The dMatern model

Parametrisation

This model is the Gaussian field with a Matérn correlation function, directly, meaning **dense matrices**. This model is intended for a low-dimension only. The correlation function is

$$Corr(d) = \frac{1}{2^{\nu-1}\Gamma(\nu)} (\kappa d)^{\nu} K_{\nu}(\kappa d), \qquad \alpha = \nu + d/2,$$

where K_{ν} is the modified Bessel function and $\Gamma(\cdot)$ is the Gamma-function. The range is defined to be

$$r = \sqrt{8\nu}/\kappa$$

which about the distance where the covariance function becomes about 0.1.

Hyperparameters

The hyperparameters are the precision parameter τ , the range r and the smoothness ν , where the internal representation are

$$\theta = (\log(\tau), \log(r), \log(\nu))$$

The latent field has marginal variance $1/\tau$ and range (as defined above) r.

We do **not recommend** to treat ν as random, and for this reason it is default fixed. You can change its value by changing the initial value.

Specification

The dmatern model is specified inside the f() function as:

```
f(idx, model="dmatern", locations = L, hyper = <hyper>)
```

where L is a matrix of the locations for which the Gaussian field is defined; row L[i,] are the coordinates for the i'th location. idx represent the location indexing the corresponding row in L, so idx = 3 means location L[idx,]. idx must be integers $1, 2, \ldots, \text{nrow}(L)$, or NA.

Hyperparameter specification and default values

```
doc Dense Matern field
```

hyper

theta1

hyperid 35101
name log precision
short.name prec
initial 3
fixed FALSE
prior pc.prec
param 1 0.01
to.theta function(x) log(x)
from.theta function(x) exp(x)

theta2

hyperid 35102

```
name log range
         short.name range
         initial 0
         fixed FALSE
         prior pc.range
         param 1 0.5
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
     theta3
         hyperid 35103
         name log nu
         short.name nu
         initial -0.693147180559945
         fixed TRUE
         prior loggamma
         param 0.5 1
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
constr FALSE
nrow.ncol FALSE
augmented FALSE
aug.factor 1
aug.constr
n.div.by
n.required TRUE
set.default.values TRUE
pdf dmatern
Example
library(INLA)
library(mvtnorm)
\# 1D example. locations are 1, 2, ..., 'n', with 'nr' replications
range = 10
n = 50
nr = 20
loc = 1:n
var = 1.0
nu = 0.5
S = matrix(0, n, n)
for(i in 1:n) {
   for(j in i:n) {
       d = sqrt((loc[i] - loc[j])^2)
```

```
S[i, j] = var * INLA:::inla.matern.cf(d, range = range, nu = nu);
        S[j, i] = S[i, j]
}
y = c(t(rmvnorm(nr, sigma = S)))
r1 = inla(y - 1 + f(idx, model = "dmatern", locations = loc, replicate = re,
                     ## placing the prior at the correct value, just for
                     ## demonstration
                     hyper = list(range = list(initial = log(range),
                                               param = c(range, 0.5))),
          data = data.frame(y, idx = rep(1:n, nr), re = rep(1:nr, each = n)),
          family = "gaussian",
          ## just this this at some high value
          control.family = list(hyper = list(
                                    prec = list(initial = 12, fixed=TRUE))))
if (FALSE)
    plot(r1, plot.random.effect = FALSE)
# 2D example. Simulate 'n' data in a [0, 1]^2 box, with 'nr' replications
range = 0.2
n = 50
nr = 20
loc = matrix(runif(2*n), ncol = 2, nrow = n)
var = 1.0
nu = 0.5
S = matrix(0, n, n)
for(i in 1:n) {
    for(j in i:n) {
        dif = loc[i, ] - loc[j, ]
        d = sqrt(sum(dif^2))
        S[i, j] = var * INLA:::inla.matern.cf(d, range = range, nu = nu);
        S[j, i] = S[i, j]
    }
}
y = c(t(rmvnorm(nr, sigma = S)))
r2 = inla(y ~ -1 + f(idx, model="dmatern", locations = loc, replicate = re,
                     ## placing the prior at the correct value, just for
                     ## demonstration
                     hyper = list(range = list(initial = log(range),
                                               param = c(range, 0.5))),
          data = data.frame(y, idx = rep(1:n, nr), re = rep(1:nr, each = n)),
          family = "gaussian",
          ## just this this at some high value
          control.family = list(hyper = list(
                                    prec = list(initial = 12, fixed=TRUE))))
if (FALSE)
    plot(r2, plot.random.effect = FALSE)
```

Notes

Note that the above definition of range, might differ from the definition in other packages. It is the same used for the SPDE-models.