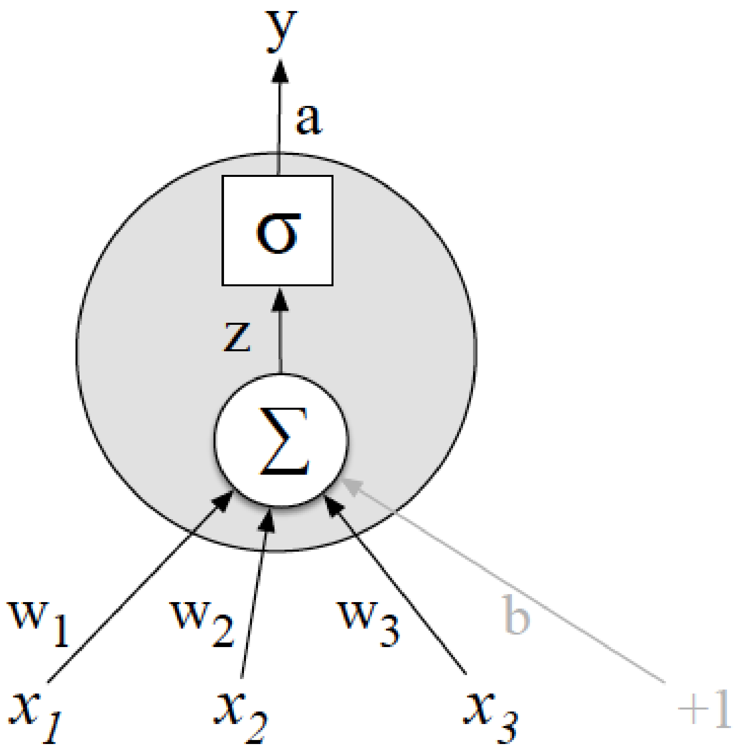
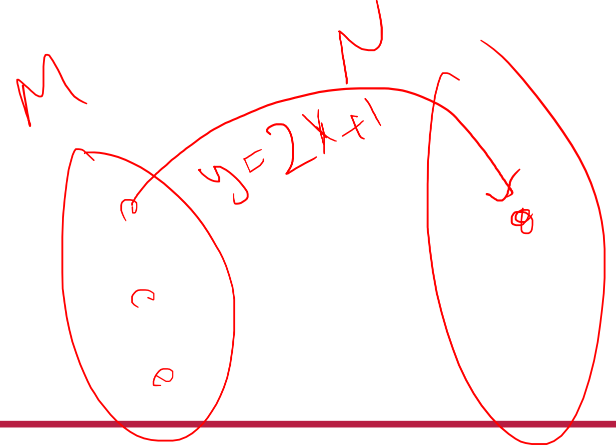
NEURAL LANGUAGE MODELS



Backgrounds

Computation ↔ Function

Computational Intelligence ↔ Learning the function



Function

$$f\!: M \longrightarrow N$$

Transferring elements of source space (M) to target space (N)

$$f\!: \mathrm{R} \longrightarrow \mathrm{R} : f(x) = y = x \times 2 + 1$$

f is known

f has two parameters (a=2, b=1)

Function

$$f: M \rightarrow N$$

Transferring elements of source space (M) to target space (N)

$$f: R \rightarrow R : \{(x, f(x)) \mid (1,1), (2,4), (3,9), \ldots\}$$

$$f(x) = ?$$

f is unknown!

Function

$$f\colon M \rightarrow N$$

Transferring elements of source space (M) to target space (N)

$$f\colon R \rightarrow R : \{(x, f(x)) \mid (1,1), (2,4), (3,9), \ldots\}$$

$$f(x) = ?$$

f is unknown! Might not exist!

Function

$$f: M \rightarrow N$$

Transferring elements of source space (M) to target space (N)

f: R → R : {(x, f(x))| (1,1), (2,4), (3,9), …}

f(x) = ?

f is unknown! If exists, there may be more than one!

Function

$$f\colon M \rightarrow N$$

Transferring elements of source space (M) to target space (N)

f: R → R : {(x, f(x))| (1,1), (2,4), (3,9), …}

f(x) = ?

f is unknown! Even more than one exists, we may not find one!

Function

$$f\colon M \rightarrow N$$

Transferring elements of source space (M) to target space (N)

f: R → R : {(x, f(x))| (1,1), (2,4), (3,9), …}

f(x) = ?

f is unknown! We assume at least one exists, we try to find it!

Transformation

$$f: M \rightarrow N$$

Transferring points (vectors) of source space (M) to target space (N)

$[T][X] = [Y]$; $X \in M$, $Y \in N$

T is a matrix that includes the parameters of f

Transformation (Linear Algebra)

$f: M \rightarrow N$

Transferring points (vectors) of source space (M) to target space (N)

$$[2\ 1]\begin{bmatrix} x \\ 1 \end{bmatrix} = [2x + 1]$$

$y = 2x + 1$

$$f(x) = 2x + 1$$

Transformation (Linear Algebra)

$f\colon M \rightarrow N$

Transferring points (vectors) of source space (M) to target space (N)

$(AB)^T = B^T A^T \rightarrow$

$$[x \; 1]\begin{bmatrix} 2 \\ 1 \end{bmatrix} = [2x + 1]$$

$$f(x) = 2x + 1$$

Transformation (Linear Algebra)

$f: M \rightarrow N$

Transferring points (vectors) of source space (M) to target space (N)

$$[x\ 1]\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \{(x, f(x)) \mid (1,1), (2,4), (3,9), \ldots\}$$

$$f(x) = ?$$

$$[2\ 1]\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \rightarrow 4$$

$$[3\ 1]\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \rightarrow 9$$

Neural Network

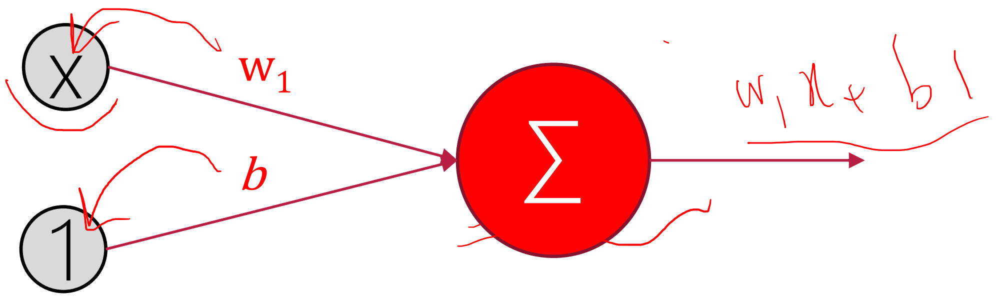
$f: M \rightarrow N$

Perceptron

Transferring points (vectors) of source space (M) to target space (N)

