Row Sketching

September 9, 2020

[1]: import numpy as np

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import matplotlib.pyplot as plt
%matplotlib inline
np.random.seed(seed=777)
#Fake some data for exhibition purpose
X = np.random.uniform(0,1,500)
X = X.reshape(-1,10)
print(X)
#This is a 50 by 10 dataset
import pandas as pd
[[0.15266373 0.30235661 0.06203641 0.45986034 0.83525338 0.92699705
 0.72698898 0.76849622 0.26920507 0.64402929]
[0.09337326 0.07968589 0.58961375 0.34334054 0.98887615 0.62647321
 0.68177928 0.55225681 0.26886006 0.37325939]
[0.2229281 0.1864426 0.39064809 0.19316241 0.61091093 0.88280845
 0.62233882 0.25311894 0.17993031 0.81640447]
[0.22537162 0.51685714 0.51849582 0.60037494 0.53262048 0.01331005
 0.52409726 0.89588471 0.76990129 0.1228517 ]
[0.29587269 0.61202358 0.72613812 0.46349747 0.76911037 0.19163103
 0.55786672 0.55077816 0.47222549 0.79188496]
[0.11524968 0.6813039 0.36233361 0.34420889 0.44951875 0.02694226
 0.41524769 0.9222317 0.09120557 0.31512178]
[0.52802224 0.32806203 0.44891554 0.01633442 0.0970269 0.69258857
 0.83594341 0.42432199 0.8487743 0.54679121]
[0.35410346 0.72724968 0.09385168 0.8928588 0.33625828 0.89183268
 0.296849
            0.30164829 0.80624061 0.83760997]
[0.63428133 0.3113273 0.02944858 0.39977732 0.51817346 0.00738845
 0.77494778 0.8544712 0.13153282 0.28767364]
[0.32658881 0.90655956 0.99955954 0.77088429 0.04284752 0.96525111
 0.97521246 0.2025168 0.67985305 0.46534506]
[0.92001748 0.72820735 0.24585653 0.01953996 0.70598881 0.77448287
 0.4729746  0.80146736  0.17539792  0.72016934]
[0.3678759  0.53209295  0.29719397  0.37429151  0.72810013  0.39850784
 0.1058295   0.39858265   0.52196395   0.1060125 ]
[0.85349239 0.51839737 0.61106603 0.82915051 0.01689229 0.770302
 0.80847474 0.8030573 0.11476295 0.88808986]
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[0.10667592 0.23981016 0.80759403 0.10198876 0.07828535 0.34543636
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- 0.83598444 0.06554275 0.6360506 0.67026503]
- [0.79456481 0.2994673 0.75812402 0.83553903 0.37388061 0.41591966
- 0.7343975 0.70654535 0.7356914 0.57297293]
- [0.94526627 0.28584853 0.65293073 0.81104054 0.40542651 0.51364871
- 0.14014905 0.15721947 0.03541432 0.49180601]
- [0.68388763 0.17620439 0.78650459 0.13184595 0.60086037 0.51830006
- 0.00414951 0.08791205 0.52267979 0.00458374]
- [0.02282895 0.93060174 0.06619459 0.12418814 0.82842641 0.36812763
- 0.04768249 0.60247133 0.37106164 0.68651169]
- [0.25305283 0.51846337 0.57930518 0.69496954 0.568014 0.57607863
- 0.97195988 0.09615256 0.15056744 0.21890239]
- $[0.18433482 \ 0.17429123 \ 0.91739598 \ 0.087738 \ \ \ 0.79970306 \ 0.83145554$
- 0.27993361 0.4459065 0.76829829 0.16288936]
- [0.77254856 0.53478201 0.15692983 0.47321536 0.70398682 0.35157667
- 0.07338461 0.48765463 0.20388045 0.78410032]
- $\hbox{\tt [0.15491643\ 0.29149967\ 0.40241112\ 0.07275669\ 0.37958101\ 0.51590022 }$
- 0.71629344 0.64766257 0.82290917 0.77098869]
- $\begin{bmatrix} 0.22900459 & 0.66012962 & 0.37327435 & 0.55183087 & 0.0804195 & 0.2109952 \end{bmatrix}$
- 0.65217012 0.02133481 0.17582667 0.19648443]
- $\hbox{\tt [0.84129635\ 0.41820739\ 0.31345148\ 0.68702801\ 0.89515608\ 0.36977154] }$
- 0.50159014 0.10483828 0.71975822 0.63448114]
- [0.80783829 0.54800581 0.02194742 0.51894304 0.09897583 0.43511542
- 0.85139725 0.44118134 0.89596741 0.12072653]
- $[0.33303632\ 0.66768574\ 0.37253643\ 0.90108999\ 0.46836287\ 0.10777418$
- 0.31245389 0.45278092 0.24713217 0.06938743]
- $[0.23291488 \ 0.09418977 \ 0.04359004 \ 0.69394711 \ 0.28812972 \ 0.01575116$
- 0.08326803 0.24411837 0.10969615 0.72659145]
- $\begin{bmatrix} 0.42820558 & 0.07101384 & 0.06750895 & 0.02519358 & 0.42825842 & 0.93007952 \end{bmatrix}$
- 0.20008667 0.39672322 0.33959378 0.9823881]
- $[0.26554444 \ 0.3623058 \ 0.70875493 \ 0.01155356 \ 0.167133 \ 0.75875134$
- 0.41508562 0.77236123 0.61117578 0.17911996]
- $\begin{bmatrix} 0.40636652 & 0.25461546 & 0.50239927 & 0.73145006 & 0.51088933 & 0.71745862 \end{bmatrix}$
- 0.76113276 0.00623583 0.70304607 0.1365286]
- $[0.95663804 \ 0.20531562 \ 0.61395393 \ 0.0095305 \ 0.06847696 \ 0.97789675$
- 0.97365789 0.32974385 0.87039916 0.17444905]
- $[0.39047352\ 0.54510419\ 0.4985748\ 0.41435601\ 0.08113125\ 0.50904101$
- 0.66225782 0.75083042 0.54691749 0.75474874]
- [0.08975803 0.80619849 0.40007355 0.16537872 0.69858091 0.52814575
- 0.44028495 0.33694007 0.14911278 0.67519679]
- [0.70946756 0.89642958 0.227859 0.17226834 0.81164764 0.39406533
- 0.73010599 0.04674746 0.77670941 0.87342282]
- $\begin{bmatrix} 0.75026214 & 0.76977188 & 0.80826695 & 0.75775349 & 0.78125569 & 0.38302591 \end{bmatrix}$
- 0.53078614 0.12339639 0.27472316 0.55060072]
- [0.23894661 0.50084472 0.19263048 0.60425696 0.35960111 0.36886074
- 0.62917861 0.78660705 0.48590931 0.69192111]
- $\begin{bmatrix} 0.87554652 & 0.81877026 & 0.40281936 & 0.57234235 & 0.80657272 & 0.57380207 \end{bmatrix}$
- 0.50132095 0.1432569 0.40690395 0.51162535]

