Gaussian

Parametrisation

The Gaussian distribution is

$$f(y) = \frac{\sqrt{s\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}s\tau (y - \mu)^2\right)$$

for continuously responses y where

 μ : is the mean

 τ : is the precision

s: is a fixed scaling, s > 0.

Link-function

The mean and variance of y are given as

$$\mu$$
 and $\sigma^2 = \frac{1}{s\tau}$

and the mean is linked to the linear predictor by

$$\mu = \eta$$

Hyperparameters

The default behaviour is to represent the precision $\tau = \kappa_1$ where

$$\theta_1 = \log \kappa_1$$

and the prior is defined on θ_1 .

The more general formulation have a second (fixed) hyperparameter θ_2 which determines a fixed offset $1/\kappa_2$, $\theta_2 = \log \kappa_2$, for the variance (scaling not included) of the response. In this case,

$$1/\tau = 1/\kappa_1 + 1/\kappa_2$$

or

$$\tau = \frac{1}{1/\kappa_1 + 1/\kappa_2}$$

In the case where $1/\kappa_2$ is zero, then $\tau = \kappa_1$ and we are back to the default behaviour. We use the convension that $1/\kappa_2$ is zero if $1/\kappa_2 <$.Machine\$double.eps,which is $\theta_2 \ge 36.05$ for common machines.

Specification

- family="gaussian"
- \bullet Required arguments: y and s (argument scale)

The scalings have default value 1.

There is also family="stdgaussian" for the case where the precision τ is fixed to 1 with no offset (but scale is still in place).

```
Hyperparameter spesification and default values
family="gaussian"
doc The Gaussian likelihoood
hyper
    theta1
         hyperid 65001
         name log precision
         short.name prec
         output.name Precision for the Gaussian observations
         output.name.intern Log precision for the Gaussian observations
         initial 4
         fixed FALSE
         prior loggamma
         param 1 5e-05
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
    theta2
         hyperid 65002
         name log precision offset
         short.name precoffset
         output.name NOT IN USE
         output.name.intern NOT IN USE
         initial 72.0873067782343
         fixed TRUE
         prior none
         param
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
survival FALSE
discrete FALSE
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\mathbf{pdf} gaussian
   family="stdgaussian"
\operatorname{doc} The stdGaussian likelihoood
hyper
survival FALSE
discrete FALSE
```

link default identity logit loga cauchit log logoffset

pdf gaussian

Example

The first example estimate the parameters in a simulated example with Gaussian responses, giving τ a Gamma-prior with parameters (1,0.01) and initial value (for the optimisations) of $\exp(2.0)$. The second example shows the use of an fixed offset in the variance.

```
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
tau = 100
scale = exp(rnorm(n))
prec = scale*tau
y = rnorm(n, mean = eta, sd = 1/sqrt(prec))
data = list(y=y, z=z)
formula = y ~ 1+z
result = inla(formula, family = "gaussian", data = data,
        control.family = list(hyper = list(
                                     prec = list(
                                             prior = "loggamma",
                                             param = c(1.0, 0.01),
                                             initial = 2))),
              scale=scale, keep=TRUE)
summary(result)
## with an offset in the variance
var0 = 1.0 ## fixed offset
var1 = 2.0
v = var0 + var1
s = sqrt(v)
x = rnorm(n)
y = 1 + x + rnorm(n, sd = s)
rr = inla(y ~ x,
         data = data.frame(y, x),
         control.family = list(
             hyper = list(precoffset = list(initial = log(1/var0)))),
         verbose = TRUE)
summary(rr)
plot(rr$internal.marginals.hyperpar[[1]], type = "1", lwd=3)
abline(v = log(1.0/var1), lwd=3, col = "blue")
```

Notes

An error is given if θ_2 is not fixed.