# Gompertz

### Parametrisation

The Gompertz distribution has log survial function

$$\log S(y) = -\frac{\mu}{\alpha} \left( \exp(\alpha y) - 1 \right)$$

for response  $y \ge 0$ ,  $\mu > 0$  and  $\alpha > 0$ . The cumulative distribution function and the density then follows as

$$F(y) = 1 - \exp\left[-\frac{\mu}{\alpha}\left(\exp(\alpha y) - 1\right)\right]$$

and

$$f(y) = \mu \exp \left[ \alpha y - \frac{\mu}{\alpha} \left( \exp(\alpha y) - 1 \right) \right].$$

## **Link-function**

The parameter  $\mu$  is linked to the linear predictor  $\eta$  as:

$$\mu = \exp(\eta)$$

# Hyperparameters

The shape parameter  $\alpha$  is represented as

$$\alpha = \exp(S\theta)$$

and the prior is defined on  $\theta$ . The constant S currently set to 0.1 to avoid numerical instabilities in the optimization, since small changes of  $\alpha$  can make a huge difference.

### **Specification**

- family="gompertz" for regression models and family="gompertz.surv" for survival models.
- Required arguments: y (to be given in a format by using inla.surv() for survival models)

#### Hyperparameter spesification and default values

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hyper

theta

```
hyperid 105101
name shape
short.name alpha
output.name.intern alpha_intern for Gompertz
output.name alpha parameter for Gompertz
initial -1
fixed FALSE
prior normal
param 0 1
to.theta function(x, sc = 0.1) log(x) / sc
```

```
from.theta function(x, sc = 0.1) exp(sc * x)
survival FALSE
discrete FALSE
link default log neglog
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hyper
    theta1
        hyperid 106101
         name shape
         short.name alpha
         output.name.intern alpha_intern for Gompertz-surv
         output.name alpha parameter for Gompertz-surv
        initial -10
         fixed FALSE
         prior normal
         param 0 1
         to.theta function(x, sc = 0.1) log(x) / sc
         from.theta function(x, sc = 0.1) exp(sc * x)
    theta2
        hyperid 106102
         name beta1
         short.name beta1
         output.name beta1 for Gompertz-Cure
         output.name.intern beta1 for Gompertz-Cure
        initial -5
         fixed FALSE
         prior normal
         param -4 100
         to.theta function(x) x
         from.theta function(x) x
    theta3
         hyperid 106103
         name beta2
         short.name beta2
         output.name beta2 for Gompertz-Cure
         output.name.intern beta2 for Gompertz-Cure
        initial 0
         fixed FALSE
         prior normal
         param 0 100
```

```
to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 106104
    name beta3
    short.name beta3
    output.name beta3 for Gompertz-Cure
    output.name.intern beta3 for Gompertz-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 106105
    name beta4
    short.name beta4
    output.name beta4 for Gompertz-Cure
    output.name.intern beta4 for Gompertz-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 106106
    name beta5
    short.name beta5
    output.name beta5 for Gompertz-Cure
    output.name.intern beta5 for Gompertz-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 106107
    name beta6
    short.name beta6
    output.name beta6 for Gompertz-Cure
    output.name.intern beta6 for Gompertz-Cure
```

```
initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 106108
    name beta7
    short.name beta7
    output.name beta7 for Gompertz-Cure
    output.name.intern beta7 for Gompertz-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 106109
    name beta8
    short.name beta8
    output.name beta8 for Gompertz-Cure
    output.name.intern beta8 for Gompertz-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 106110
    name beta9
    short.name beta9
    output.name beta9 for Gompertz-Cure
    output.name.intern beta9 for Gompertz-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta11
    hyperid 106111
```

```
name beta10
         short.name beta10
         output.name beta10 for Gompertz-Cure
         output.name.intern beta10 for Gompertz-Cure
         initial 0
         fixed FALSE
         prior normal
         param 0 100
         to.theta function(x) x
         from.theta function(x) x
survival TRUE
discrete FALSE
link default log neglog
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Example
In the following example we estimate the parameters in a simulated case
library(flexsurv)
library(INLA)
n <- 1000
alpha <- 1.0
intercept <- 1.1</pre>
beta <- 1.2
x \leftarrow rnorm(n, sd = 0.2)
eta <- intercept + beta*x
mu <- exp(eta)</pre>
event <- rep(1,n)</pre>
y <- rgompertz(n, rate = mu, shape = alpha)
r \leftarrow inla(y ~1 + x,
           family ="gompertz", data=data.frame(y, x))
r.surv <- inla(inla.surv(y, event) ~ 1 + x,</pre>
                family ="gompertzsurv", data=data.frame(y, event, x))
## should be 'small'
print(r$mlik - r.surv$mlik)
```

Notes