$$[x\ 1]\begin{bmatrix} w_1 \\ b \end{bmatrix} = y$$

$$f(x) = y$$



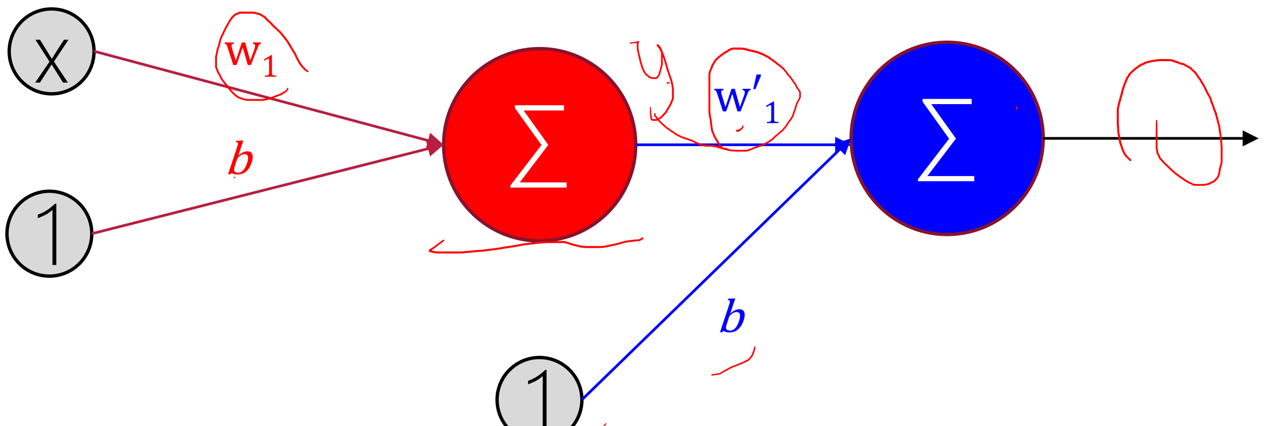
Neural Network

$$g: (f: M \rightarrow N) \rightarrow O$$

Transferring points (vectors) of source space (M) to target space (O)

$$[[X\ 1]\begin{bmatrix} w_1 \\ b \end{bmatrix}\ 1]\begin{bmatrix} w'_1 \\ b \end{bmatrix} = z$$

$$(g \circ f)(x) = g(f(x)) = z$$



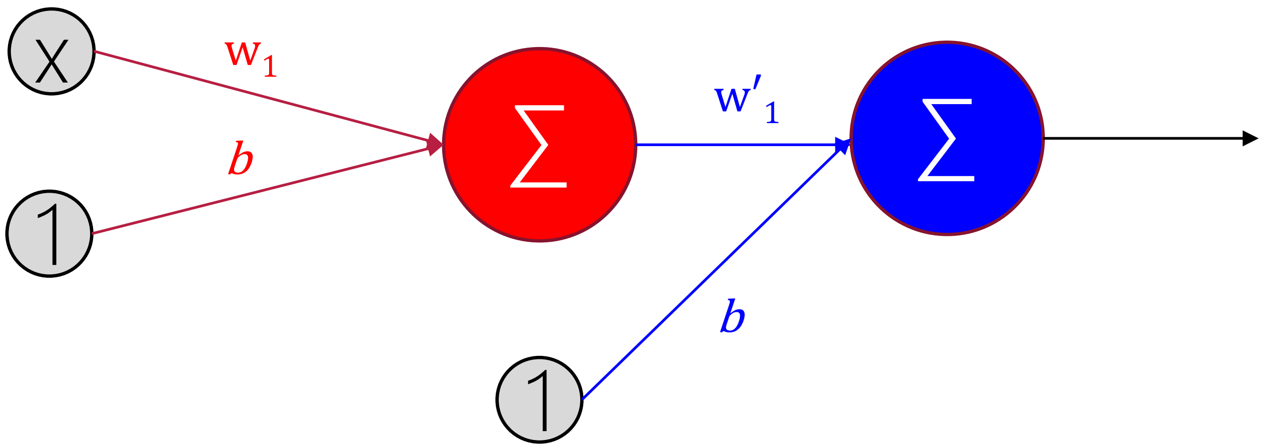
Neural Network

$$g\colon (f\colon M \to N) \to O$$

Transferring points (vectors) of source space (M) to target space (O)

$$[y\ 1]\begin{bmatrix} w'_1 \\ b \end{bmatrix} = z$$

$$g(f(x)) = g(y) = z$$



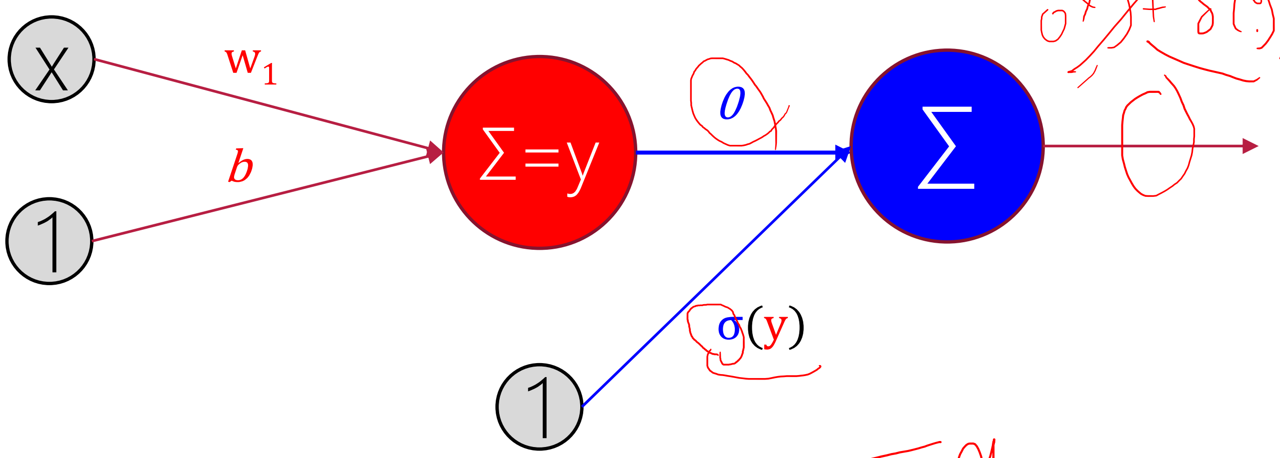
Neural Network

$$g\colon (f\colon M \to N) \to O$$

Transferring points (vectors) of source space (M) to target space (O)

$$[y\ \ 1]\begin{bmatrix} 0 \\ \sigma(y) \end{bmatrix} = y$$

$$g(f(x)) = g(y) = z$$



$$[[x\ \ 1]\begin{bmatrix} w_1 \\ b \end{bmatrix}\ \ 1]\begin{bmatrix} 0 \\ \sigma(x \times w_1 + b) \end{bmatrix} = z$$

