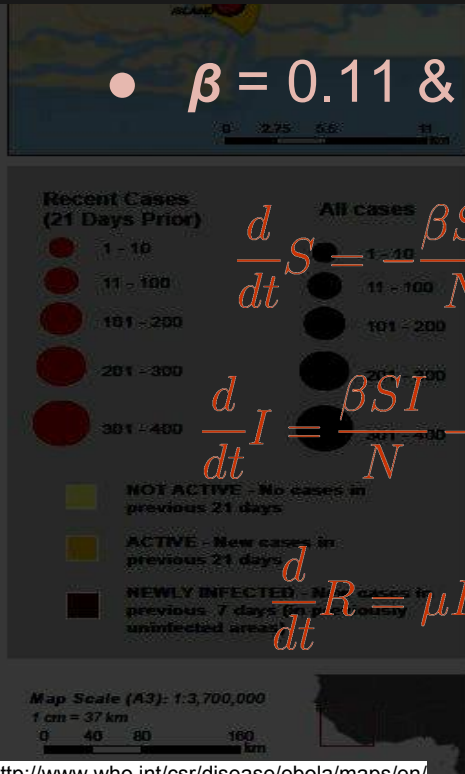
Modeling Sierra Leone (2)

- $\beta = 0.08$ & $\mu = 0.03$



Modeling Sierra Leone (3)

