

Sentiment analysis Logistic Regression

Logistic Regression (LR)

a neural network can be viewed as a series of logistic regression classifiers stacked on top of each other.

Generative vs. Discriminative

NB is a Generative Model

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)} = \frac{P(E|H)P(H)}{\sum_{H \in \mathcal{H}} P(E|H)P(H)}$$

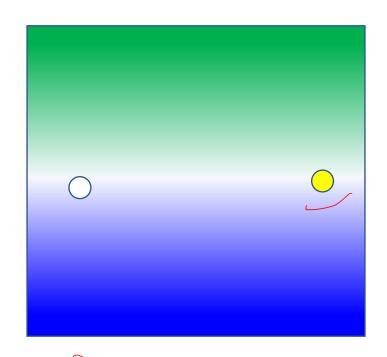
If we choose H, P(H), how can we generate instances of event/evidences P(E|H) If we are in H4: Winter and in Canada, generate a day \rightarrow it would be mostly no sun!

LR is Discriminative!

Don't care about <u>hypothesis</u> and their probabilities.

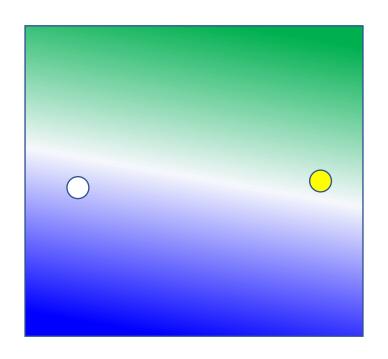
Try to maximize the distance of data from different classes.

Try to <u>discriminate</u> the most!



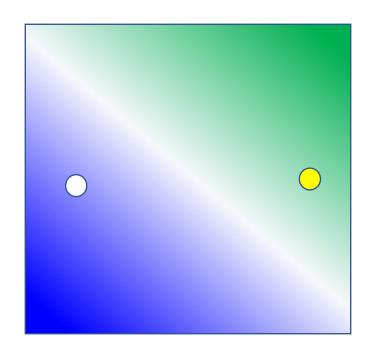
$$P(+|x_{+})=0.50 P(-|x_{+})=0.50$$

 $P(-|x_{-})=0.50 P(+|x_{-})=0.50$



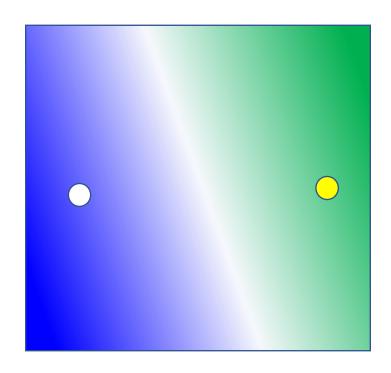
$$P(+|x_{+})=0.55 P(-|x_{+})=0.45$$

 $P(-|x_{-})=0.55 P(+|x_{-})=0.45$

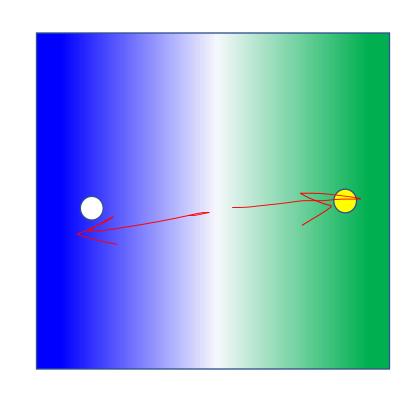


$$P(+|x_{+})=0.65 P(-|x_{+})=0.35$$

 $P(-|x_{-})=0.65 P(+|x_{-})=0.35$



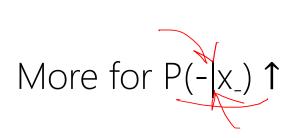
 $P(+|x_{+})=0.75 P(-|x_{+})=0.25$ $P(-|x_{-})=0.75 P(+|x_{-})=0.25$

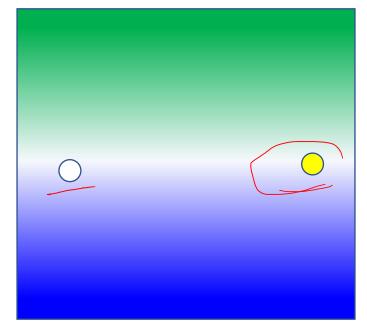


 $P(+|x_{+})=0.85 P(-|x_{+})=0.15$ $P(-|x_{-})=0.85 P(+|x_{-})=0.15$

(6N/WC 55. 6[]

- (I) <u>Iterative</u> Process
- (II) Optimization = Discriminate classes the most

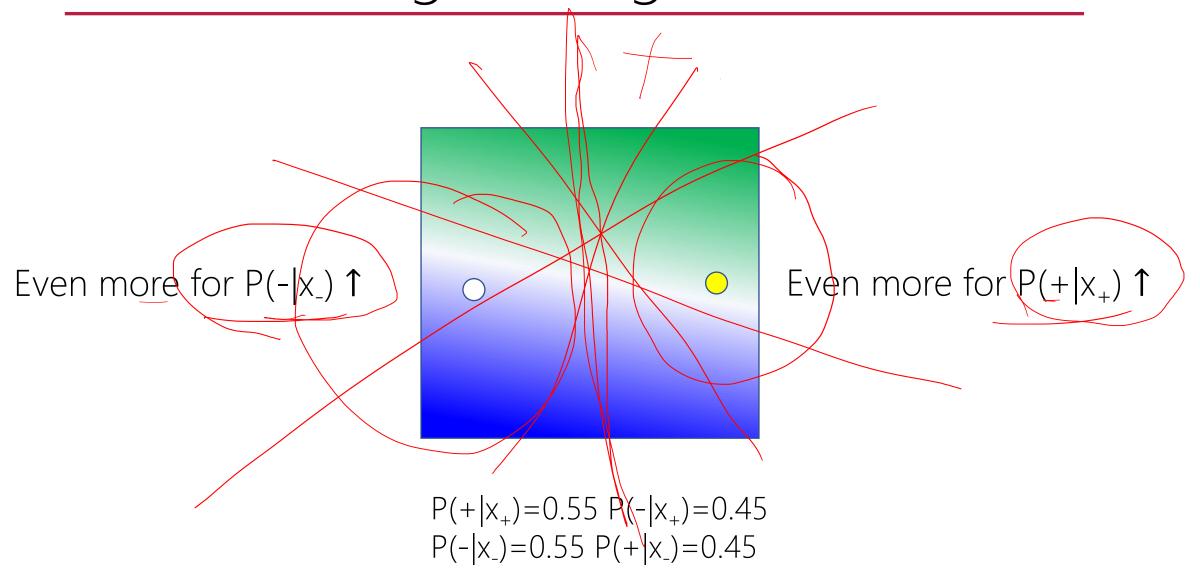




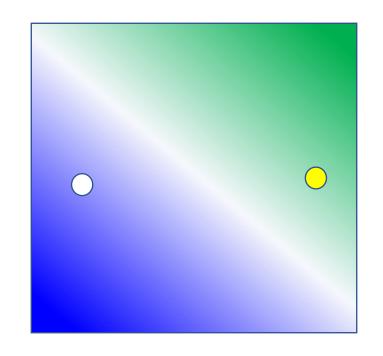
More for $P(+|x_+) \uparrow$

$$P(+|x_{+})=0.50 P(-|x_{+})=0.50$$

 $P(-|x_{-})=0.50 P(+|x_{-})=0.50$



Even more for $P(-|x_{-}|)$

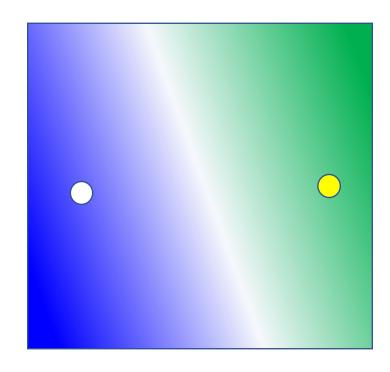


Even more for $P(+|x_+) \uparrow$

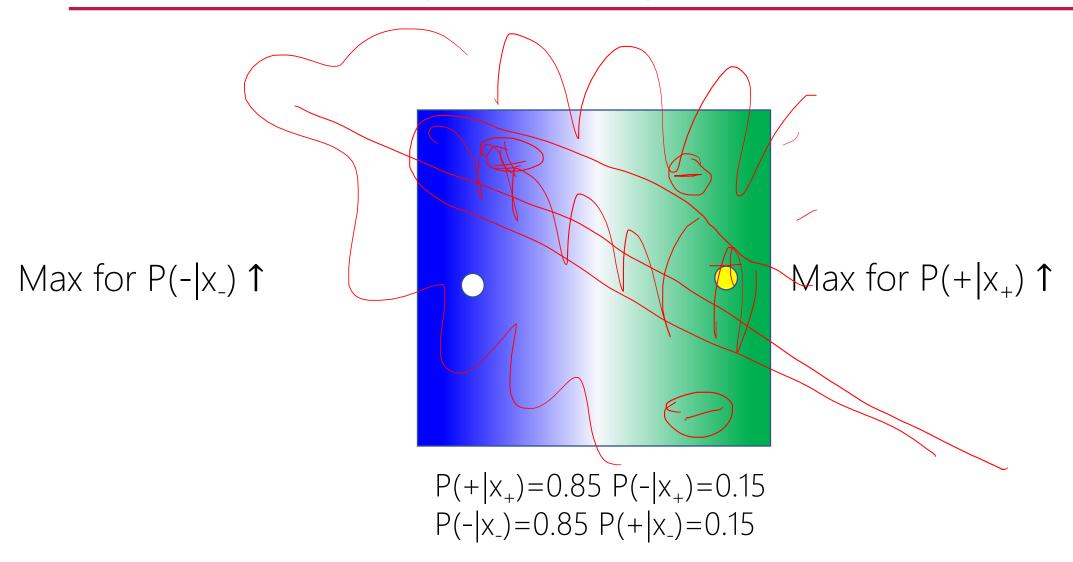
$$P(+|x_{+})=0.65 P(-|x_{+})=0.35$$

 $P(-|x_{-})=0.65 P(+|x_{-})=0.35$

Even more for $P(-|x_{-}|) \uparrow$



 $P(+|x_{+})=0.75 P(-|x_{+})=0.25$ $P(-|x_{-})=0.75 P(+|x_{-})=0.25$ Even more for $P(+|x_+)$ 1



