Zero-inflated models: Poisson, Binomial, negative Binomial and BetaBinomial

Parametrisation

There is support two types of zero-inflated models, which we name type 0 and type 1. These are defined for both the Binomial, the Poisson, the censored Poisson, the negative Binomial and BetaBinomial likelihood. For simplicity we will describe only the Poisson as the other cases are similar.

Type 0

The (type 0) likelihood is defined as

$$\text{Prob}(y \mid \ldots) = p \times 1_{[y=0]} + (1-p) \times \text{Poisson}(y \mid y > 0)$$

where p is a hyperparameter where

$$p = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and θ is the internal representation of p; meaning that the initial value and prior is given for θ. This is model is called `zeroinflatedpoisson0` (and `zeroinflatedbinomial0`).

Type 1

The (type 1) likelihood is defined as

$$\text{Prob}(y \mid \ldots) = p \times 1_{[y=0]} + (1-p) \times \text{Poisson}(y)$$

where p is a hyperparameter where

$$p = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and θ is the internal representation of p; meaning that the initial value and prior is given for θ. This is model is called `zeroinflatedpoisson1` (and `zeroinflatedbinomial1`).

Link-function

As for the Poisson, the Binomial the negative Binomial and the BetaBinomial.

Hyperparameters

For Poisson and the Binomial, there is one hyperparameter; where

$$p = \frac{\exp(\theta)}{1 + \exp(\theta)}$$

and the prior and initial value is is given for θ.

For the negative Binomial and BetaBinomial, there are two hyperparameters. The overdispersion parameter n for the negative Binomial is represented as

$$\theta_1 = \log(n)$$

and the prior is defined on θ_1. The zero-inflation parameter p, is represented as

$$p = \frac{\exp(\theta_2)}{1 + \exp(\theta_2)}$$

and the prior and initial value is is given for θ_2. For the BetaBinomial it is similar.

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Specification

- family="zeroinflatedbinomial0"

- family="zeroinflatedbinomial1"

- family="zeroinflatednbinomial0"

- family="zeroinflatednbinomial1"

- family="zeroinflatedpoisson0"

- family="zeroinflatedpoisson1"

- family="zeroinflatedcenpoisson0"

- family="zeroinflatedcenpoisson1"

- family="zeroinflatedbetabinomial0"

- family="zeroinflatedbetabinomial1"

- Required arguments: As for the Binomial, the negative Binomial, BetaBinomial and Poisson likelihood.

Hyperparameter spesification and default values

Zeroinflated Binomial Type 0

doc Zero-inflated Binomial, type 0

hyper

    theta

        hyperid 90001
        name logit probability
        short.name prob
        output.name zero-probability parameter for zero-inflated binomial_0
        output.name.intern intern zero-probability parameter for zero-inflated binomial_0
        initial -1
        fixed FALSE
        prior gaussian
        param -1 0.2
        to.theta function(x) log(x / (1 - x))
        from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete FALSE

link default logit loga cauchit probit cloglog ccloglog loglog robit sn

pdf zeroinflated

Zeroinflated Binomial Type 1

doc `Zero-inflated Binomial, type 1`

hyper

    theta

        hyperid `91001`
        name `logit probability`
        short.name `prob`
        output.name `zero-probability parameter for zero-inflated binomial_1`
        output.name.intern `intern zero-probability parameter for zero-inflated binomial_1`
        initial `-1`
        fixed `FALSE`
        prior `gaussian`
        param `-1 0.2`
        to.theta `function(x) log(x / (1 - x))`
        from.theta `function(x) exp(x) / (1 + exp(x))`

survival `FALSE`

discrete `FALSE`

link `default logit loga cauchit probit cloglog ccloglog loglog robit sn`

pdf `zeroinflated`

Zeroinflated NegBinomial Type 0

doc `Zero inflated negBinomial, type 0`

hyper

    theta1

        hyperid `95001`
        name `log size`
        short.name `size`
        output.name `size for nbinomial_0 zero-inflated observations`
        output.name.intern `log size for nbinomial_0 zero-inflated observations`
        initial `2.30258509299405`
        fixed `FALSE`
        prior `pc.mgamma`
        param `7`
        to.theta `function(x) log(x)`
        from.theta `function(x) exp(x)`

    theta2

        hyperid `95002`
        name `logit probability`
        short.name `prob`
        output.name `zero-probability parameter for zero-inflated nbinomial_0`

output.name.intern `intern zero-probability parameter for zero-inflated nbinomial_0`