x

f(x) = 2

log x

x √2

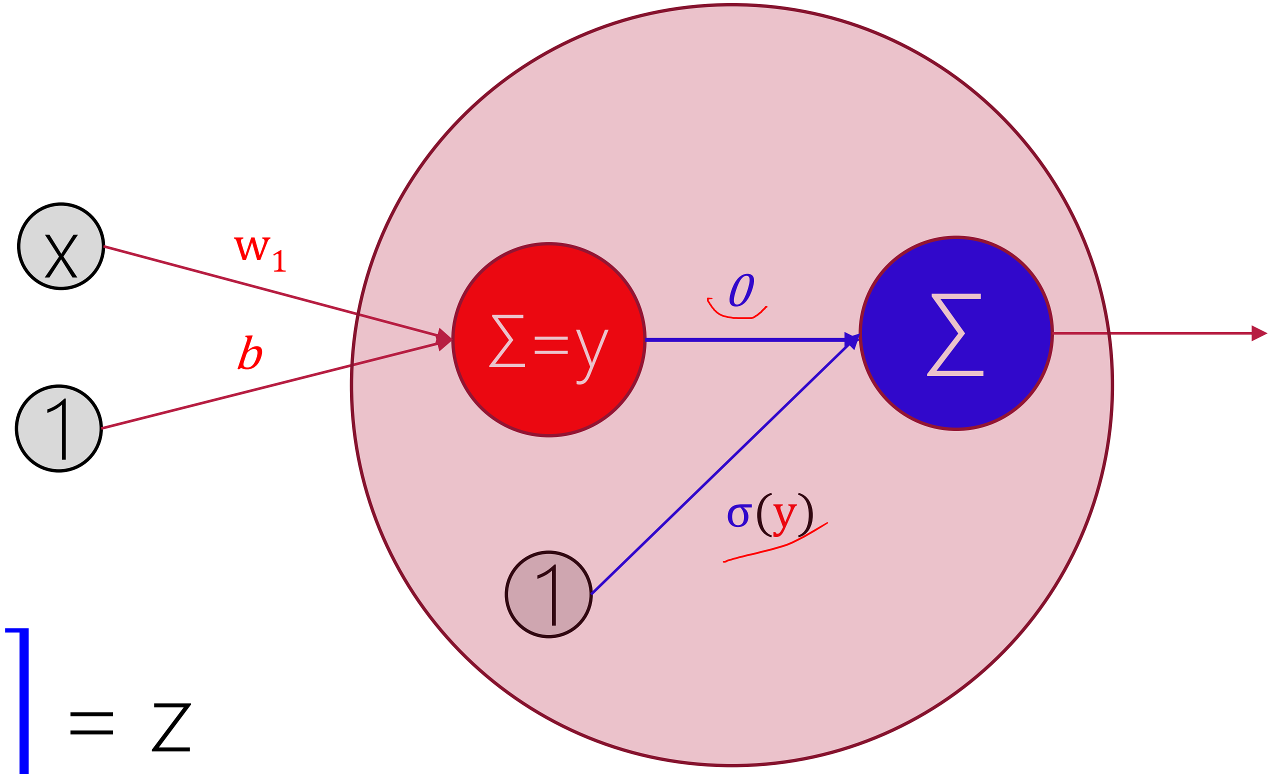
Neural Network

$$g\colon (f\colon M \to N) \to O$$

Transferring points (vectors) of source space (M) to target space (O)

$$[y\ \ 1]\begin{bmatrix} 0 \\ \sigma(y) \end{bmatrix} = y$$

$$g(f(x)) = g(y) = z$$



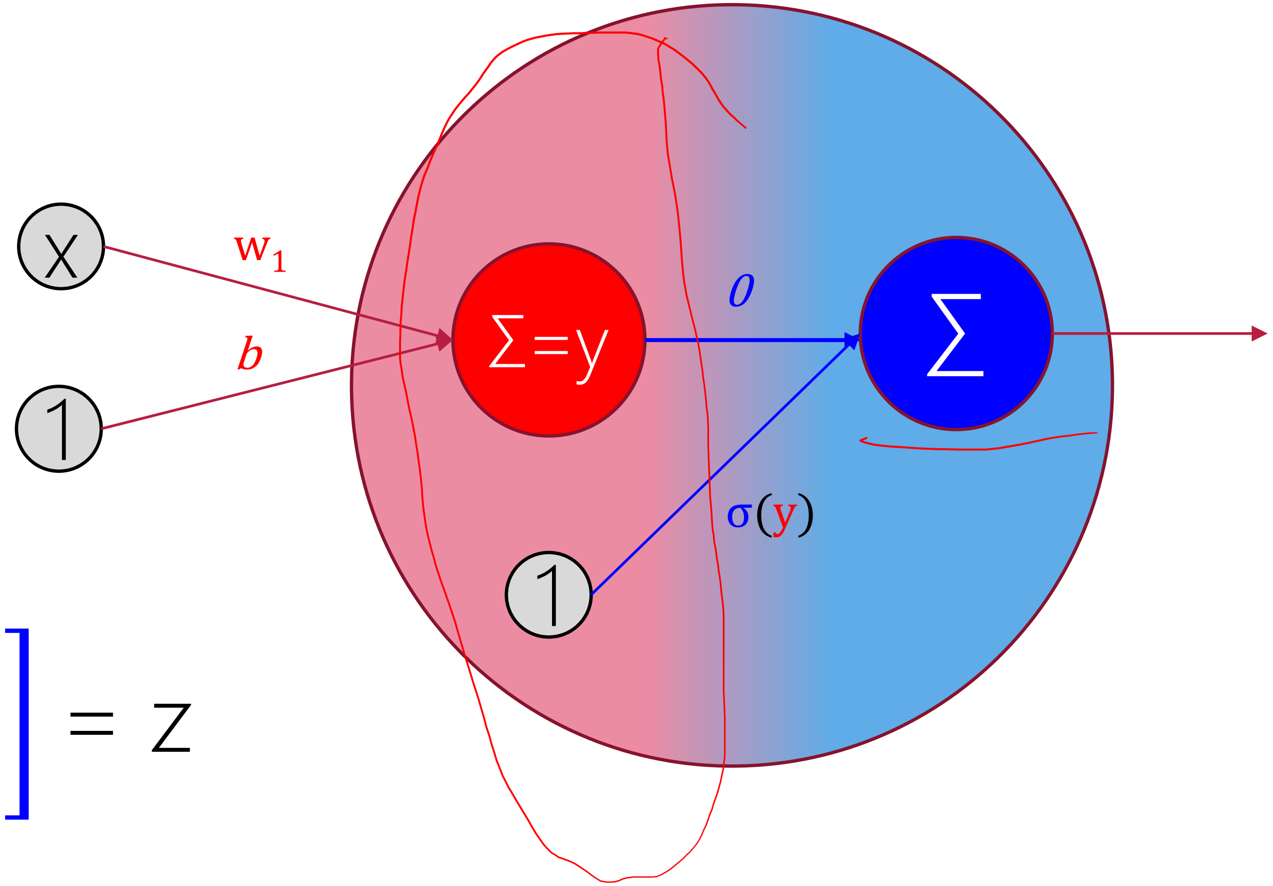
$$[[x\ \ 1]\begin{bmatrix} w_1 \\ b \end{bmatrix}\ \ 1]\begin{bmatrix} 0 \\ \sigma(x \times w_1 + b) \end{bmatrix} = z$$

