

## Gaussian

### Parametrisation

The Gaussian distribution is

$$f(y) = \frac{\sqrt{s\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}s\tau(y-\mu)^2\right)$$

for continuously responses  $y$  where

$\mu$ : is the the mean

$\tau$ : is the precision

$s$ : is a fixed scaling,  $s > 0$ .

### Link-function

The mean and variance of  $y$  are given as

$$\mu \quad \text{and} \quad \sigma^2 = \frac{1}{s\tau}$$

and the mean is linked to the linear predictor by

$$\mu = \eta$$

### Hyperparameters

The default behaviour is to represent the precision  $\tau = \kappa_1$  where

$$\theta_1 = \log \kappa_1$$

and the prior is defined on  $\theta_1$ .

The more general formulation have a second (fixed) hyperparameter  $\theta_2$  which determines a fixed offset  $1/\kappa_2$ ,  $\theta_2 = \log \kappa_2$ , for the variance (scaling not included) of the response. In this case,

$$1/\tau = 1/\kappa_1 + 1/\kappa_2$$

or

$$\tau = \frac{1}{1/\kappa_1 + 1/\kappa_2}$$

In the case where  $1/\kappa_2$  is zero, then  $\tau = \kappa_1$  and we are back to the default behaviour. We use the convension that  $1/\kappa_2$  is zero if  $1/\kappa_2 < \text{Machine}\$double.eps$ , which is  $\theta_2 \geq 36.05$  for common machines.

### Specification

- `family="gaussian"`
- Required arguments:  $y$  and  $s$  (argument `scale`)

The scalings have default value 1.

There is also `family="stdgaussian"` for the case where the precision  $\tau$  is fixed to 1 with no offset (but `scale` is still in place).

## Hyperparameter specification and default values

family="gaussian"

doc The Gaussian likelihood

hyper

theta1

hyperid 65001

name log precision

short.name prec

output.name Precision for the Gaussian observations

output.name.intern Log precision for the Gaussian observations

initial 4

fixed FALSE

prior loggamma

param 1 5e-05

to.theta function(x) log(x)

from.theta function(x) exp(x)

theta2

hyperid 65002

name log precision offset

short.name preoffset

output.name NOT IN USE

output.name.intern NOT IN USE

initial 72.0873067782343

fixed TRUE

prior none

param

to.theta function(x) log(x)

from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default identity logit loga cauchit log logoffset

pdf gaussian

family="stdgaussian"

doc The stdGaussian likelihood

hyper

survival FALSE

discrete FALSE

link default identity logit loga cauchit log logoffset

pdf gaussian

## Example

The first example estimate the parameters in a simulated example with Gaussian responses, giving  $\tau$  a Gamma-prior with parameters (1,0.01) and initial value (for the optimisations) of exp(2.0). The second example shows the use of an fixed offset in the variance.

```
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
tau = 100
scale = exp(rnorm(n))
prec = scale*tau
y = rnorm(n, mean = eta, sd = 1/sqrt(prec))

data = list(y=y, z=z)
formula = y ~ 1+z
result = inla(formula, family = "gaussian", data = data,
              control.family = list(hyper = list(
                                prec = list(
                                  prior = "loggamma",
                                  param = c(1.0,0.01),
                                  initial = 2))),
                                scale=scale, keep=TRUE)
summary(result)

## with an offset in the variance
var0 = 1.0 ## fixed offset
var1 = 2.0
v = var0 + var1
s = sqrt(v)
x = rnorm(n)
y = 1 + x + rnorm(n, sd = s)
rr = inla(y ~ x,
          data = data.frame(y, x),
          control.family = list(
            hyper = list(precoffset = list(initial = log(1/var0)))),
          verbose = TRUE)
summary(rr)
plot(rr$internal.marginals.hyperpar[[1]], type = "l", lwd=3)
abline(v = log(1.0/var1), lwd=3, col = "blue")
```

## Notes

An error is given if  $\theta_2$  is not fixed.