initial `-1`

fixed `FALSE`

prior `gaussian`

param `-1 0.2`

to.theta `function(x) log(x / (1 - x))`

from.theta `function(x) exp(x) / (1 + exp(x))`

survival `FALSE`

discrete `FALSE`

link `default log`

pdf `zeroinflated`

Zeroinflated NegBinomial Type 1

doc `Zero inflated negBinomial, type 1`

hyper

    theta1

        hyperid `96001`

        name `log size`

        short.name `size`

        output.name `size for nbinomial_1 zero-inflated observations`

        output.name.intern `log size for nbinomial_1 zero-inflated observations`

        initial `2.30258509299405`

        fixed `FALSE`

        prior `pc.mgamma`

        param `7`

        to.theta `function(x) log(x)`

        from.theta `function(x) exp(x)`

    theta2

        hyperid `96002`

        name `logit probability`

        short.name `prob`

        output.name `zero-probability parameter for zero-inflated nbinomial_1`

        output.name.intern `intern zero-probability parameter for zero-inflated nbinomial_1`

        initial `-1`

        fixed `FALSE`

        prior `gaussian`

        param `-1 0.2`

        to.theta `function(x) log(x / (1 - x))`

        from.theta `function(x) exp(x) / (1 + exp(x))`

survival `FALSE`

discrete `FALSE`

link `default log`

pdf `zeroinflated`

Zeroinflated BetaBinomial Type 0

doc `Zero-inflated Beta-Binomial, type 0`

hyper

    theta1

        hyperid `88001`
        name `overdispersion`
        short.name `rho`
        output.name `rho for zero-inflated betabinomial_0`
        output.name.intern `rho_intern for zero-inflated betabinomial_0`
        initial `0`
        fixed `FALSE`
        prior `gaussian`
        param `0 0.4`
        to.theta `function(x) log(x / (1 - x))`
        from.theta `function(x) exp(x) / (1 + exp(x))`

    theta2

        hyperid `88002`
        name `logit probability`
        short.name `prob`
        output.name `zero-probability parameter for zero-inflated betabinomial_0`
        output.name.intern `intern zero-probability parameter for zero-inflated betabinomial_`
        initial `-1`
        fixed `FALSE`
        prior `gaussian`
        param `-1 0.2`
        to.theta `function(x) log(x / (1 - x))`
        from.theta `function(x) exp(x) / (1 + exp(x))`

survival `FALSE`

discrete `TRUE`

link `default logit loga cauchit probit cloglog ccloglog loglog robit sn`

pdf `zeroinflated`

Zeroinflated BetaBinomial Type 1

doc `Zero-inflated Beta-Binomial, type 1`

hyper

    theta1

        hyperid `89001`
        name `overdispersion`
        short.name `rho`
        output.name `rho for zero-inflated betabinomial_1`

        output.name.intern rho_intern for zero-inflated betabinomial_1

        initial 0

        fixed FALSE

        prior gaussian

        param 0 0.4

        to.theta function(x) log(x / (1 - x))

        from.theta function(x) exp(x) / (1 + exp(x))

    theta2

        hyperid 89002

        name logit probability

        short.name prob

        output.name zero-probability parameter for zero-inflated betabinomial_1

        output.name.intern intern zero-probability parameter for zero-inflated betabinomial_

        initial -1

        fixed FALSE

        prior gaussian

        param -1 0.2

        to.theta function(x) log(x / (1 - x))

        from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete TRUE

link default logit loga cauchit probit cloglog ccloglog loglog robit sn

pdf zeroinflated

Zeroinflated Poisson Type 0

doc Zero-inflated Poisson, type 0

hyper

    theta

        hyperid 85001

        name logit probability

        short.name prob

        output.name zero-probability parameter for zero-inflated poisson_0

        output.name.intern intern zero-probability parameter for zero-inflated poisson_0

        initial -1

        fixed FALSE

        prior gaussian

        param -1 0.2

        to.theta function(x) log(x / (1 - x))

        from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

Zeroinflated Poisson Type 1

doc `Zero-inflated Poisson, type 1`

hyper

    theta

        hyperid `86001`
        name `logit probability`
        short.name `prob`
        output.name `zero-probability parameter for zero-inflated poisson_1`
        output.name.intern `intern zero-probability parameter for zero-inflated poisson_1`
        initial `-1`
        fixed `FALSE`
        prior `gaussian`
        param `-1 0.2`
        to.theta `function(x) log(x / (1 - x))`
        from.theta `function(x) exp(x) / (1 + exp(x))`

