

# LogLogistic likelihood

## Parametrisation

The LogLogistic distribution has cumulative distribution function

$$F_0(y) = \frac{1}{1 + \lambda y^{-\alpha}}, \quad y > 0$$

if `variant=0`, or

$$F_1(y) = \frac{1}{1 + (\lambda y)^{-\alpha}}, \quad y > 0$$

if `variant=1`, where

$\alpha > 0$  is a shape parameter, and

$\lambda > 0$  is a scale parameter.

## Link-functions

The parameter  $\lambda$  is linked to the linear predictor, by default as

$$\lambda = \exp(\eta)$$

## Hyperparameters

The  $\alpha$  parameter is represented as

$$\theta = \log \alpha$$

and the prior is defined on  $\theta$ .

## Specification

- `family="loglogistic"` (regression) or `"loglogistic.surv"` (survival)
- `variant=0` (default) or 1, choosing between parameterisation  $F_0$  or  $F_1$ .
- Required arguments:  $y$  (regression) or an `inla.surv`-object using `inla.surv()` (for survival data)

## Hyperparameter specification and default values

### Regression:

`doc` The loglogistic likelihood

`hyper`

`theta`

`hyperid` 80001

`name` log alpha

`short.name` alpha

`output.name` alpha for loglogistic observations

`output.name.intern` log alpha for loglogistic observations

`initial` 1

`fixed` FALSE

```
prior loggamma
param 25 25
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

survival FALSE

discrete FALSE

link default log neglog

pdf loglogistic

**Survival:**

doc The loglogistic likelihood (survival)

hyper

theta1

```
hyperid 80011
name log alpha
short.name alpha
output.name alpha for loglogisticsurv observations
output.name.intern log alpha for loglogisticsurv observations
initial 1
fixed FALSE
prior loggamma
param 25 25
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

theta2

```
hyperid 80012
name beta1
short.name beta1
output.name beta1 for logLogistic-Cure
output.name.intern beta1 for logLogistic-Cure
initial -5
fixed FALSE
prior normal
param -4 100
to.theta function(x) x
from.theta function(x) x
```

theta3

```
hyperid 80013
name beta2
short.name beta2
output.name beta2 for logLogistic-Cure
output.name.intern beta2 for logLogistic-Cure
```

```

    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 80014
    name beta3
    short.name beta3
    output.name beta3 for logLogistic-Cure
    output.name.intern beta3 for logLogistic-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 80015
    name beta4
    short.name beta4
    output.name beta4 for logLogistic-Cure
    output.name.intern beta4 for logLogistic-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 80016
    name beta5
    short.name beta5
    output.name beta5 for logLogistic-Cure
    output.name.intern beta5 for logLogistic-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 80017

```

```

name beta6
short.name beta6
output.name beta6 for logLogistic-Cure
output.name.intern beta6 for logLogistic-Cure
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta8
  hyperid 80018
  name beta7
  short.name beta7
  output.name beta7 for logLogistic-Cure
  output.name.intern beta7 for logLogistic-Cure
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta9
  hyperid 80019
  name beta8
  short.name beta8
  output.name beta8 for logLogistic-Cure
  output.name.intern beta8 for logLogistic-Cure
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta10
  hyperid 80020
  name beta9
  short.name beta9
  output.name beta9 for logLogistic-Cure
  output.name.intern beta9 for logLogistic-Cure
  initial 0
  fixed FALSE
  prior normal
  param 0 100

```

```

    to.theta function(x) x
    from.theta function(x) x
theta11
    hyperid 80021
    name beta10
    short.name beta10
    output.name beta10 for logLogistic-Cure
    output.name.intern beta10 for logLogistic-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x

survival TRUE

discrete FALSE

link default log neglog

pdf loglogistic

```

## Example

In the following example we estimate the parameters in a simulated case

```

rloglogistic = function(n, lambda, alpha, variant=0)
{
  u = runif(n)
  if (variant == 0) {
    y = (lambda/(1.0/u - 1.0))^(1.0/alpha)
  } else if (variant == 1) {
    y = (1.0/(1.0/u - 1.0))^(1.0/alpha) / lambda
  } else {
    stop("ERROR")
  }
}

n = 1000
alpha = 2.1
x = c(scale(runif(n)))
eta = 1.1+2.2*x
lambda = exp(eta)

for(variant in 0:1) {

  print(paste("variant=", variant))
  y = rloglogistic(n, lambda = lambda,
                   alpha = alpha,
                   variant = variant)

```

```

formula = y ~ 1 + x
r=inla(formula,
      family ="loglogistic",
      data=data.frame(y, x),
      control.family = list(variant = variant))
print("REGRESSION")
print(summary(r))

event = rep(1,n)
formula=inla.surv(y,event) ~ 1 + x
r=inla(formula,
      family ="loglogisticsurv",
      data = list(y=y, event=event, x=x),
      control.family = list(variant = variant))
print("SURVIVAL")
print(summary(r))
}

```

## Notes

- Loglogisticsurv model can be used for right censored, left censored, interval censored data. If the observed times  $y$  are large/huge, then this can cause numerical overflow in the likelihood routine. If you encounter this problem, try to scale the observations, `time = time / max(time)` or similar.