

# The modal-Gamma-distribution

## Parametrisation

The modal-Gamma-distribution has the following density

$$\pi(y) = \frac{b^a}{\Gamma(a)} y^{a-1} \exp(-by), \quad a > 1, \quad b > 0, \quad y > 0.$$

where the mode  $m = (a - 1)/b$  is the target of the regression model instead of the mean or quantile.

**Note the constraint**  $a > 1$ .

With precision

$$\tau = (s\phi)/m^2$$

for fixed scaling  $s$  and precision parameter  $\phi$ , this complete the specification. The mapping back to the  $(a, b)$ -parameterisation is then  $\phi' = s\phi$ ,  $\delta = \left(\sqrt{\phi'(\phi' + 4)} + \phi'\right)/2$ , then

$$a = 1 + \delta \quad \text{and} \quad b = \delta/m$$

## Link-function

The linear predictor  $\eta$  is linked to the mode  $m$  using a default log-link

$$m = \exp(\eta)$$

## Hyperparameter

The hyperparameter is the precision parameter  $\phi$ , which is represented as

$$\phi = \exp(\theta)$$

and the prior is defined on  $\theta$ .

## Specification

- `family="mgamma"` for regression models and `family="mgamma.surv"` for survival models.
- Required arguments: for `mgamma.surv`,  $y$  (to be given in a format by using `inla.surv()`), and for `mgamma`,  $y$  and  $s$  (default value 1).

The scalings  $s$  is **not** used for `family="mgamma.surv"`.

## Hyperparameter specification and default values

`doc` The modal Gamma likelihood

`hyper`

`theta`

`hyperid` 58002

`name` precision parameter

`short.name` prec

`output.name` Precision-parameter for the modal Gamma observations

`output.name.intern` Intern precision-parameter for the modal Gamma observations

`initial` 4.60517018598809

```

    fixed FALSE
    prior loggamma
    param 1 0.01
    to.theta function(x) log(x)
    from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default log

pdf mgamma

doc The modal Gamma likelihood (survival)

hyper

  theta1
    hyperid 58121
    name precision parameter
    short.name prec
    output.name Precision-parameter for the modal Gamma surv observations
    output.name.intern Intern precision-parameter for the modal Gamma surv observations
    initial 0
    fixed FALSE
    prior loggamma
    param 1 0.01
    to.theta function(x) log(x)
    from.theta function(x) exp(x)

  theta2
    hyperid 58122
    name beta1
    short.name beta1
    output.name beta1 for modal Gamma-Cure
    output.name.intern beta1 for modal Gamma-Cure
    initial -7
    fixed FALSE
    prior normal
    param -4 100
    to.theta function(x) x
    from.theta function(x) x

  theta3
    hyperid 58123
    name beta2
    short.name beta2
    output.name beta2 for modal Gamma-Cure
    output.name.intern beta2 for modal Gamma-Cure

```

```

    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 58124
    name beta3
    short.name beta3
    output.name beta3 for modal Gamma-Cure
    output.name.intern beta3 for modal Gamma-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 58125
    name beta4
    short.name beta4
    output.name beta4 for Ga mma-Cure
    output.name.intern beta4 for modal Gamma-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 58126
    name beta5
    short.name beta5
    output.name beta5 for modal Gamma-Cure
    output.name.intern beta5 for modal Gamma-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 58127

```

```

name beta6
short.name beta6
output.name beta6 for modal Gamma-Cure
output.name.intern beta6 for modal Gamma-Cure
initial 0
fixed FALSE
prior normal
param 0 100
to.theta function(x) x
from.theta function(x) x
theta8
  hyperid 58128
  name beta7
  short.name beta7
  output.name beta7 for modal Gamma-Cure
  output.name.intern beta7 for modal Gamma-Cure
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta9
  hyperid 58129
  name beta8
  short.name beta8
  output.name beta8 for modal Gamma-Cure
  output.name.intern beta8 for modal Gamma-Cure
  initial 0
  fixed FALSE
  prior normal
  param 0 100
  to.theta function(x) x
  from.theta function(x) x
theta10
  hyperid 58130
  name beta9
  short.name beta9
  output.name beta9 for modal Gamma-Cure
  output.name.intern beta9 for modal Gamma-Cure
  initial 0
  fixed FALSE
  prior normal
  param 0 100

```

```

    to.theta function(x) x
    from.theta function(x) x
  theta11
    hyperid 58131
    name beta10
    short.name beta10
    output.name beta10 for modal Gamma-Cure
    output.name.intern beta10 for modal Gamma-Cure
    initial 0
    fixed FALSE
    prior normal
    param 0 100
    to.theta function(x) x
    from.theta function(x) x

survival TRUE

discrete FALSE

link default log neglog

pdf agamma

```

## Example

In the following example we estimate the parameters in a simulated example.

```

n <- 10^4
phi <- 2.2
x <- rnorm(n)
m <- exp(1.1 + 1.2 * x)
delta <- (sqrt(phi * (phi + 4.0)) + phi) / 2.0

y <- rgamma(n, shape = 1 + delta, rate = delta / m)
r <- inla(y ~ 1 + x,
  data = data.frame(y, x),
  family = "mgamma",
  control.compute = list(cpo = T),
  verbose = TRUE)

Y <- inla.surv(y, 1)
rr <- inla(Y ~ 1 + x,
  data = list(Y = Y, x = x),
  family = "mgammasurv",
  control.compute = list(cpo = T),
  verbose = TRUE)

```

## Notes

None.