

## Poisson

### Parametrisation

The Poisson distribution is

$$\text{Prob}(y) = \frac{\lambda^y}{y!} \exp(-\lambda)$$

for responses  $y = 0, 1, 2, \dots$ , where

$\lambda$ : the expected value.

### Link-function

The mean and variance of  $y$  are given as

$$\mu = \lambda \quad \text{and} \quad \sigma^2 = \lambda$$

and the mean is linked to the linear predictor by

$$\lambda(\eta) = E \exp(\eta)$$

where  $E > 0$  is a known constant (or  $\log(E)$  is the offset of  $\eta$ ).

### Hyperparameters

None.

### Specification

- `family="poisson"`
- Required arguments: (integer-valued)  $y$  and  $E$

There is an alternative expert-version,

- `family="xpoisson"`
- Required arguments:  $y$  and  $E$

which allows the Poisson likelihood to be evaluated for a real-numbered response  $y \geq 0$ , in cases where this is known to make sense. Note that  $y!$  is computed using the integer part of  $y$ . Please be aware of its impact on the marginal likelihood, information criterias and cross-validation scores.

There is limiting version,

- `family="npoisson"`
- Required arguments:  $y$  and  $E$

which approximate the discrete Poisson with a continuous Normal approximation. Please be aware of its impact on the marginal likelihood, information criterias and cross-validation scores.

## Example

In the following example we estimate the parameters in a simulated example with Poisson responses.

```
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
E = sample(1:10, n, replace=TRUE)
lambda = E*exp(eta)
y = rpois(n, lambda = lambda)

data = list(y=y,z=z)
formula = y ~ 1+z
result = inla(formula, family = "poisson", data = data, E=E)
summary(result)
```

## Notes

This likelihood also accept  $E = 0$  and in this case  $\log(E)$  is *defined* to be 0. Non-integer values of  $y \geq 0$  is accepted although the normalising constant of the likelihood is then wrong (but its a constant only).

For the quantile-link, then `model="quantile"` is applied to  $\lambda$  only and this is then scaled with  $E$ . A more population version, can be achived moving the constant  $E$  into the linear predictor by

```
~ offset(log(E)) + ...
```

Note there is no link-model `pquantile` for the Poisson, as this would disable the  $E$  argument.