More general Gaussian likelihoods: GGaussian and GGaussianS

Parameterisation

The Gaussian distribution is

$$f(y) = \frac{\sqrt{\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\tau (y - \mu)^2\right)$$

for continuously responses y where

μ: is the the mean

τ: is the precision

These likelihood families generalise the normal procedure. We allow for two linear predictors in the model. One needs to be "simple", only consists of fixed effects, while the other one is general and defined via the formula.

For family="ggaussian", then

$$\text{link}(\mu) = \text{formula} \qquad \text{link.simple}(\tau) = \text{simple}$$

while for family="ggaussianS", then

$$\text{link}(\tau) = \text{formula} \qquad \text{link.simple}(\mu) = \text{simple}$$

The default link is the *identity* for the mean and the *log* for the precision. We will describe each model separately, as the specifications are a little different.

Family *ggaussian*

Link-function

The mean is given by the linear predictor η from the formula

$$\text{link}(\mu) = \eta$$

where the default is the *identity*-link. The precision is given as

$$\text{link.simple}(\frac{1}{s}\tau) = \beta_1 z_1 + \beta_2 z_2 + \ldots + \beta_m z_m, \qquad m \leq 10$$

where s is a **fixed** scaling (or often log-offset) and similar to the argument *scale* for family="gaussian". For the log-link, the precision is expressed as

$$\tau = s \times \exp\left(\beta_1 z_1 + \beta_2 z_2 + \ldots + \beta_m z_m\right)$$

Note: there is no default intercept in the simple model, so it is common to set $z_1 = 1$.

Hyperparameters

The hyperparameters in the model are $\theta_1 = \beta_1, \ldots, \theta_m = \beta_m$, and the priors are defined on $\theta_1, \ldots, \theta_m$.

Note: The default prior for θ_1 is similar to the default prior for the precision for the Gaussian family, please change if you do not want this feature.

Specification

- `family="ggaussian"`

- Required arguments: **inla.mdata**-object that defines the response, s and covariates. The **inla.mdata** object is defined as

$$\text{inla.mdata}(y, s, z_1, \ldots, z_m)$$

  where each argument are vectors of the same length.

- **Note:** The scaling argument s **MUST** be given as it has no default value. Often we can just use $s = 1$ as it will auto-expand.

- **Note:** $m = 0$ is allowed, which means that $\tau = s$ with with log link.simple.

Hyperparameter spesification and default values

`family="ggaussian"`

doc Generalized Gaussian

hyper

    theta1

        hyperid 66501

        name beta1

        short.name beta1

        output.name beta1 for ggaussian observations

        output.name.intern beta1 for ggaussian observations

        initial 4

        fixed FALSE

        prior normal

        param 9.33 0.61

        to.theta function(x) x

        from.theta function(x) x

    theta2

        hyperid 66502

        name beta2

        short.name beta2

        output.name beta2 for ggaussian observations

        output.name.intern beta2 for ggaussian observations

        initial 0

        fixed FALSE

        prior normal

        param 0 10

        to.theta function(x) x

        from.theta function(x) x

    theta3

        hyperid 66503

name beta3

short.name beta3

output.name beta3 for ggaussian observations

output.name.intern beta3 for ggaussian observations

initial 0

fixed FALSE

prior normal

param 0 10

to.theta function(x) x

from.theta function(x) x

theta4

hyperid 66504

name beta4

short.name beta4

output.name beta4 for ggaussian observations

output.name.intern beta4 for ggaussian observations

initial 0

fixed FALSE

prior normal

param 0 10

to.theta function(x) x

from.theta function(x) x

theta5

hyperid 66505

name beta5

short.name beta5

output.name beta5 for ggaussian observations

output.name.intern beta5 for ggaussian observations

initial 0

fixed FALSE

prior normal

param 0 10

to.theta function(x) x

from.theta function(x) x

theta6

hyperid 66506

name beta6

short.name beta6

output.name beta6 for ggaussian observations

output.name.intern beta6 for ggaussian observations

initial 0

fixed FALSE

prior normal

param 0 10

to.theta `function(x) x`

from.theta `function(x) x`

theta7

    hyperid `66507`

    name `beta7`

    short.name `beta7`

    output.name `beta7 for ggaussian observations`

    output.name.intern `beta7 for ggaussian observations`

    initial `0`

    fixed `FALSE`

    prior `normal`

    param `0 10`

    to.theta `function(x) x`

    from.theta `function(x) x`

theta8

    hyperid `66508`

    name `beta8`

    short.name `beta8`

    output.name `beta8 for ggaussian observations`

    output.name.intern `beta8 for ggaussian observations`

    initial `0`

    fixed `FALSE`

    prior `normal`

    param `0 10`

    to.theta `function(x) x`

    from.theta `function(x) x`

theta9

    hyperid `66509`

    name `beta9`

    short.name `beta9`

    output.name `beta9 for ggaussian observations`

    output.name.intern `beta9 for ggaussian observations`

    initial `0`

    fixed `FALSE`

    prior `normal`

    param `0 10`

    to.theta `function(x) x`

    from.theta `function(x) x`

theta10

    hyperid `66510`

    name `beta10`

    short.name `beta10`

    output.name `beta10 for ggaussian observations`

    output.name.intern `beta10 for ggaussian observations`

> **initial** `0`
> **fixed** `FALSE`
> **prior** `normal`
> **param** `0 10`
> **to.theta** `function(x) x`
> **from.theta** `function(x) x`

status `experimental`

survival `FALSE`

discrete `FALSE`

link `default identity`

link.simple `default log`

pdf `ggaussian`

Example

```
n <- 1000
x <- rnorm(n)
xx <- rnorm(n)
eta <-  0.1 + 1.1 * x + 2.2 * xx

s <- runif(n)
z <- rnorm(n)
zz <- rnorm(n)
eta.prec <- 1 + 0.55 * z + 1.1 * zz

y <- eta + 1/sqrt(s * exp(eta.prec)) * rnorm(n)
Y <- inla.mdata(y, s, 1, z, zz)
r <- inla(Y ~ 1 + x + xx,
          data = list(Y = Y, x = x, xx = xx, z = z, zz = zz, s = s),
          family = "ggaussian")
summary(r)
```

