# Besag model for spatial effects

#### **Parametrization**

The besag model for random vector  $\mathbf{x} = (x_1, \dots, x_n)$  is defined as

$$x_i|x_j, i \neq j, \tau \sim \mathcal{N}(\frac{1}{n_i} \sum_{i \sim j} x_j, \frac{1}{n_i \tau})$$
 (1)

where  $n_i$  is the number of neighbours of node  $i, i \sim j$  indicates that the two nodes i and j are neighbours.

## Hyperparameters

The precision parameter  $\tau$  is represented as

$$\theta_1 = \log \tau$$

and the prior is defined on  $\theta_1$ .

## Specification

The besag model is specified inside the f() function as

```
f(<whatever>,model="besag",graph=<graph>,
  hyper=<hyper>, adjust.for.con.comp = TRUE,
  scale.model = FALSE)
```

The neighbourhood structure of x is passed to the program through the graph argument.

If the option adjust.for.con.comp=TRUE then the model is adjusted if the graph has more than one connected component. This adjustment can be disabled setting this option to FALSE. If adjust.for.con.comp=TRUE then constr=TRUE is interpreted as a sum-to-zero constraint on each connected component in the graph and the rankdef parameter is set to the number of connected components.

The logical option scale.model determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is FALSE so that the model is not scaled.

#### Hyperparameter spesification and default values

```
\operatorname{doc} The Besag area model (CAR-model)
```

# hyper

#### theta

```
hyperid 8001
name log precision
short.name prec
prior loggamma
param 1 5e-05
initial 4
fixed FALSE
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

```
constr TRUE
```

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required TRUE

set.default.values TRUE

pdf besag

### Example

For examples of application of this model see the Bym, Munich, Zambia or Scotland examples in Volume I.

# Details on the implementation

This gives some details of the implementation, which depends on the following variables

**nc1** Number of connected components in the graph with size 1. These nodes, *singletons*, have no neighbours.

**nc2** Number of connected components in the graph with size  $\geq 2$ .

scale.model The value of the logical flag, if the model should be scaled or not. (Default FALSE)

adjust.for.com.comp The value of the logical flag if the constr=TRUE option should be reinterpreted.

```
The case (scale.model==FALSE && adjust.for.con.comp == FALSE)
```

The option constr=TRUE is interpreted as a sum-to-zero constraint over the whole graph. Singletons are given a uniform distribution on  $(-\infty, \infty)$  before the constraint.

```
The case (scale.model==TRUE && adjust.for.con.comp == FALSE)
```

The option constr=TRUE is interpreted as a sum-to-zero constraint over the whole graph. Let  $Q = \tau R$  be the standard precision matrix from the besag-model with precision parameter  $\tau$ . Then R, except the singletons, are scaled so that the geometric mean of the marginal variances is 1, and R is modified so that singletons have a standard Normal distribution.

```
The case (scale.model==FALSE && adjust.for.con.comp == TRUE)
```

The option constr=TRUE is interpreted as one sum-to-zero constraint over each of the nc2 connected components of size  $\geq 2$ . Singletons are given a uniform distribution on  $(-\infty, \infty)$ .

The case (scale.model==TRUE && adjust.for.con.comp == TRUE)

The option constr=TRUE is interpreted as nc2 sum-to-zero constraints for each of the connected components of size  $\geq 2$ . Let  $Q = \tau R$  be the standard precision matrix from the besag-model with precision parameter  $\tau$ . Then R, are scaled so that the geometric mean of the marginal variances in each connected component of size  $\geq 2$  is 1, and modified so that singletons have a standard Normal distribution.

## Notes

The term  $\frac{1}{2}\log(|R|^*)$  of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here R is the precision matrix with a unit precision parameter.