Neural Network

$$h: M \rightarrow O$$

Transferring points (vectors) of source space (M) to target space (O)

$$h(x) = \sigma(x \times w_1 + b)$$



$$[[x\ 1]\begin{bmatrix} w_1 \\ b \end{bmatrix}\ \ 1]\begin{bmatrix} 0 \\ \sigma(x \times w_1 + b) \end{bmatrix} = z$$

Neural Network

$$h: M \rightarrow O$$

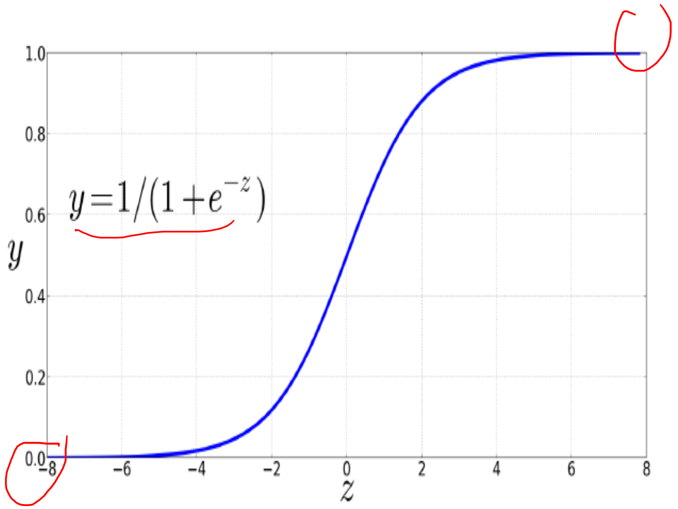
Transferring points (vectors) of source space (M) to target space (O)

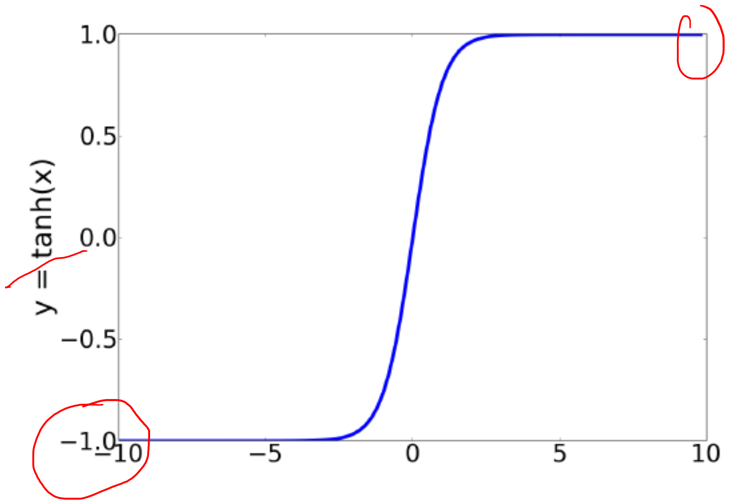
$$h(x) = \sigma(x \times w_1 + b)$$

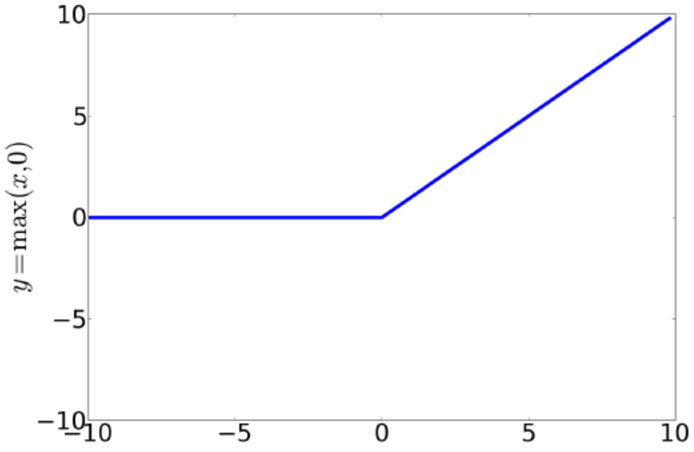
σ is a known *non*-linear function: Sigmoid, ReLU, Tanh

$$[[x\ 1]\begin{bmatrix} w_1 \\ b \end{bmatrix} \quad 1] \begin{bmatrix} 0 \\ \sigma(x \times w_1 + b) \end{bmatrix} = z$$

Neural Network: Activation Function σ







Neural Network

$$h: M=\{0,1\}^2 \rightarrow O=\{0,1\}$$

M = {(0,0), (0,1), (1,0), (1,1)}

O = {0, 1}

h = {((0,0), 0), ((0,1), 0), ((1,0), 0), ((1,1), 1)}

AND

$$[[x_1 \quad x_2 \quad 1]\begin{bmatrix} w_1 \\ w_2 \\ b \end{bmatrix} 1] \begin{bmatrix} 0 \\ \sigma(x_1 \times w_1 + x_2 \times w_2 + b) \end{bmatrix} = z$$

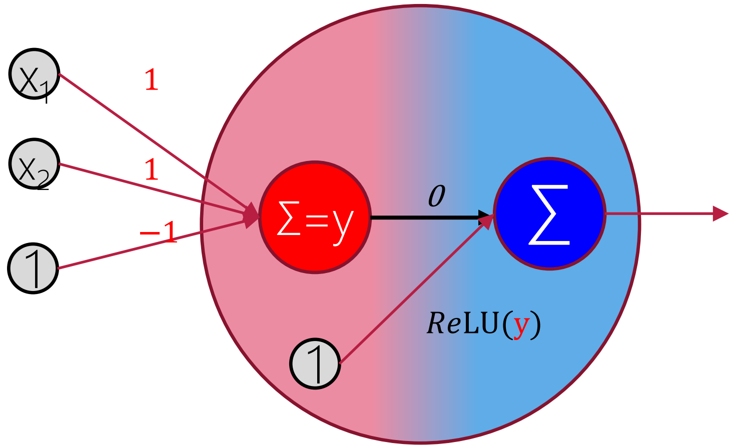
Neural Network

AND: *M*={0,1}² → *O*={0,1}

$M = \{(0,0), (0,1), (1,0), (1,1)\}$

$O = \{0, 1\}$

$h = \{((0,0), 0), ((0,1), 0), ((1,0), 0), ((1,1), 1)\}$



[0 1]

0+0=0

$$\left[\begin{bmatrix} x_1 & x_2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}\right] \left[\begin{matrix} \boldsymbol{0} \\ max(0, x_1 \times 1 + x_2 \times 1 + -1) \end{matrix}\right] = z$$

