Stochastic volatility models

The data consist in 945 observed logarithms of the daily difference of the dollar-pound exchange rate from October 1st, to June 28th, 1985. We analyse this data set using a univariate stochastic volatility model ([Taylor, 1986]). The likelihood of the data, conditional on the latent variables is:

$$y_t|\eta_t \sim \mathcal{N}(0, \exp(\eta_t)), \quad t = 0, \ldots, n_d - 1 \tag{1}$$

and the model for the latent variables:

$$\eta_t = \mu + f_t \quad t = 0, \quad , n_\eta - 1 \tag{2}$$

where μ is an unknown common mean with vague Gaussian prior and $\boldsymbol{f} = (f_0, \ldots, f_{n_\eta - 1})$ is modelled as an auto regressive process of order 1 (AR1) with persistence parameter $\phi \in (-1, 1)$ to ensure stationarity, and precision parameter λ_f. The model has two hyperparameters, $(\log \lambda_f, \phi)$. We re-parametrise the persistence parameter ϕ as

$$\kappa = \text{logit}\left(\frac{\phi + 1}{2}\right)$$

and assign the following prior distributions

$$\begin{aligned} &\log \lambda_f \sim \text{LogGamma}(1, 0.0005) \\ &\kappa \sim \mathcal{N}(0, 1/0.0001) \end{aligned}$$

Student-t distribution

An alternative model for the response variable y_t is a Student-t. This allows heavier tail, a feature which is often observed in financial time series. The observation model in equation (1) then becomes

$$y_t = \exp(\eta_t/2)\ \mathcal{T}_t(\nu) \quad t = 1, \ldots, T \tag{3}$$

where $\mathcal{T}_t(\nu)$ is a random variable having a Student-t distribution having ν degree of freedom and standardised so that its variance is 1 for any value of $\nu > 2$.

Student-t NIG distribution

Yet another model is the *normal inverse Gaussian* (NIG) distribution, for which

$$y_t = \exp(\eta_t/2)\ NIG, \quad t = 1, \ldots, T \tag{4}$$

where NIG is a standardised NIG distribution with two parameters, which (essentially) are skewness and shape-parameters.

References

[Taylor, 1986] Taylor, S. J. (1986). *Modelling Stochastic volatility.* John Wiley.