Censored Poisson

Parametrisation

The Poisson distribution is

$$\text{Prob}(y) = \frac{\lambda^y}{y!}\exp(-\lambda)$$

for responses $y = 0, 1, 2, \ldots$, where λ is the expected value. The cencored version is that reponse response in the interval $L \le y \le H$ are cencored (and reported as $y = L$, say), whereas other values are reported as is. This is often due to privacy issue, for example using $L = 1$ and $H = 5$, for example.

The "cenpoisson" probability distribution is then, for $y = 0, 1, \ldots,$

$$\text{Prob}^*(y) = \begin{cases} \sum_{z=L}^{H} \frac{\lambda^z}{z!}\exp(-\lambda) & L \le y \le H \\ \frac{\lambda^y}{y!}\exp(-\lambda) & \text{otherwise} \end{cases}$$

Link-function

The mean-parameter is λ and is linked to the linear predictor by

$$\lambda(\eta) = E\exp(\eta)$$

where $E > 0$ is a known constant (or $\log(E)$ is the offset of η).

Hyperparameters

None.

Specification

- `family="cenpoisson"`
- Required arguments: y, E, L and H (family-argument `cenpoisson.I=c(L,H)`).

Example

In the following example we estimate the parameters in a simulated example with Poisson responses.

```
n=100
a = 0
b = 1
x = rnorm(n, sd = 0.5)
eta = a + b*x
interval = c(1, 4)
E = sample(1:10, n, replace=TRUE)
lambda = E*exp(eta)
y = rpois(n, lambda = lambda)

censored = (y >= interval[1] & y <= interval[2])
y[censored] = interval[1]

r = (inla(y ~ 1 + x,
          family = "cenpoisson",
          control.family = list(cenpoisson.I = interval),
          data = data.frame(y, x),
          E=E))
```

```
summary(r)
```

Notes

For censored values, then y must be one of the values in the interval.