Linear Algebra - HW 5

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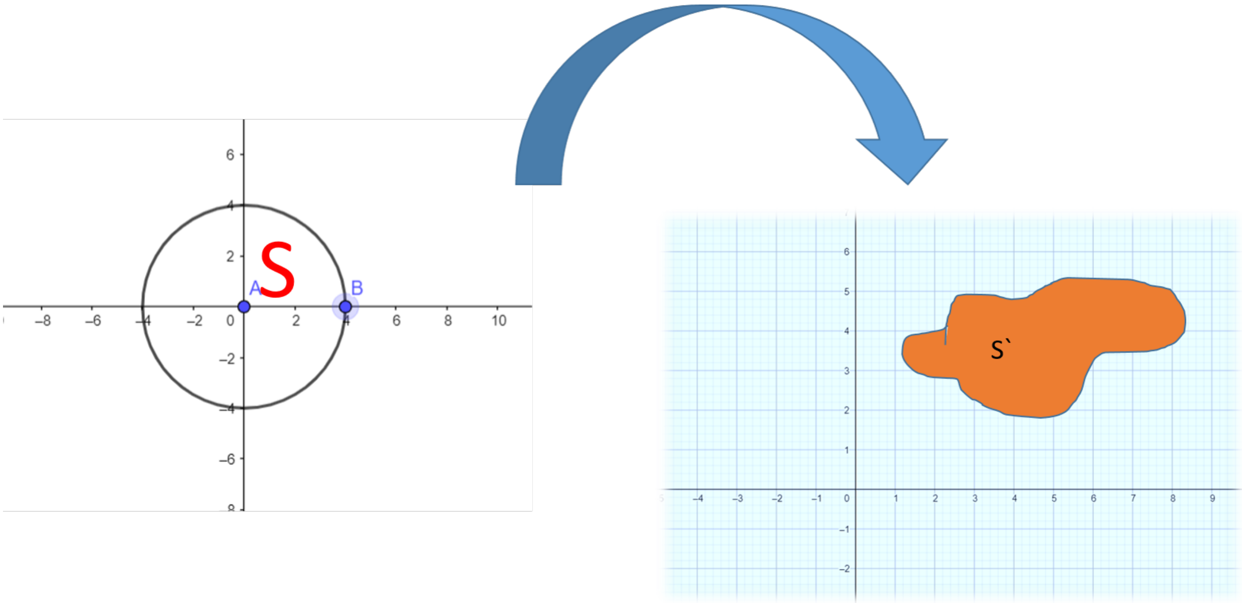
1 Question 1

۱) اگر مساحت سطح محصور توسط معادله $x^2 + y^2 = 4$ را بدانیم و آن را با ماتریس $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ به نقاط دیگری نگاشت کنیم. مساحت سطح محصور شده توسط مجموعه نقاط جدید را بدست آورید.



$$S = \pi r^2 = 16\pi$$
$$S' = S * \left\| det \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\| \Rightarrow$$
$$S' = S * \|ad - bc\| \Rightarrow 16\pi \, \|ad - bc\|$$

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2 Question 2

۲) درستی رابطه $\det\left(\mathbf{I}_K \otimes \mathbf{A}\right) = \det\left(\mathbf{A}\right)^K$ را بر اساس رابطه (۸.۵۶) کتاب اثبات کنید (تمرین ۵ کتاب).

رابطه (۸.۵۶) کتاب: $\det\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{bmatrix} = \det\left(\mathbf{A}\right)\det\left(\mathbf{D}\right)$

As we know , $I_n = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}_{n\times n}$

And we also know that $A \otimes I_n = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & A \end{pmatrix}_{n\times n}$

We also know as a fact that $det \begin{pmatrix} A & 0 \\ 0 & D \end{pmatrix} = det(A)det(D) \Rightarrow$ so

$$det \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & A \end{pmatrix}_{n\times n} = \underbrace{det(A) * det(A) * .... * det(A)}_{\text{n times}} = det(A)^n$$

3 Question 3

۳) فرض کنید که $\mathbf{A}$ و B ماتریس‌هایی ۶×۶ بعدی بوده و می‌دانیم که $\det(\mathbf{AB}^2) = 72$ و $\det(\mathbf{A}^2\mathbf{B}^2) = 144$ می‌باشند. دترمینان ماتریس‌های $\mathbf{A}$, $2\mathbf{A}$, $\mathbf{AB}$ را محاسبه کنید (تمرین ۷ کتاب).

$$det(AB^2) = 72 \Rightarrow det(A)det(B)^2 = 72$$

$$det(A^2B^2) = 144 \Rightarrow det(A)^2 * det(B)^2 = 144$$

$$det(A) = ?, det(2A) = ?, det(AB) = ?$$

$$\frac{det(A^2B^2)}{det(AB^2)} = \frac{144}{72} \Rightarrow \frac{det(A)\cancel{det(A)det(B)^2}}{\cancel{det(A)det(B)^2}} \Rightarrow det(A) = 2$$

$$det(A)det(B)^2 = 72 \Rightarrow 2 * det(B)^2 = 72 \Rightarrow det(B) = 6$$

$$det(2A) = 2det(A) = 2 * 2 = 4 \Rightarrow det(2A) = 4$$

$$det(AB) = det(A)det(B) = 6 * 2 = 12 \Rightarrow det(AB) = 12$$

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4 Question 4

۴) دترمینان ماتریس‌های زیر را محاسبه نمایید (تمرین ۹ کتاب).