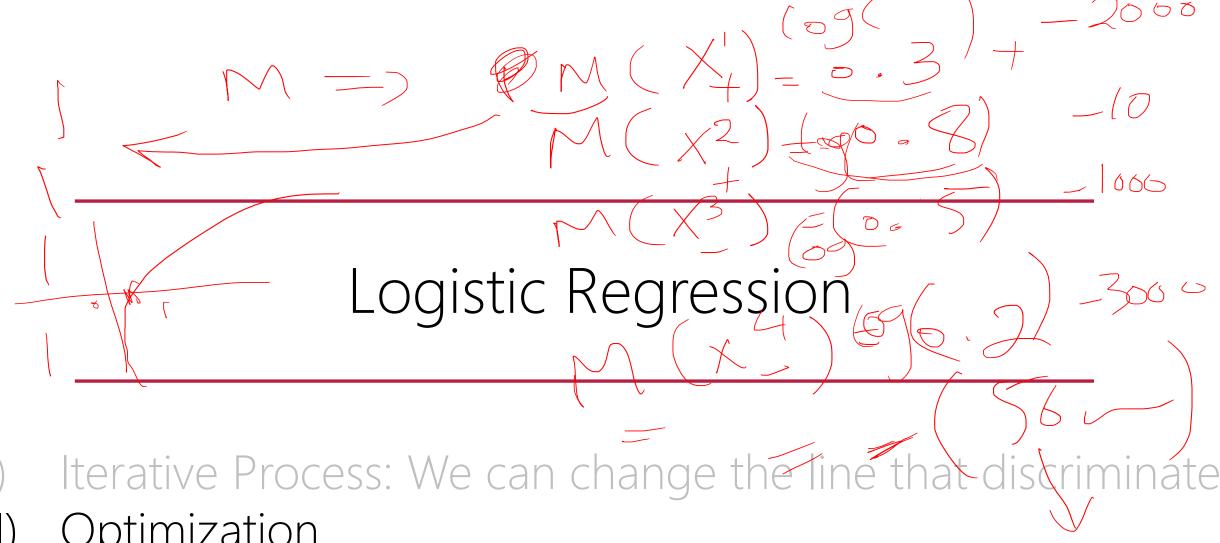
- (I) Iterative Process: We can change the line that discriminate
- (II) Optimization = Discriminate classes the most
 - (I) Maximizing the $P(+|x_+) + P(-|x_-)$

- (I) What is that line?
- (II) What if we have more than two inputs?



- (I) Iterative Process: We can change the line that discriminate
- (II) Optimization

(I)
$$Max (\prod_{x \in \{+\}} P(+|x_+) \nearrow \prod_{x \in \{-\}} P(-|x_-))$$



Optimization

(I)
$$Max(\sum_{x \in \{+\}} Log P(+|x_+) + \sum_{x \in \{-\}} Log P(-|x_-))$$

- (I) Iterative Process: We can change the line that discriminate
- (II) Optimization

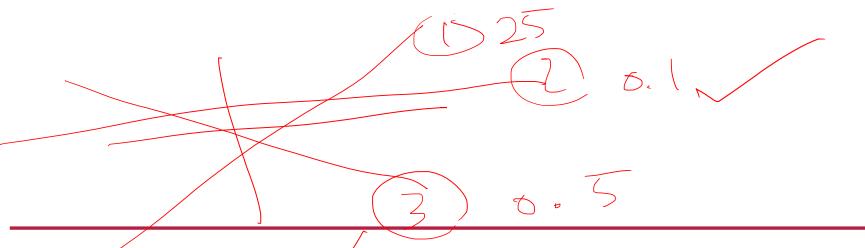
(I)
$$Min_{-}(\sum_{x \in \{+\}} Log P(+|x_+) + \sum_{x \in \{-\}} Log P(-|x_-))$$

$$c = \{-, +\} \rightarrow y = \{0, 1\}$$

- (I) Iterative Process: We can change the line that discriminate
- (II) Optimization
 - (I) $\frac{\text{Min}}{-\sum_{(x,y)\in D}} [(y)\text{Log } P(y|x_y) + (1-y)\text{ Log } P(y|x_y)]$

Logistic Regression $c = \{-, +\} \rightarrow y = \{0, 1\}$

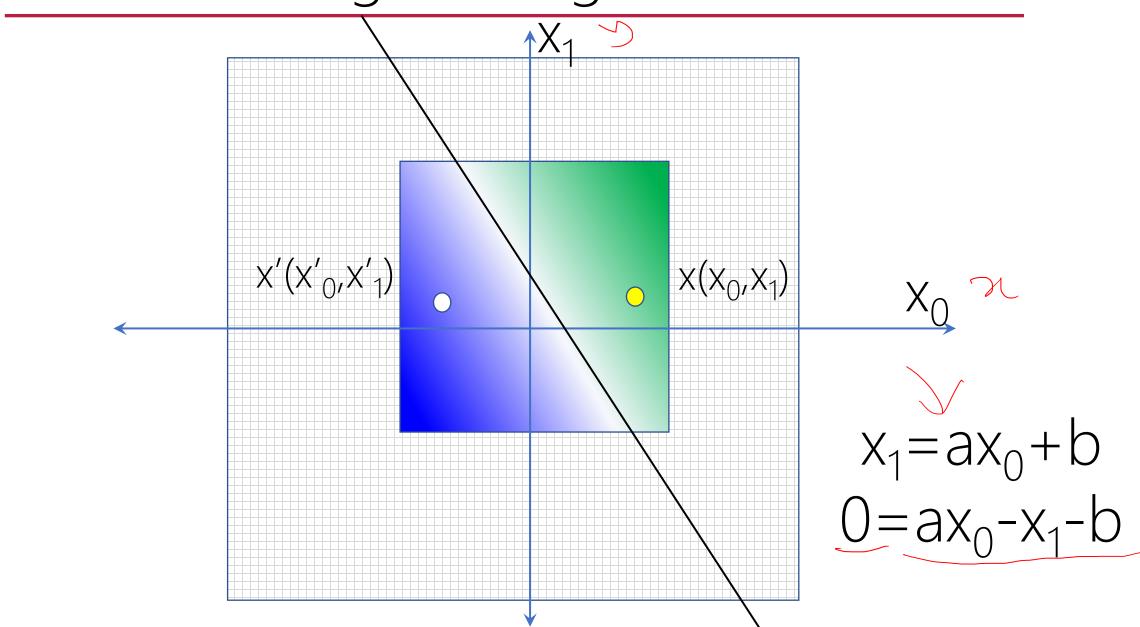
- (I) Iterative Process: We can change the line that discriminate
- (II) Optimization
- (I) $Min = \sum_{(x,y) \in D} Log P(y|x_y)]_{7}$