1+0 −1

0

God (Oracle) told the weights

Neural Network

$OR: M=\{0,1\}^2 \rightarrow O=\{0,1\}$

$M = \{(0,0), (0,1), (1,0), (1,1)\}$

$O = \{0, 1\}$

$h = \{((0,0), 0), ((0,1), 1), ((1,0), 1), ((1,1), 1)\}$

$4 \leftarrow w_1, w_2, b$

$$[[x_1 \quad x_2 \quad 1]\begin{bmatrix} w_1 \\ w_2 \\ b \end{bmatrix} 1] \begin{bmatrix} 0 \\ \sigma(x_1 \times w_1 + x_2 \times w_2 + b) \end{bmatrix} = z$$

sigmoid

Ask God (Oracle)

Neural Network

$XOR: M=\{0,1\}^2 \rightarrow O=\{0,1\}$

M = {(0,0), (0,1), (1,0), (1,1)}

O = {0, 1}

h = {((0,0), 0), ((0,1), 1), ((1,0), 1), ((1,1), 0)}

$$[[x_1 \quad x_2 \quad 1]\begin{bmatrix} w_1 \\ w_2 \\ b \end{bmatrix} 1] \begin{bmatrix} 0 \\ \sigma(x_1 \times w_1 + x_2 \times w_2 + b) \end{bmatrix} = z$$

Even God (Oracle) cannot tell

Neural Network

$XOR: M=\{0,1\}^2 \rightarrow O=\{0,1,2\}^2 \rightarrow P=\{0,1\}$

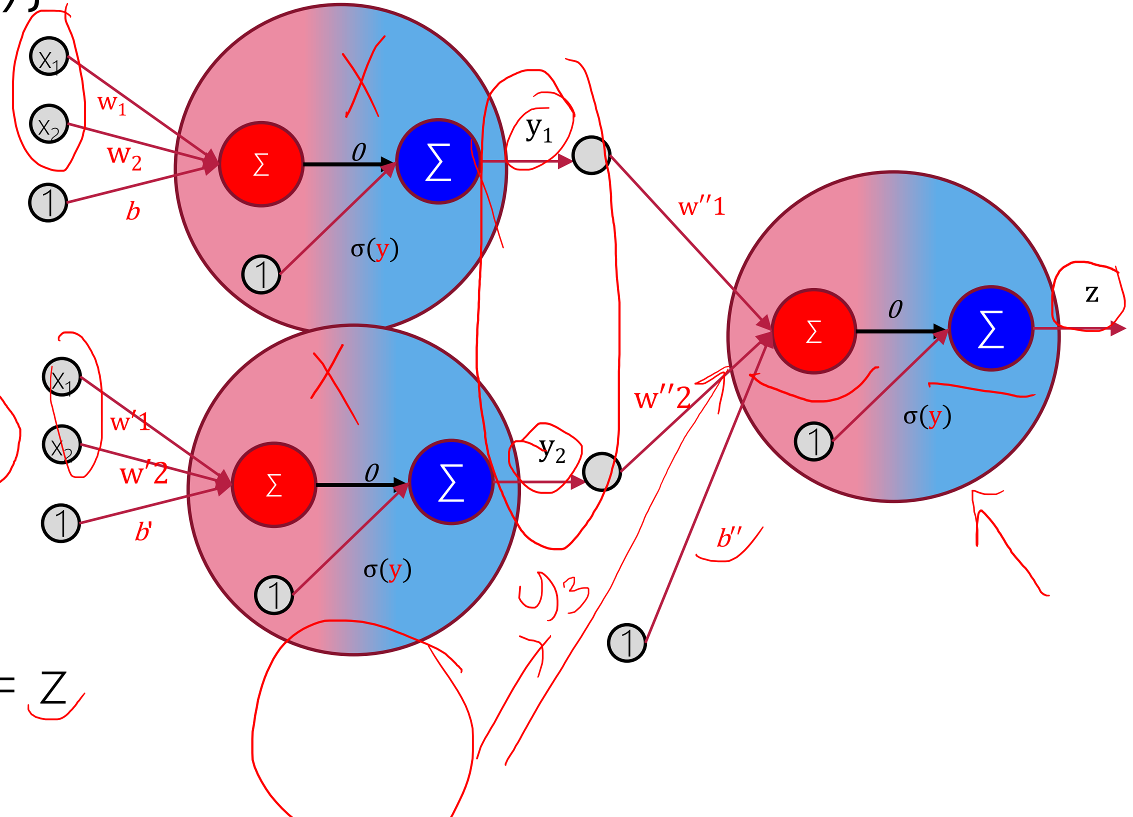
$M = \{(0,0), (0,1), (1,0), (1,1)\} \rightarrow O = \{(0,0), (0,1), \ldots, (2,2)\} \rightarrow P = \{0, 1\}$

$h = \{((0,0), 0), ((0,1), 1), ((1,0), 1), ((1,1), 0)\}$

$$[[x_1\ \ x_2\ \ 1]\begin{bmatrix} w_1 \\ w_2 \\ b \end{bmatrix} 1]\begin{bmatrix} 0 \\ \sigma(x_1 \times w_1 + x_2 \times w_2 + b) \end{bmatrix} = y_1$$

$$[[x_1\ \ x_2\ \ 1]\begin{bmatrix} w'_1 \\ w'_2 \\ b' \end{bmatrix} 1]\begin{bmatrix} 0 \\ \sigma(x_1 \times w'_1 + x_2 \times w'_2 + b') \end{bmatrix} = y_2$$

$$[[y_1\ \ y_2\ \ 1]\begin{bmatrix} w''_1 \\ w''_2 \\ b'' \end{bmatrix} 1]\begin{bmatrix} 0 \\ \sigma(x_1 \times w''1 + x_2 \times w''2 + b'') \end{bmatrix} = z$$



Neural Network

$$XOR: M=\{0,1\}^2 \rightarrow O=\{0,1,2\}^2 \rightarrow P=\{0,1\}$$

$$M = \{(0,0), (0,1), (1,0), (1,1)\} \rightarrow O = \{(0,0), (0,1), \ldots, (2,2)\} \rightarrow P = \{0, 1\}$$

$$h = \{((0,0), 0), ((0,1), 1), ((1,0), 1), ((1,1), 0)\}$$

$$[[x_1 \;\; x_2 \;\; 1]\begin{bmatrix} w_1 & w'1 \\ w_2 & w'2 \\ b & b' \end{bmatrix} \; 1] =$$

$$[x_1 \times w_1 + x_2 \times w_2 + b \quad x_1 \times w'1 + x_2 \times w'2 + b' \quad 1]\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \sigma(x_1 \times w_1 + x_2 \times w_2 + b) & \sigma(x_1 \times w'1 + x_2 \times w'2 + b') \end{bmatrix} =$$

$$[\underbrace{\sigma(x_1 \times w_1 + x_2 \times w_2 + b)}_{\alpha} \; \underbrace{\sigma(x_1 \times w'1 + x_2 \times w'2 + b')}_{\beta} \;\; 1]\begin{bmatrix} w''_1 \\ w''_2 \\ b'' \end{bmatrix} \; 1] \begin{bmatrix} 0 \\ \sigma(\alpha \times w''1 + \beta \times w''2 + b'') \end{bmatrix} = z$$

