Evolutionary Dynamics

With slides

Big Question: Why do we cooperate?

- We want to come up with a model that can help us understand cooperating behaviors
- More specifically, why do cooperating behaviors survive through evolution?
- We will start simple and build up to something more general

Fitness

- We have all heard of the phrase “survival of the fittest”, and it has something to do with reproduction.
- For example, fitness could influence the number of offspring an individual have, the chance of it reproducing under some conditions etc.
- So how can we model the “fitness” (r) of cooperators?

$$r = ?$$

Cooperators and Defectors

- Suppose there are two types of behaviors: **cooperators** and **defectors**:
- Cooperators are individuals that help others by sharing their own resources.
- Defectors are individuals that would rather keep things to themselves.
- For example:
  - If I am a cooperator and I interact with another cooperator, I will give them my resource (paying a cost) but I also get something in return (receiving a benefit).
  - If I am a cooperator and I interact with another defector, I will give them my resource but I get nothing in return.

Payoff Matrix

- We can characterize these interactions in a matrix known as the **payoff matrix.**
- Average benefit = b
- Average cost = c

	Cooperator	Defector
Cooperator	b - c	-c
Defector	b	0

Payoff Matrix

- When a cooperator interacts with another cooperator, it receives a benefit (b) and pays a cost (-c).

	Cooperator	Defector
Cooperator	$\boxed{b - c}$	-c
Defector	b	0

Payoff Matrix

- When a cooperator interacts with a defector, the cooperator doesn't receive anything but still pays the cost.

	Cooperator	Defector
Cooperator	b - c	$\boxed{-c}$
Defector	b	0

Payoff Matrix

- When a defector interacts with a cooperator it receives a benefit from the cooperator and pays nothing.

	Cooperator	Defector
Cooperator	b - c	-c
Defector	$\boxed{b}$	0

Payoff Matrix

- When defectors interact nothing is gained or lost.

	Cooperator	Defector
Cooperator	b - c	-c
Defector	b	$\boxed{0}$

Payoff Matrix

- We call the ratio between the number of cooperators or defectors to the total number of the population **frequency**.
- Suppose the frequency of cooperators and defectors in a population is f_c and f_d
- We can calculate the average payoff (p) for each cooperator and defector by:

	Cooperator	Defector
Cooperator	b - c	-c
Defector	b	0

$$p_c = f_c \cdot (b - c) + f_d \cdot (-c)$$
$$p_d = f_c \cdot (b) + f_d \cdot (0)$$

Payoff Equations

$$p_c = f_c \cdot (b - c) + f_d \cdot (-c)$$
$$p_d = f_c \cdot (b) + f_d \cdot (0)$$

After some simplifications, we see:

$$p_c = f_c \cdot (b) - c \cdot (f_c + f_d)$$
$$p_d = f_c \cdot (b)$$

Because cooperators and defectors make up the entire population, $f_c + f_d = 1$ and we have:

$$p_c = f_c \cdot (b) - c$$
$$p_d = f_c \cdot (b)$$

Payoff Equations

$$p_c = f_c \cdot (b) - c$$
$$p_d = f_c \cdot (b)$$

Since payoff is relative, we don't lose any information by dividing both equations by c.

$$p_c = f_c \cdot (\frac{b}{c}) - 1$$
$$p_d = f_c \cdot (\frac{b}{c})$$

- This also make sense because it isn't the exact benefit or cost that matter but their ratio.

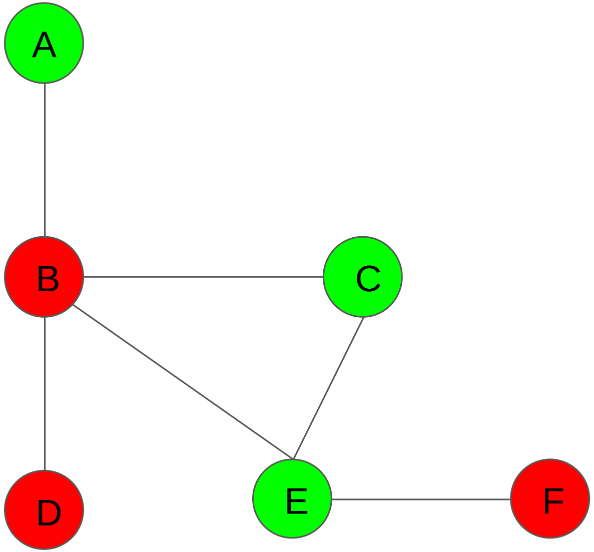
Fitness Equation

$$r = 1 + \delta p$$

- In this equation p is the payoff and delta is a non-negative real number known as the selection constant
- By having delta, we can finetune how much do payoff matter in interactions.
- For example, when delta = 0 (**neutral drift**), payoff don't matter at all to the fitness of the individual. When delta is small (known as **weak selection**) payoff matters a little, and when delta is big (**strong selection**) payoff matters a lot.
- 1 is added as a technicality so we can normalize fitness as probability later in our model