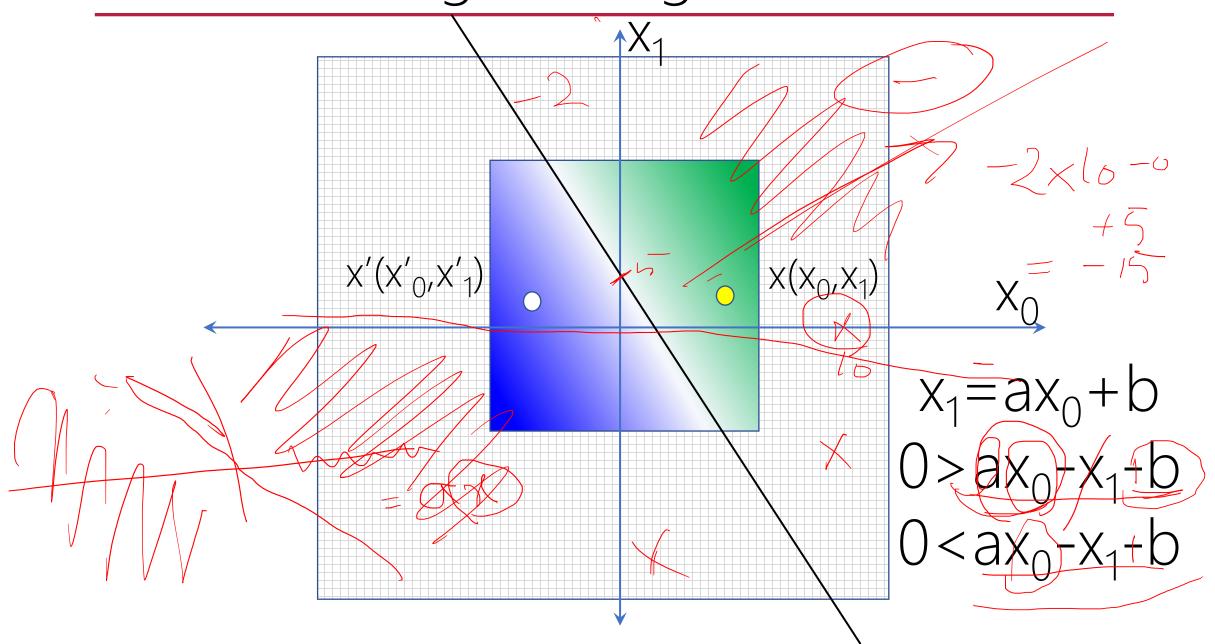


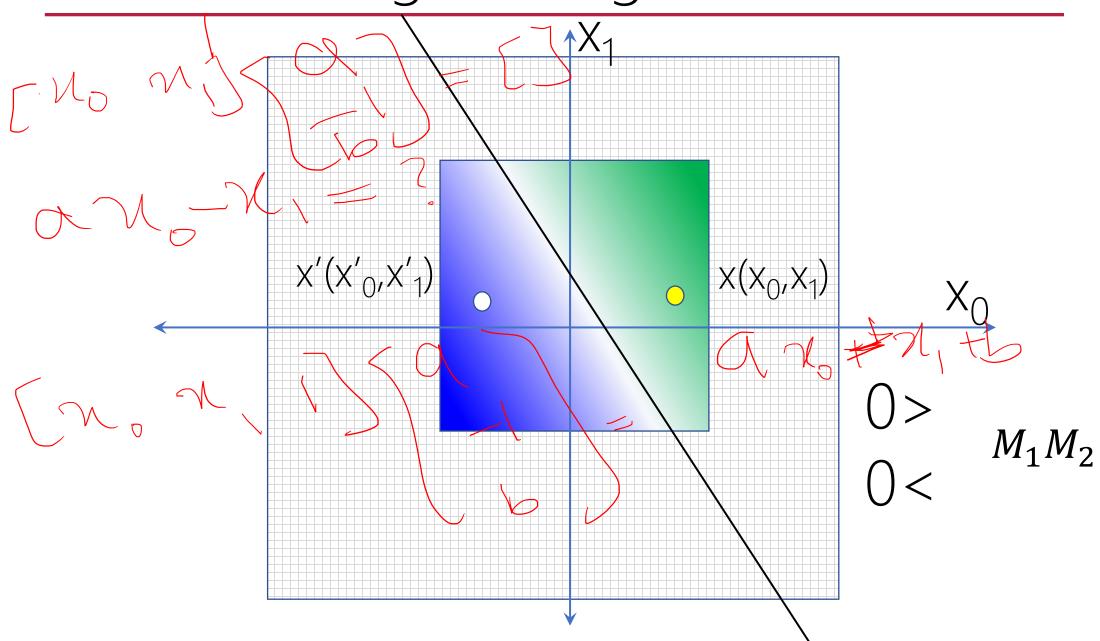
Logistic Regression

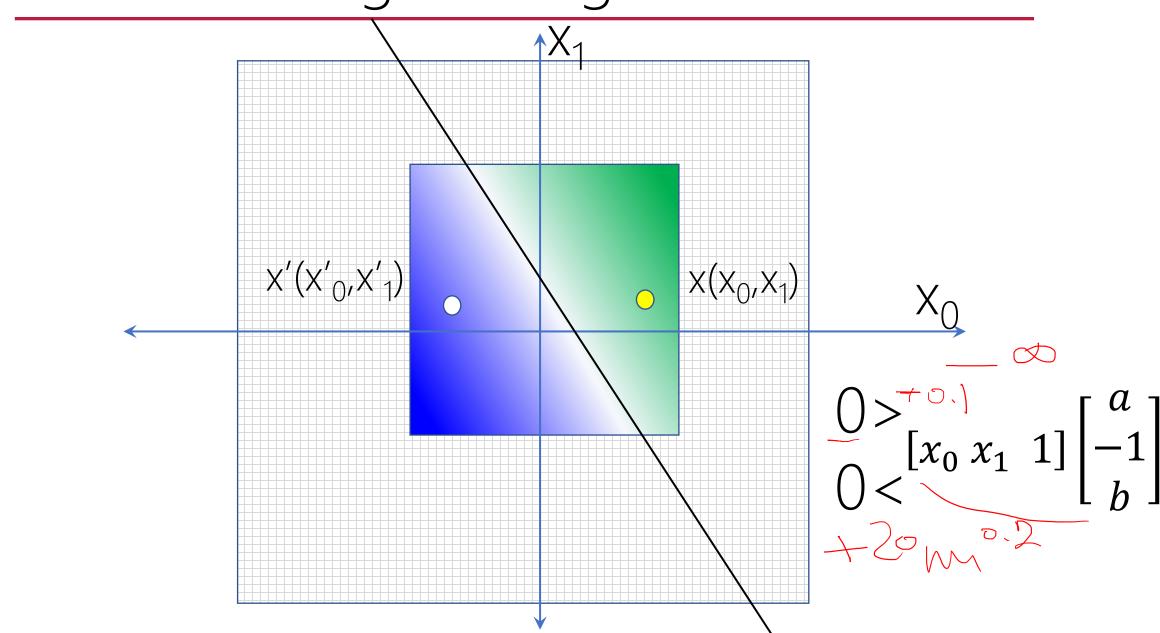
$$c = \{-, +\} \rightarrow y = \{0, 1\}$$

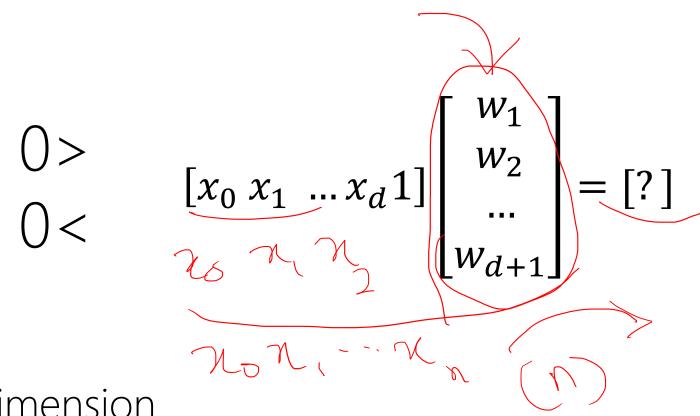
- Iterative Process: We can change the line that discriminate
- $Min \sum_{(x,y)\in D} [(x) Log P(+|x_+) + (1-y) Log P(-|x_-)]$











- $d = 1 \rightarrow Line in 2-dimension$
- $d = 2 \rightarrow Plane in 3 dimension$
- $d = n \rightarrow Hyperplane in (n+1) dimension$

Parametric vs. non-Parametric

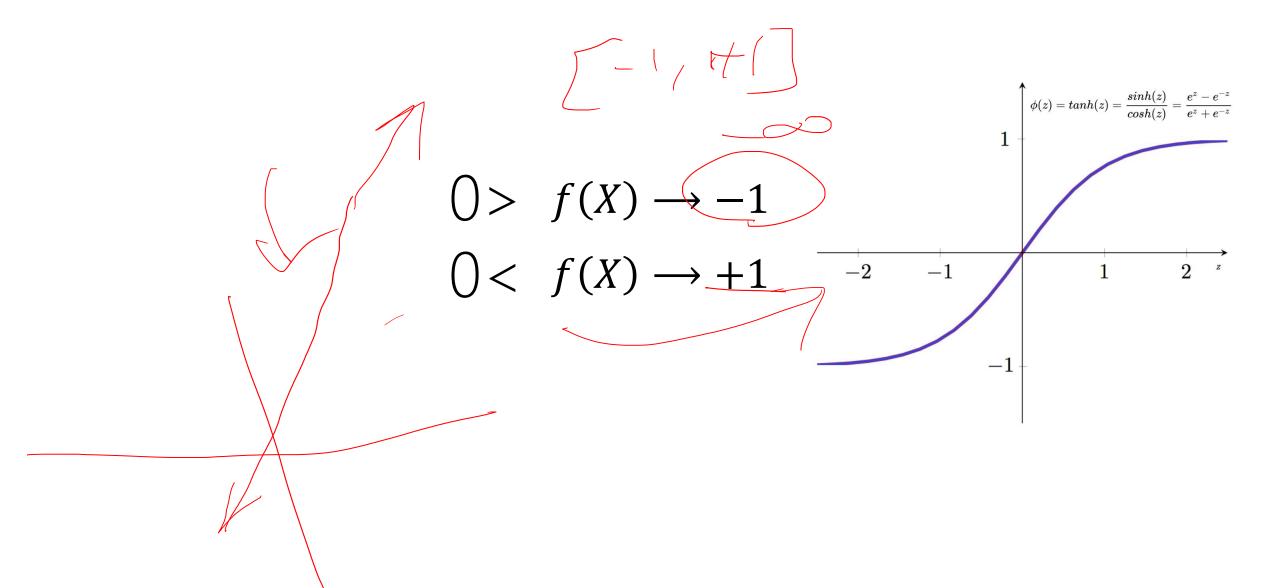
LR vs. Naïve Bayes

(True Bayesian Inference is Parametric, Why?)

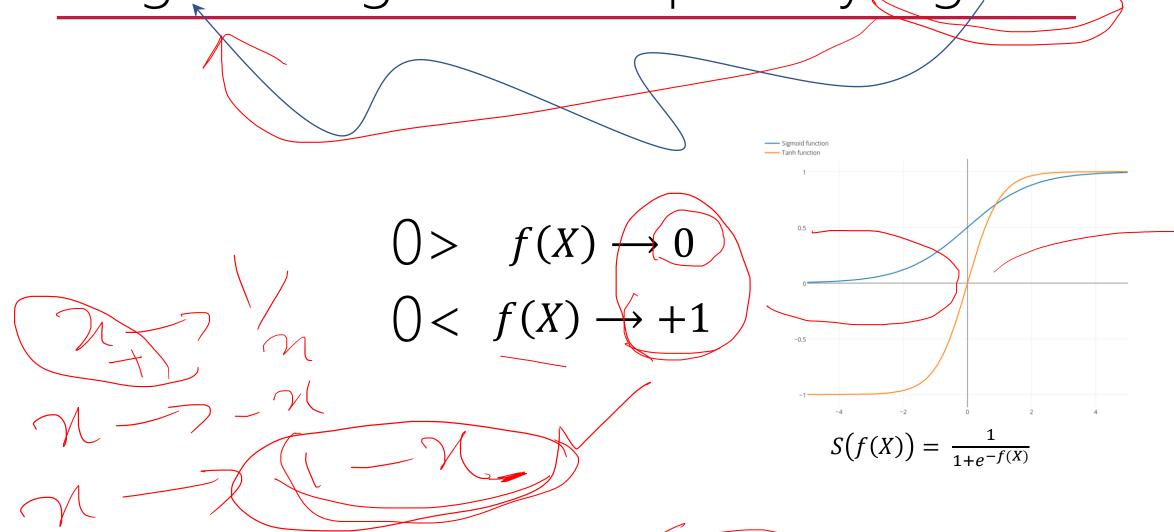
$$0 > f(X) \to -\infty$$
$$0 < f(X) \to +\infty$$

- $d = 1 \rightarrow Line in 2-dimension$
- $d = 2 \rightarrow Plane in 3 dimension$
- $d = n \rightarrow Hyperplane in (n+1) dimension$

Logistic Regression: Squish by Tanh

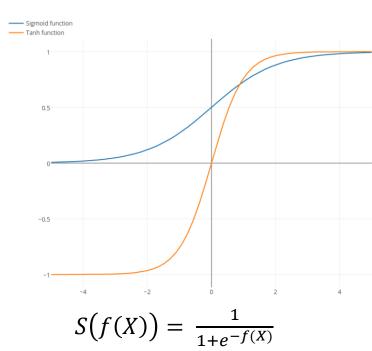


Logistic Regression: Squish by Sigmoid



Becomes very similar to probability values!

Logistic Regression: Squish



Becomes very similar to probability values!

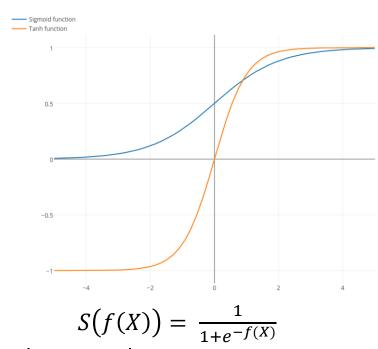
But only for positive class (+)

$$y=1 \rightarrow P(y|x) = P(+|x) = Sigmoid(x)$$

Logistic Regression: Squish

$$\bigcirc > f(X) \longrightarrow 0$$

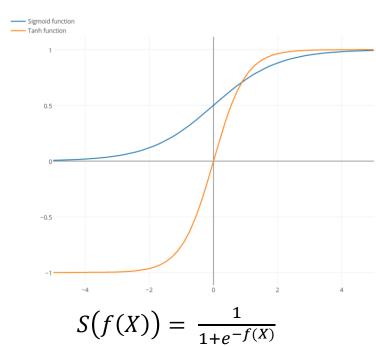
$$\bigcirc < f(X) \longrightarrow +1$$



Becomes very similar to probability values! For negative class (-)?

Logistic Regression: Squish

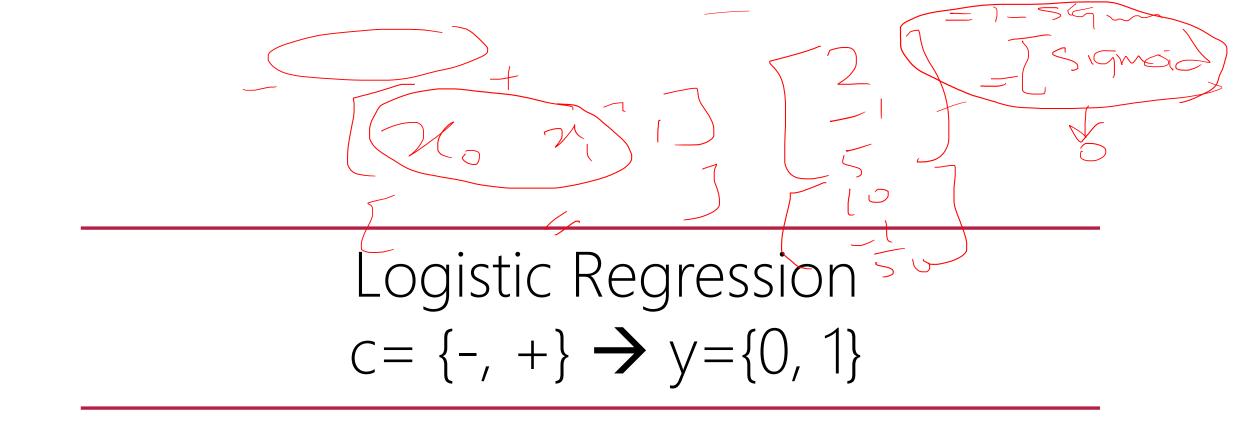
$$\begin{array}{ccc}
O > & f(X) \longrightarrow 0 \\
O < & f(X) \longrightarrow +1
\end{array}$$



Becomes very similar to probability values!

