# Bym model for spatial effects

## Parametrization

This model is simply a union of the besag model $u$ and a iid model $v$, so that

$$x = \begin{pmatrix} v + u \\ u \end{pmatrix}$$

Note that the length of $x$ is $2n$ if the length of $u$ (and $v$) is $n$. The benefite is that this allows to get the posterior marginals of the sum of the spatial and iid model; otherwise it offers no advantages.

## Hyperparameters

The hyperparameters are the precision $\tau_1$ of the iid model ($v$) and the precision $\tau_2$ of the besag model ($u$). The precision parameters are represented as

$$\theta = (\theta_1, \theta_2) = (\log \tau_1, \log \tau_2)$$

and the prior is defined on $\theta$.

## Specification

The bym model is specified inside the `f()` function as

```
f(<whatever>,model="bym",graph=<graph>,
  hyper=<hyper>, adjust.for.con.comp = TRUE,
  scale.model = FALSE)
```

The neighbourhood structure of `x` is passed to the program through the `graph` argument.

The option `adjust.for.con.comp` adjust the model if the graph has more than one connected compoment, and this adjustment can be disabled setting this option to `FALSE`. This means that `constr=TRUE` is interpreted as a sum-to-zero constraint on *each* connected component and the `rankdef` parameter is set accordingly.

The logical option `scale.model` determine if the besag-model-part of the model $u$ should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is `FALSE` so that the model is not scaled.

### Hyperparameter spesification and default values

**doc** `The BYM-model (Besag-York-Mollier model)`

**hyper**

**theta1**

- **hyperid** `10001`
- **name** `log unstructured precision`
- **short.name** `prec.unstruct`
- **prior** `loggamma`
- **param** `1 5e-04`
- **initial** `4`
- **fixed** `FALSE`
- **to.theta** `function(x) log(x)`
- **from.theta** `function(x) exp(x)`

**theta2**

**hyperid** `10002`
**name** `log spatial precision`
**short.name** `prec.spatial`
**prior** `loggamma`
**param** `1 5e-04`
**initial** `4`
**fixed** `FALSE`
**to.theta** `function(x) log(x)`
**from.theta** `function(x) exp(x)`

**constr** `TRUE`

**nrow.ncol** `FALSE`

**augmented** `TRUE`

**aug.factor** `2`

**aug.constr** `2`

**n.div.by**

**n.required** `TRUE`

**set.default.values** `TRUE`

**pdf** `bym`

### Example

For examples of application of this model see the `Bym` example in Volume I.

## Details on the implementation

This gives some details of the implementation, which depends on the following variables

**nc1** Number of connected components in the graph with size 1. These nodes, *singletons*, have no neighbours.

**nc2** Number of connected components in the graph with size $\geq 2$.

**scale.model** The value of the logical flag, if the model should be scaled or not. (Default FALSE)

**adjust.for.con.comp** The value of the logical flag if the `constr=TRUE` option should be reinterpreted.

#### The case `(scale.model==FALSE && adjust.for.con.comp == FALSE)`

The option `constr=TRUE` is interpreted as a sum-to-zero constraint over the whole graph. Singletons are given a uniform distribution on $(-\infty, \infty)$ before the constraint.

**The case** `(scale.model==TRUE && adjust.for.con.comp == FALSE)`

The option `constr=TRUE` is interpreted as a sum-to-zero constraint over the whole graph. Let $Q = \tau R$ be the standard precision matrix from the `besag`-model with precision parameter $\tau$. Then $R$, except the singletons, are scaled so that the geometric mean of the marginal variances is 1, and $R$ is modified so that singletons have a standard Normal distribution.

**The case** `(scale.model==FALSE && adjust.for.con.comp == TRUE)`

The option `constr=TRUE` is interpreted as one sum-to-zero constraint over each of the `nc2` connected components of size $\geq 2$. Singletons are given a uniform distribution on $(-\infty, \infty)$.

**The case** `(scale.model==TRUE && adjust.for.con.comp == TRUE)`

The option `constr=TRUE` is interpreted as `nc2` sum-to-zero constraints for each of the connected components of size $\geq 2$. Let $Q = \tau R$ be the standard precision matrix from the `besag`-model with precision parameter $\tau$. Then $R$, are scaled so that the geometric mean of the marginal variances in each connected component of size $\geq 2$ is 1, and modified so that singletons have a standard Normal distribution.

## Notes

The term $\frac{1}{2}\log(|R|^*)$ of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here $R$ is the precision matrix with a unit precision parameter for the Besag part of the model.