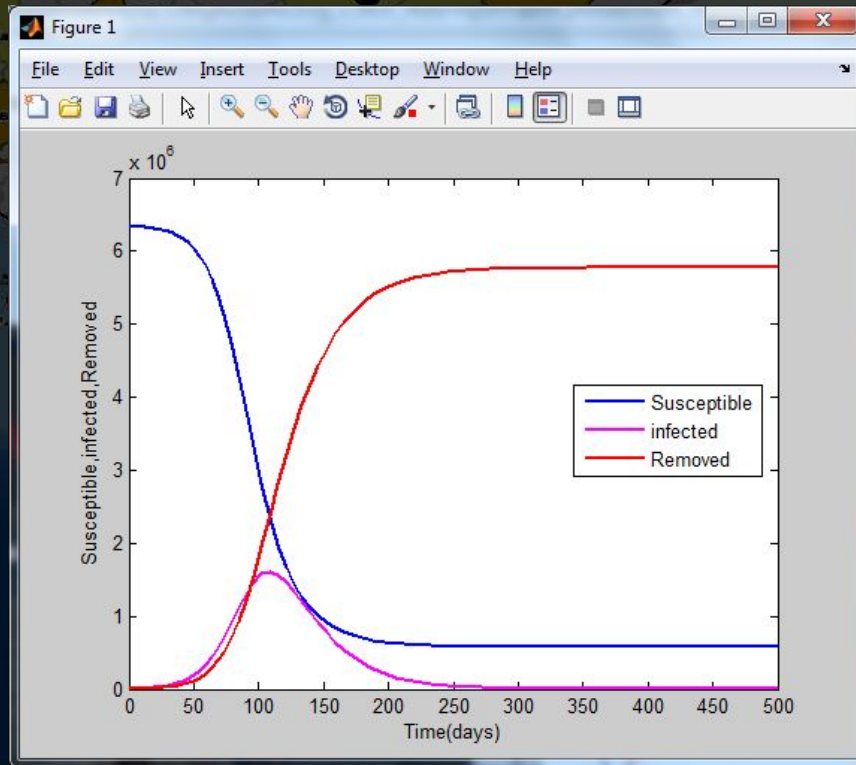
- $\beta = 0.11$ & $\mu = 0.04$



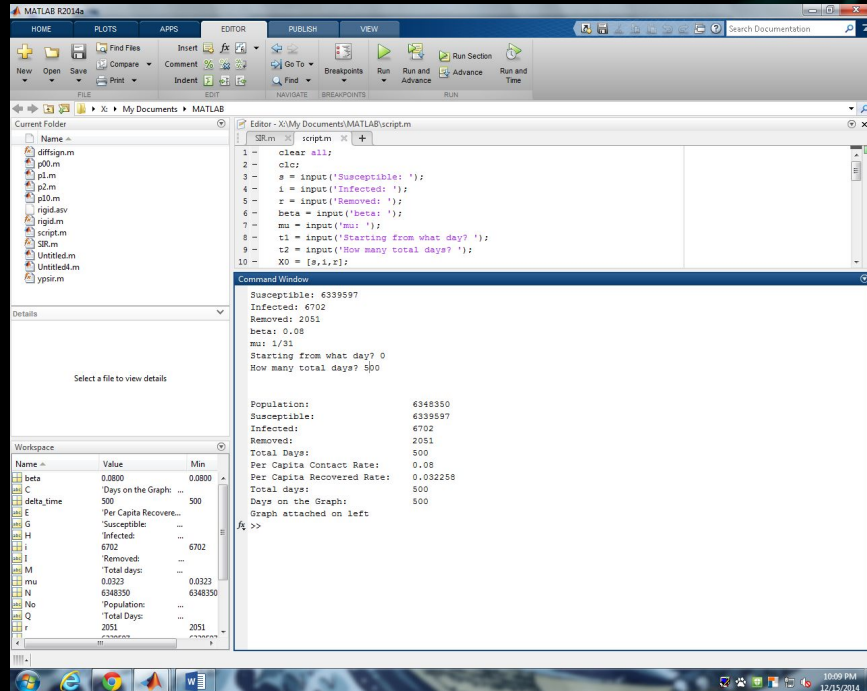
$$\frac{dS}{dt} = -\frac{\beta SI}{N}$$

$$\frac{dI}{dt} = \frac{\beta SI}{N} - \mu I$$

$$\frac{dR}{dt} = \mu I$$



Program Executions



Current Folder: X:\My Documents\MATLAB

```
1 = clear all;
2 = cloc;
3 = a = input('Susceptible: ');
4 = i = input('Infected: ');
5 = r = input('Removed: ');
6 = beta = input('beta: ');
7 = mu = input('mu: ');
8 = t1 = input('Starting from what day? ');
9 = t2 = input('How many total days? ');
10 = XO = [a,i,r];
```

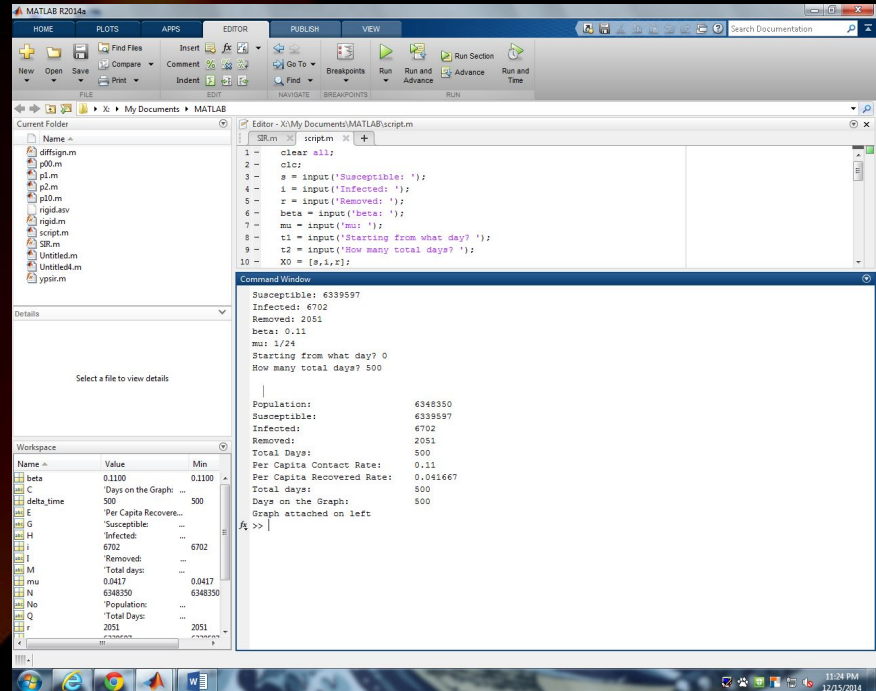
Command Window

Susceptible: 6339597
Infected: 6702
Removed: 2051
beta: 0.08
mu: 1/01
Starting from what day? 0
How many total days? 500

