Likelihood fmri

Parametrisation

This is special parameterisation of the non-central χ_ν distribution. Let $\{x_i\}$ are iid Gaussians with mean μ and precision τ, then

$$z = \sqrt{\sum_{i=1}^{\nu} \tau x_i^2}$$

is non-central χ-distribution with (integer and fixed by design) $\nu > 0$ degrees of freedom, and non-centrality parameter

$$\rho = \sqrt{\nu \tau \mu^2}.$$

The observation y is $y = z/\sqrt{\tau}$ and we're interested in the underlying true signal

$$\lambda = \rho/\sqrt{\tau} = \sqrt{\nu \mu^2}$$

Link-function

The linkfunction is given as

$$\log(\lambda) = \eta$$

where η is the linear predictor.

Hyperparameters

The hyperparameters are $\theta = (\theta_1, \theta_2)$, where

$$\tau = \exp(\theta_1)$$

is the precision, and

$$\nu = \theta_2$$

For technical reasons, ν is implemented as a hyper-parameter, but is required to be fixed. Hence, the initial value for θ_2 defines the (fixed) value for ν.

The prior is given on θ_1.

Specification

- `family="fmri"` or `family="fmrisurv"`
- Required arguments: y (and optional `scale` for `fmri` to scale τ)

Hyperparameter spesification and default values

doc `fmri distribution (special nc-chi)`

hyper

theta1

hyperid `103101`
name `precision`
short.name `prec`
output.name `Precision for fmri`
output.name.intern `Log precision for fmri`

initial 0
fixed FALSE
prior loggamma
param 10 10
to.theta function(x) log(x)
from.theta function(x) exp(x)

theta2

hyperid 103202
name dof
short.name df
output.name NOT IN USE
output.name.intern NOT IN USE
initial 4
fixed TRUE
prior normal
param 0 1
to.theta function(x) x
from.theta function(x) x

survival FALSE

discrete FALSE

link default log

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hyper

theta1

hyperid 104101
name precision
short.name prec
output.name Precision for fmrisurv
output.name.intern Log precision for fmrisurv
initial 0
fixed FALSE
prior loggamma
param 10 10
to.theta function(x) log(x)
from.theta function(x) exp(x)

theta2

hyperid 104201
name dof
short.name df
output.name NOT IN USE

output.name.intern NOT IN USE

initial 4

fixed TRUE

prior normal

param 0 1

to.theta function(x) x

from.theta function(x) x

survival TRUE

discrete FALSE

link default log

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Example

In the following example we estimate the parameters in a simulated example.

```
n <- 300
x <- rnorm(n, sd = 0.3)
df <- 1
prec <- 3
eta <- 1 + x
lambda <- exp(eta)
y <- sqrt(rchisq(n, df = df, ncp = prec * lambda^2) /prec)

r <- inla(y ~ 1 + x,
          data = data.frame(y, x),
          family = "fmri",
          control.family = list(hyper = list(df = list(initial = df))),
          control.inla = list(cmin = 0,
                              int.strategy = "eb",
                              strategy = "adaptive"),
          verbose = TRUE)
summary(r)

## 'cmin=0' seems to be required only for initial values that can give
## 'crazy' values. We can rerun without this re-starting at the prev fit,
## to validate
r$.args$control.inla$cmin <- -Inf
r$.args$control.inla$int.strategy <- "auto"
rr <- inla.rerun(r)
summary(rr)
```

Notes

Thanks to LS for providing all the details and a robust implementation of this likelihood.