Occupancy likelihood

Parametrisation

This is a specialized likelihood to for occupancy models.

Details

An observation is an vector $y = (y_1, \ldots, y_m)$ of binary observations, each depending on spesific covariates, with additional zero-inflation. If there fewer than m observations, like m' < m, then "observations not observed" must be set to NA. The likelihood for one observation(-vector) is

$$f(y) = \phi \prod_{i=1}^{m} p_i^{y_i} (1 - p_i)^{1 - y_i} + (1 - \phi) 1_{[y_i = 0, \forall i]}$$

with the convension that if y_i =NA, the contribution from y_i is ignored. Further,

$$logit(p_i) = x_i^T \beta$$

and $x_i = (x_{i1}, \dots, x_{ik})$ are the k > 0 covariate associated to y_i , and $\beta = (\beta_1, \dots, \beta_k)$ are the regression coefficients. The linear predictor from the formula η , goes into ϕ , as

$$logit(\phi) = \eta$$

Link-function

The link-function for the p_i -model is given by argument link.simple in the control.family-argument. The link-function for the ϕ -model is given as normal. Both defaults to the logit-link.

Hyperparameters

The regression coefficients β are treated as hyperparameters, and k is maximum 10. An intercept must be defined manually by adding a constant covariate vector.

Specification

- family="occupancy"
- Required arguments: A matrix Y with the observations and a matrix X with the covariates.

The matrix Y is $n \times m$, where m is then the maximum number of observations over all locations. If there fewer than m observations in one location, then NA must be added to reach m.

The matrix X stored the covariates. Since there are k covariate for each observation, then the dimension of X is $n \times mk$. For the ith observation(-vector), then the ith row of X is $(x_{i1}, x_{i2}, \ldots, x_{im})$, where x_{ij} is the covariate vector for jth observation at location i. In a single observation is NA, the corresponding covariate(-vectors) is not used (but it still needs to be given).

Here is a simple example with n=5 locations, maximum m=3 observations, and k=2 covariates.

```
> Y
     [,1] [,2] [,3]
[1,]
         1
              1
                  NA
[2,]
        0
              1
[3,]
        0
              0
                    0
[4,]
        0
              1
                    0
[5,]
       NA
              0
                    0
> round(dig=3,X)
             [,2] [,3]
                           [,4] [,5]
                                        [,6]
     [,1]
[1,]
         1 - 0.129
                      1 - 0.248
                                          NA
[2,]
         1 -0.030
                        0.151
                                      0.100
                                   1
[3,]
        1 -0.148
                      1 -0.061
                                   1 0.023
[4,]
        1 -0.055
                      1 0.081
                                   1 -0.112
[5,]
       NA
               NA
                      1
                        0.164
                                   1 0.027
```

For Y[1,1] the covariates are (1,-0.129), for Y[1,2] the covariates are (1,-0.248), for Y[1,3] is NA hence not used, for Y[2,1] the covariates are (1,-0.030), etc. We pass both Y and X in the inla.mdata() in the formula, as

```
inla.mdata(Y,X) \sim ...
```

Hyperparameter spesification and default values

```
doc Occupancy likelihood
hyper
     theta1
         hyperid 56601
         name beta1
         short.name beta1
         output.name beta1 for occupancy observations
         output.name.intern beta1 for occupancy observations
         initial -2
         fixed FALSE
         prior normal
         param -2 10
         to.theta function(x) x
         from.theta function(x) x
     theta2
         hyperid 56602
         name beta2
         short.name beta2
         output.name beta2 for occupancy observations
         output.name.intern beta2 for occupancy observations
         initial 0
         fixed FALSE
         prior normal
         param 0 10
```

to.theta function(x) x

```
from.theta function(x) x
theta3
    hyperid 56603
    name beta3
    short.name beta3
    output.name beta3 for occupancy observations
    output.name.intern beta3 for occupancy observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 56604
    name beta4
    short.name beta4
    output.name beta4 for occupancy observations
    output.name.intern beta4 for occupancy observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 56605
    name beta5
    short.name beta5
    output.name beta5 for occupancy observations
    output.name.intern beta5 for occupancy observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 56606
    name beta6
    short.name beta6
    output.name beta6 for occupancy observations
    output.name.intern beta6 for occupancy observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
```

```
theta7
    hyperid 56607
    name beta7
    short.name beta7
    output.name beta7 for occupancy observations
    output.name.intern beta7 for occupancy observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 56608
    name beta8
    short.name beta8
    output.name beta8 for occupancy observations
    output.name.intern beta8 for occupancy observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 56609
    name beta9
    short.name beta9
    output.name beta9 for occupancy observations
    output.name.intern beta9 for occupancy observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 56610
    name beta10
    short.name beta10
    output.name beta10 for occupancy observations
    output.name.intern beta10 for occupancy observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
```

discrete TRUE
link default logit
link.simple default logit
pdf occupancy

Example

```
n <- 1000
m <- 3
nc <- 2
beta <- c(-1, rnorm(nc-1, sd = 0.2))
Y <- matrix(NA, n, m)
X <- matrix(NA, n, m*nc)</pre>
z <- rnorm(n, mean = 0, sd = 0.3)
eta \leftarrow 3 + z
p.obs <- 1/(1 + \exp(-eta))
for (i in 1:n) {
    is.zero <- rbinom(1, size = 1, prob = 1 - p.obs[i])</pre>
    nyy <- sample(2:m, 1)</pre>
    for(j in 1:m) {
         off <- (j-1) * nc
         if (j <= nyy) {
             X[i, off + 1:nc] \leftarrow c(1, rnorm(nc-1, sd = 0.1))
             eeta <- sum(X[i, off + 1:nc] * beta)
             p <- 1/(1 + exp(-eeta))
             if (is.zero) {
                 Y[i, j] <- 0
             } else {
                 Y[i, j] \leftarrow rbinom(1, size = 1, prob = p)
             }
         } else {
             X[i, off + 1:nc] <- rep(NA, nc)</pre>
             Y[i, j] \leftarrow NA
         }
    }
r \leftarrow inla(inla.mdata(Y, X) ~ 1 + z,
           family = "occupancy",
           data = list(Y = Y, X = X, z = z),
           control.fixed = list(prec.intercept = 1),
           control.inla = list(cmin = 0.0))
summary(r)
cbind(beta, r$summary.hyperpar[, "mean"])
```