Generalized Pareto distribution

Parametrisation

The generalized Pareto (GP) distribution with positive shape parameter has cumulative distribution function

$$F(y; \sigma, \xi) = 1 - \left(1 + \xi \frac{y}{\sigma}\right)^{-1/\xi}, \quad y > 0,$$

for a continuous response y where

 ξ : is the tail parameter, $\xi > 0$

 σ : is the scale parameter, $\sigma > 0$

The limit for $\xi \downarrow 0$ is $F(y; \sigma, 0) = 1 - \exp(-x/\sigma)$.

Link function

The linear predictor η controls the α quantile of the GP

$$P(y \le q_{\alpha}) = \alpha$$

and $q_{\alpha} = \exp(\eta)$. The scaling σ , is then a function of (q_{α}, ξ) , as

$$\sigma = \frac{\xi \exp(\eta)}{(1 - \alpha)^{-\xi} - 1}$$

Hyperparameters

The GP model has one hyperparameter. The tail $\xi > 0$ is represented as

$$\xi = \xi_{\text{low}} + (\xi_{\text{high}} - \xi_{\text{low}}) \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and the prior is defined on θ , with constant low and high values. The prior is FIXED to pc.gevtail, see inla.doc("pc.gevtail") for more info.

Specification

- family="gp"
- Required arguments: y and the quantile α .

The quantile is given as control.family=list(control.link=list(quantile= α)).

Hyperparameter spesification and default values

```
doc Generalized Pareto likelihood
```

```
hyper
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```
theta
         hyperid 101201
         name tail
         short.name xi
         output.name Tail parameter for the gp observations
         output.name.intern Intern tail parameter for the gp observations
         initial -4
         fixed FALSE
         prior pc.gevtail
         param 7 0 0.5
         to.theta function(x, interval = c(REPLACE.ME.low, REPLACE.ME.high)) log(-(interval
         from.theta function(x, interval = c(REPLACE.ME.low, REPLACE.ME.high)) interval[1]
survival FALSE
discrete TRUE
link default quantile
pdf genPareto
Example
rgp = function(n, sigma, eta, alpha, xi = 0.001)
{
    if (missing(sigma)) {
        stopifnot(!missing(eta) && !missing(alpha))
        sigma = exp(eta) * xi / ((1.0 - alpha)^(-xi) -1.0)
    return (sigma / xi * (runif(n)^(-xi) -1.0))
}
n = 300
x = runif(n) - 0.5
eta = 1+x
alpha = 0.95
xi = 0.3
y = rgp(n, eta = eta, alpha = alpha, xi=xi)
```

Notes

The prior for ξ is fixed to pc.gevtail or one that respect an given interval similarly. Note that the default prior (and the internal representation of it and its name) changed 2019/10/12.