

# Intercept-slope model

## Parametrization

The intercept-slope model is a convenient re-implementation of a commonly used construct, where

$$(a, b)$$

is bi-variate Gaussian with a Wishart prior for the precision matrix<sup>1</sup>, and various forms of

$$\gamma(a + bz), \tag{1}$$

where  $z$  is a covariate and  $\gamma$  is a (random) scaling, goes into the linear predictor. Replicates of  $(a, b)$  is indexed by *subject*,  $i = 1, \dots, n$ , and the various scaling of Eq. 1 by *strata*  $j = 1, \dots, m$ , leading to a model for (a subset of)

$$\{\gamma_j(a_i + b_i z_{ij}), \quad i = 1, \dots, n, \quad j = 1, \dots, m\},$$

as not all combinations need to be present.

## Hyperparameters

The hyperparameters are  $(\theta_1, \theta_2, \theta_3)$  as in the model “iid2d” (related to the precisions of  $a$  and  $b$ , and their correlation), and  $\theta_4 = \gamma_1, \dots, \theta_{13} = \gamma_{10}$ . Since  $m$  is defined in the input, only  $\gamma_1, \dots, \gamma_m$  are used.  $m$  is limited to  $m \leq 10$ . **Please note** that all  $\gamma_i$ ’s are by default **fixed** to 1.

## Specification

The is specified as

```
f(idx, model="intslope", hyper = ...,
  precision = exp(14),
  args.intslope = list(subject=i, strata=j, covariate = z))
```

The definition of the model is through the `args.intslope` argument, where `i` and `j` are factors/integers and `z` is numerical, all with same length  $N$ , say. The argument `idx`, index which row that is used for the linear predictor, hence values of `idx` must take integer values in the interval 1 to  $N$ . The precision argument, defines the tiny small noise added to each  $\gamma(a + bz)$  to avoid a singular joint model. The `subject` and `strata` argument, is converted internally into integers 1, 2, ..., using

```
subject = as.numerical(as.factor(subject))
strata = as.numerical(as.factor(strata))
```

and the results is shown after this conversion.

## Hyperparameter specification and default values

**doc** Intecept-slope model with Wishart-prior

**hyper**

```
theta1
hyperid 16101
name log precision1
```

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<sup>1</sup>The documentation for the model “iid2d” gives the details of the definition of the parameterization of the precision matrix and the Wishart-prior.

```

    short.name prec1
    initial 4
    fixed FALSE
    prior wishart2d
    param 4 1 1 0
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta2
    hyperid 16102
    name log precision2
    short.name prec2
    initial 4
    fixed FALSE
    prior none
    param
    to.theta function(x) log(x)
    from.theta function(x) exp(x)
theta3
    hyperid 16103
    name logit correlation
    short.name cor
    initial 4
    fixed FALSE
    prior none
    param
    to.theta function(x) log((1 + x) / (1 - x))
    from.theta function(x) 2 * exp(x) / (1 + exp(x)) - 1
theta4
    hyperid 16104
    name gamma1
    short.name g1
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 16105
    name gamma2
    short.name g2
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 16106

```

```

    name gamma3
    short.name g3
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 16107
    name gamma4
    short.name g4
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 16108
    name gamma5
    short.name g5
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 16109
    name gamma6
    short.name g6
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 16110
    name gamma7
    short.name g7
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
theta11

```

```

    hyperid 16111
    name gamma8
    short.name g8
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
  theta12
    hyperid 16112
    name gamma9
    short.name g9
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x
  theta13
    hyperid 16113
    name gamma10
    short.name g10
    initial 1
    fixed TRUE
    prior normal
    param 1 36
    to.theta function(x) x
    from.theta function(x) x

  constr FALSE
  nrow.ncol FALSE
  augmented FALSE
  aug.factor 1
  aug.constr
  n.div.by
  n.required FALSE
  set.default.values TRUE
  pdf intslope

```

## Example

```
library(mvtnorm)
n = 300
idx = 1:n
nstrata = 3
strata = sample(1:nstrata, n, replace=TRUE)
nsubject = n %% nstrata
subject = sample(1:nsubject, n, replace=TRUE)
z = rnorm(n)
gam = c(1, 1 + rnorm(nstrata-1, sd = 0.2))

rho = sqrt(3)/2
Sigma = matrix(c(1/1, NA, NA, 1/2), 2, 2)
Sigma[1,2] = Sigma[2,1] = rho*sqrt(Sigma[1,1]*Sigma[2,2])

ab = rmvnorm(nsubject, sigma=Sigma)
a = ab[,1]
b = ab[,2]
s = 0.01
y = gam[strata] * (a[subject] + z * b[subject]) + rnorm(n, s = 0.01)
r = inla(y ~ -1 + f(idx, model = "intslope",
  args.intslope = list(subject = subject,
    strata = strata,
    covariates = z),
  ## this is for nstrata = 3
  hyper = list(gamma1 = list(fixed = TRUE),
    gamma2 = list(fixed = FALSE),
    gamma3 = list(fixed = FALSE))),
  data = list(y = y,
    idx = idx,
    subject = subject,
    strata = strata,
    z = z),
  control.family = list(hyper = list(
    prec = list(initial = log(1/s^2),
      fixed=TRUE))))))

summary(r)
```

## Notes

- With  $n_s = \max(\text{subject})$ , the internal storage of this model is

$$(\gamma_{j_1}(a_{i_1} + z_1 b_{i_1}), \dots, \gamma_{j_N}(a_{i_N} + z_N b_{i_N}), a_1, \dots, a_{n_s}, b_1, \dots, b_{n_s}),$$

i.e. a vector of length  $N + 2n_s$ .