Neural Network

$$XOR: M=\{0,1\}^2 \rightarrow O=\{0,1,2\}^2 \rightarrow P=\{0,1\}$$

$$M = \{(0,0), (0,1), (1,0), (1,1)\} \rightarrow O = \{(0,0), (0,1), \ldots, (2,2)\} \rightarrow P = \{0, 1\}$$

$$h = \{((0,0), 0), ((0,1), 1), ((1,0), 1), ((1,1), 0)\}$$

$$g(f(x_1, x_2)) = ?$$

God (Oracle)

$$[[x_1 \ \ x_2 \ \ 1]\begin{bmatrix} w_1 & w'1 \\ w_2 & w'2 \\ b & b' \end{bmatrix} \ 1] = [x_1 \ \ x_2 \ \ 1]\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$[(x_1 \times 1 + x_2 \times 1 + 0) \quad (x_1 \times 1 + x_2 \times 1 + -1) \quad 1]\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \mathrm{ReLU}(x_1 + x_2) & \mathrm{ReLU}(x_1 + x_2 - 1) \end{bmatrix} =$$

$$[\mathrm{ReLU}(x_1 + x_2) \ \ \mathrm{ReLU}(x_1 + x_2 - 1) \ \ 1]\begin{bmatrix} w''_1 \\ w''_2 \\ b'' \end{bmatrix} 1] = [\mathrm{ReLU}(x_1 + x_2) \ \ \mathrm{ReLU}(x_1 + x_2 - 1) \ \ 1]\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} 1]$$

Neural Network

$$XOR: M=\{0,1\}^2 \rightarrow O=\{0,1,2\}^2 \rightarrow P=\{0,1\}$$

$$M = \{(0,0), (0,1), (1,0), (1,1)\} \rightarrow O = \{(0,0), (0,1), \ldots, (2,2)\} \rightarrow P = \{0, 1\}$$

$$(0,0) \rightarrow (0,0) \rightarrow (0)$$
$$(0,1) \rightarrow (1,0) \rightarrow (1)$$
$$(1,0) \rightarrow (1,0) \rightarrow (1)$$
$$(1,1) \rightarrow (2,1) \rightarrow (0)$$

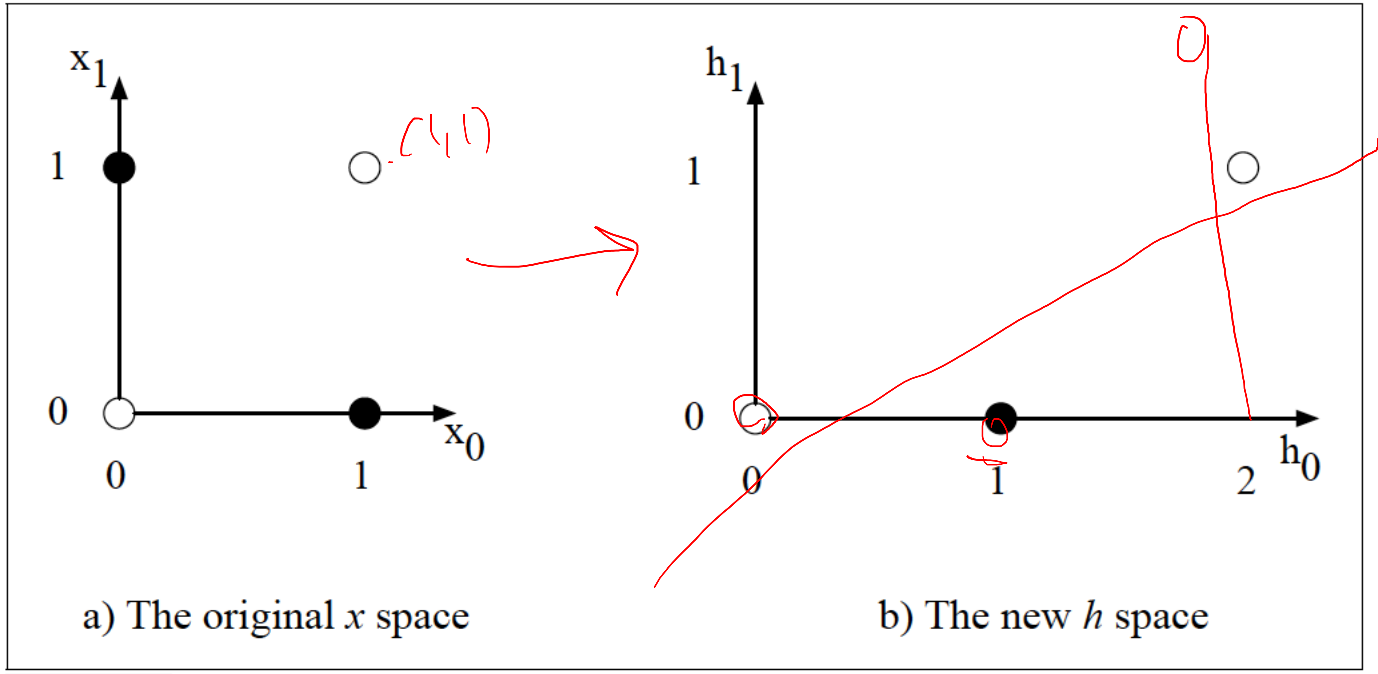


Figure 7.7 The hidden layer forming a new representation of the input. Here is the representation of the hidden layer, h, compared to the original input representation x. Notice that the input point [0 1] has been collapsed with the input point [1 0], making it possible to linearly separate the positive and negative cases of XOR. After Goodfellow et al. (2016).

https://www.facebook.com/watch/?extid=NS-UNK-UNK-UNK-IOS_GK0T-GK1C&v=473881334464404