Graph Model

- We can study what happen to cooperator-defector populations overtime by modeling it using a graph.
- Each vertex represents a cooperator (green) or defector (red).
- A vertex only interacts with its neighbors.



Calculating Payoff On Graph

- Frequency is with respect to the degree of the vertex.

$$p_A = f_c \cdot (\frac{b}{c}) - 1 = 0 \cdot (\frac{b}{c}) - 1 = -1$$

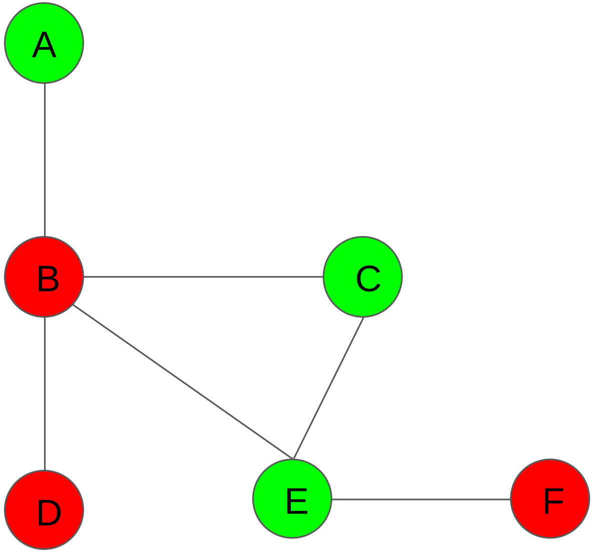
$$p_B = f_c \cdot (\frac{b}{c}) = \frac{3}{4} \cdot \frac{b}{c}$$

$$p_C = f_c \cdot (\frac{b}{c}) - 1 = \frac{1}{2} \cdot \frac{b}{c}$$

$$p_D = f_c \cdot (\frac{b}{c}) = 0$$

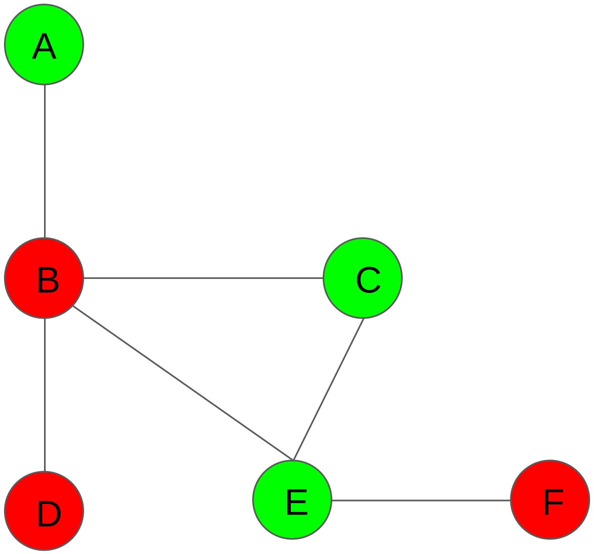
$$p_E = f_c \cdot (\frac{b}{c}) - 1 = \frac{1}{3} \cdot \frac{b}{c}$$

