Zero-inflated models: Beta-Binomial

Parameterisation

There is support for a further zero-inflated model of type 2 (see zero-inflated.pdf), the zero-inflated beta-binomial. It is only defined for type 2.

Type 2

The likelihood is defined as

$$\text{Prob}(y \mid \ldots) = p \times 1_{[y=0]} + (1 - p) \times \text{Beta-binomial}(y)$$

where:

$$p = 1 - \left(\frac{\exp(x)}{1 + \exp(x)} \right)^{\alpha}$$

Link-function

As for the Binomial (see Zero-inflated.pdf).

Hyperparameters

The Beta-binomial distribution has two arguments (β_1 & β_2) which we assume are a (specific) function of an underlying hyperparameter (δ) & x. There is a further hyperparameter, α, governing zero-inflation where:

The parameter controlling the degree of overdispersion, δ, is represented as

$$\theta_1 = \log(\delta)$$

and the prior is defined on θ_1.

The zero-inflation parameter α, is represented as

$$\theta_2 = \log(\alpha)$$

and the prior and initial value is is given for θ_2.

Specification

- family="zeroinflatedbetabinomial2"
- Required arguments: As for the zero-inflated-nbinomial2 likelihood.

Hyperparameter spesification and default values

doc Zero inflated Beta-Binomial, type 2

hyper

    theta1

        hyperid 94001
        name log alpha
        short.name a

        output.name zero-probability parameter for zero-inflated betabinomial_2

        output.name.intern intern zero-probability parameter for zero-inflated betabinomial_

        initial 0.693147180559945

        fixed FALSE

        prior gaussian

        param 0.693147180559945 1

        to.theta function(x) log(x)

        from.theta function(x) exp(x)

    theta2

        hyperid 94002

        name beta

        short.name b

        output.name overdispersion parameter for zero-inflated betabinomial_2

        output.name.intern intern overdispersion parameter for zero-inflated betabinomial_2

        initial 0

        fixed FALSE

        prior gaussian

        param 0 1

        to.theta function(x) log(x)

        from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default logit loga cauchit probit cloglog ccloglog loglog robit sn

pdf zeroinflated

Example

In the following we estimate the parameters in a simulated example.

Example-zero-inflated-beta-binomial2.R

```
nx = 1000                # number of x's to consider
n.trial = 20             # size of each binomial trial
x = rnorm(nx)            # generating x

delta = 10                 #hyperparameter 1
p = exp(1+x)/(1+exp(1+x))    #hyperparameter 2
alpha = 2                    #ZI parameter
q = p^alpha                  #prob presence

beta_1=delta*p                   #beta-bin parameter 1
beta_2=delta*(1-p)               #beta-bin parameter 2
rb = rbeta(nx, beta_1, beta_2, ncp = 0)


y = rep(0,nx)                       #generating data
```

```
abs.pres = rbinom(nx,1,q)
y[abs.pres==1] = rbinom( sum(abs.pres>0), n.trial, rb[abs.pres==1])

formula = y ~ x +1
r = inla(formula, data = data.frame(x,y), family = "zeroinflatedbetabinomial2",
         control.family = list(hyper=list(a = list(prior = "flat", param=numeric(0)),
                                           b = list(prior = "flat", param=numeric(0)))),
         Ntrials = rep(n.trial, nx),
         verbose=TRUE)
```