Autoregressive model of order 1 (AR1)

Parametrization

The autoregressive model of order 1 (AR1) for the Gaussian vector $\mathbf{x} = (x_1, \ldots, x_n)$ is defined as:

$$\begin{aligned} x_1 &\sim \mathcal{N}(0, (\tau(1-\rho^2))^{-1}) \\ x_i &= \rho\, x_{i-1} + \epsilon_i; \qquad \epsilon_i \sim \mathcal{N}(0, \tau^{-1}) \qquad i = 2, \ldots, n \end{aligned}$$

where

$$|\rho| < 1$$

Hyperparameters

The precision parameter κ is represented as

$$\theta_1 = \log(\kappa)$$

where κ is the *marginal* precision,

$$\kappa = \tau(1-\rho^2).$$

The parameter ρ is represented as

$$\theta_2 = \log\left(\frac{1+\rho}{1-\rho}\right)$$

and the prior is defined on $\theta = (\theta_1, \theta_2)$.

Specification

The AR1 model is specified inside the `f()` function as

```
f(<whatever>, model="ar1", values=<values>, hyper = <hyper>)
```

The (optional) argument `values` is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See the example for RW1 for an application.

Hyperparameter spesification and default values

doc `Auto-regressive model of order 1 (AR(1))`

hyper

theta1

hyperid `14001`
name `log precision`
short.name `prec`
prior `loggamma`
param `1 5e-05`
initial `4`
fixed `FALSE`
to.theta `function(x) log(x)`
from.theta `function(x) exp(x)`

theta2

hyperid `14002`

name logit lag one correlation
short.name rho
prior normal
param 0 0.15
initial 2
fixed FALSE
to.theta function(x) log((1 + x) / (1 - x))
from.theta function(x) 2 * exp(x) / (1 + exp(x)) - 1

theta3

hyperid 14003
name mean
short.name mean
prior normal
param 0 1
initial 0
fixed TRUE
to.theta function(x) x
from.theta function(x) x

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values FALSE

pdf ar1

Example

In this exaple we implement an ar1 model observed with Poisson counts

```
#simulate data
n = 100
rho = 0.8
prec = 10
## note that the marginal precision would be
marg.prec = prec * (1-rho^2)

E=sample(c(5,4,10,12),size=n,replace=T)
eta = as.vector(arima.sim(list(order = c(1,0,0), ar = rho), n = n,sd=sqrt(1/prec)))
y=rpois(n,E*exp(eta))
data = list(y=y, z=1:n, E=E)
```

```
## fit the model
formula = y~f(z,model="ar1")
result = inla(formula,family="poisson", data = data, E=E)
```

Notes

A third hyperparameter θ_3 is **experimental**, and the *mean* of the AR1 process. By default this parameter is fixed to the be zero.