# Weibull

#### Parametrisation

The Weibull distribution is (variant=0)

$$f(y) = \alpha y^{\alpha - 1} \lambda \exp(-\lambda y^{\alpha}), \qquad \alpha > 0, \qquad \lambda > 0$$

and (variant=1)

$$f(y) = \alpha y^{\alpha - 1} \lambda^{\alpha} \exp(-(\lambda y)^{\alpha}), \qquad \alpha > 0, \qquad \lambda > 0$$

where

 $\alpha$ : shape parameter.

#### Link-function

The parameter  $\lambda$  is linked to the linear predictor as:

$$\lambda = \exp(\eta)$$

### Hyperparameters

The  $\alpha$  parameter is represented as

$$\alpha = \exp(S\theta)$$

and the prior is defined on  $\theta$ . The constant S currently set to 0.1 to avoid numerical instabilities in the optimization, since small changes of  $\alpha$  can make a huge difference.

### **Specification**

- family="weibull" for regression and family="weibull.surv" for survival
- Required arguments: y (to be given using inla.surv() for survival models), and variant=0 (default) or 1 to define the parameterisation.

### Hyperparameter spesification and default values

#### weibull

```
doc The Weibull likelihood
hyper
    theta
        hyperid 79001
        name log alpha
        short.name alpha
        output.name alpha parameter for weibull
        output.name.intern alpha_intern for weibull
        initial -2
        fixed FALSE
        prior pc.alphaw
        param 5
        to.theta function(x, sc = 0.1) log(x) / sc
        from.theta function(x, sc = 0.1) exp(sc * x)
```

```
survival FALSE
    discrete FALSE
    {f link} default log neglog quantile
    pdf weibull
weibullsurv
    {
m doc} The Weibull likelihood (survival)
    hyper
         theta
             hyperid 79101
            name log alpha
            short.name alpha
             output.name alpha parameter for weibullsurv
             output.name.intern alpha_intern for weibullsurv
            initial -2
            \mathbf{fixed} \ \mathsf{FALSE}
             prior pc.alphaw
             param 5
             to.theta function(x, sc = 0.1) log(x) / sc
             from.theta function(x, sc = 0.1) exp(sc * x)
         theta2
             hyperid 79102
            name beta1
             short.name beta1
             output.name beta1 for Weibull-Cure
             output.name.intern beta1 for Weibull-Cure
            initial -7
             fixed FALSE
             prior normal
             param -4 100
             to.theta function(x) x
            from.theta function(x) x
         theta3
             hyperid 79103
             name beta2
             short.name beta2
             output.name beta2 for Weibull-Cure
             output.name.intern beta2 for Weibull-Cure
             initial 0
             fixed FALSE
             prior normal
             param 0 100
             to.theta function(x) x
             from.theta function(x) x
```

```
theta4
   hyperid 79104
   name beta3
   short.name beta3
   output.name beta3 for Weibull-Cure
   output.name.intern beta3 for Weibull-Cure
   initial 0
   fixed FALSE
   prior normal
   param 0 100
   to.theta function(x) x
   from.theta function(x) x
theta5
   hyperid 79105
   name beta4
   short.name beta4
   output.name beta4 for Weibull-Cure
   output.name.intern beta4 for Weibull-Cure
   initial 0
   fixed FALSE
   prior normal
   param 0 100
   to.theta function(x) x
   from.theta function(x) x
theta6
   hyperid 79106
   name beta5
   short.name beta5
   output.name beta5 for Weibull-Cure
   output.name.intern beta5 for Weibull-Cure
   initial 0
   fixed FALSE
   prior normal
   param 0 100
   to.theta function(x) x
   from.theta function(x) x
theta7
   hyperid 79107
   name beta6
   short.name beta6
   output.name beta6 for Weibull-Cure
   output.name.intern beta6 for Weibull-Cure
   initial 0
   fixed FALSE
   prior normal
```

```
param 0 100
   to.theta function(x) x
   from.theta function(x) x
theta8
   hyperid 79108
   name beta7
   short.name beta7
   output.name beta7 for Weibull-Cure
   output.name.intern beta7 for Weibull-Cure
   initial 0
   fixed FALSE
   prior normal
   param 0 100
   to.theta function(x) x
   from.theta function(x) x
theta9
   hyperid 79109
   name beta8
   short.name beta8
   output.name beta8 for Weibull-Cure
   output.name.intern beta8 for Weibull-Cure
   initial 0
   fixed FALSE
   prior normal
   param 0 100
   to.theta function(x) x
   from.theta function(x) x
theta10
   hyperid 79110
   name beta9
   short.name beta9
   output.name beta9 for Weibull-Cure
   output.name.intern beta9 for Weibull-Cure
   initial 0
   fixed FALSE
   prior normal
   param 0 100
   to.theta function(x) x
   from.theta function(x) x
theta11
   hyperid 79111
   name beta10
   short.name beta10
   output.name beta10 for Weibull-Cure
   output.name.intern beta10 for Weibull-Cure
```

```
initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
survival TRUE
discrete FALSE
link default log neglog quantile
pdf weibull
```

# Example

In the following example we estimate the parameters in a simulated case

```
n = 1000
alpha = 1.1
beta = 2.2
x = c(scale(runif(n)))
eta = 1+beta*x
lambda = exp(eta)
for(variant in 0:1) {
    y = rweibull(n,
                 shape= alpha,
                 scale= if (variant == 0)
                             lambda^(-1/alpha)
                        else
                             1/lambda)
    print(paste("VARIANT=", variant))
    event = rep(1,n)
    data = list(y=y, event=event, x=x)
    formula=inla.surv(y,event)~ x
    r=inla(formula,
           family ="weibullsurv",
           data=data,
           control.family = list(list(variant = variant)))
    print("SURV")
    print(summary(r))
    formula= y ~ x
    r=inla(formula,
           family ="weibull",
           data=data,
           control.family = list(list(variant = variant)))
    print("REGRESSION")
    print(summary(r))
}
```

# Notes

• Weibullsurv model can be used for right censored, left censored, interval censored data. If the observed times y are large/huge, then this can cause numerical overflow in the likelihood routine. If you encounter this problem, try to scale the observatios, time = time / max(time) or similar.