Random walk model of order 2 (RW2)

Parametrization

The random walk model of order 2 (RW2) for the Gaussian vector $\mathbf{x} = (x_1, \dots, x_n)$ is constructed assuming independent second-orderincrements:

$$\Delta^2 x_i = x_i - 2 \ x_{i+1} + x_{i+2} \sim \mathcal{N}(0, \tau^{-1})$$

The density for **x** is derived from its n-2 second-order increments as

$$\pi(\mathbf{x}|\tau) \propto \tau^{(n-2)/2} \exp\left\{-\frac{\tau}{2} \sum (\Delta^2 x_i)^2\right\}$$

$$= \tau^{(n-2)/2} \exp\left\{-\frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x}\right\}$$
(1)

where $\mathbf{Q} = \tau \mathbf{R}$ and \mathbf{R} is the structure matrix reflecting the neighbourhood structure of the model. It is also possible to define a *cyclic* version of the RW2 model.

Hyperparameters

The precision parameter τ is represented as

$$\theta = \log \tau$$

and the prior is defined on θ .

Specification

The RW2 model is specified inside the f() function as

```
f(<whatever>, model="rw2", values=<values>,
    cyclic=FALSE, scale.model = FALSE)
```

The (optional) argument values is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See the example for RW1 for an application.

The logical option scale.model determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is FALSE so that the model is not scaled.

Hyperparameter spesification and default values

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hyper

theta

hyperid 5001
name log precision
short.name prec
prior loggamma
param 1 5e-05
initial 4
fixed FALSE
to.theta function(x) log(x)

```
constr TRUE
nrow.ncol FALSE
augmented FALSE
aug.factor 1
aug.constr
n.div.by
n.required FALSE
set.default.values FALSE
min.diff 0.001
pdf rw2
Example
n=100
z=seq(0,6,length.out=n)
y=sin(z)+rnorm(n,mean=0,sd=0.5)
data=data.frame(y=y,z=z)
formula=y~f(z,model="rw2")
result=inla(formula,data=data,family="gaussian")
```

from.theta function(x) exp(x)

Notes

- The RW2 is a intrinsic with rank deficiency 2.
- The RW2 model for irregular locations are supported although not described here.
- The term $\frac{1}{2}\log(|R|^*)$ of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it.