$$p_F = f_c \cdot (\frac{b}{c}) = \frac{b}{c}$$



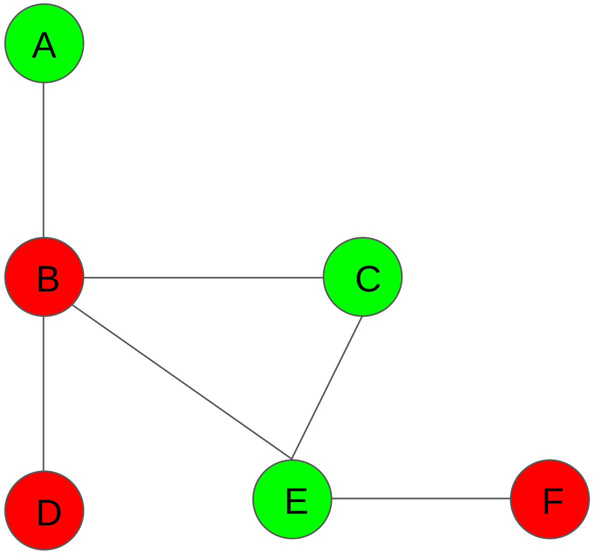
Graph Model

- We can simulate the passage of time by using death-birth update.
- This means for each timestep in the interaction:
  - We uniformly randomly choose a vertex to die
  - A neighbor of the vertex will be chosen to reproduce. The probability of being chosen is proportional to the vertex's fitness
  - The vertex chosen to die is now replaced by the vertex that reproduced

