Generalized Poisson

The generalized Poisson distribution is given by

$$f(y|\lambda, w) = \frac{\lambda(\lambda + wy)^{y-1}}{y!} \exp(-(\lambda + wy))$$

for $y = 0, 1, 2, \ldots$ and where $\lambda > 0$ and $\max(-1, -\lambda/4) \le w \le 1$. The mean and variance of y are

$$\mu = \lambda (1 - w)^{-1}$$
 and $\sigma^2 = \lambda (1 - w)^{-3} = \mu (1 - w)^{-2}$.

Since the dispersion parameter w influence the mean as well as the variance, we will use the following parameterisation (Consul and Jain (1973), Zamani and Ismail(2012))

$$w = \frac{\varphi \mu^{p-1}}{1 + \varphi \mu^{p-1}},$$

which gives the following density

$$f(y|\mu,\varphi,p) = \frac{\mu(\mu + \varphi\mu^{p-1}y)^{y-1}}{(1 + \varphi\mu^{p-1})^y y!} \exp\left(-\frac{\mu + \varphi\mu^{p-1}y}{1 + \varphi\mu^{p-1}}\right)$$

for $y = 0, 1, 2, \dots$ We assume $\varphi \ge 0$.

Link-function

The mean and variance of y are given as

$$E(y|.) = \mu$$
 and $Var(y|.) = \mu (1 + \varphi \mu^{p-1})^2$

and the mean is linked to the linear predictor by

$$\mu = E \exp(\eta)$$

Hyperparameters

The overdispersion parameter $\varphi \geq 0$ is represented as

$$\varphi = \exp(\theta_1)$$

The "shape" parameter p is represented as

$$p = \theta_2$$

Note that $\theta_2 = 1$ and fixed = TRUE, default. The prior is defined on $\theta = (\theta_1, \theta_2)$.

Specification

• family="gpoisson"

Hyperparameter spesification and default values doc The generalized Poisson likelihood hyper theta1 hyperid 56001 name overdispersion short.name phi output.name Overdispersion for gpoisson output.name.intern Log overdispersion for gpoisson initial 0 fixed FALSE prior loggamma param 1 1 to.theta function(x) log(x) from.theta function(x) exp(x) theta2 hyperid 56002 name p short.name p output.name Parameter p for gpoisson output.name.intern Parameter p_intern for gpoisson initial 1 fixed TRUE prior normal param 1 100 to.theta function(x) x from.theta function(x) x survival FALSE discrete TRUE link default log logoffset pdf gpoisson Example In the following example we estimate the parameters in a simulated example with generalized Poisson responses. dgpoisson = function(y, mu, phi, p) $a = mu + phi * mu^(p-1.0) * y;$

 $b = 1. + phi * mu^(p-1.0);$

return (d)

 $d = \exp(\log(mu) + (y-1.0)*\log(a) -$

y*log(b) - lfactorial(y) - a/b)

```
}
rgpoisson = function(n, mu, phi, p)
    stopifnot(length(mu) == 1)
    s = sqrt(mu*(1+phi*mu^(p-1))^2)
    low = as.integer(max(0, mu - f*s))
    high = as.integer(mu + f*s)
    prob = dgpoisson(low:high, mu, phi, p)
    y = sample(low:high, n, replace=TRUE,
            prob = prob)
    return (y)
}
n = 1000
phi = 1
mu = exp(1 + 5*(1:n)/n)
y = numeric(n)
for(i in 1:n) {
    y[i] = rgpoisson(1, mu[i], phi, p)
}
idx = (1:n)/n
r = inla(y ~ 1 + idx, data = data.frame(y, idx),
        family = "gpoisson")
```

Notes

The parameter p is default fixed to be 1. Allowing it to be estimated jointly with the overdispersion parameter, please note the following.

- \bullet The parameter p and the overdispersion parameter are strongly correlated when estimated jointly.
- You may want to chose an informative prior for p, as the shape of the likelihood might not be want you expect for "extreme" p.
- You may experience problems in the numerical optimization (fail to converge); a more informative prior (if available) for p will help with this issue.

References

- Consul, P. C. and Jain, G.C (1973) A generalization of Poisson distribution. Technometrics 15, 791-799.
- Zamani, H. and Ismail, N. (2011). Functional form for the generalized Poisson regression model. Communication in Statistics Theory and Methods (IN PRESS).