For negative class (-)

$$P(+|x) + P(-|x) = 1 \rightarrow P(-|x) = 1 - P(+|x)$$



- (I) Iterative Process: We can change the that discriminate
- (II) Optimization

(I)
$$\lim_{x \to \infty} -\sum_{(x,y) \in D} \text{Log} P(y|x_y)] \rightarrow \begin{cases} y = 1: P(+|x_+|) = \underbrace{Sigmoid(f(x_+))} \\ y = 0: P(-|x_-|) = 1 - \underbrace{Sigmoid(f(x_+))} \\ \end{cases}$$

Logistic Regression P ~ f

- (I) Iterative Process: We can change the $\frac{f}{f}$ that discriminate
- (II) Optimization

(I) Min
$$-\sum_{(X,Y)\in D} Log P(Y|X_y)$$
 $\rightarrow \begin{cases} y = 1: P(+|x_+) = Sigmoid(f(x_+)) \\ y = 0: P(-|x_-) = 1 - Sigmoid(f(x_+)) \end{cases}$

Logistic Regression

Optimization: $Min - \sum_{(X,Y) \in D} Log P(y|X_y)$ $\Rightarrow \begin{cases} y = 1: P(+|x_+) = Sigmoid(f(x_+)) \\ y = 0: P(-|x_-) = 1 - Sigmoid(f(x_+)) \end{cases}$

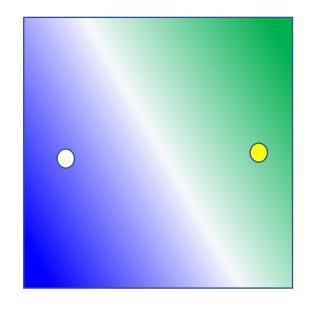
- Derivatives per function weights (Gradients)
 - o Update on each input data
 - o Update on batches of input data (Stochastic Gradient Descend)
 - o Update on multiple rounds (epoch) of ALL data



Logistic Regression

Optimization: Min
$$-\sum_{(x,y)\in D} \text{Log } P(y|x_y)$$
] $\rightarrow \begin{cases} y = 1: P(+|x_+) = Sigmoid(f(x_+)) \\ y = 0: P(-|x_-) = 1 - Sigmoid(f(x_+)) \end{cases}$

Function f is linear function of weights

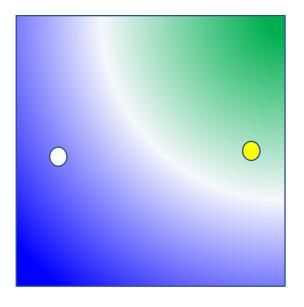


$$[x_0 \ x_1 \ \dots x_d 1] \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_{d+1} \end{bmatrix} = [?]$$

Logistic Regression

Optimization: Min
$$-\sum_{(x,y)\in D} \text{Log } P(y|x_y)$$
] $\rightarrow \begin{cases} y = 1: P(+|x_+) = Sigmoid(f(x_+)) \\ y = 0: P(-|x_-) = 1 - Sigmoid(f(x_+)) \end{cases}$

Can we have f as a non-linear function of weights?

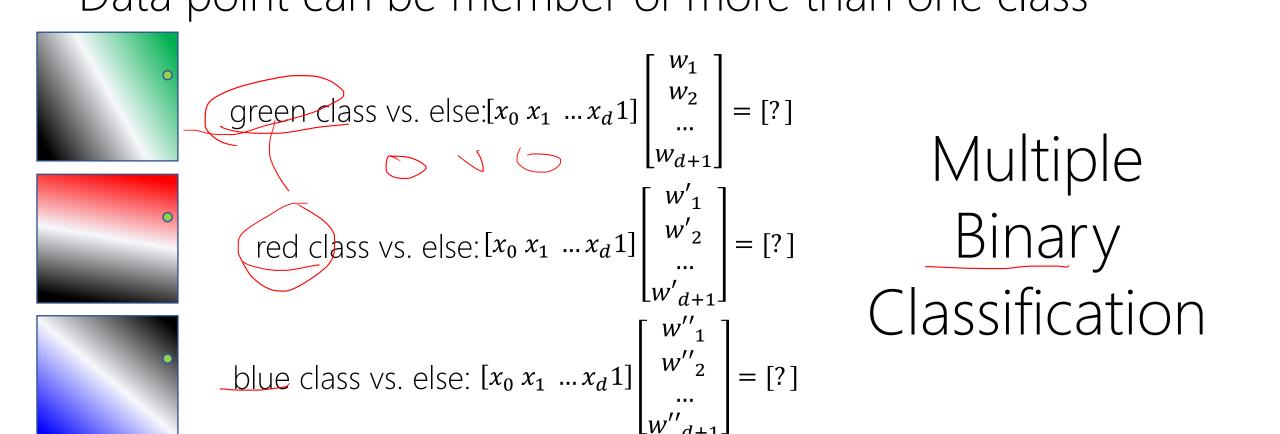


Multiclass with Logistic Regression

Multi-target (Multi-label)

Optimization: Min $-\sum_{(x,y)\in D} \text{Log } P(y|x_y)$

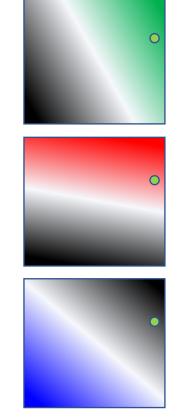
Data point can be member of more than one class



Multi-target (Multi-label)

Optimization: Min $-\sum_{(x,y)\in D} \text{Log } P(y|x_y)$]

Data point can be member of more than one class



(COM) (Sy, 13, (So, 13))

Multinomial Logistic Regression

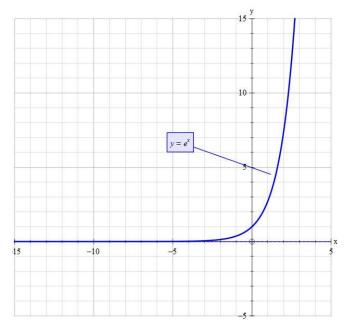
Logistic Regression -> Softmax Regression

Softmax Regression

```
softmax(\mathbf{z} = [z_1, z_2, ..., z_d]) = \frac{1}{\sum_{i=1}^{d} e^{z_i}} [e^{z_1}, e^{z_2}, ..., e^{z_d}]
0 < \frac{e^{z_j}}{\sum_{i=1}^{d} e^{z_i}}
```

```
0 \le \frac{\sum_{i=1}^{d} e^{z_i}}{\sum_{i=1}^{d} e^{z_i}} \le 1
\sum_{j=1}^{d} \frac{e^{z_j}}{\sum_{i=1}^{d} e^{z_i}} = 1
```





Multi-target (Multi-label)

Optimization: Min $-\sum_{(x,y)\in D} \text{Log } P(y|x_y)$

```
 [x_0 \ x_1 \ ... x_d 1] \begin{bmatrix} w_1 & w'_1 & w''_1 \\ w_2 & w'_2 & w''_2 \\ ... & ... \\ w_{d+1} w'_{d+1} w''_{d+1} \end{bmatrix} = [1.5, -1.4, 10] \Rightarrow Softmax 
 \Rightarrow [2.0342e - 04, 1.1193e - 05, 9.9979e - 01] 
                                                                                                                        torch.softmax(torch.as_tensor([1.5, -1.4, 10.0]).view(-1), dim=0)
```

$$[x_0 \ x_1 \ ... x_d 1] \begin{bmatrix} w_1 & w'_1 & w''_1 \\ w_2 & w'_2 & w''_2 \\ ... & ... & ... \\ w_{d+1} w'_{d+1} w''_{d+1} \end{bmatrix} = [1.5, -1.4, 10]$$
 $\Rightarrow sigmoid = [0.8176, 0.1978, 1.0000]$ $\Rightarrow softmax \Rightarrow [0.3652, 0.1965, 0.4383]$

```
torch.sigmoid(torch.as_tensor([1.5, -1.4, 10.0]).view(-1))
Out[14]: tensor([0.8176, 0.1978, 1.0000])
        torch.softmax(torch.sigmoid(torch.as_tensor([1.5, -1.4, 10.0]).view(-1)), dim=0)
Out[15]: tensor([0.3652, 0.1965, 0.4383])
```

Multi-target (Multi-label)

Optimization: Min $-\sum_{(x,y)\in D} \text{Log } P(y|x_y)$

Multitarget: $(x, [1, 0, 1]) \stackrel{?}{\leftrightarrow} (x, [0.8176, 0.1978, 1.0])$

$$(x,\{c1\}) \stackrel{?}{\leftrightarrow} (x,[0.8176,\blacksquare,\blacksquare])$$

$$(x,\{c2\}) \stackrel{?}{\leftrightarrow} (x,[\blacksquare,0.1978,\blacksquare]) \stackrel{?}{\leftrightarrow} (x,[\blacksquare,1-0.1978,\blacksquare])$$

$$(x,\{c3\}) \stackrel{?}{\leftrightarrow} (x,[\blacksquare,\blacksquare,1.0])$$

$$- \left[\text{Log P}(\text{c1}|\text{x}_{\text{c1}}) + \text{Log P}(\text{c2}|\text{x}_{\text{c2}}) + \text{Log P}(\text{c3}|\text{x}_{\text{c3}}) \right] \\ - \left[\text{Log (0.81)} + \text{Log (1-0.19)} + \text{Log (1.0)} \right]$$

Multiclass

Optimization: Min $-\sum_{(x,y)\in D} \text{Log } P(y|x_y)$

Multiclass:
$$(x, [0, 1, 0]) \stackrel{?}{\leftrightarrow} (x, [0.3652, 0.1965, 0.4383])$$

 $(x, \{1\}) \stackrel{?}{\leftrightarrow} (x, [0.3652, \blacksquare, \blacksquare]) \stackrel{?}{\leftrightarrow} (x, [1 - 0.3652, \blacksquare, \blacksquare])$
 $(x, \{2\}) \stackrel{?}{\leftrightarrow} (x, [\blacksquare, 0.1965, \blacksquare])$
 $(x, \{3\}) \stackrel{?}{\leftrightarrow} (x, [\blacksquare, \blacksquare, 0.4383]) \stackrel{?}{\leftrightarrow} (x, [\blacksquare, \blacksquare, 1 - 0.4383])$