Population: 6348350
Susceptible: 6339597
Infected: 6702
Removed: 2051
Total Days: 500
Per Capita Contact Rate: 0.08
Per Capita Recovered Rate: 0.032258
Total days: 500
Days on the Graph: 500
Graph attached on left

Workspace

Name	Value	Min
beta	0.0800	0.0800
C	Days on the Graph: ...	
delta_time	500	
E	'Per Capita Recovered Rate: ...	
G	'Susceptible: ...	
H	'Infected: ...	
I	6702	6702
M	'Removed: ...	
mu	0.0323	0.0323
N	6348350	6348350
No	'Population: ...	
Q	2051	2051



Current Folder: X:\My Documents\MATLAB

```
1 = clear all;
2 = cloc;
3 = a = input('Susceptible: ');
4 = i = input('Infected: ');
5 = r = input('Removed: ');
6 = beta = input('beta: ');
7 = mu = input('mu: ');
8 = t1 = input('Starting from what day? ');
9 = t2 = input('How many total days? ');
10 = XO = [a,i,r];
```

Command Window

Susceptible: 6339597
Infected: 6702
Removed: 2051
beta: 0.11
mu: 1/24
Starting from what day? 0
How many total days? 500

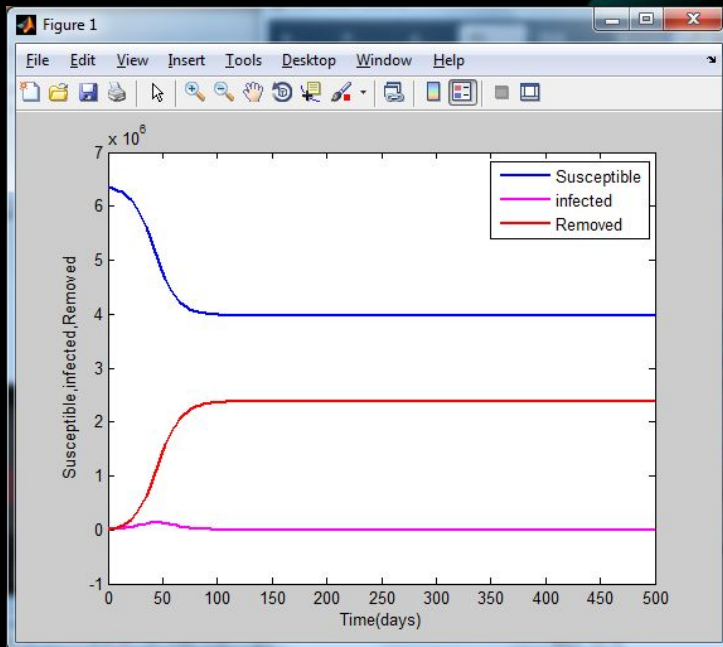
Population: 6348350
Susceptible: 6339597
Infected: 6702
Removed: 2051
Total Days: 500
Per Capita Contact Rate: 0.11
Per Capita Recovered Rate: 0.041667
Total days: 500
Days on the Graph: 500
Graph attached on left

Workspace

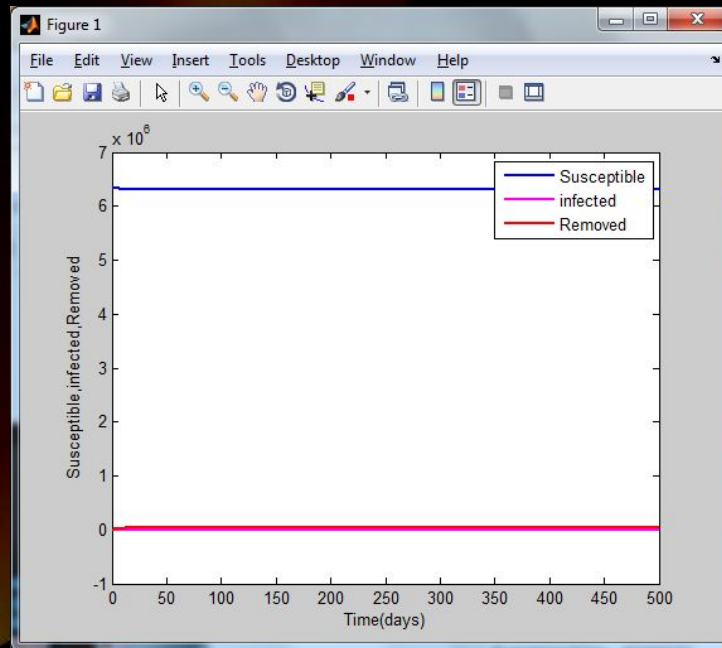
Name	Value	Min
beta	0.1100	0.1100
C	Days on the Graph: ...	
delta_time	500	
E	'Per Capita Recovered Rate: ...	
G	'Susceptible: ...	
H	'Infected: ...	
I	6702	6702
M	'Removed: ...	
mu	0.0417	0.0417
N	6348350	6348350
No	'Population: ...	
Q	2051	2051

