Box-Cox Gaussian

Parametrisation

The Gaussian distribution is

$$f(y) = \frac{\sqrt{s\tau}}{\sqrt{2\pi}} y^{\lambda-1} \exp\left(-\frac{1}{2} s \left(f_\lambda(y) - \mu\right)^2\right)$$

for continuously responses $y > 0$ where

μ: is the the mean

τ: is the precision

s: is a fixed scaling, $s > 0$.

and the Box-Cox transformation is

$$f_\lambda(y) = \frac{y^\lambda - 1}{\lambda} - \frac{\mu^\lambda - 1}{\lambda}$$

Link-function

The mean and variance of y are given as

$$\mu \quad \text{and} \qquad \sigma^2 = \frac{1}{s\tau}$$

and the mean is linked to the linear predictor by

$$\mu = \eta$$

Hyperparameters

The precision is τ and

$$\theta_1 = \log \tau$$

and the prior is defined on θ_1.

The Box-Cox transformation-parameter λ is $\theta_2 = \lambda$ and the prior is defined on θ_2.

Specification

- `family="bcgaussian"`
- Required arguments: y and s (argument `scale`)

The scalings have default value 1.

Hyperparameter spesification and default values

Example

```
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
tau = 100
```

```
scale = exp(rnorm(n))
prec = scale*tau
y = rnorm(n, mean = eta, sd = 1/sqrt(prec))

data = list(y=y, z=z)
formula = y ~ 1+z
result = inla(formula, family = "gaussian", data = data,
        control.family = list(hyper = list(
                                    prec = list(
                                            prior = "loggamma",
                                            param = c(1.0,0.01),
                                            initial = 2))),
              scale=scale, keep=TRUE)
summary(result)

## with an offset in the variance
var0 = 1.0 ## fixed offset
var1 = 2.0
v = var0 + var1
s = sqrt(v)
x = rnorm(n)
y = 1 + x + rnorm(n, sd = s)
rr = inla(y ~ x,
         data = data.frame(y, x),
         control.family = list(
             hyper = list(precoffset = list(initial = log(1/var0)))),
         verbose = TRUE)
summary(rr)
plot(rr$internal.marginals.hyperpar[[1]], type = "l", lwd=3)
abline(v = log(1.0/var1), lwd=3, col = "blue")
```

Notes