$$\mathbf{A} = \begin{bmatrix} 0 & 5 & 1 & 4 \\ 0 & 1 & 0 & 0 \\ 8 & 1 & 7 & 5 \\ 2 & 1 & 9 & 4 \end{bmatrix} \quad , \quad \mathbf{B} = \begin{bmatrix} 4 & 3 & 7 & 0 \\ 3 & 6 & 9 & 4 \\ 1 & 4 & 3 & 9 \\ 4 & 0 & 1 & 1 \end{bmatrix}$$

4.1 Part I

I would like to calculate the determinant using the second row due to having the most number of zeros which makes my calculations a lot easier.

$$A = \begin{pmatrix} 0 & 5 & 1 & 4 \\ 0 & 1 & 0 & 0 \\ 8 & 1 & 7 & 5 \\ 2 & 1 & 9 & 4 \end{pmatrix}$$

$$det(A) = (-1)det(A_{22}) + 0 = -det(A_{22})$$

$$A' = A_{22} = \begin{pmatrix} 0 & 1 & 4 \\ 8 & 7 & 5 \\ 2 & 9 & 4 \end{pmatrix}$$

$$det(A_{22}) = 0 * \cdots - (1)det(A'_{12}) + 4 * det(A'_{13}) \Rightarrow$$

$$= (-1)\begin{vmatrix} 8 & 5 \\ 2 & 4 \end{vmatrix} + (4)\begin{vmatrix} 8 & 7 \\ 2 & 9 \end{vmatrix} = (32-10)*(-1) + 4(72-14) = -22 + 4(58) = 232 - 22 = 210$$

4.2 Part II

I would like to calculate the determinant using the second row due to having the most number of zeros which makes my calculations a lot easier.

$$B = \begin{pmatrix} 4 & 2 & 7 & 0 \\ 3 & 6 & 9 & 4 \\ 1 & 4 & 3 & 9 \\ 4 & 0 & 1 & 1 \end{pmatrix}$$

$$det(B) = (4)det(B_{11}) - (3)det(B_{12}) + (7)det(B_{13})$$

$$det(B_{11}) =? \Rightarrow B_{11} = \begin{pmatrix} 6 & 9 & 4 \\ 4 & 3 & 9 \\ 0 & 1 & 1 \end{pmatrix} \Rightarrow$$

$$= (6)\begin{vmatrix} 3 & 9 \\ 1 & 1 \end{vmatrix} + (-4)\begin{vmatrix} 9 & 4 \\ 1 & 1 \end{vmatrix} = (6 * -6) - 4 * 5 = -36 - 20 = -56 \Rightarrow$$

$$det(B_{11}) = -56$$

$$det(B_{12}) =? \Rightarrow B_{12} = \begin{pmatrix} 3 & 9 & 4 \\ 1 & 3 & 9 \\ 4 & 1 & 1 \end{pmatrix} \Rightarrow$$

$$= (3)\begin{vmatrix} 3 & 9 \\ 1 & 1 \end{vmatrix} + (-9)\begin{vmatrix} 1 & 9 \\ 4 & 1 \end{vmatrix} + (4)\begin{vmatrix} 1 & 3 \\ 4 & 1 \end{vmatrix} = 3(-6) - 9(-35) + 4(-11) = 253 \Rightarrow$$

$$det(B_{12}) = 253$$

$$det(B_{13}) =? \Rightarrow B_{13} = \begin{pmatrix} 3 & 6 & 4 \\ 1 & 4 & 9 \\ 4 & 0 & 1 \end{pmatrix} \Rightarrow$$

$$= (4)\begin{vmatrix} 6 & 4 \\ 4 & 9 \end{vmatrix} + (1)\begin{vmatrix} 3 & 6 \\ 1 & 4 \end{vmatrix} = 4(56 - 16) + (12 - 6) = 158 \Rightarrow$$

$$det(B_{13}) = 158$$

$$det(B) = 4(-56) - 3(253) + 7(158) = 123$$

5 Question 5

$\det\left(\mathbf{A}^{Adj}\right)=\det\left(\mathbf{A}\right)^{n-1}$ ۵) اگر $\mathbf{A}$ یک ماتریس n بعدی باشد، نشان دهید (تمرین ۱۴ کتاب):

Prove that $det(A^{adj}) = det(A)^{n-1}$

$$A^{-1} = \frac{A^{adj}}{det(A)} \Rightarrow$$

$$A^{adj} = A^{-1} * det(A) \xrightarrow{\text{*A from left}}$$

$$A * A^{adj} = A * A^{-1} * det(A) \Rightarrow$$

$$A * A^{adj} = I_n * det(A) \xrightarrow{\text{det() from both sides}}$$

$$det(A * A^{adj}) = det(I_n * det(A))$$

We know that $det(\alpha I_n) = \alpha^n$

$$det(A * A^{adj}) = det(I_n * det(A)) \Rightarrow$$

$$det(A) * det(A^{adj}) = det(A)^n \Rightarrow$$

$$det(A^{adj}) = det(A)^{n-1}$$

End of this HomeWork.
This file is written in LaTeX