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[0.18927863 0.34074641 0.1228205 0.69749297 0.96862159 0.6271846
      0.31281466 0.53188689 0.89087163 0.59373093]
     [0.88013472 0.61303218 0.93233281 0.18296368 0.02774407 0.33227895
      0.01076536 0.44528887 0.93149479 0.201871 ]
     [0.54213979 0.92942292 0.84216478 0.078738
                                                 0.75564842 0.4047021
      0.53665957 0.04737449 0.68841986 0.55807444]
     [0.03669628 0.75678572 0.32432037 0.01432732 0.94748355 0.51850623
      0.68308831 0.55382561 0.05646545 0.51749431]
     [0.36943643 0.05924653 0.10640852 0.86190162 0.16830811 0.06004371
      0.21744526 0.20526956 0.5349881 0.18730338]
     [0.49325975 0.65963597 0.59545756 0.64787981 0.2240036 0.52404835
      0.93018037 0.62463147 0.21829653 0.70607196]
     [0.30926068 0.29041254 0.52918557 0.26915884 0.6525739 0.46158807
      0.08430998 0.73240877 0.37039744 0.59914316]
     [0.96407415 0.54916536 0.80262017 0.16681119 0.02511028 0.77530006
      0.31921348 0.79156353 0.21402362 0.80138723
     [0.52418399 0.67885827 0.94021637 0.82493606 0.34068255 0.80114599
      0.07084921 0.17493708 0.42903926 0.52804127]
     [0.32702651 0.47491164 0.23533784 0.98811328 0.06705414 0.72075662
      0.45687396 0.11821332 0.13410829 0.45344758]
     [0.03750877 0.77659848 0.46021742 0.94319951 0.83279599 0.34511277
      0.96941911 0.98689797 0.1267323 0.09586372]
     [0.10259792 0.45599757 0.23395138 0.83984091 0.75213683 0.30652171
      0.55532686 0.78316191 0.07237307 0.61426566]
     [0.44044365 0.1243283 0.53103499 0.62764022 0.28191159 0.26811501
      0.9924446  0.54324407  0.69513388  0.71643219]]
[2]: n = np.shape(X)[0]
     p = np.shape(X)[1]
     r = 0.25/np.power(np.log10(n),1/p)
     print(n,p,r)
```

50 10 0.23709447706328904

