

# Censored Poisson

## Parametrisation

The Poisson distribution is

$$\text{Prob}(y) = \frac{\lambda^y}{y!} \exp(-\lambda)$$

for responses  $y = 0, 1, 2, \dots$ , where  $\lambda$  is the expected value. The censored version is that response in the interval  $L \leq y \leq H$  are censored (and reported as  $y = L$ , say), whereas other values are reported as is. This is often due to privacy issue, for example using  $L = 1$  and  $H = 5$ , for example.

The “cenpoisson” probability distribution is then, for  $y = 0, 1, \dots$ ,

$$\text{Prob}^*(y) = \begin{cases} \sum_{z=L}^H \frac{\lambda^z}{z!} \exp(-\lambda) & L \leq y \leq H \\ \frac{\lambda^y}{y!} \exp(-\lambda) & \text{otherwise} \end{cases}$$

## Link-function

The mean-parameter is  $\lambda$  and is linked to the linear predictor by

$$\lambda(\eta) = E \exp(\eta)$$

where  $E > 0$  is a known constant (or  $\log(E)$  is the offset of  $\eta$ ).

## Hyperparameters

None.

## Specification

- family="cenpoisson"
- Required arguments:  $y$ ,  $E$ ,  $L$  and  $H$  (family-argument `cenpoisson.I=c(L,H)`).

## Example

In the following example we estimate the parameters in a simulated example with Poisson responses.

```
n=100
a = 0
b = 1
x = rnorm(n, sd = 0.5)
eta = a + b*x
interval = c(1, 4)
E = sample(1:10, n, replace=TRUE)
lambda = E*exp(eta)
y = rpois(n, lambda = lambda)

censored = (y >= interval[1] & y <= interval[2])
y[censored] = interval[1]

r = (inla(y ~ 1 + x,
  family = "cenpoisson",
  control.family = list(cenpoisson.I = interval),
  data = data.frame(y, x),
  E=E))
```

`summary(r)`

## Notes

For censored values, then  $y$  must be one of the values in the interval.