survival `FALSE`

discrete `FALSE`

link `default log`

pdf `zeroinflated`

Zeroinflated Censored Poisson Type 0

doc `Zero-inflated censored Poisson, type 0`

hyper

    theta

        hyperid `87101`
        name `logit probability`
        short.name `prob`
        output.name `zero-probability parameter for zero-inflated poisson_0`
        output.name.intern `intern zero-probability parameter for zero-inflated poisson_0`
        initial `-1`
        fixed `FALSE`
        prior `gaussian`
        param `-1 0.2`
        to.theta `function(x) log(x / (1 - x))`
        from.theta `function(x) exp(x) / (1 + exp(x))`

survival `FALSE`

discrete `FALSE`

link `default log`

pdf `zeroinflated`

Zeroinflated Censored Poisson Type 1

doc `Zero-inflated censored Poisson, type 1`

hyper

> **theta**
>
>> **hyperid** `87201`
>> **name** `logit probability`
>> **short.name** `prob`
>> **output.name** `zero-probability parameter for zero-inflated poisson_1`
>> **output.name.intern** `intern zero-probability parameter for zero-inflated poisson_1`
>> **initial** `-1`
>> **fixed** `FALSE`
>> **prior** `gaussian`
>> **param** `-1 0.2`
>> **to.theta** `function(x) log(x / (1 - x))`
>> **from.theta** `function(x) exp(x) / (1 + exp(x))`

survival `FALSE`

discrete `FALSE`

link `default log`

pdf `zeroinflated`

Example

In the following example we estimate the parameters in a simulated example for both type 0 and type 1.

Poisson

```
## type 0
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
p = 0.2
E = sample(c(1,5,10,15), size=n, replace=TRUE)
lambda = E*exp(eta)

## first sample y|y>0
y = rpois(n, lambda = lambda)
is.zero = (y == 0)
while(sum(is.zero) > 0)
{
    y[is.zero] = rpois(sum(is.zero), lambda[is.zero])
    is.zero = (y == 0)
}
```

```
## then set some of these to zero
y[ rbinom(n, size=1, prob=p) == 1 ] = 0

data = list(y=y,z=z)
formula = y ~ 1+z
result0 = inla(formula, family = "zeroinflatedpoisson0", data = data, E=E)
summary(result0)

## type 1
y = rpois(n, lambda = lambda)
y[ rbinom(n, size=1, prob=p) == 1 ] = 0
data = list(y=y,z=z)
formula = y ~ 1+z
result1 = inla(formula, family = "zeroinflatedpoisson1", data = data, E=E)
summary(result1)
```

Binomial

```
## type 0
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
p = 0.2
Ntrials = sample(c(1,5,10,15), size=n, replace=TRUE)
prob = exp(eta)/(1 + exp(eta))

y = rbinom(n, size = Ntrials, prob = prob)
is.zero = (y == 0)
while(sum(is.zero) > 0)
{
    y[is.zero] = rbinom(sum(is.zero), size = Ntrials[is.zero], prob = prob[is.zero])
    is.zero = (y == 0)
}
y[ rbinom(n, size=1, prob=p) == 1 ] = 0
data = list(y=y,z=z)
formula = y ~ 1+z
result0 = inla(formula, family = "zeroinflatedbinomial0", data = data, Ntrials = Ntrials)
summary(result0)

## type 1
y = rbinom(n, size = Ntrials, prob = prob)
y[ rbinom(n, size=1, prob=p) == 1 ] = 0
data = list(y=y,z=z)
formula = y ~ 1+z
result1 = inla(formula, family = "zeroinflatedbinomial1", data = data, Ntrials=Ntrials)
summary(result1)
```

Advanced example

In the following example we estimate the parameters in a simulated example for a type0 likelihood, where one linear predictor enters the zero-probability and one other linear predictor enters the non-zero Poisson for example. The same trick can be used for other models of type0. The trick is that the likelihood

$$p^* 1_{[y=0]} + (1 - p^*)P(y|y > 0)$$

can be reformulated as a Bernoulli likelihood for the "class"-variable

$$z = \begin{cases} 1, & \text{if } y = 0 \\ 0, & \text{if } y > 0. \end{cases}$$