Family *ggaussianS*

Link-function

This is the swapped version. The mean is given by a simple linear predictor

$$\text{link.simple}(\mu) = \text{off} + \beta_1 z_1 + \beta_2 z_2 + \ldots + \beta_m z_m, \qquad m \leq 10$$

where "off" is a fixed offset and where link.simple is the *identity*-link by default. **Note:** there is no default intercept in the simple model, so it is common to set $z_1 = 1$. The precision is specified in the formula

$$\text{link}(\tau) = \text{formula}$$

using the log-link as default.

Hyperparameters

The hyperparameters in the model are $\theta_1 = \beta_1, \ldots, \theta_m = \beta_m$, and the priors are defined on $\theta_1, \ldots, \theta_m$.

Specification

- family="ggaussianS"

- Required arguments: **inla.mdata**-object that defines the response, the offset "off" and covariates. The **inla.mdata** object is defined as

$$\text{inla.mdata}(y, \text{off}, z_1, \ldots, z_m)$$

  where each argument are vectors of the same length.

- **Note:** The offset argument **MUST** be given as it has no default value. Often we can just use off= 0 as it will auto-expand.

- **Note:** $m = 0$ is allowed, which means that $\mu = $ off with with identity link.simple.

Hyperparameter spesification and default values

family="ggaussianS"

doc Generalized GaussianS

hyper

    theta1

        hyperid 66601
        name beta1
        short.name beta1
        output.name beta1 for ggaussianS observations
        output.name.intern beta1 for ggaussianS observations
        initial 0
        fixed FALSE
        prior normal
        param 0 0.001
        to.theta function(x) x
        from.theta function(x) x

theta2

    hyperid 66602

    name beta2

    short.name beta2

    output.name beta2 for ggaussianS observations

    output.name.intern beta2 for ggaussianS observations

    initial 0

    fixed FALSE

    prior normal

    param 0 0.001

    to.theta function(x) x

    from.theta function(x) x

theta3

    hyperid 66603

    name beta3

    short.name beta3

    output.name beta3 for ggaussianS observations

    output.name.intern beta3 for ggaussianS observations

    initial 0

    fixed FALSE

    prior normal

    param 0 0.001

    to.theta function(x) x

    from.theta function(x) x

theta4

    hyperid 66604

    name beta4

    short.name beta4

    output.name beta4 for ggaussianS observations

    output.name.intern beta4 for ggaussianS observations

    initial 0

    fixed FALSE

    prior normal

    param 0 0.001

    to.theta function(x) x

    from.theta function(x) x

theta5

    hyperid 66605

    name beta5

    short.name beta5

    output.name beta5 for ggaussianS observations

    output.name.intern beta5 for ggaussianS observations

    initial 0

    fixed FALSE

prior normal

param 0 0.001

to.theta function(x) x

from.theta function(x) x

theta6

hyperid 66606

name beta6

short.name beta6

output.name beta6 for ggaussianS observations

output.name.intern beta6 for ggaussianS observations

initial 0

fixed FALSE

prior normal

param 0 0.001

to.theta function(x) x

from.theta function(x) x

theta7

hyperid 66607

name beta7

short.name beta7

output.name beta7 for ggaussianS observations

output.name.intern beta7 for ggaussianS observations

initial 0

fixed FALSE

prior normal

param 0 0.001

to.theta function(x) x

from.theta function(x) x

theta8

hyperid 66608

name beta8

short.name beta8

output.name beta8 for ggaussianS observations

output.name.intern beta8 for ggaussianS observations

initial 0

fixed FALSE

prior normal

param 0 0.001

to.theta function(x) x

from.theta function(x) x

theta9

hyperid 66609

name beta9

short.name beta9

          output.name beta9 for ggaussianS observations

          output.name.intern beta9 for ggaussianS observations

          initial 0

          fixed FALSE

          prior normal

          param 0 0.001

          to.theta function(x) x

          from.theta function(x) x

      theta10

          hyperid 66610

          name beta10

          short.name beta10

          output.name beta10 for ggaussianS observations

          output.name.intern beta10 for ggaussianS observations

          initial 0

          fixed FALSE

          prior normal

          param 0 0.001

          to.theta function(x) x

          from.theta function(x) x

status experimental

survival FALSE

discrete FALSE

link default log

link.simple default identity

pdf ggaussian

Example

```
n <- 1000
x <- rnorm(n)
xx <- rnorm(n)
off <- runif(n)
z <- rnorm(n)
zz <- rnorm(n)

mean <- off + 0.1 + 1.1 * z + 2.2 * zz
prec <- exp(1 + 0.55 * x + 1.1 * xx)

y <- mean + (1/sqrt(prec)) * rnorm(n)
Y <- inla.mdata(y, off, 1, z, zz)
r <- inla(Y ~ 1 + x + xx,
          data = list(Y = Y, off = off, x = x, xx = xx, z = z, zz = zz),
          family = "ggaussianS")
summary(r)
```