Multiclass

Optimization: Min $-\sum_{(x,y)\in D} \text{Log } P(y|x_y)$

Multiclass: $(x, [0, 1, 0]) \stackrel{?}{\leftrightarrow} (x, [0.3652, 0.1965, 0.4383])$

$$(x,\{1\}) \xrightarrow{?} (x,[0.3652,\blacksquare,\blacksquare]) \xrightarrow{?} (x,[1-0.3652,\blacksquare,\blacksquare])$$

$$(x,\{2\}) \xrightarrow{?} (x,[\blacksquare,0.1965,\blacksquare])$$

$$(x,\{3\}) \xrightarrow{?} (x,[\blacksquare,\blacksquare,0.4383]) \xrightarrow{?} (x,[\blacksquare,\blacksquare,1-0.4383])$$

If x belongs to one class, it does not belong to other classes Also, in softmax, if we increase one element, it reduces other (sum=1)

Multiclass

Optimization: Min $-\sum_{(x,y)\in D} \text{Log } P(y|x_y)$

Multiclass:
$$(x, [0, 1, 0]) \stackrel{?}{\leftrightarrow} (x, [0.3652, 0.1965, 0.4383])$$

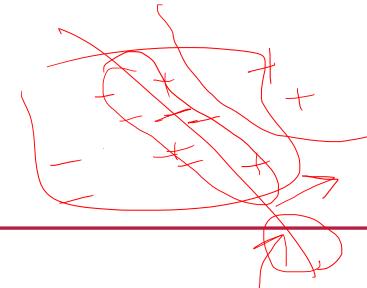
$$-\left([0, 1, 0] \log \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = +\infty$$

$$-\left([0, 1, 0] \log \begin{bmatrix} 0.3652 \\ 0.1965 \\ 0.4383 \end{bmatrix}\right) = ?$$

$$-\left([0, 1, 0] \log \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = 0$$

Evaluation

Curves



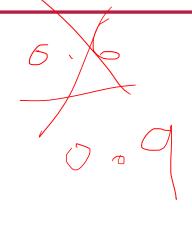
Threshold-based Model

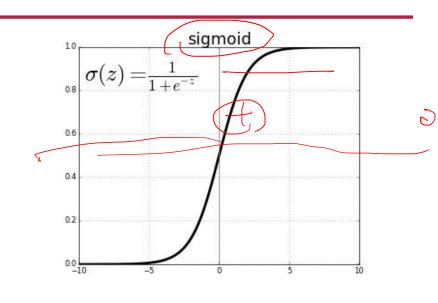
$$P(+|x) = 1 - P(-|x)$$

$$P(x) = Sigmoid(f(x))$$

$$P(x) \ge \delta \longrightarrow x$$
 is positive

$$P(x) < \delta \rightarrow x$$
 is negative





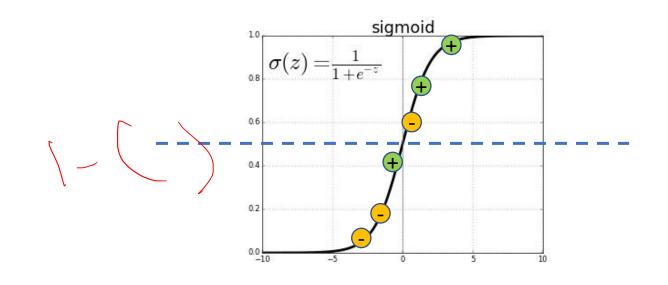
Threshold-based Model δ =0.5

$$P(+|x) = 1 - P(-|x)$$

$$P(x) = Sigmoid(f(x))$$

$$P(x) \ge 0.5 \longrightarrow x$$
 is positive

$$P(x) < 0.5 \rightarrow x$$
 is negative



Threshold-based Model

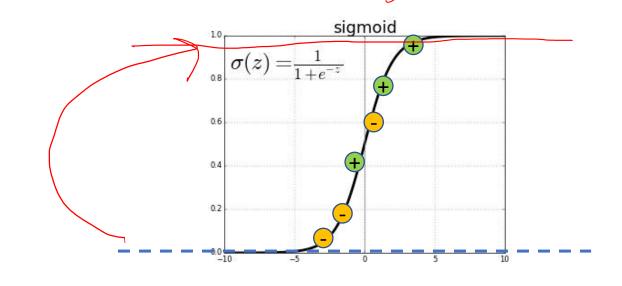
$$\delta$$
=0.0 \longrightarrow Biased Model \longrightarrow All are positives

P(+|x) = 1 - P(-|x)

$$P(x) = Sigmoid(f(x))$$

 $P(x) \ge 0.0 \longrightarrow x$ is positive

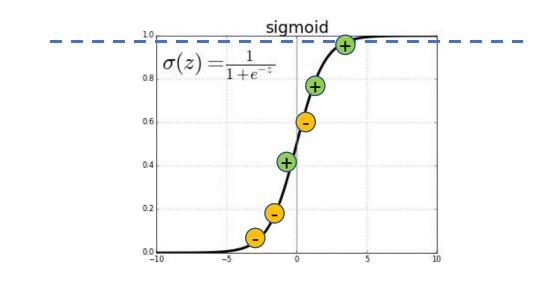
 $P(x) < 0.0 \rightarrow x$ is negative



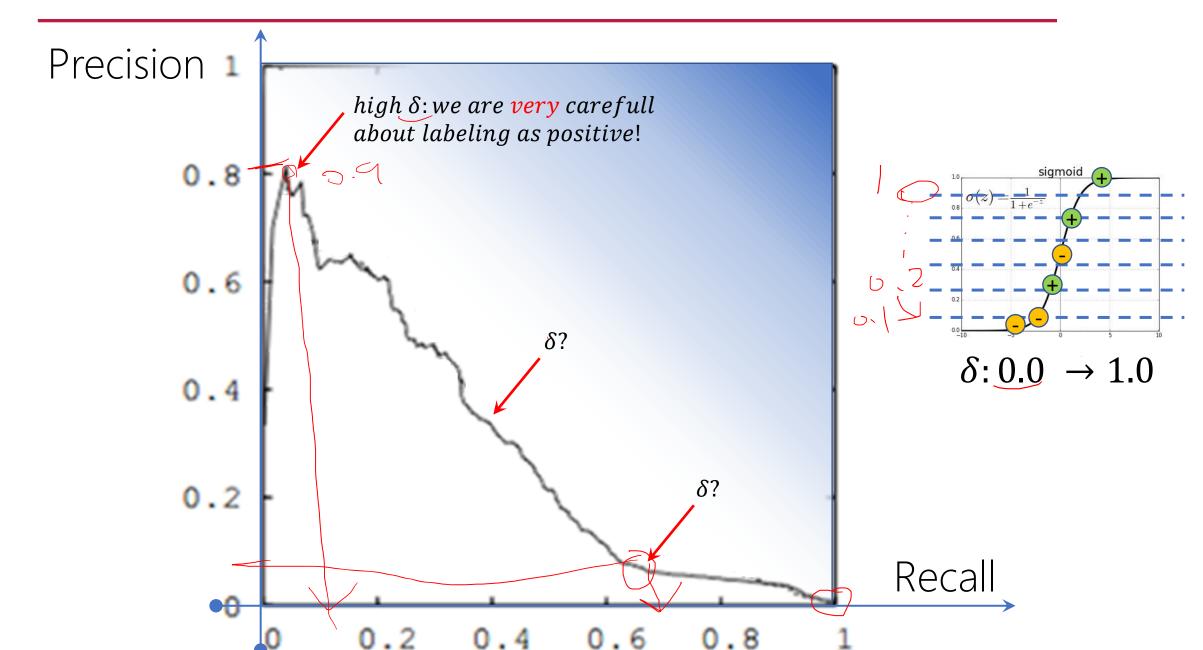
Threshold-based Model δ =1.0 \rightarrow Biased Model \rightarrow All are negatives

$$P(+|x) = 1 - P(-|x)$$

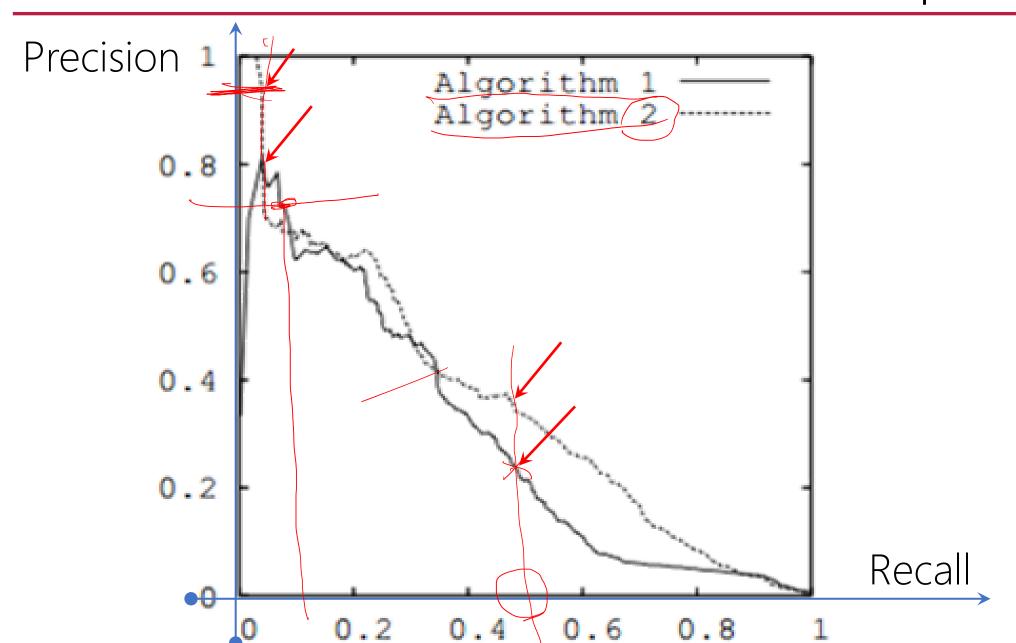
 $P(x) = Sigmoid(f(x))$
 $P(x) \ge 1.0 \longrightarrow x$ is positive
 $P(x) < 1.0 \longrightarrow x$ is negative



Precision-Recall Curve: Best δ



Precision-Recall Curve: Model Comparison



Receiver Operating Characteristic ROC

The ROC curve was first developed by electrical engineers and radar engineers during World War II for detecting enemy objects in battlefield!

Missile attack vs. passenger airplane!

Recall aka True Positive Rate (TPR)

What percentage of positives are captured.

