Bym2 model for spatial effects

Parametrization

This model is a reparameterisation of the BYM-model, which is a union of the besag model u^* and a iid model v^* , so that

$$x = \begin{pmatrix} v^* + u^* \\ u^* \end{pmatrix}$$

where both u^* and v^* has a precision (hyper-)parameter. The length of x is 2n if the length of u^* (and v^*) is n. The BYM2 model uses a different parameterisation of the hyperparameters where

$$x = \begin{pmatrix} \frac{1}{\sqrt{\tau}} \left(\sqrt{1 - \phi} \ v + \sqrt{\phi} \ u \right) \\ u \end{pmatrix}$$

where both u and v are standardised to have (generalised) variance equal to one. The marginal precision is then τ and the proportion of the marginal variance explained by the spatial effect (u) is ϕ .

Hyperparameters

The hyperparameters are the margainal precision τ and the mixing parameter ϕ . The marginal precision τ is represented as

$$\theta_1 = \log(\tau)$$

and the mixing parameter as

$$\theta_2 = \log\left(\frac{\phi}{1 - \phi}\right)$$

and the prior is defined on $\theta = (\theta_1, \theta_2)$.

Specification

The bym2 model is specified inside the f() function as

```
f(<whatever>, model="bym2", graph=<graph>,
    hyper=<hyper>, adjust.for.con.comp = TRUE)
```

The neighbourhood structure of \mathbf{x} is passed to the program through the graph argument.

The option adjust.for.com.comp adjust the model if the graph has more than one connected component, and this adjustment can be disabled setting this option to FALSE. This means that constr=TRUE is interpreted as a sum-to-zero constraint on *each* connected component and the rankdef parameter is set accordingly.

Hyperparameter spesification and default values

 $\operatorname{\mathbf{doc}}$ The BYM-model with the PC priors

hyper

theta1

hyperid 11001 name log precision short.name prec prior pc.prec

```
param 1 0.01
         initial 4
         fixed FALSE
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
     theta2
         hyperid 11002
         name logit phi
         short.name phi
         prior pc
         param 0.5 0.5
         initial -3
         fixed FALSE
         to.theta function(x) log(x / (1 - x))
         from.theta function(x) exp(x) / (1 + exp(x))
constr TRUE
nrow.ncol FALSE
augmented TRUE
aug.factor 2
aug.constr 2
n.div.by
n.required TRUE
set.default.values TRUE
pdf bym2
```

Example

Details on the implementation

This gives some details of the implementation, which depends on the following variables

nc1 Number of connected components in the graph with size 1. These nodes, *singletons*, have no neighbours.

nc2 Number of connected components in the graph with size ≥ 2 .

scale.model The value of the logical flag, if the model should be scaled or not. (Default FALSE)

adjust.for.con.comp The value of the logical flag if the constr=TRUE option should be reinterpreted.

```
The case (scale.model==FALSE && adjust.for.con.comp == FALSE)
```

The option constr=TRUE is interpreted as a sum-to-zero constraint over the whole graph. Singletons are given a uniform distribution on $(-\infty, \infty)$ before the constraint.

The case (scale.model==TRUE && adjust.for.con.comp == FALSE)

The option constr=TRUE is interpreted as a sum-to-zero constraint over the whole graph. Let $Q = \tau R$ be the standard precision matrix from the besag-model with precision parameter τ . Then R, except the singletons, are scaled so that the geometric mean of the marginal variances is 1, and R is modified so that singletons have a standard Normal distribution.

The case (scale.model==FALSE && adjust.for.con.comp == TRUE)

The option constr=TRUE is interpreted as one sum-to-zero constraint over each of the nc2 connected components of size ≥ 2 . Singletons are given a uniform distribution on $(-\infty, \infty)$.

The case (scale.model==TRUE && adjust.for.con.comp == TRUE)

The option constr=TRUE is interpreted as nc2 sum-to-zero constraints for each of the connected components of size ≥ 2 . Let $Q = \tau R$ be the standard precision matrix from the besag-model with precision parameter τ . Then R, are scaled so that the geometric mean of the marginal variances in each connected component of size ≥ 2 is 1, and modified so that singletons have a standard Normal distribution.

Notes

The term $\frac{1}{2}\log(|R|^*)$ of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here R is the precision matrix for the standardised Besag part of the model.

The generic PC-prior for ϕ is available as prior="pc" and parameters param="c(u, alpha)", where $\operatorname{Prob}(\phi \leq u) = \alpha$. If $\alpha < 0$ or $\alpha > 1$, then it is set to a value close to the minimum value of α allowed. This prior depends on the graph and its computational cost is $\mathcal{O}(n^3)$.