```
[3]: #Row algorithm
    m = 1 #pointer for exemplar
    row = np.copy(X[0,:])
    exemplars = [np.nan]*n
    exemplars[0] = row
    members = [ [] for _ in range(n) ]

#print(exemplars)
    #print(members)

def myRange(start,stop,step):
    if start == stop:
        return ([stop])
```

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else:
        return (np.arange(start,stop,step))
def EuclideanDistance(a,b):
    if np.shape(a) != np.shape(b):return(-1)
    dist = 0
    for k in range(np.shape(a)[0]):
        #Try L^2 distance first.
        dist = dist + np.power(a[k] - b[k],2)
    dist = np.sqrt(dist)
    return(dist)
#print('test EuclideanDistance,', EuclideanDistance(np.array([0,0]),np.
\rightarrow array([1,2]))
print('r=',r)
#VERBOSE = True for debugging
VERBOSE = False
for i in myRange(1,n,1):
    if VERBOSE: print('Handling point ',i,' out of ',n)
    newExemplar = True
    row = np.copy(X[i-1,:]) #This should be i-th row, just i-1 since python_{\bot}
\rightarrowstarts couting from 0-th row.
    for j in myRange(1,m,1):
        d = EuclideanDistance(row, exemplars[j-1])
        if VERBOSE:print('The',i,'-th point to the ',j,'-th exemplar has L^2_

→distance', d)
        if d<r:
            members[j-1].append(i)
            newExemplar = False
            break
    if newExemplar:
        if VERBOSE:
            print('New exemplar created. Current # of exemplars:',m)
            print(exemplars)
            print(members)
        m = m + 1
        exemplars [m-1] = np.copy(row)
        members[m-1] = [i]
#print(exemplars)
#print(members)
```

r= 0.23709447706328904

```
[4]: #Reduced dataset
X_reduced = np.empty([0,np.shape(X)[1]])
for k in range(n):
    if np.any( np.isnan(exemplars[k]) ):continue
    #print(exemplars[k])
    X_reduced = np.vstack((X_reduced, exemplars[k]))

#plot first two dimensions of original and reduced datasets.
X_original = np.asarray(X)
plt.figure(0)
plt.scatter(X_original[:,0], X_original[:,1])
plt.title('Original dataset')

X_reduced = np.asarray(X_reduced)
plt.figure(1)
plt.scatter(X_reduced[:,0], X_reduced[:,1],color='orange')
plt.title(f'Row Algorithm processed dataset, r={r}')
```

[4]: Text(0.5, 1.0, 'Row Algorithm processed dataset, r=0.23709447706328904')