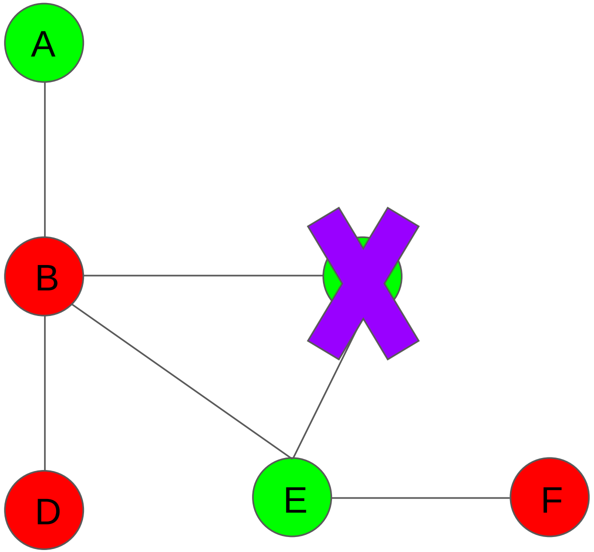


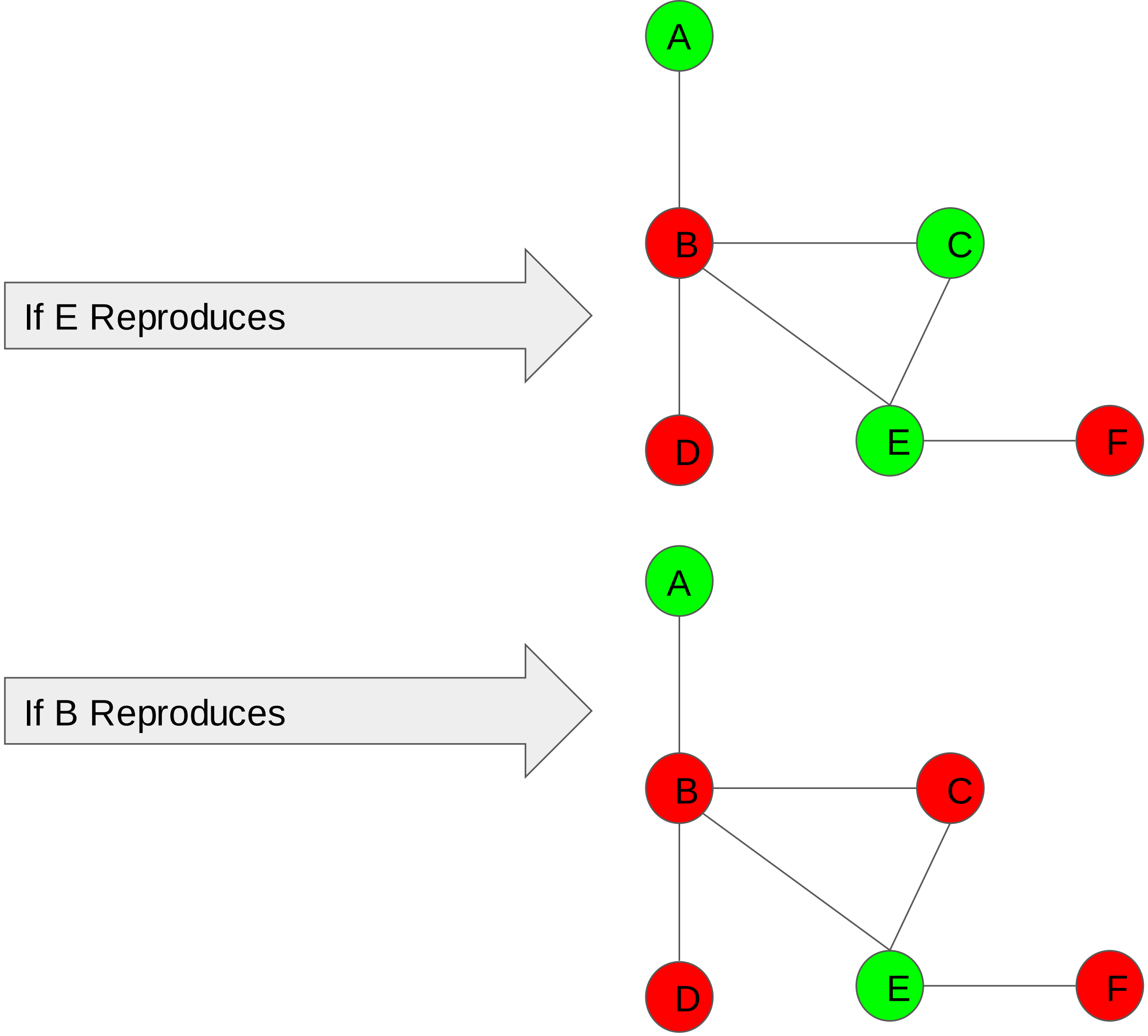
Probability of Reproducing

- For example, if C is chosen to die at the current timestep then:
- Prob(B replacing C) = $r_B / (r_B+r_E)$
- Prob(E replacing C) = $r_E / (r_B+r_E)$



