Linear Algebra - HW 4

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Contents

1 Question 1 **1**

2 Question 2 **2**

2.1 Part I 2

2.2 Part II 3

3 Question 3 **4**

4 Question 4 **5**

1 Question 1

۱. **تمرین ۴ کتاب:** قضیه ۶ را اثبات کنید.

قضیه ۶: کرنل هر تبدیل خطی، یک زیر فضا است.

$Ker(T(v)) = \{x \mid x \in V, T(x) = 0\}$

$T : v \Rightarrow W$

In order to prove that a Kernel is a subspace, we should prove two ,main features of a kernel :

- It's closed under vector addition.
- It's closed under multiplication.

Now proving the first feature :

Suppose : $x, y \in Ker(T)$ this means that : $T(x) = 0$ & $T(y) = 0$

can we imply that $(x+y) \in Ker(T)$??

$T(x+y) = T(x) + T(y) = 0 + 0 = 0 \in Ker(T) \Rightarrow x + y \in Ker(T)$ ✓

Now Proving the second feature :

Assume: $x \in K(T)$ Then $\Rightarrow T(\alpha x) = \alpha T(x) = \alpha 0 = 0 \in K(T) \Rightarrow$ The second feature is also proved ✓

Now We can say that the Kernel of a Linear Transformation is a subspace.

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2 Question 2

تمرین ۱۰ کتاب: نشان دهید که ماتریس $\mathbf{C_v} \overset{\text{def}}{=} \mathbf{I} - \frac{\mathbf{v}\mathbf{v}^T}{\mathbf{v}^T\mathbf{v}}$ یک ماتریس متقارن بوده و $\mathbf{C_v}^2 = \mathbf{C_v}$ می‌باشد.

2.1 Part I

Assume : $\alpha = VV^T$ and $\beta = V^TV$

$\beta^T = \alpha$ and $\alpha^T = \beta$

$(x/y)^T = y^T/x^T$

$C_v = I - \frac{\alpha}{\beta} \Rightarrow C_v^T = I^T - \frac{(\beta)^T}{(\alpha)^T} \Rightarrow C_v^T = I^T - \frac{(\alpha)}{(\beta)} = I - \frac{\alpha}{\beta}$

As we can see C_v^T is equal with C_v which means : $C_v^T = C_v \Rightarrow C_v$ is a parallel matrix.

2.2 Part II

$C_v = I - \frac{(VV^T)}{V^TV} \Rightarrow$ As we already know $\left(V^TV\right) = \|V\|^2$ so :

$$C_v = I - \frac{(VV^T)}{V^TV} = I - \frac{(VV^T)}{\|V\|^2} = \frac{I\|V\|^2 - VV^T}{\|V\|^2}$$

$$C_v^2 = \frac{\|V\|^2 I^2 + (VV^T)^2 - 2\|V\|^2 I(VV^T)}{\|V\|^4} \Rightarrow$$

$$C_v^2 = \frac{\|V\|^4 I}{\|V\|^4} + \frac{(VV^T)(VV^T) - 2\|V\|^2 (VV^T)}{\|V\|^4} \Rightarrow$$

$$C_v^2 = I + \frac{V(V^TV)V^T - 2\|V\|^2 (VV^T)}{\|V\|^4} \Rightarrow$$

$$C_v^2 = I + \frac{V(\|V\|^2)V^T - 2\|V\|^2 (VV^T)}{\|V\|^4} \Rightarrow$$

$$C_v^2 = I + \frac{\|V\|^2 (VV^T) - 2\|V\|^2 (VV^T)}{\|V\|^4} \Rightarrow$$

$$C_v^2 = I + \frac{-\|V\|^2 (VV^T)}{\|V\|^4} \Rightarrow$$

$$C_v^2 = I - \frac{\|V\|^2 (VV^T)}{\|V\|^4} \Rightarrow$$

$$C_v^2 = I - \frac{\|V\|^2 (VV^T)}{\|V\|^4} \Rightarrow$$

$$C_v^2 = I - \frac{(VV^T)}{\|V\|^2} = C_v \Rightarrow$$

$$C_v^2 = I - \frac{(VV^T)}{V^TV} = C_v$$

3 Question 3

قسمت الف تمرین ۳۴ کتاب: تبدیل‌های $T:\mathbb{R}^2 \to \mathbb{R}^2$ و $S:\mathbb{R}^2 \to \mathbb{R}^2$ به صورت زیر بر بردار

$\mathbf{x}=[x\ y]$ عمل می‌کنند. تبدیل‌های $S \circ T$ و $T \circ S$ را بدست آورید.

$$T(\mathbf{x})=\begin{bmatrix} x+y \\ x-y \end{bmatrix} \quad , \quad S(\mathbf{x})=\begin{bmatrix} x-y \\ x+y \end{bmatrix}$$

$X = [x, y]$

$S(X) = \begin{pmatrix} x-y \\ x+y \end{pmatrix}$

$T(X) = \begin{pmatrix} x+y \\ x-y \end{pmatrix}$

$$ToS : \begin{pmatrix} (x-y)+(x+y) \\ (x-y)-(x+y) \end{pmatrix} = \begin{pmatrix} 2x \\ -2y \end{pmatrix}$$

$$SoT : \begin{pmatrix} (x+y)-(x-y) \\ (x+y)+(x-y) \end{pmatrix} = \begin{pmatrix} 2y \\ 2x \end{pmatrix}$$

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4 Question 4

تمرین ۳۷ کتاب: تبدیل خطی $T:\mathbb{R}^2 \to \mathbb{R}^2$ را پیدا کنید که برای آن $T(\mathbf{i}_1)=\mathbf{i}_1-3\mathbf{i}_2$ و $T(\mathbf{i}_2)=\mathbf{i}_2$ باشد.

$T(i_1) = i_1 - 3i_2 \Rightarrow T(i_1) = \binom{1}{-3}$

$T(i_2) = i_2 \Rightarrow T(i_2) = \binom{0}{1}$

$A = [T(i_1)\ T(i_2)] = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}$

testing :

$T(x) = Ax \Rightarrow T(i_1) = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \checkmark$

$T(x) = Ax \Rightarrow T(i_2) = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \checkmark$

End of this HomeWork.
This file is written in LaTeX