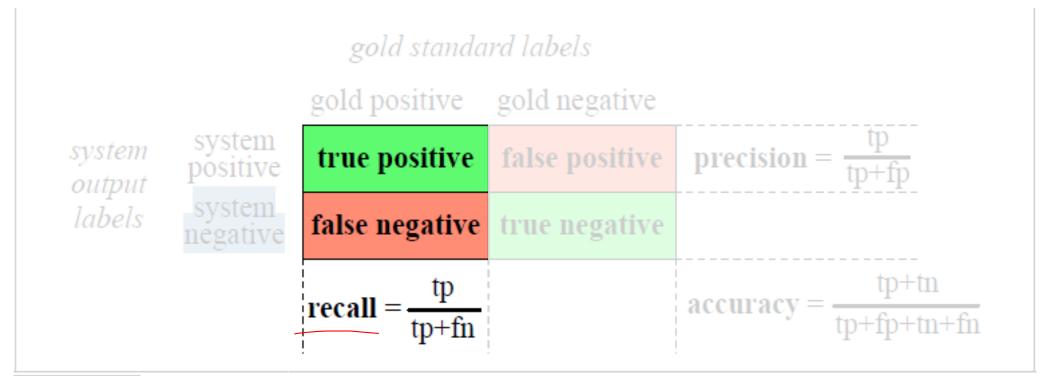


Figure 4.4 Contingency table

False Positive Rate (FPR)

What percentage are *incorrectly* captured as positives!

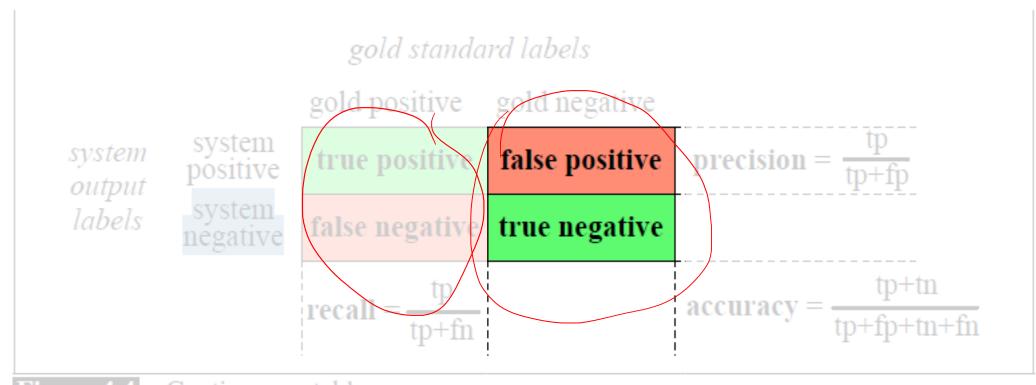
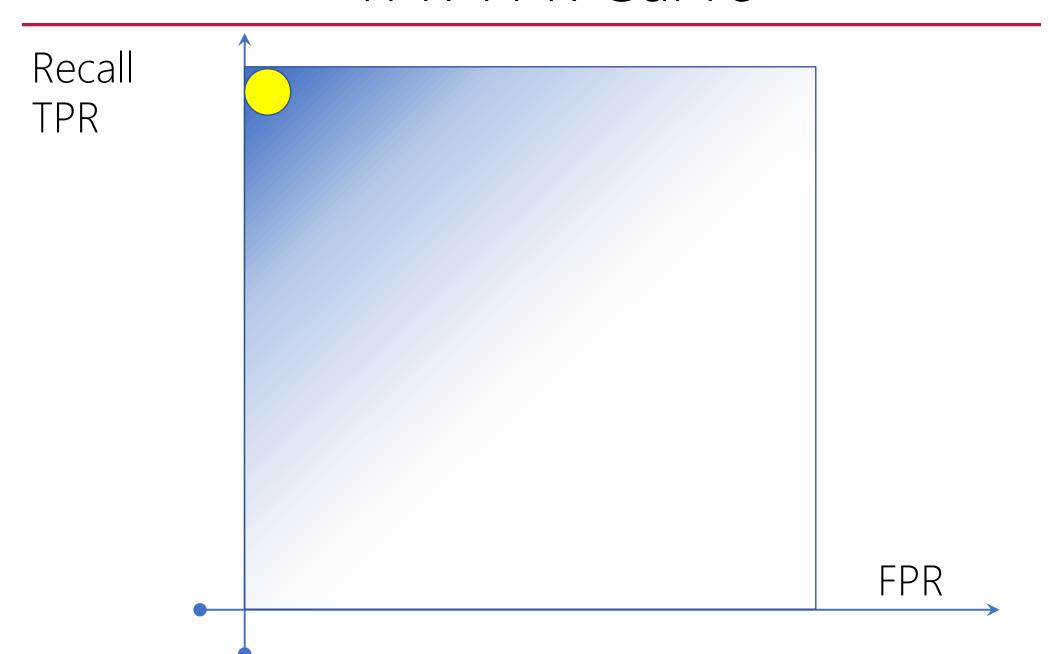


Figure 4.4 Contingency table

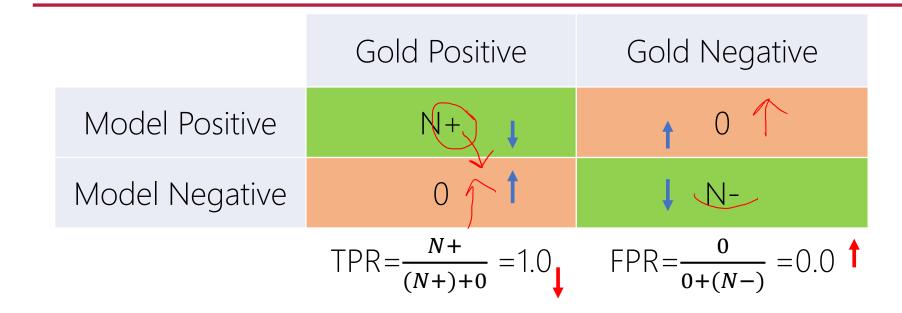
Perfect Classifier

	Gold Positive	Gold Negative
Model Positive	N+	0
Model Negative	0	N-
	$TPR = \frac{N+}{(N+)+0} = 1.0$	$FPR = \frac{0}{0 + (N -)} = 0.0$

TPR-FPR Curve



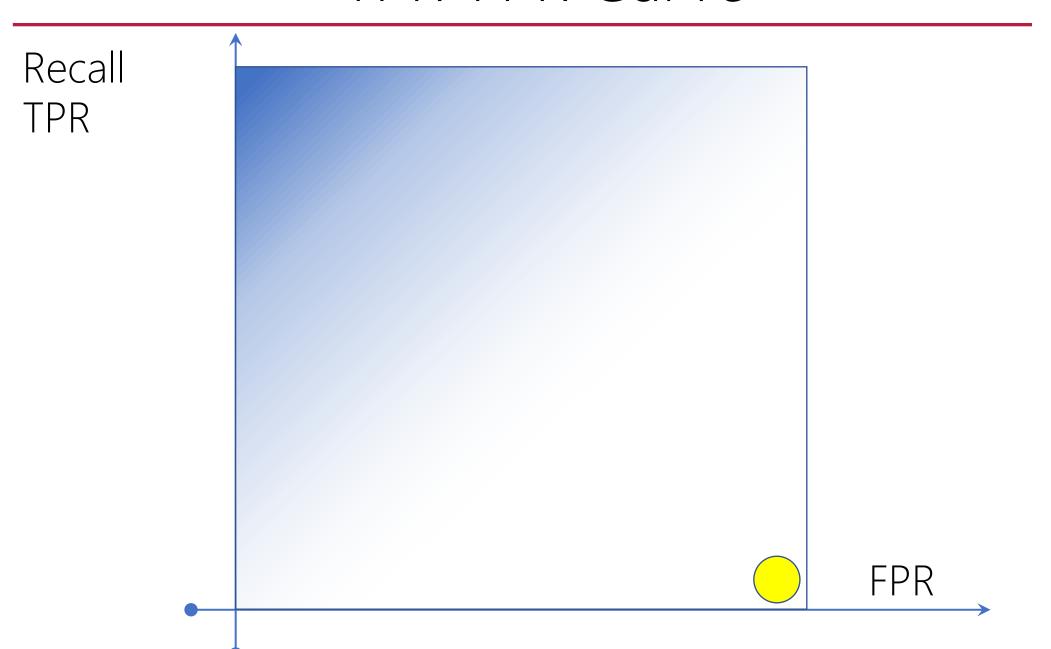
Perfect Classifier



Worst Classifier

	Gold Positive	Gold Negative
Model Positive	0	N-
Model Negative	N+	0
	$TPR = \frac{0}{(N+)+0} = 0.0$	$FPR = \frac{N-}{0+(N-)} = 1.0$

TPR-FPR Curve



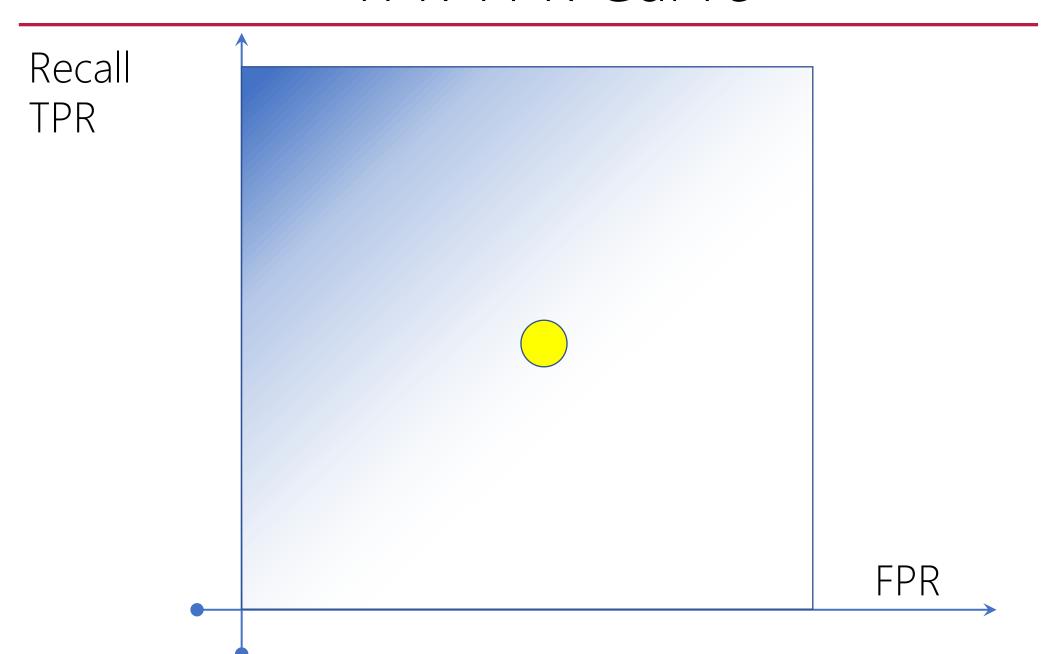
Uniformly Random Classifier

	Gold Positive	Gold Negative
Model Positive	?	?
Model Negative	?	?

