# The Beta-distribution

## Parametrisation

The Beta-distribution has the following density

$$\pi(y) = \frac{1}{B(a,b)} y^{a-1} (1-y)^{b-1}, \qquad 0 < y < 1, \quad a > 0, \quad b > 0$$

where B(a, b) is the Beta-function

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

and  $\Gamma(x)$  is the Gamma-function. The (re-)parameterisation used is

$$\mu = \frac{a}{a+b}, \qquad 0 < \mu < 1$$

and

$$\phi = a + b, \qquad \phi > 0,$$

as it makes

$$E(y) = \mu$$
 and  $Var(y) = \frac{\mu(1-\mu)}{1+\phi}$ .

The parameter  $\phi$  is known as the *precision parameter*, since for fixed  $\mu$ , the larger  $\phi$  the smaller the variance of y. The parameters  $\{a,b\}$  are given as  $\{\mu,\phi\}$  as follows,

$$a = \mu \phi$$
 and  $b = -\mu \phi + \phi$ .

In some applications then observations close to 0 or 1, are censored and represented as exactly 0 and 1. For this, we introduced a censor value  $0 < \delta < 1/2$  and treat all  $y \le \delta$  or  $y \ge 1 - \delta$  as censored observations. By default, no censoring is applied  $(\delta = 0)$ .

#### **Link-function**

The linear predictor  $\eta$  is linked to the mean  $\mu$  using a default logit-link

$$\mu = \frac{\exp(\eta)}{1 + \exp(\eta)}.$$

### Hyperparameter

The hyperparameter is the precision parameter  $\phi$ , which is represented as

$$\phi = s_i \exp(\theta)$$

where  $s = (s_i) > 0$  is a fixed scaling, and the prior is defined on  $\theta$ .

### Specification

- family="beta"
- Required argument: y
- Optional argument: s (argument scale, default all 1, s > 0)
- Optional argument: truncation limit  $0 \le \delta < 1/2$  (argument beta.trunctation,  $\delta = 0$  means no trunctation).

```
Hyperparameter spesification and default values
```

```
doc The Beta likelihood
hyper
     theta
         hyperid 61001
         name precision parameter
         short.name phi
         output.name precision parameter for the beta observations
         output.name.intern intern precision-parameter for the beta observations
         initial 2.30258509299405
         fixed FALSE
         prior loggamma
         param 1 0.1
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
survival FALSE
discrete FALSE
link default logit loga cauchit probit cloglog ccloglog loglog
pdf beta
Example
In the following example we estimate the parameters in a simulated example.
n = 1000
w = runif(n, min = 0.25, max = 0.75)
phi = 5 * w
z = rnorm(n, sd=0.2)
eta = 1 + z
mu = exp(eta)/(1+exp(eta))
a = mu * phi
b = -mu * phi + phi
y = rbeta(n, a, b)
formula = y \sim 1 + z
r = inla(formula, data = data.frame(y, z, w),
         family = "beta", scale = w)
summary(r)
   In this example we add truncation.
## the precision parameter in the beta distribution
phi = 5
## generate simulated data
```

```
n = 1000
z = rnorm(n, sd=.2)
eta = 1 + z
mu = exp(eta)/(1+exp(eta))
a = mu * phi
b = -mu * phi + phi
y = rbeta(n, a, b)
## this is the censoring
cens <- 0.05
y[y \le cens] <- 0
y[y \ge 1-cens] <- 1
## estimate the model
formula = y \sim 1 + z
r = inla(formula, data = data.frame(y, z), family = "beta",
         control.family = list(beta.censor.value = cens))
summary(r)
```

### Notes

None.