where p^* is the probability for success, and zero-inflated type0 likelihood (with fixed $p = 0$) for those $y > 0$. Since p^* and the linear predictor in P is separated into two likelihoods, we can apply one linear predictor to each one, hence extend the basic model to cases where p^* also depends on a linear predictor. Here is a small simulated example doing this.

```
require(INLA)


n = 100
a = 0.5
b = 1.5
x1 = rnorm(n, sd = 0.5)

eta.z = -a - b*x1
z = rbinom(n, 1, inla.link.logit(eta.z, inverse=TRUE))
n.y = sum(z)

x2 = rnorm(n.y, sd = 0.5)
eta.y = a + b*x2
lambda = exp(eta.y)
y = rpois(n.y, lambda)

is.zero = (y == 0)
while(sum(is.zero) > 0)
{
   y[is.zero] = rpois(sum(is.zero), lambda[is.zero])
   is.zero = (y == 0)
}

Y = matrix(NA, n + n.y, 2)
Y[1:n, 1] = z
Y[n + 1:n.y, 2] = y

form = Y ~ 0 + mu.z + mu.y + cov.z + cov.y
ldat = list(
        Y=Y,
        mu.z=rep(1:0, c(n, n.y)),
        mu.y=rep(0:1, c(n, n.y)),
        cov.z=c(x1, rep(NA,n.y)),
        cov.y=c(rep(NA, n), x2))
```

```
res <- inla(form, data=ldat,
            family=c('binomial', 'zeroinflatedpoisson0'),
            control.family=list(
                    list(),
                    list(hyper = list(
                                prob = list(
                                        initial = -20,
                                        fixed = TRUE)))))
round(res$summary.fix, 4)
```

Notes

None.

Extentions

There are some extentions available which currently is only implemented for the cases where its needed/requested.

Type 2 Is like Type 1 but where (for the Poisson)

$$p = 1 - \left(\frac{E \exp(x)}{1 + E \exp(x)} \right)^{\alpha}$$

where $\alpha > 0$ is the hyperparameter instead of p (and $E \exp(x)$ is the mean). Available for Poisson as `zeroinflatedpoisson2`, for binomial as `zeroinflatedbinomial2` and for the negative binomial as `zeroinflatednbinomial2`.

The internal representation is $\theta = \log(\alpha)$ and prior is defined on $\log(\alpha)$.

Zeroinflated Poisson Type 2

doc Zero-inflated Poisson, type 2

hyper

    theta

        hyperid 87001
        name log alpha
        short.name a
        output.name zero-probability parameter for zero-inflated poisson_2
        output.name.intern intern zero-probability parameter for zero-inflated poisson_2
        initial 0.693147180559945
        fixed FALSE
        prior gaussian
        param 0.693147180559945 1
        to.theta function(x) log(x)
        from.theta function(x) exp(x)

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

Zeroinflated Binomial Type 2

doc `Zero-inflated Binomial, type 2`

hyper

    theta

        hyperid `92001`
        name `alpha`
        short.name `alpha`
        output.name `zero-probability parameter for zero-inflated binomial_2`
        output.name.intern `intern zero-probability parameter for zero-inflated binomial_2`
        initial `-1`
        fixed `FALSE`
        prior `gaussian`
        param `-1 0.2`
        to.theta `function(x) log(x)`
        from.theta `function(x) exp(x)`

survival `FALSE`

discrete `FALSE`

link `default logit loga cauchit probit cloglog ccloglog loglog robit sn`

pdf `zeroinflated`

Zeroinflated Negative Binomial Type 2

doc `Zero inflated negBinomial, type 2`

hyper

    theta1

        hyperid `99001`
        name `log size`
        short.name `size`
        output.name `size for nbinomial zero-inflated observations`
        output.name.inter `log size for nbinomial zero-inflated observations`
        initial `2.30258509299405`
        fixed `FALSE`
        prior `pc.mgamma`
        param `7`
        to.theta `function(x) log(x)`
        from.theta `function(x) exp(x)`

    theta2

        hyperid `99002`
        name `log alpha`
        short.name `a`
        output.name `parameter alpha for zero-inflated nbinomial2`

output.name.intern `parameter alpha.intern for zero-inflated nbinomial2`

initial `0.693147180559945`

fixed `FALSE`

prior `gaussian`

param `2 1`

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

survival `FALSE`

discrete `FALSE`

link `default log`

pdf `zeroinflated`

Zeroinflated Negative Binomial Type 1 Strata 2

doc `Zero inflated negBinomial, type 1, strata 2`

hyper

theta1

hyperid `97001`

name `log size`

short.name `size`

output.name `size for zero-inflated nbinomial_1_strata2`

output.name.intern `log size for zero-inflated nbinomial_1_strata2`

initial `2.30258509299405`

fixed `FALSE`

prior `pc.mgamma`

param `7`

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta2

hyperid `97002`

name `logit probability 1`

short.name `prob1`

output.name `zero-probability1 for zero-inflated nbinomial_1_strata2`

output.name.intern `intern zero-probability1 for zero-inflated nbinomial_1_strata2`