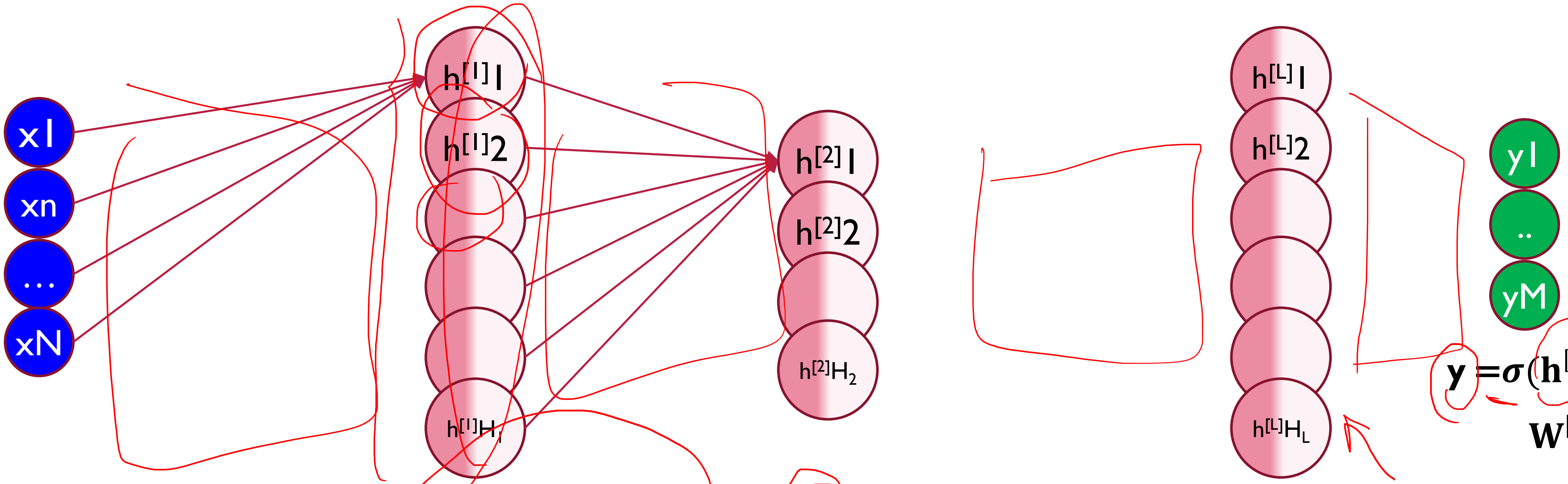
Universal Approximation Theorem

http://neuralnetworksanddeeplearning.com/chap4.html

Be carful, it shows the existence (power) of neural nets, but it does not show which architecture is the function!

How many transformation (#layers)?
To what intermediate space (#nodes)?

Neural Network: Feed-forward



$$\mathbf{y} = \sigma(\mathbf{h}^{[L]}\mathbf{W}^{[L+1]} + \mathbf{b}^{[L+1]})$$

$\mathbf{W}^{[L]}: H_1 \times M$

$$\mathbf{h}^{[1]} = \sigma(\mathbf{x}\mathbf{W}^{[1]} + \mathbf{b}^{[1]})$$

$$\mathbf{h}^{[2]} = \sigma(\mathbf{h}^{[1]}\mathbf{W}^{[2]} + \mathbf{b}^{[2]})$$

$$\mathbf{h}^{[L]} = \sigma(\mathbf{h}^{[L-1]}\mathbf{W}^{[L]} + \mathbf{b}^{[L]})$$

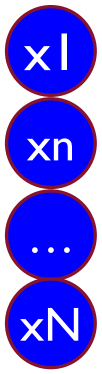
$\mathbf{x}: 1 \times N$

$\mathbf{W}^{[1]}: N \times H_1$

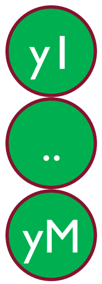
$\mathbf{W}^{[2]}: H_1 \times H_2$

$\mathbf{W}^{[L]}: H_{L-1} \times HL$

Neural Network: Feed-forward Train



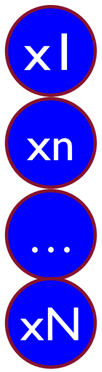
The only known is the data (input: X, output: Y)
Given the input point X, the net should land it to Y.



If the net should land it to f(X) = Y' → error!
Correct the net to make Y' close to Y.

How?

Neural Network: Feed-forward Train

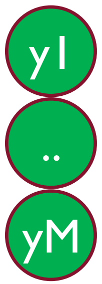


How?

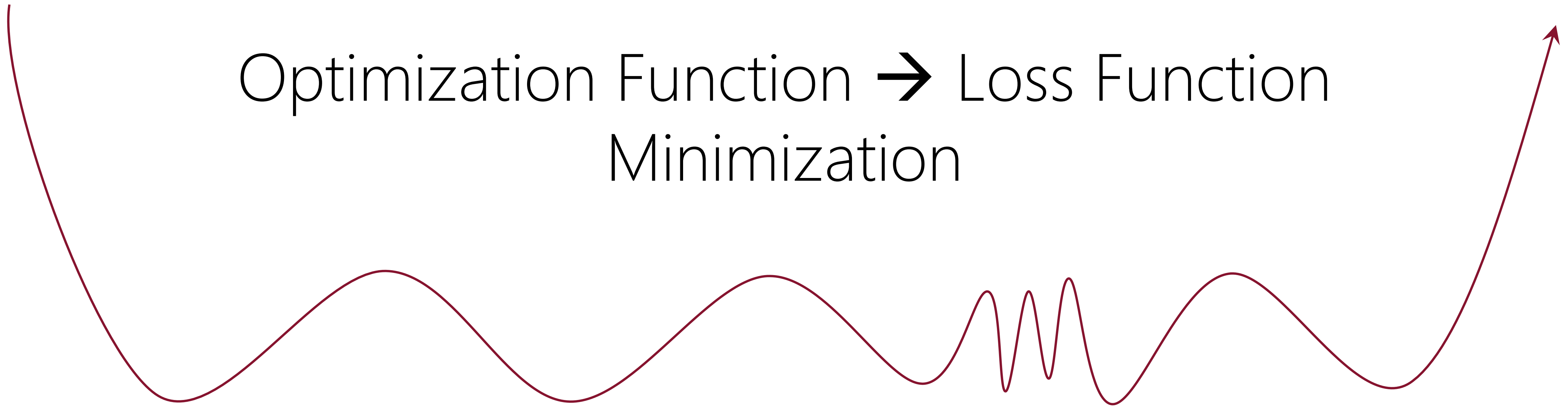
By minimizing the overall error.

What is error?

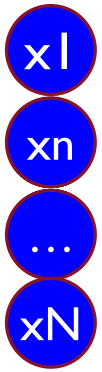
e = distance (Y, f(X)); for all (X, Y)



Optimization Function → Loss Function Minimization



Neural Network: Gradient

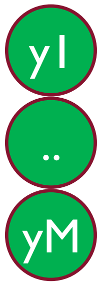


How?

By minimizing the overall error.

