Extented Generalised Pareto Distribution (EGP)

Parametrisation

The EGP distribution as cumulative distribution function

$$F(y) = \left(1 - \left(1 + \xi \frac{y}{\sigma}\right)^{-1/\xi}\right)^{\kappa}$$

- ξ is the tail-parameter
- $\sigma > 0$ is the scale-parameter
- $\kappa > 0$ is the shape-parameter

Link-function

The linear predictor η controls the α -quantile of EGP

$$\operatorname{Prob}(y \leq q_{\alpha}) = \alpha.$$

 σ can be expressed in terms of the quantile

$$\sigma = \frac{\xi q_{\alpha}}{(1 - \alpha^{1/\kappa})^{-\xi} - 1}$$

and the quantile is linked to the linear predictor as

$$q_{\alpha} = \exp(\eta).$$

Hyperparameters

There are two hyperparameters, the tail and shape parameter, represented as (θ_1, θ_2) . The tail-parameter is in a predefined range (ξ_L, ξ_H)

$$\xi = \xi_L + (\xi_H - \xi_L) \frac{1}{1 + \exp(-\theta_1)}$$

and the prior is defined on θ_1 . The shape-parameter is

$$\kappa = \exp(\theta_2)$$

and the prior is defined on θ_2 .

Specification

- family="egp"
- \bullet Required arguments: the observations y
- The quantile level α

Hyperparameter spesification and default values

```
doc Exteneded Generalized Pareto likelihood
hyper
     theta1
         hyperid 101211
         name tail
         short.name xi
         output.name Tail parameter for egp observations
         output.name.intern Intern tail parameter for egp observations
         initial 0
         fixed FALSE
         prior pc.egptail
         param 5 -0.5 0.5
         to.theta function(x, interval = c(REPLACE.ME.low, REPLACE.ME.high)) log(-(interval[1] - x) /
         from.theta function(x, interval = c(REPLACE.ME.low, REPLACE.ME.high)) interval[1] + (interval
     theta2
         hyperid 101212
         name shape
         short.name kappa
         output.name Shape parameter for the egp observations
         output.name.intern Intern shape parameter for the egp observations
         initial 0
         fixed FALSE
         prior loggamma
         param 100 100
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
status experimental
survival FALSE
discrete FALSE
link default quantile
pdf egp
```

Example

```
n <- 3000
for (xi in c(-0.3, 0.3)) {
    xi.intern <- inla.models()$likelihood$egp$hyper$theta1$to.theta(xi, interval=c(-0.5, 0.5))</pre>
    alpha <- 0.9
    kappa <- 1.1
    kappa.intern <- inla.models()$likelihood$egp$hyper$theta2$to.theta(kappa)</pre>
    a <- ((1-alpha^(1/kappa))^(-xi) -1)
    x \leftarrow rnorm(n, sd = 0.3)
    eta <- 0.9 + 1.1 * x
    q <- exp(eta)
    sigma \leftarrow xi * q / a
    y <- - (sigma / xi) * (1- (1-runif(n)^(1/kappa))^(-xi))</pre>
    r <- inla(y ~ 1 + x,
              family = "egp",
              control.family = list(
                   control.link = list(quantile = alpha),
                   hyper = list(tail = list(
                                    ##initial = xi.intern,
                                     fixed = !TRUE,
                                    prior = "pc.egptail",
                                    param = c(5, -0.5, 0.5)),
                                 shape = list(
                                    ##initial = kappa.intern,
                                    fixed = !TRUE,
                                    prior = "loggamma",
                                    param = c(100, 100))),
              data = data.frame(y, x),
              verbose = !TRUE)
    print(summary(r))
```