

## qLogLogistic likelihood

### Parametrisation

The LogLogistic distribution has cumulative distribution function

$$F_0(y) = \frac{1}{1 + \lambda y^{-\alpha}}, \quad y > 0$$

if `variant=0`, or

$$F_1(y) = \frac{1}{1 + (\lambda y)^{-\alpha}}, \quad y > 0$$

if `variant=1`, where

$\alpha > 0$  is a shape parameter, and

$\lambda > 0$  is a scale parameter.

The  $\lambda$  is defined implicitly through the quantile, as

$$F_0(y_q) = q, \quad \text{or} \quad F_1(y_q) = q, \quad 0 < q < 1$$

and the linear predictor is defined on  $y_q$ .

### Link-functions

The parameter  $\lambda$  is linked to the linear predictor, implicitly through

$$y_q = \exp(\eta)$$

### Hyperparameters

The  $\alpha$  parameter is represented as

$$\theta = \log \alpha$$

and the prior is defined on  $\theta$ .

### Specification

- `family="qloglogistic"` (regression) or `family="qloglogistic.surv"` (survival)
- `variant=0` (default) or 1, choosing between parameterisation  $F_0$  or  $F_1$ .
- Required arguments:  $y$  (regression) or an `inla.surv`-object using `inla.surv()` (for survival data), and `quantile=q`.

### Hyperparameter specification and default values

#### Regression:

`doc` A quantile loglogistic likelihood

`hyper`

`theta`

`hyperid 60011`

`name log alpha`

```

short.name alpha
output.name alpha for qloglogistic observations
output.name.intern log alpha for qloglogistic observations
initial 1
fixed FALSE
prior loggamma
param 25 25
to.theta function(x) log(x)
from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default log neglog

pdf qloglogistic

```

#### Survival:

```
doc A quantile loglogistic likelihood (survival)
```

#### hyper

##### theta1

```

hyperid 60021
name log alpha
short.name alpha
output.name alpha for qloglogisticsurv observations
output.name.intern log alpha for qloglogisticsurv observations
initial 1
fixed FALSE
prior loggamma
param 25 25
to.theta function(x) log(x)
from.theta function(x) exp(x)

```

##### theta2

```

hyperid 60022
name beta1
short.name beta1
output.name beta1 for qlogLogistic-Cure
output.name.intern beta1 for logLogistic-Cure
initial -5
fixed FALSE
prior normal
param -4 100
to.theta function(x) x
from.theta function(x) x

```

##### theta3

```

hyperid 60023
name beta2
short.name beta2
output.name beta2 for qlogLogistic-Cure
output.name.intern beta2 for logLogistic-Cure
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta4
hyperid 60024
name beta3
short.name beta3
output.name beta3 for qlogLogistic-Cure
output.name.intern beta3 for qlogLogistic-Cure
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta5
hyperid 60025
name beta4
short.name beta4
output.name beta4 for qlogLogistic-Cure
output.name.intern beta4 for qlogLogistic-Cure
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta6
hyperid 60026
name beta5
short.name beta5
output.name beta5 for qlogLogistic-Cure
output.name.intern beta5 for qlogLogistic-Cure
initial 0
fixed FALSE
prior normal

```

```

    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 60027
    name beta6
    short.name beta6
    output.name beta6 for qlogLogistic-Cure
    output.name.intern beta6 for qlogLogistic-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 60028
    name beta7
    short.name beta7
    output.name beta7 for qlogLogistic-Cure
    output.name.intern beta7 for qlogLogistic-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 60029
    name beta8
    short.name beta8
    output.name beta8 for qlogLogistic-Cure
    output.name.intern beta8 for qlogLogistic-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 60030
    name beta9
    short.name beta9
    output.name beta9 for qlogLogistic-Cure

```

```

    output.name.intern beta9 for qlogLogistic-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
  theta11
    hyperid 60031
    name beta10
    short.name beta10
    output.name beta10 for qlogLogistic-Cure
    output.name.intern beta10 for qlogLogistic-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x

survival TRUE

discrete FALSE

link default log neglog

pdf qloglogistic

```

## Example

In the following example we estimate the parameters in a simulated case

```

lam_loglogistic = function(yq, alpha, q, variant = 0)
{
  if (variant == 0) {
    lambda = yq^alpha * (1/q-1)
  } else if (variant == 1) {
    lambda = 1/yq * (1/(1/q-1))^(1/alpha)
  } else
    stop("ERR")
  return (lambda)
}

```

```

rloglogistic = function(n, lambda, alpha, variant=0)
{
  u = runif(n)
  if (variant == 0) {
    y = (lambda/(1.0/u - 1.0))^(1.0/alpha)
  } else if (variant == 1) {

```

```

        y = (1.0/(1.0/u -1.0))^(1.0/alpha) / lambda
    } else {
        stop("ERROR")
    }
}

n = 500
alpha = 2.1
x = c(scale(runif(n)))
eta = 1.1+2.2*x
yq = exp(eta)

for(variant in 0:1) {
  for(q in c(0.2, 0.8)) {

    print(paste("variant=", variant, "quantile=", q))
    lambda = lam_loglogistic(yq, alpha, q, variant=variant)
    y = rloglogistic(n,
                     lambda = lambda,
                     alpha = alpha,
                     variant = variant)

    formula = y ~ 1 + x
    rr=inla(formula,
            family ="qloglogistic",
            data=data.frame(y, x),
            control.family = list(list(variant = variant, control.link = list(quantile = q))
    print("REGRESSION")
    print(summary(rr))

    event = rep(1,n)
    formula=inla.surv(y,event) ~ 1 + x
    r=inla(formula,
           family ="qloglogisticsurv",
           data = list(y=y, event=event, x=x),
           control.family = list(list(variant = variant, control.link = list(quantile = q))
    print("SURVIVAL")
    print(summary(r))
  }
}

```

## Notes

- Loglogisticsurv model can be used for right censored, left censored, interval censored data. If the observed times  $y$  are large/huge, then this can cause numerical overflow in the likelihood routine. If you encounter this problem, try to scale the observatios, `time = time / max(time)` or similar.