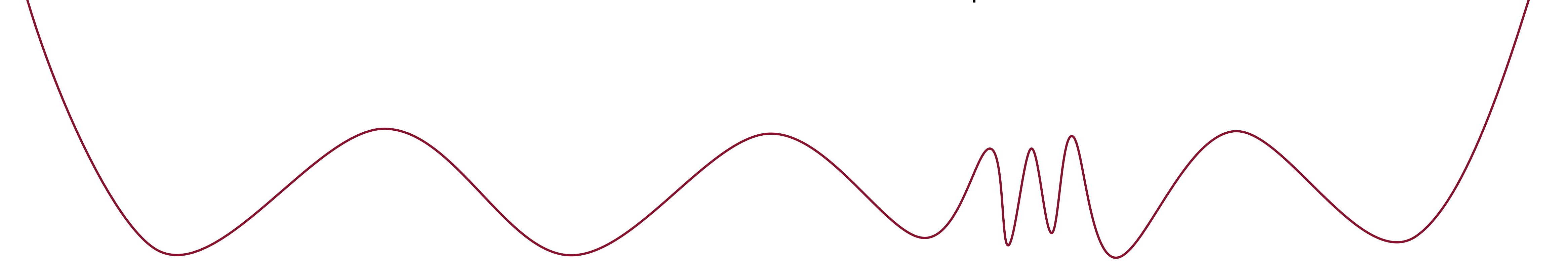
What is error?

e = distance (Y, f(X)); for all (X, Y)

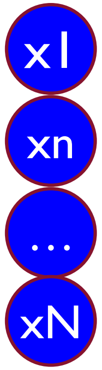


We have to change f such that Y and f(X) get closer and closer!

Y and X is constant in this procedure.



Neural Network: Gradient

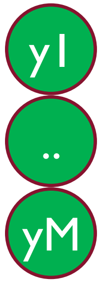


How?

By minimizing the overall error.

What is error?

$e = \text{distance}(Y, f(X))$; for all (X, Y)

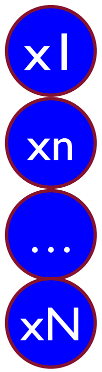


We have to change parameters such that Y and f(X) get closer and closer!

Y and X is constant in this procedure.

$e(\boldsymbol{\Theta}) = \text{distance}(Y, f(X; \boldsymbol{\Theta}))$

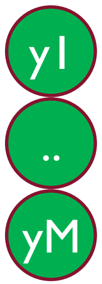
Neural Network: Gradient



$$f(x) = 3x+1 \rightarrow f(x;\, \boldsymbol{\Theta}) = f(x;\, [3,1]) = 3x+1$$

$$f(x;\, [w_1,w_2]) = w_1\, x + w_2$$

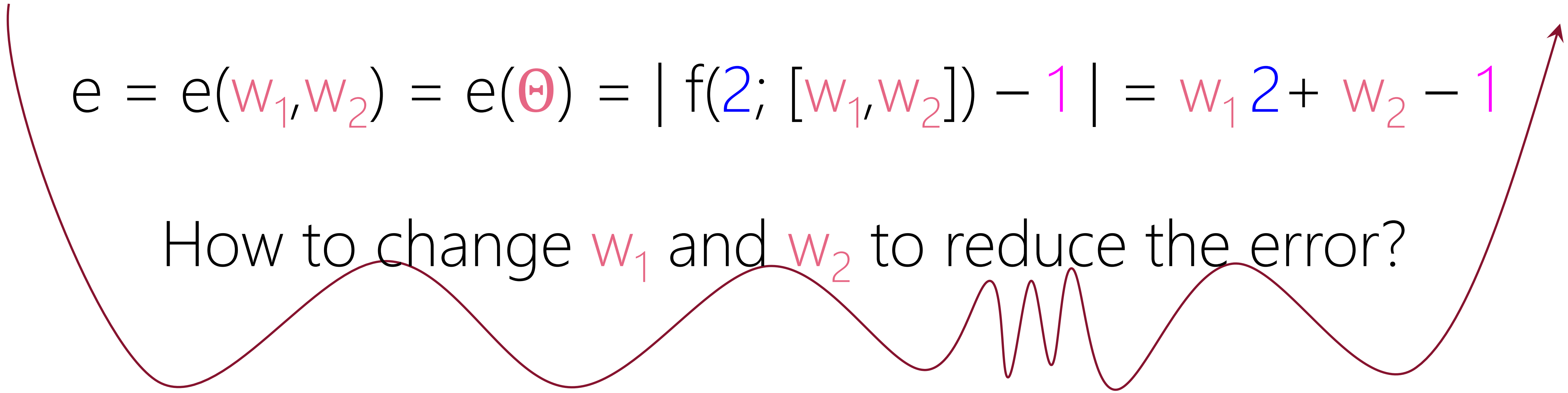
$$(x,\, y) = (2,\, 1)$$



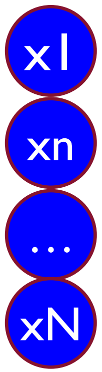
$$f(2;\, [w_1,w_2]) = w_1\, 2 + w_2$$

$$e = e(w_1,w_2) = e(\boldsymbol{\Theta}) = |\, f(2;\, [w_1,w_2]) - 1 \,| = w_1\, 2 + w_2 - 1$$

How to change w_1 and w_2 to reduce the error?



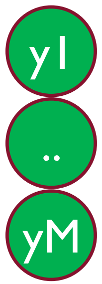
Neural Network: Gradient Descent



$$f(x) = 3x+1 \rightarrow f(x;\, \Theta) = f(x;\, [3,1]) = 3x+1$$

$$f(x;\, [w_1,w_2]) = w_1 x + w_2$$

$$(x,\, y) = (2,\, 1)$$



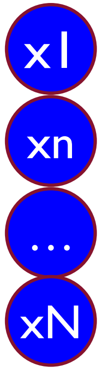
$$f(2;\, [w_1,w_2]) = w_1 2 + w_2$$

$$e = e(w_1,w_2) = e(\Theta) = |\, f(2;\, [w_1,w_2]) - 1 \,| = w_1 2 + w_2 - 1$$

$$w_1 = w_1 - \eta \frac{\partial e}{\partial w_1} = w_1 - 0.1 \times 2 = w_1 - 0.2$$

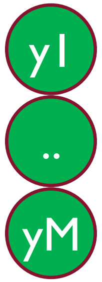
$$w_2 = w_2 - \eta \frac{\partial e}{\partial w_2} = w_2 - 0.1 \times 1 = w_1 - 0.1$$

Neural Network: Gradient Descent

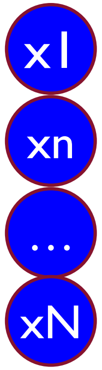


$$f(x;\,\boldsymbol{\Theta}) = f(x;\,[2.8, 0.9]) = 2.8x + 0.9 =$$
$$f(x;\,\boldsymbol{\Theta}) = f(x;\,[2.6, 0.8]) = 2.6x + 0.8 =$$
$$f(x;\,\boldsymbol{\Theta}) = f(x;\,[2.4, 0.7]) = 2.4x + 0.7 =$$

...

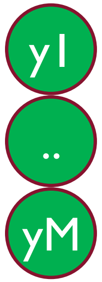


Neural Network: Gradient Descent



Convex Error Function

Many Parameters (all matrices' elements)



Backpropagation!

Computation Graph

Forward Pass

Backward Differentiation.