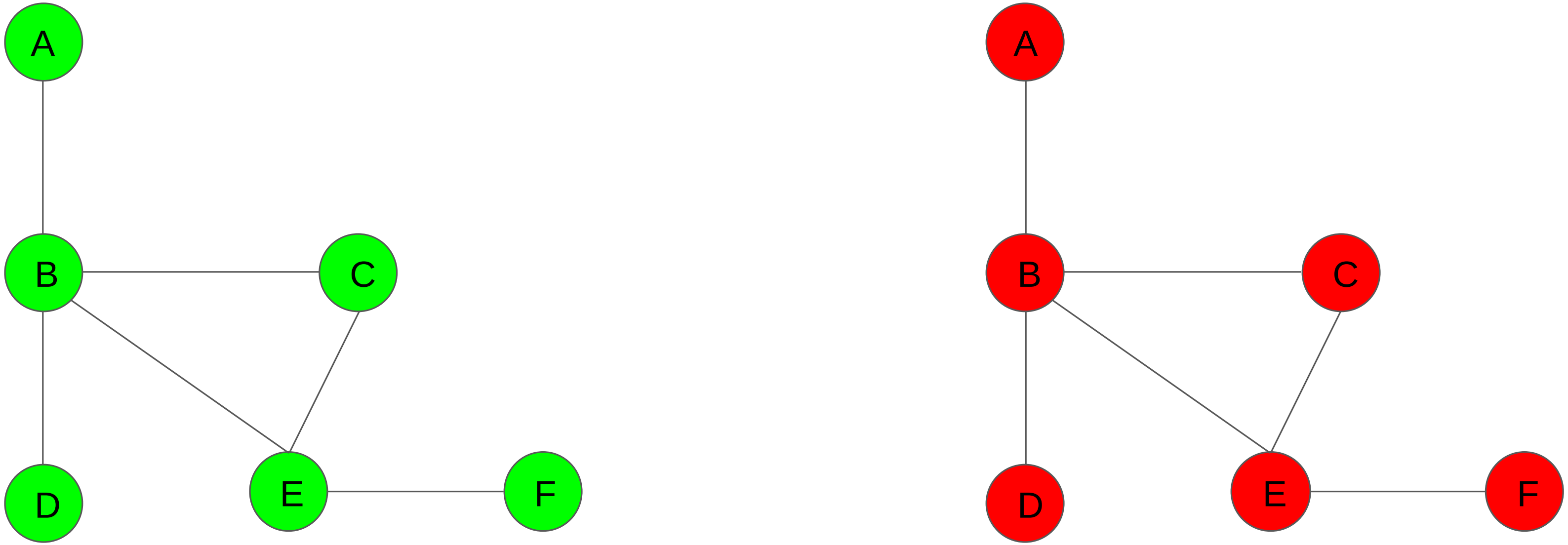
Graph Model





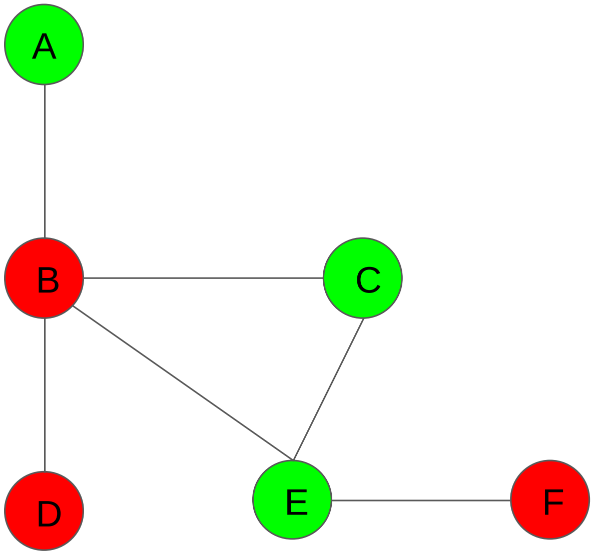
Graph Model

- We repeat the death-birth update until we reach an absorbing state (when every vertex is the same)



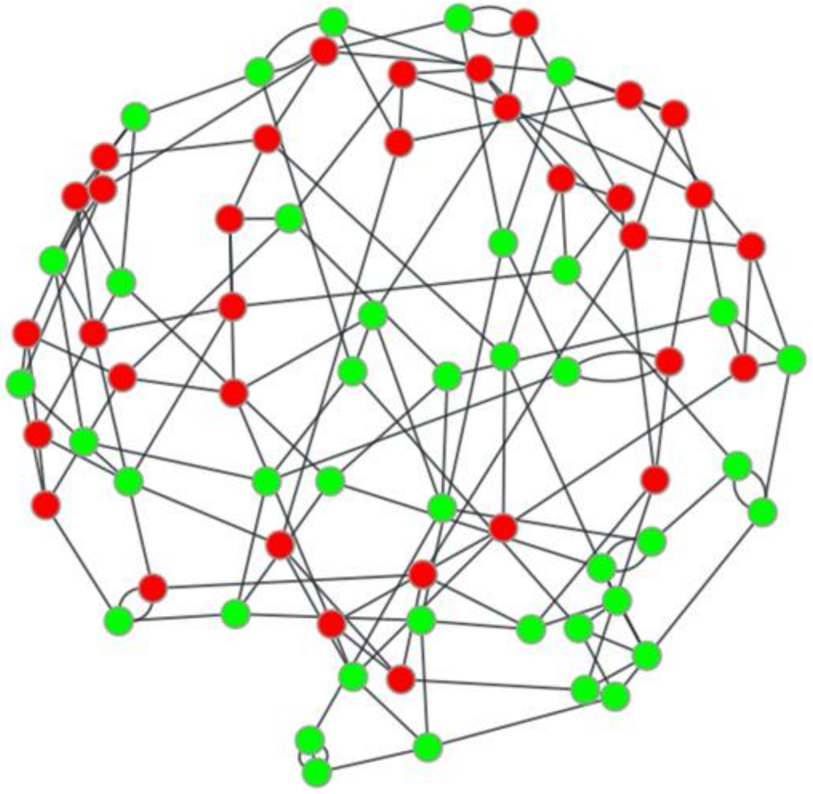
Fixation Probability

- A natural question arises: Given a starting configuration with *k* cooperators, what is the probability that the cooperators will take over the whole graph?
- This probability is called the **fixation probability**.