Graphs with Different inputs

$$\beta = 0.5 \text{ \& } \mu = 0.4$$



$$\beta = 0.5 \text{ \& } \mu = 0.6$$

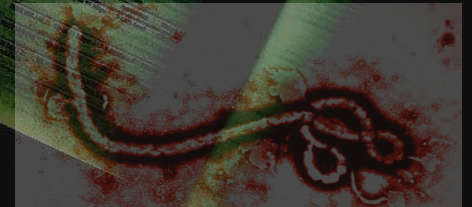


$$R_n \leq 1,$$

No Epidemic
will occur

Further Research

- Quarantine
- Assumptions
 - Variables



Conclusion

- Based on our assumptions and the use differential equations, we predict about 1.5 million people in Sierra Leone will be infected by the Ebola virus in between 100~120 days if epidemic starts on 14 December 2014.
- Model can not be used when Infected population is 0 at time, $t > 0$
 - There must exist at least one Infected individual at time t .
- Epidemic scenarios will vary depending on \mathbf{S} , \mathbf{I} , \mathbf{R} , and the rate constants.
- Epidemic can only occur when $\beta > \mu$.

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} S \\ I \end{pmatrix} &= \begin{pmatrix} 0 & -\beta SI \\ 0 & \beta SI - \mu I \end{pmatrix} \begin{pmatrix} S \\ I \end{pmatrix} \\ \frac{d}{dt} S &= -\frac{\beta SI}{N} & \frac{d}{dt} I &= \frac{\beta SI}{N} - \mu I & \frac{d}{dt} R &= \mu I \end{aligned}$$



Work Cited

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http://www4.ncsu.edu/eos/users/w/white/www/white/mamac/Par_Id_SIR.pdf

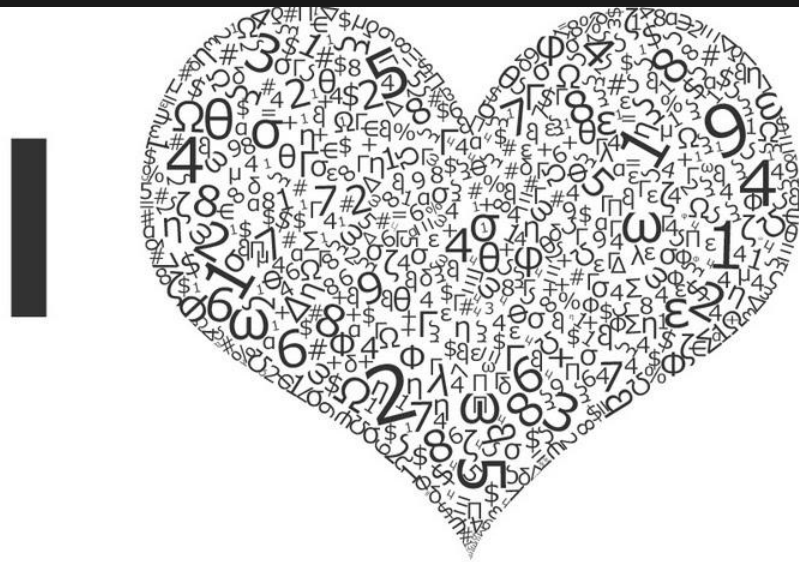
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http://www.huffingtonpost.com/2014/08/02/ebola-symptoms-infection-virus_n_5639456.html

<http://currents.plos.org/outbreaks/article/estimating-the-reproduction-number-of-zaire-ebolavirus-ebov-during-the-2014-outbreak-in-west-africa/>

Thank you!



Math.