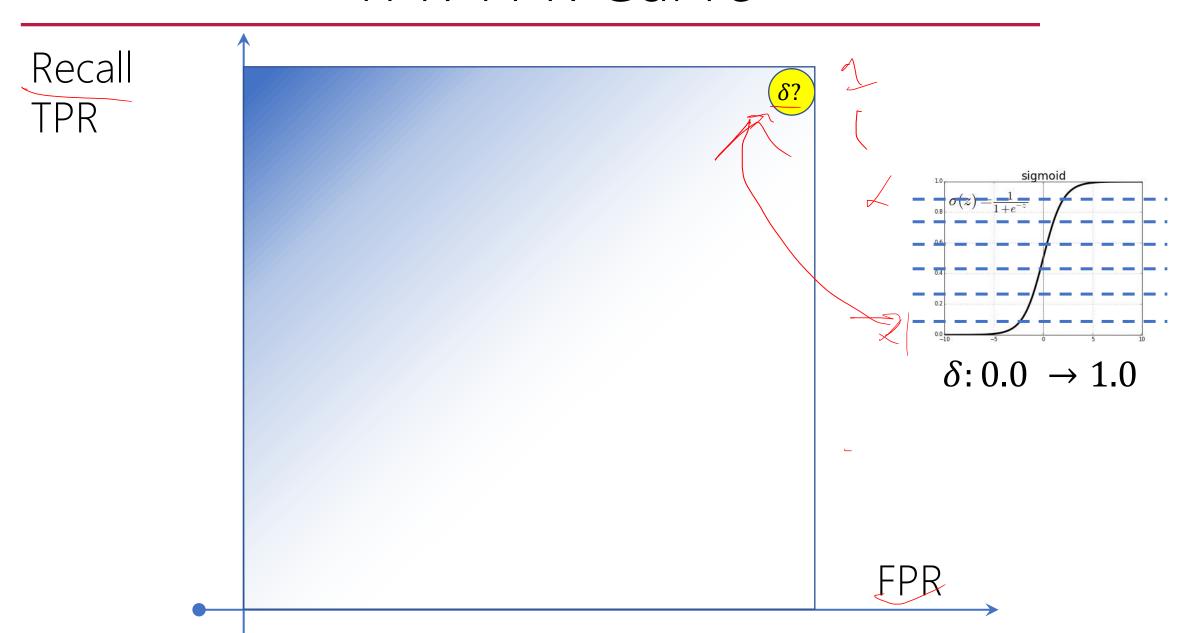
TPR=0.5

FPR=0.5

TPR-FPR Curve



TPR-FPR Curve

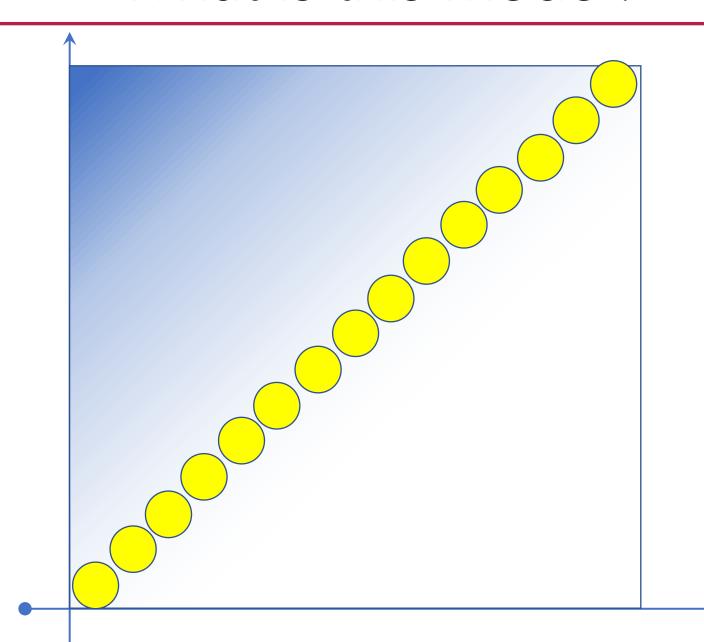


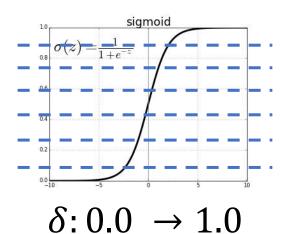
TPR-FPR Curve



What is this model?

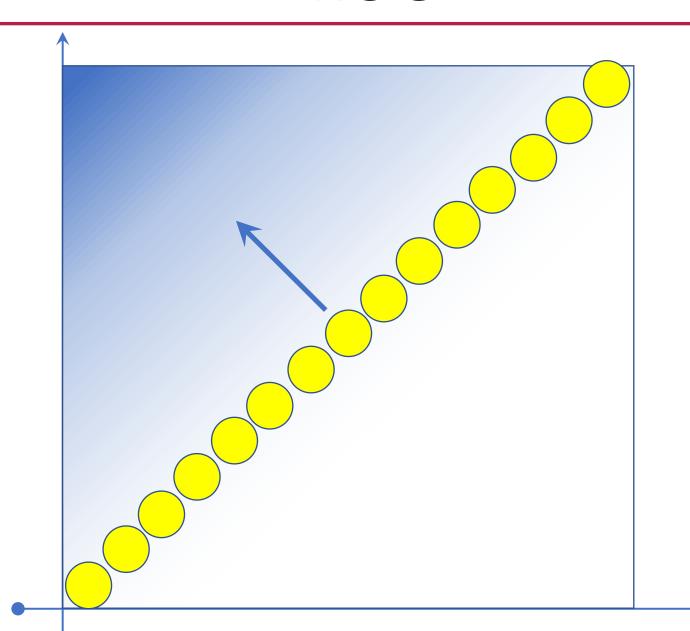
Recall TPR

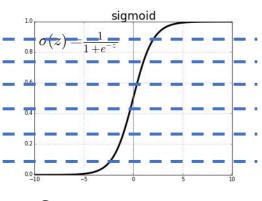




ROC

Recall TPR

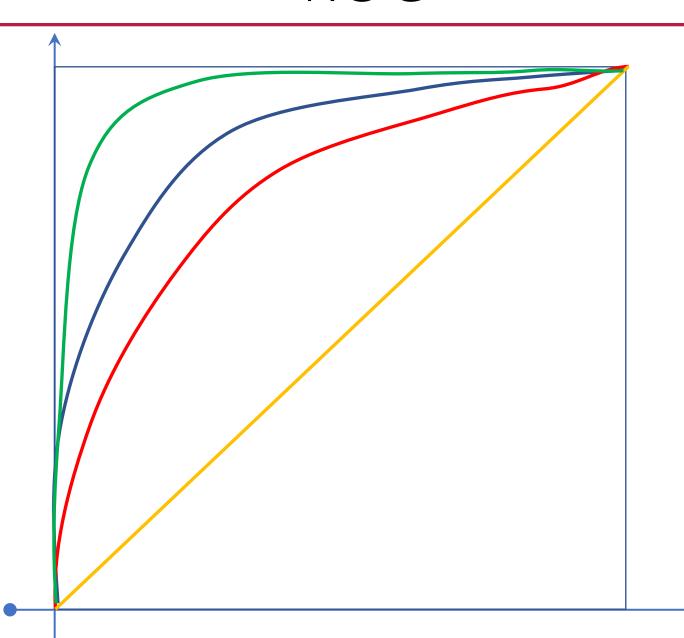


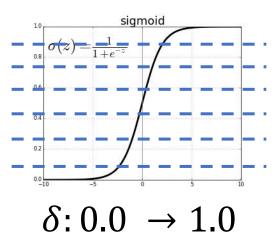


 $\delta: 0.0 \rightarrow 1.0$

ROC

Recall TPR





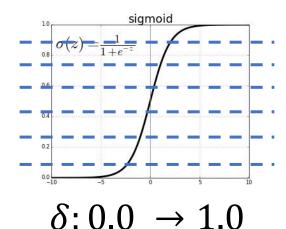
Area Under Curve (AUC): Single Real Point

Recall TPR

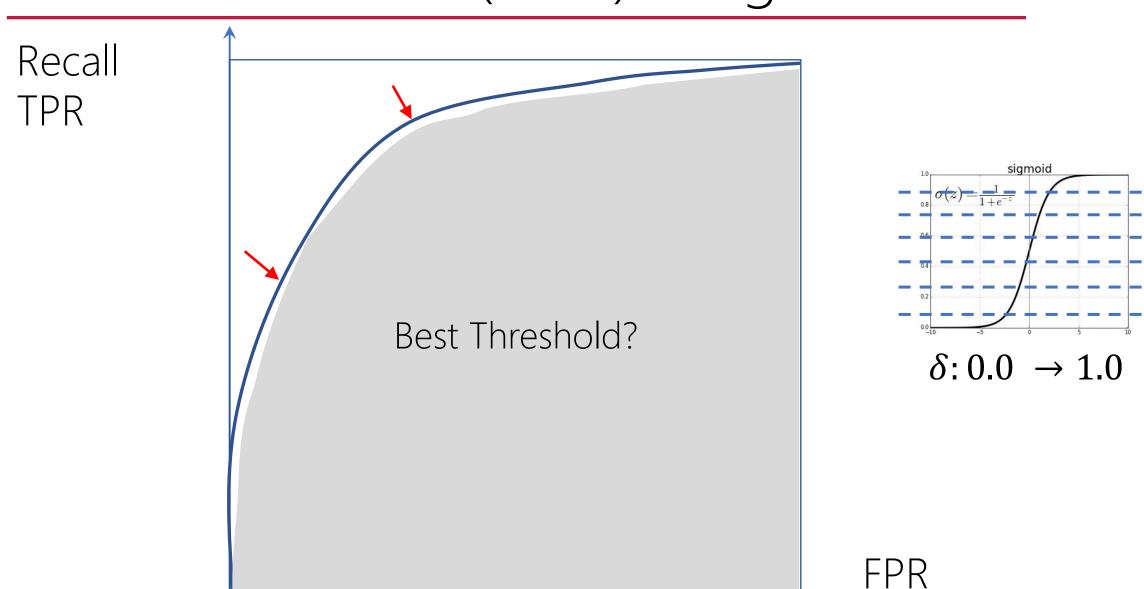


Max = ?

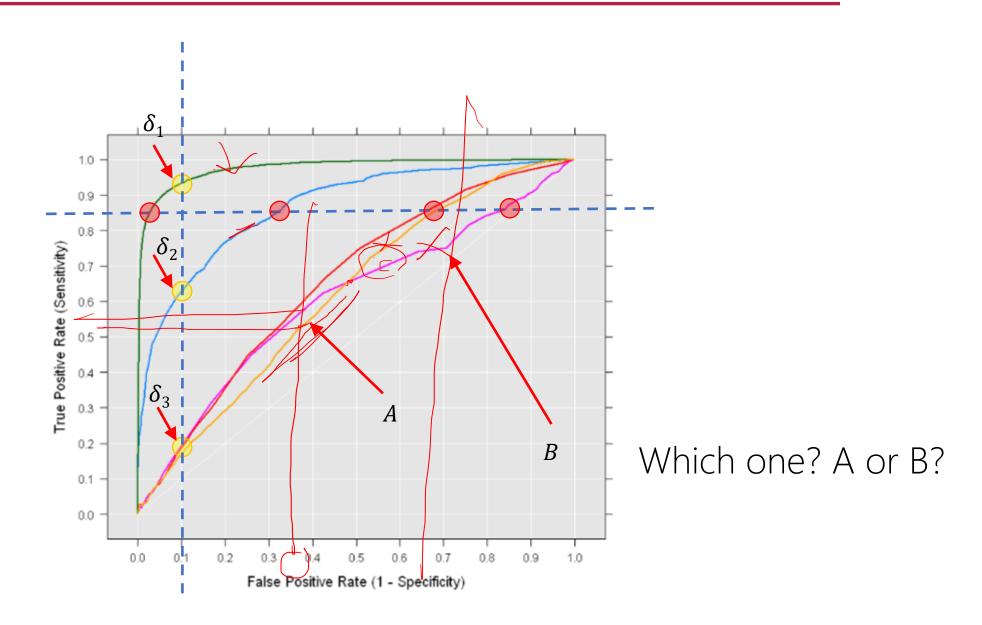
Min = ?



