Random walk model of order 2 (RW2)

Parametrization

The random walk model of order 2 (RW2) for the Gaussian vector $\mathbf{x} = (x_1, \ldots, x_n)$ is constructed assuming independent second-orderincrements:

$$\Delta^2 x_i = x_i - 2\ x_{i+1} + x_{i+2} \sim \mathcal{N}(0, \tau^{-1})$$

The density for $\mathbf{x}$ is derived from its $n - 2$ second-order increments as

$$\pi(\mathbf{x}|\tau) \quad \propto \quad \tau^{(n-2)/2} \exp\left\{-\frac{\tau}{2}\sum(\Delta^2 x_i)^2\right\} \tag{1}$$

$$= \quad \tau^{(n-2)/2} \exp\left\{-\frac{1}{2}\mathbf{x}^T \mathbf{Q}\mathbf{x}\right\} \tag{2}$$

where $\mathbf{Q} = \tau\mathbf{R}$ and $\mathbf{R}$ is the structure matrix reflecting the neighbourhood structure of the model.

It is also possible to define a *cyclic* version of the RW2 model.

Hyperparameters

The precision parameter τ is represented as

$$\theta = \log\tau$$

and the prior is defined on θ.

Specification

The RW2 model is specified inside the `f()` function as

```
 f(<whatever>, model="rw2", values=<values>,
   cyclic=FALSE, scale.model = FALSE)
```

The (optional) argument `values` is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See the example for RW1 for an application.

The logical option `scale.model` determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is `FALSE` so that the model is not scaled.

Hyperparameter spesification and default values

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hyper

    theta

        hyperid 5001

        name log precision

        short.name prec

        prior loggamma

        param 1 5e-05

        initial 4

        fixed FALSE

        to.theta function(x) log(x)

from.theta `function(x) exp(x)`

constr `TRUE`

nrow.ncol `FALSE`

augmented `FALSE`

aug.factor `1`

aug.constr

n.div.by

n.required `FALSE`

set.default.values `FALSE`

min.diff `1e-04`

pdf `rw2`

Example

```
n=100
z=seq(0,6,length.out=n)
y=sin(z)+rnorm(n,mean=0,sd=0.5)
data=data.frame(y=y,z=z)

formula=y~f(z,model="rw2")
result=inla(formula,data=data,family="gaussian")
```

Notes

- The RW2 is a intrinsic with rank deficiency 2.

- The RW2 model for irregular locations are supported although not described here.

- The term $\frac{1}{2}\log(|R|^*)$ of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it.