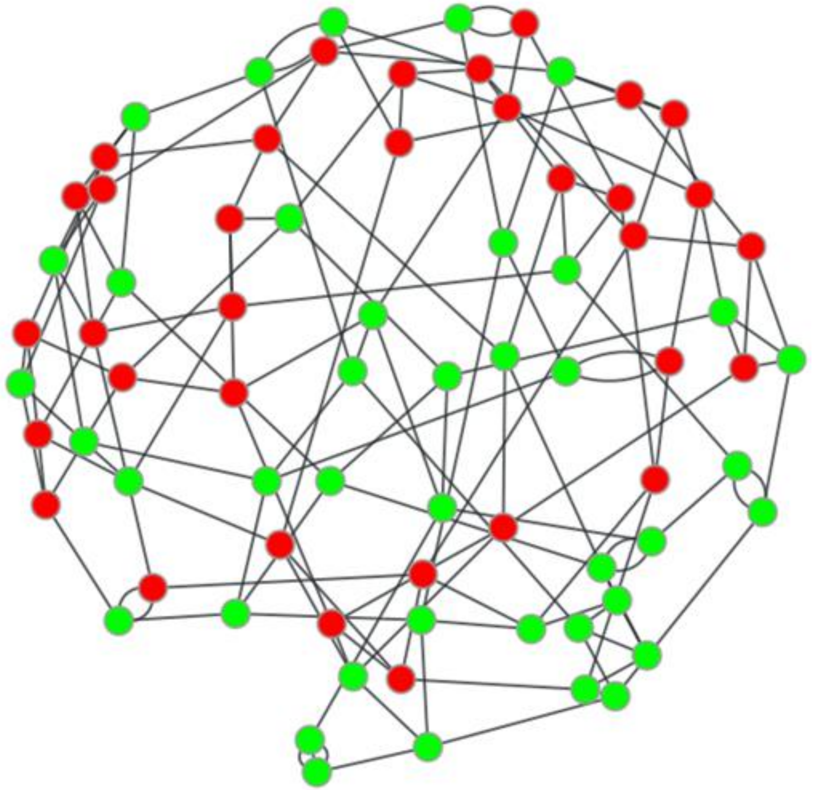
Fixation Probability

- Finding this fixation probability is difficult in general, but on a regular graph, under neutral drift, we can compute it by hand.
- A regular graph is a graph where the degree of every vertex is the same.



Fixation Probability

- The fixation probability of *k* cooperators under neutral drift of a regular graph with *N* vertices is *k/N*
- Using *k/N* as a baseline, what can we do to increase cooperators' fixation probability?



Fixation Probability

- Intuitively, if we can increase the probability of cooperator reproducing, then we can increase their fixation probability.
- This happen when the b/c ratio in the payoff equation increases
- The threshold b/c that allows cooperators to have a higher fixation probability compared to the neutral drift fixation probability (if it exist) is referred to as the **critical ratio**.

Research Development

- In 2006 Ohtsuki et al. estimated the critical ratio of a regular graph under weak selection to be the degree of the graph.
- In 2016 this result was made precise in a joint paper by 7 mathematicians, including professor Gabor Lippner from NEU. The exact critical ratio of a regular graph is (n-2) / ((n/d) - 2) where n is the total number of vertices and d is the degree.

A simple rule for the evolution of cooperation on graphs and social networks

Hisashi Ohtsuki[1,2], Christoph Hauert[2], Erez Lieberman[2,3] & Martin A. Nowak[2]

Evolutionary dynamics on any population structure

Benjamin Allen[1,2,3], Gabor Lippner[4], Yu-Ting Chen[5], Babak Fotouhi[1,6], Naghmeh Momeni[1,7], Shing-Tung Yau[3,8], and Martin A. Nowak[1,8,9]

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Strong selection

- The result presented before this slide relied on the assumption that the graph is under weak selection (delta is small).
- How can we improve the fixation probability for cooperators in strong selection?
- Currently there are not much literature on this topic and we can only find a 2017 paper that studies a particular type of graph.

Evolutionary games on cycles with strong selection

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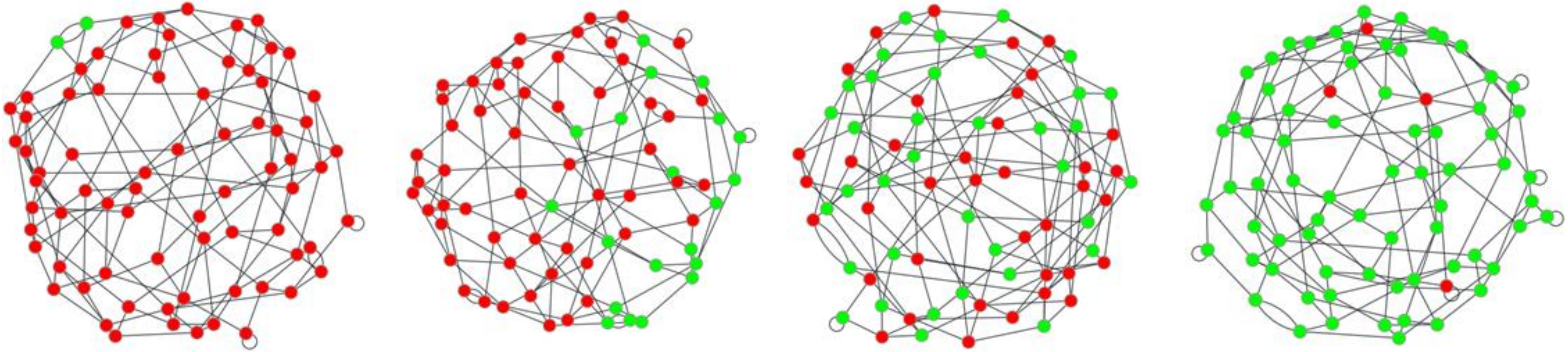
Current State of Research

