Skew-Normal likelihood

Parametrisation

The standardised Skew-Normal distribution is

$$f(z) = \frac{2}{\omega_{\alpha}} \phi \left(\frac{z - \xi_{\alpha}}{\omega_{\alpha}} \right) \Phi \left(\alpha \frac{z - \xi_{\alpha}}{\omega_{\alpha}} \right)$$

where ω_{α} and ξ_{α} are so that the mean is zero and variance is one, and they depends both on the skewness parameter α .

The skew-normal likelihood is defined as the density wrt y, where

$$z = (y - \eta)\sqrt{w\tau} \sim f(z)$$

and

 η : is the the linear predictor

 τ : is the precision

w: is a fixed scale or weight, w > 0,

Link-function

The mean equals the linear predictor

$$\mu = \eta$$

Hyperparameters

The precision is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on θ_1 .

The (standardised) skewness γ , is represented as

$$\gamma = 0.988(2\frac{\exp(\theta_2)}{1 + \exp(\theta_2)} - 1)$$

and the prior is defined on θ_2 . The standardised skewness depends on α as

$$\gamma = \frac{4 - \pi}{2} \frac{\left(\delta\sqrt{2/\pi}\right)^3}{\left(1 - 2\delta^2/\pi\right)^{3/2}}, \qquad \delta = \frac{\alpha}{\sqrt{1 + \alpha^2}}$$

Specification

- family="sn"
- Required arguments: y and w (keyword scale, and w = 1 by default).

```
Hyperparameter specification and default values
doc The Skew-Normal likelihoood
hyper
     theta1
         hyperid 74001
         name log precision
         short.name prec
         output.name precision for skew-normal observations
         {\bf output.name.intern}\ {\tt log}\ {\tt precision}\ {\tt for}\ {\tt skew-normal}\ {\tt observations}
         initial 4
         fixed FALSE
         prior loggamma
         param 1 5e-05
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
     theta2
         hyperid 74002
         name logit skew
         short.name skew
         output.name Skewness for skew-normal observations
         output.name.intern Intern skewness for skew-normal observations
         initial 0.00123456789
         fixed FALSE
         prior pc.sn
         param 10
         to.theta function(x, skew.max = 0.988) log((1 + x / skew.max) / (1 - x / skew.max))
         from.theta function(x, skew.max = 0.988) skew.max * (2 * exp(x) / (1 + exp(x)) - 1)
survival FALSE
discrete FALSE
link default identity
pdf sn
Example
library(sn)
set.seed(246)
n = 300
x = rnorm(n, sd = 1)
eta = 1+x
skewness = 0.25
y = numeric(n)
prec <- 100
```

for(i in 1:n) {

Notes

- \bullet This implementation replaces older ones ("sn" and "sn2") from 16^{th} September 2020.
- A N(a,0) prior is interpreted as a constant prior with density equal to one.