Area Under Curve (AUC): Single Real Point



ROC: Model Comparison

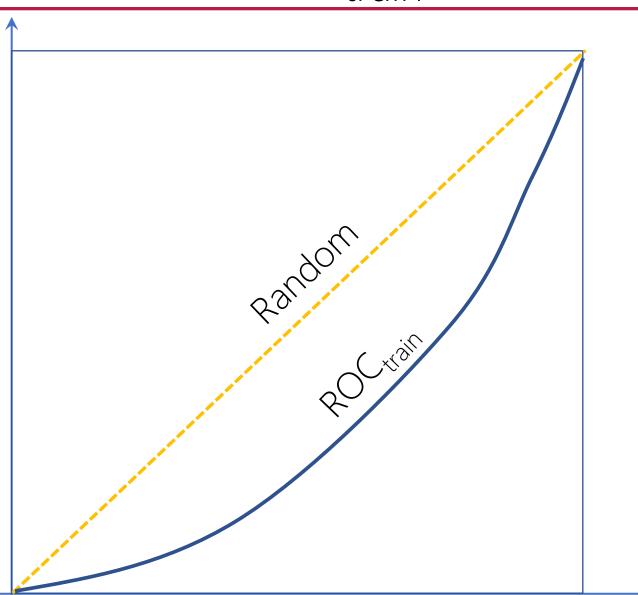


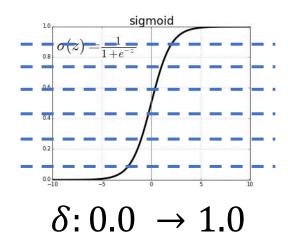
ROC_{train}

Recall TPR

The model couldn't learn anything from training set!

 $ROC_{Test} = ?$



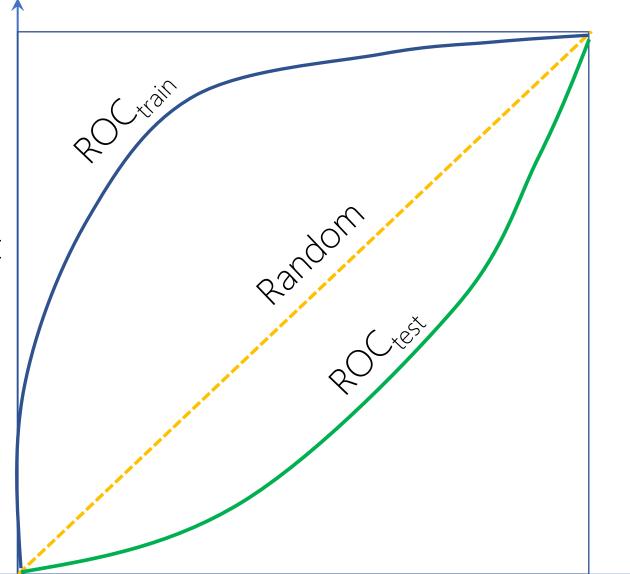


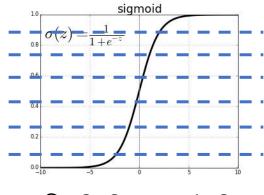
ROC_{test}

Recall TPR

The model performs well on train set. It means it learnt!

But performs poor in test set. It only know train set. Cannot generalize!





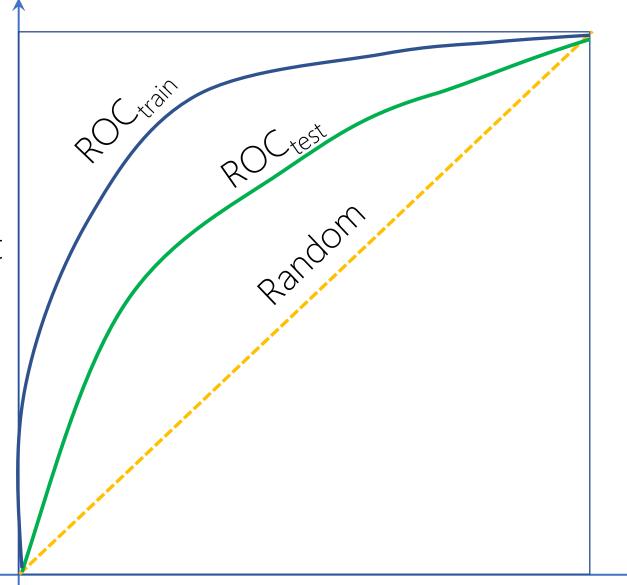
 δ : 0.0 \rightarrow 1.0

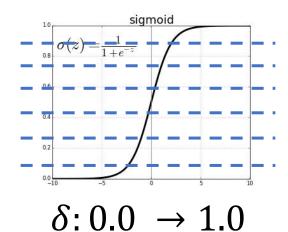
ROC_{test}

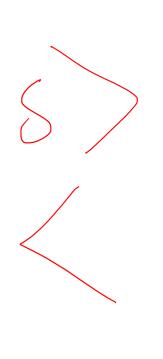
Recall TPR

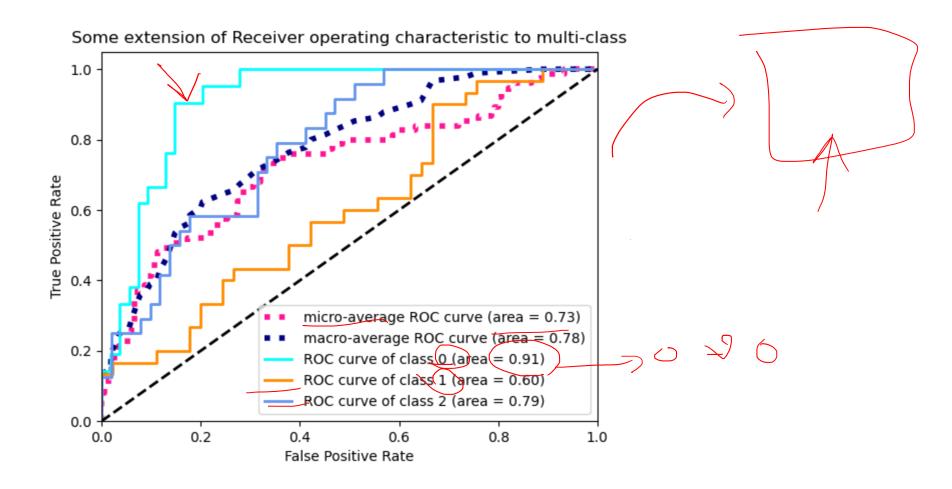
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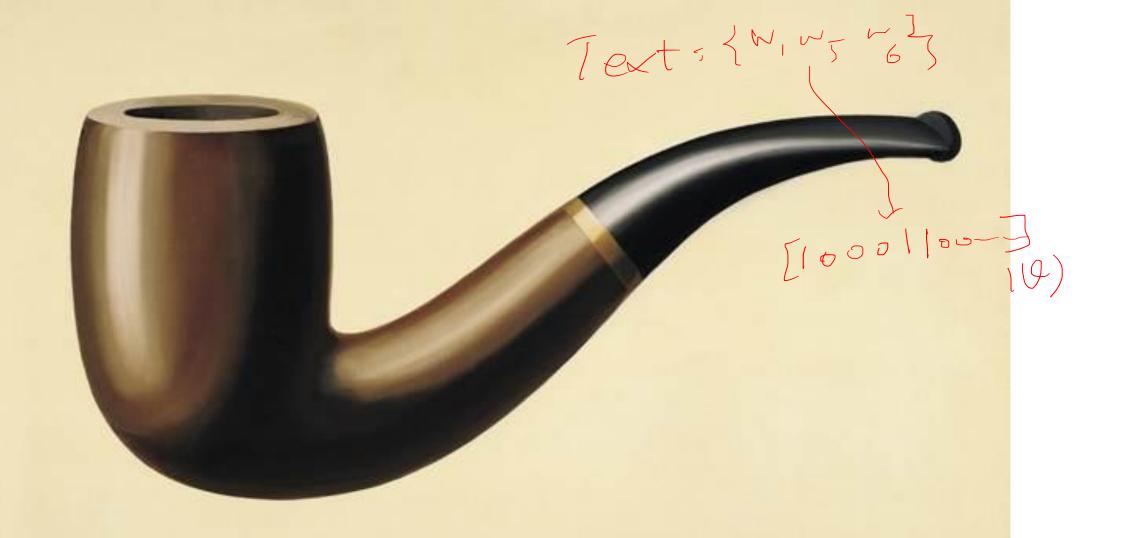




So Far, the LR model and its output

How to input text as X to LR?

X is a vector! How to map text into yector?



Ceci n'est pas une pipe.

Word vector space mo

The Treachery of Images



Artist René Magritte

Year 1929

Medium Oil on canvas

Movement Surrealism

Dimensions 60.33 cm × 81.12 cm (23.75 in

× 31.94 in)

Location Los Angeles County Museum of Art^[1]

- Phonetics and Phonology knowledge about linguistic sounds

- Morphology

knowledge of the formation and internal structure of words

- Syntax

knowledge of the structural relationships between words

- Semantics

knowledge of meaning

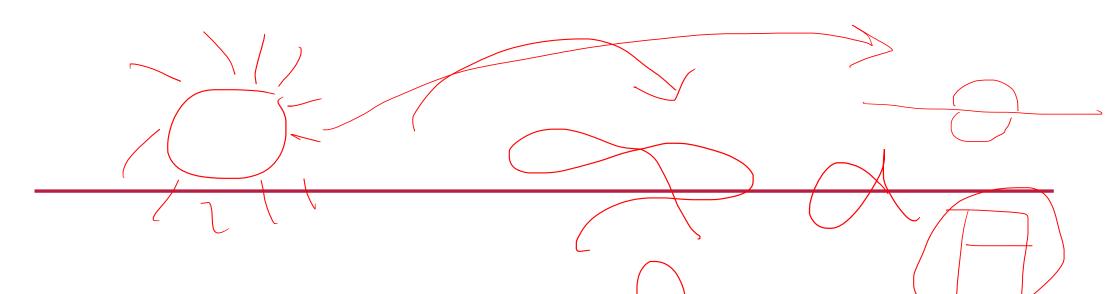
- Pragmatics

knowledge of the relationship of meaning to the goals & intentions of the speaker

- Discourse

knowledge about linguistic units larger than a single utterance

Task: Engaging in Natural Language Communication



Semiotics: The Science of Symbols

Semantics: Relation between signs and things to which they refer: meaning; sense

Syntactics: Relations among signs in formal structures

Pragmatics: Relation between signs and sign-using agents