

## Skew-Normal likelihood

### Parametrisation

The standardised Skew-Normal distribution is

$$f(z) = \frac{2}{\omega_\alpha} \phi\left(\frac{z - \xi_\alpha}{\omega_\alpha}\right) \Phi\left(\alpha \frac{z - \xi_\alpha}{\omega_\alpha}\right)$$

where  $\omega_\alpha$  and  $\xi_\alpha$  are so that the mean is zero and variance is one, and they depends both on the skewness parameter  $\alpha$ .

The skew-normal likelihood is defined as the density wrt  $y$ , where

$$z = (y - \eta)\sqrt{w\tau} \sim f(z)$$

and

$\eta$ : is the the linear predictor

$\tau$ : is the precision

$w$ : is a fixed scale or weight,  $w > 0$ ,

### Link-function

The mean equals the linear predictor

$$\mu = \eta$$

### Hyperparameters

The precision is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on  $\theta_1$ .

The (standardised) skewness  $\gamma$ , is represented as

$$\gamma = 0.988 \left( 2 \frac{\exp(\theta_2)}{1 + \exp(\theta_2)} - 1 \right)$$

and the prior is defined on  $\theta_2$ . The standardised skewness depends on  $\alpha$  as

$$\gamma = \frac{4 - \pi}{2} \frac{\left(\delta \sqrt{2/\pi}\right)^3}{(1 - 2\delta^2/\pi)^{3/2}}, \quad \delta = \frac{\alpha}{\sqrt{1 + \alpha^2}}$$

### Specification

- `family="sn"`
- Required arguments:  $y$  and  $w$  (keyword `scale`, and  $w = 1$  by default).

## Hyperparameter specification and default values

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hyper

theta1

```
hyperid 74001
name log precision
short.name prec
output.name precision for skew-normal observations
output.name.intern log precision for skew-normal observations
initial 4
fixed FALSE
prior loggamma
param 1 5e-05
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

theta2

```
hyperid 74002
name logit skew
short.name skew
output.name Skewness for skew-normal observations
output.name.intern Intern skewness for skew-normal observations
initial 0.00123456789
fixed FALSE
prior pc.sn
param 10
to.theta function(x, skew.max = 0.988) log((1 + x / skew.max) / (1 - x / skew.max))
from.theta function(x, skew.max = 0.988) skew.max * (2 * exp(x) / (1 + exp(x)) - 1)
```

status experimental

survival FALSE

discrete FALSE

link default identity

pdf sn

## Example

```
library(sn)
set.seed(246)
n = 300
x = rnorm(n, sd = 1)
eta = 1+x
skewness = 0.25
y = numeric(n)
prec <- 100
```

```

for(i in 1:n) {
  ## map moments to sn-parameters c(xi, omega, alpha)
  param = INLA::inla.sn.reparam(moments = c(eta[i], 1/prec, skewness))
  y[i] = rsn(1, xi=param$xi, omega = param$omega, alpha = param$alpha)
}

r = inla(y ~ 1+x,
  family = "sn",
  data = data.frame(y, x),
  control.family = list(
    hyper = list(prec = list(
      prior = "pc.prec",
      param = c(3, 0.01))))))

summary(r)

```

## Notes

- This implementation replaces older ones ("sn" and "sn2") from 16<sup>th</sup> September 2020.
- A  $N(a, 0)$  prior is interpreted as a constant prior with density equal to one.