- We conjecture that connectivity (edges between cooperators and defectors) influences the fixation probability under strong selection. But now in neutral drift.
- To verify, we first calculated the fixation probability empirically by running simulations on different starting configuration of cooperators and connectivity under neutral drift. So we can use it to compare later.

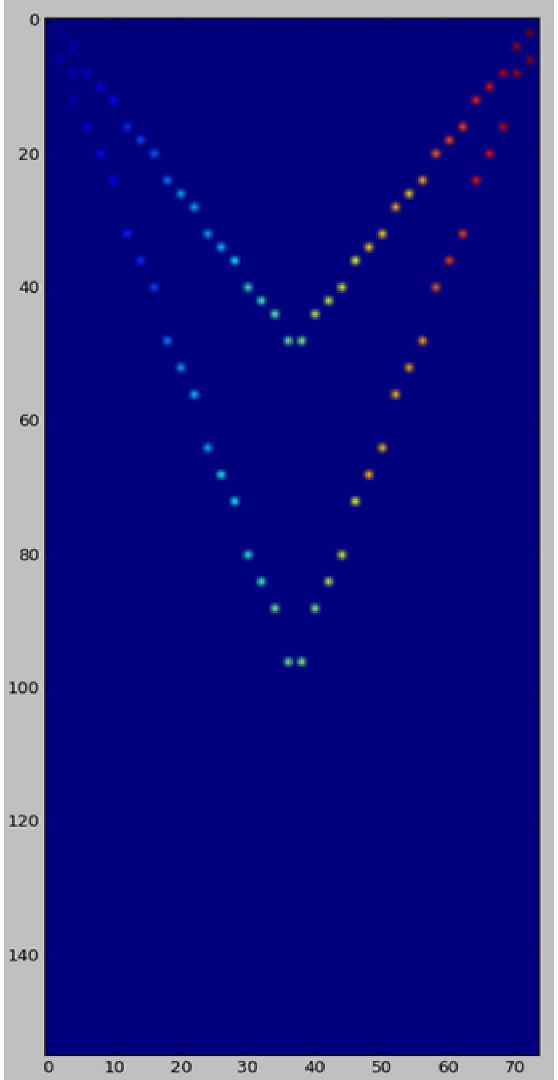


Simulation Method

- The graph we are using to simulate is 4-regular with 74 vertices.
- To save time, we start with even number of cooperators.
- For each number of starting cooperators, we find the fixation probability when the connectivity is at:
  - ⅓ of maximum possible connectivity (rounded down)
  - ⅔ of maximum possible connectivity (rounded down)
- For example, when there are 2 cooperators, the maximum possible of connectivity it can have is 8, and we find the fixation probability at connectivity 2 and 6.
- To find the fixation probability of cooperator at a configuration, we run 500 iterations, and count how many times the cooperators succeeded in taking over the entire population.

Data we got

- X-axis: Initial cooperators.
- Y-axis: Initial connectivity.
- Color: More red = higher fixation probability.
- As the number of initial cooperator increases, so does the fixation probability.
- An increase in connectivity does not have a significant impact on fixation probability.



Next Step

- Each data point takes on average 30 minutes to complete and the graph took around 2 weeks to finish.
- Unfortunately in strong selection, the runtime is even slower and we weren’t able to produce enough data.
- For next step we plan to use the NEU Research Cluster Service so we can get more data quickly.

Thank you!

- Jonier for being an awesome mentor for the past two months!
- Emily for providing very helpful feedbacks.
- REU for giving me this wonderful opportunity.
- NEU faculty and fellow REU students for making this a fun and learning experience!