More general Gaussian likelihoods: GGaussian and GGaussianS

Parameterisation

The Gaussian distribution is

$$f(y) = \frac{\sqrt{\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\tau\,(y-\mu)^2\right)$$

for continuously responses y where

μ**:** is the the mean

τ**:** is the precision

These likelihood families generalise the normal procedure. We allow for two linear predictors in the model. One needs to be "simple", only consists of fixed effects, while the other one is general and defined via the formula.

For family="ggaussian", then

$$\text{link}(\mu) = \text{formula} \qquad \text{link.simple}(\tau) = \text{simple}$$

while for family="ggaussianS", then

$$\text{link}(\tau) = \text{formula} \qquad \text{link.simple}(\mu) = \text{simple}$$

The default link is the *identity* for the mean and the *log* for the precision. We will describe each model separately, as the specifications are a little different.

Family *ggaussian*

Link-function

The mean is given by the linear predictor η from the formula

$$\text{link}(\mu) = \eta$$

where the default is the *identity*-link. The precision is given as

$$\text{link.simple}(\frac{1}{s}\tau) = \beta_1 z_1 + \beta_2 z_2 + \ldots + \beta_m z_m, \qquad m \leq 10$$

where s is a **fixed** scaling (or often log-offset) and similar to the argument *scale* for family="gaussian". For the log-link, the precision is expressed as

$$\tau = s \times \exp\left(\beta_1 z_1 + \beta_2 z_2 + \ldots + \beta_m z_m\right)$$

Note: there is no default intercept in the simple model, so it is common to set $z_1 = 1$.

Hyperparameters

The hyperparameters in the model are $\theta_1 = \beta_1, \ldots, \theta_m = \beta_m$, and the priors are defined on $\theta_1, \ldots, \theta_m$.

Note: The default prior for θ_1 is similar to the default prior for the precision for the Gaussian family, please change if you do not want this feature.

Specification

- `family="ggaussian"`
- Required arguments: **inla.mdata**-object that defines the response, s and covariates. The **inla.mdata** object is defined as

$$\text{inla.mdata}(y, s, z_1, \ldots, z_m)$$

  where each argument are vectors of the same length.
- **Note:** The scaling argument s **MUST** be given as it has no default value. Often we can just use $s = 1$ as it will auto-expand.
- **Note:** $m = 0$ is allowed, which means that $\tau = s$ with with log link.simple.

Hyperparameter spesification and default values

`family="ggaussian"`

doc `Generalized Gaussian`

hyper

theta1

hyperid `66501`
name `beta1`
short.name `beta1`
output.name `beta1 for ggaussian observations`
output.name.intern `beta1 for ggaussian observations`
initial `4`
fixed `FALSE`
prior `normal`
param `0 10`
to.theta `function(x) x`
from.theta `function(x) x`

theta2

hyperid `66502`
name `beta2`
short.name `beta2`
output.name `beta2 for ggaussian observations`
output.name.intern `beta2 for ggaussian observations`
initial `0`
fixed `FALSE`
prior `normal`
param `0 10`
to.theta `function(x) x`
from.theta `function(x) x`

theta3

hyperid `66503`

name beta3
short.name beta3
output.name beta3 for ggaussian observations
output.name.intern beta3 for ggaussian observations
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x

theta4

hyperid 66504
name beta4
short.name beta4
output.name beta4 for ggaussian observations
output.name.intern beta4 for ggaussian observations
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x

theta5

hyperid 66505
name beta5
short.name beta5
output.name beta5 for ggaussian observations
output.name.intern beta5 for ggaussian observations
initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x

theta6

hyperid 66506
name beta6
short.name beta6
output.name beta6 for ggaussian observations
output.name.intern beta6 for ggaussian observations
initial 0
fixed FALSE
prior normal
param 0 10

to.theta `function(x) x`

from.theta `function(x) x`

theta7

hyperid `66507`

name `beta7`

short.name `beta7`

output.name `beta7 for ggaussian observations`

output.name.intern `beta7 for ggaussian observations`

initial `0`

fixed `FALSE`

prior `normal`

param `0 10`

to.theta `function(x) x`

from.theta `function(x) x`

theta8

hyperid `66508`

name `beta8`

short.name `beta8`

output.name `beta8 for ggaussian observations`

output.name.intern `beta8 for ggaussian observations`

initial `0`

fixed `FALSE`

prior `normal`

param `0 10`

to.theta `function(x) x`

from.theta `function(x) x`

theta9

hyperid `66509`

name `beta9`

short.name `beta9`

output.name `beta9 for ggaussian observations`

output.name.intern `beta9 for ggaussian observations`

initial `0`

fixed `FALSE`

prior `normal`

param `0 10`

to.theta `function(x) x`

from.theta `function(x) x`

theta10

hyperid `66510`

name `beta10`

short.name `beta10`

output.name `beta10 for ggaussian observations`

output.name.intern `beta10 for ggaussian observations`

initial 0
fixed FALSE
prior normal
param 0 10
to.theta function(x) x
from.theta function(x) x

status experimental

survival FALSE

discrete TRUE

link default identity

link.simple default log

pdf ggaussian

Example

```
n <- 1000
x <- rnorm(n)
xx <- rnorm(n)
eta <-  0.1 + 1.1 * x + 2.2 * xx

s <- runif(n)
z <- rnorm(n)
zz <- rnorm(n)
eta.prec <- 1 + 0.55 * z + 1.1 * zz

y <- eta + 1/sqrt(s * exp(eta.prec)) * rnorm(n)
Y <- inla.mdata(y, s, 1, z, zz)
r <- inla(Y ~ 1 + x + xx,
          data = list(Y = Y, x = x, xx = xx, z = z, zz = zz, s = s),
          family = "ggaussian")
summary(r)
```

