

# Skew-Normal stochastic volatility likelihood

## Parametrisation

The standardised Skew-Normal distribution is

$$f(z) = \frac{2}{\omega_\alpha} \phi\left(\frac{z - \xi_\alpha}{\omega_\alpha}\right) \Phi\left(\alpha \frac{z - \xi_\alpha}{\omega_\alpha}\right)$$

where  $\omega_\alpha$  and  $\xi_\alpha$  are so that the mean is zero and variance is one, and they are functions of the skewness parameter  $\alpha$ . The skewness  $s$  is the standardised skewness (standardized third central moment), which is a function of  $\alpha$

The skew-normal stochastic likelihood is defined as the density wrt  $y$ , where  $y$  is Skew-Normal distributed with zero mean, skewness  $s$  and variance

$$\text{variance} = \exp(\eta) + 1/\tau$$

and

$\eta$ : is the linear predictor

$\tau$ : is an offset in the variance

## Link-function

The variance depends on the linear predictor

$$\mu = \exp(\eta) + 1/\tau$$

## Hyperparameters

The (standardised) skewness  $s$ , is represented as

$$\gamma = 0.988(2 \frac{\exp(\theta_1)}{1 + \exp(\theta_1)} - 1)$$

and the prior is defined on  $\theta_1$ .

The offset in the variance,  $1/\tau$  is represented as

$$\tau = \exp(\theta_2)$$

and the prior is defined on  $\theta_2$ . (By default  $\theta_2$  is fixed to a high value which makes  $1/\tau \approx 0$ .)

## Specification

- `family="stochvolnsn"`
- Required arguments:  $y$ .

## Hyperparameter specification and default values

`doc` The SkewNormal stochvol likelihood

`hyper`

`theta1`

`hyperid 82101`

```

name logit skew
short.name skew
output.name Skewness for stochvol_sn observations
output.name.intern Intern skewness for stochvol_sn observations
initial 0.00123456789
fixed FALSE
prior pc.sn
param 10
to.theta function(x, skew.max = 0.988) log((1 + x / skew.max) / (1 - x / skew.max))
from.theta function(x, skew.max = 0.988) skew.max * (2 * exp(x) / (1 + exp(x)) - 1)
theta2
  hyperid 82102
  name log precision
  short.name prec
  output.name Offset precision for stochvol_sn
  output.name.intern Log offset precision for stochvol_sn
  initial 500
  fixed TRUE
  prior loggamma
  param 1 0.005
  to.theta function(x) log(x)
  from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default log

pdf stochvol_sn

```

## Example

```

library(sn)
n <- 1000
x <- scale(arima.sim(n, model= list(ar = 0.95)))
skew <- 0.2
y <- numeric(n)
for(i in 1:n) {
  variance <- exp(x[i])
  par <- unlist(INLA::inla.sn.reparam(moments = c(0, variance, skew)))
  y[i] <- rsn(1, dp = par)
}

r = inla(y ~ 1 + f(idx, model="ar1",
  hyper = list(
    prec = list(prior = "pc.prec",
      param = c(0.5, 0.01)),
    rho = list(prior = "pc.cor1",

```

```
                                param = c(0.8, 0.5))))),  
control.fixed = list(prec.intercept = 1),  
data = data.frame(y, idx=1:n),  
family = "stochvolnsn",  
control.inla = list(cmin = 0, b.strategy="skip"),  
num.threads = "3:1",  
verbose = TRUE)  
summary(r)
```

## Notes

This implementation is similar to `family="sn"`, see also that documentation.