Family *ggaussianS*

Link-function

This is the swapped version. The mean is given by a simple linear predictor

$$\text{link.simple}(\mu) = \text{off} + \beta_1 z_1 + \beta_2 z_2 + \ldots + \beta_m z_m, \qquad m \leq 10$$

where "off" is a fixed offset and where link.simple is the *identity*-link by default. **Note:** there is no default intercept in the simple model, so it is common to set $z_1 = 1$. The precision is specified in the formula

$$\text{link}(\tau) = \text{formula}$$

using the log-link as default.

Hyperparameters

The hyperparameters in the model are $\theta_1 = \beta_1, \ldots, \theta_m = \beta_m$, and the priors are defined on $\theta_1, \ldots, \theta_m$.

Specification

- `family="ggaussianS"`
- Required arguments: **inla.mdata**-object that defines the response, the offset "off" and covariates. The **inla.mdata** object is defined as

  $$\text{inla.mdata}(y, \text{off}, z_1, \ldots, z_m)$$

  where each argument are vectors of the same length.
- **Note:** The offset argument **MUST** be given as it has no default value. Often we can just use off= 0 as it will auto-expand.
- **Note:** $m = 0$ is allowed, which means that $\mu = \text{off}$ with with identity link.simple.

Hyperparameter spesification and default values

`family="ggaussianS"`

doc `Generalized GaussianS`

hyper

theta1

hyperid `66601`
name `beta1`
short.name `beta1`
output.name `beta1 for ggaussianS observations`
output.name.intern `beta1 for ggaussianS observations`
initial `0`
fixed `FALSE`
prior `normal`
param `0 0.001`
to.theta `function(x) x`
from.theta `function(x) x`

theta2

hyperid 66602
name beta2
short.name beta2
output.name beta2 for ggaussianS observations
output.name.intern beta2 for ggaussianS observations
initial 0
fixed FALSE
prior normal
param 0 0.001
to.theta function(x) x
from.theta function(x) x

theta3

hyperid 66603
name beta3
short.name beta3
output.name beta3 for ggaussianS observations
output.name.intern beta3 for ggaussianS observations
initial 0
fixed FALSE
prior normal
param 0 0.001
to.theta function(x) x
from.theta function(x) x

theta4

hyperid 66604
name beta4
short.name beta4
output.name beta4 for ggaussianS observations
output.name.intern beta4 for ggaussianS observations
initial 0
fixed FALSE
prior normal
param 0 0.001
to.theta function(x) x
from.theta function(x) x

theta5

hyperid 66605
name beta5
short.name beta5
output.name beta5 for ggaussianS observations
output.name.intern beta5 for ggaussianS observations
initial 0
fixed FALSE

prior normal
param 0 0.001
to.theta function(x) x
from.theta function(x) x

theta6
hyperid 66606
name beta6
short.name beta6
output.name beta6 for ggaussianS observations
output.name.intern beta6 for ggaussianS observations
initial 0
fixed FALSE
prior normal
param 0 0.001
to.theta function(x) x
from.theta function(x) x

theta7
hyperid 66607
name beta7
short.name beta7
output.name beta7 for ggaussianS observations
output.name.intern beta7 for ggaussianS observations
initial 0
fixed FALSE
prior normal
param 0 0.001
to.theta function(x) x
from.theta function(x) x

theta8
hyperid 66608
name beta8
short.name beta8
output.name beta8 for ggaussianS observations
output.name.intern beta8 for ggaussianS observations
initial 0
fixed FALSE
prior normal
param 0 0.001
to.theta function(x) x
from.theta function(x) x

theta9
hyperid 66609
name beta9
short.name beta9

output.name beta9 for ggaussianS observations
output.name.intern beta9 for ggaussianS observations
initial 0
fixed FALSE
prior normal
param 0 0.001
to.theta function(x) x
from.theta function(x) x

theta10

hyperid 66610
name beta10
short.name beta10
output.name beta10 for ggaussianS observations
output.name.intern beta10 for ggaussianS observations
initial 0
fixed FALSE
prior normal
param 0 0.001
to.theta function(x) x
from.theta function(x) x

status experimental

survival FALSE

discrete TRUE

link default log

link.simple default identity

pdf ggaussian

Example

```
n <- 1000
x <- rnorm(n)
xx <- rnorm(n)
off <- runif(n)
z <- rnorm(n)
zz <- rnorm(n)

mean <- off + 0.1 + 1.1 * z + 2.2 * zz
prec <- exp(1 + 0.55 * x + 1.1 * xx)

y <- mean + (1/sqrt(prec)) * rnorm(n)
Y <- inla.mdata(y, off, 1, z, zz)
r <- inla(Y ~ 1 + x + xx,
          data = list(Y = Y, off = off, x = x, xx = xx, z = z, zz = zz),
          family = "ggaussianS")
summary(r)
```