initial `-1`

fixed `FALSE`

prior `gaussian`

param `-1 0.2`

to.theta `function(x) log(x / (1 - x))`

from.theta `function(x) exp(x) / (1 + exp(x))`

theta3

hyperid `97003`

name logit probability 2

short.name prob2

output.name zero-probability2 for zero-inflated nbinomial_1_strata2

output.name.intern intern zero-probability2 for zero-inflated nbinomial_1_strata2

initial -1

fixed FALSE

prior gaussian

param -1 0.2

to.theta function(x) log(x / (1 - x))

from.theta function(x) exp(x) / (1 + exp(x))

theta4

hyperid 97004

name logit probability 3

short.name prob3

output.name zero-probability3 for zero-inflated nbinomial_1_strata2

output.name.intern intern zero-probability3 for zero-inflated nbinomial_1_strata2

initial -1

fixed TRUE

prior gaussian

param -1 0.2

to.theta function(x) log(x / (1 - x))

from.theta function(x) exp(x) / (1 + exp(x))

theta5

hyperid 97005

name logit probability 4

short.name prob4

output.name zero-probability4 for zero-inflated nbinomial_1_strata2

output.name.intern intern zero-probability4 for zero-inflated nbinomial_1_strata2

initial -1

fixed TRUE

prior gaussian

param -1 0.2

to.theta function(x) log(x / (1 - x))

from.theta function(x) exp(x) / (1 + exp(x))

theta6

hyperid 97006

name logit probability 5

short.name prob5

output.name zero-probability5 for zero-inflated nbinomial_1_strata2

output.name.intern intern zero-probability5 for zero-inflated nbinomial_1_strata2

initial -1

fixed TRUE

prior gaussian

param -1 0.2

```
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))
```

theta7

```
    hyperid 97007
    name logit probability 6
    short.name prob6
    output.name zero-probability6 for zero-inflated nbinomial_1_strata2
    output.name.intern intern zero-probability6 for zero-inflated nbinomial_1_strata2
    initial -1
    fixed TRUE
    prior gaussian
    param -1 0.2
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))
```

theta8

```
    hyperid 97008
    name logit probability 7
    short.name prob7
    output.name zero-probability7 for zero-inflated nbinomial_1_strata2
    output.name.intern intern zero-probability7 for zero-inflated nbinomial_1_strata2
    initial -1
    fixed TRUE
    prior gaussian
    param -1 0.2
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))
```

theta9

```
    hyperid 97009
    name logit probability 8
    short.name prob8
    output.name zero-probability8 for zero-inflated nbinomial_1_strata2
    output.name.intern intern zero-probability8 for zero-inflated nbinomial_1_strata2
    initial -1
    fixed TRUE
    prior gaussian
    param -1 0.2
    to.theta function(x) log(x / (1 - x))
    from.theta function(x) exp(x) / (1 + exp(x))
```

theta10

```
    hyperid 97010
    name logit probability 9
    short.name prob9
    output.name zero-probability9 for zero-inflated nbinomial_1_strata2
    output.name.intern intern zero-probability9 for zero-inflated nbinomial_1_strata2
```

        initial -1

        fixed TRUE

        prior gaussian

        param -1 0.2

        to.theta function(x) log(x / (1 - x))

        from.theta function(x) exp(x) / (1 + exp(x))

    theta11

        hyperid 97011

        name logit probability 10

        short.name prob10

        output.name zero-probability10 for zero-inflated nbinomial_1_strata2

        output.name.intern intern zero-probability10 for zero-inflated nbinomial_1_strata2

        initial -1

        fixed TRUE

        prior gaussian

        param -1 0.2

        to.theta function(x) log(x / (1 - x))

        from.theta function(x) exp(x) / (1 + exp(x))

survival FALSE

discrete FALSE

link default log

pdf zeroinflated

Zeroinflated Negative Binomial Type 1 Strata 3

doc Zero inflated negBinomial, type 1, strata 3

hyper

    theta1

        hyperid 98001

        name logit probability

        short.name prob

        output.name zero-probability for zero-inflated nbinomial_1_strata3

        output.name.intern intern zero-probability for zero-inflated nbinomial_1_strata3

        initial -1

        fixed FALSE

        prior gaussian

        param -1 0.2

        to.theta function(x) log(x / (1 - x))

        from.theta function(x) exp(x) / (1 + exp(x))

    theta2

        hyperid 98002

        name log size 1

short.name `size1`

output.name `size1 for zero-inflated nbinomial_1_strata3`

output.name.intern `log_size1 for zero-inflated nbinomial_1_strata3`

initial `2.30258509299405`

fixed `FALSE`

prior `pc.mgamma`

param `7`

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta3

hyperid `98003`

name `log size 2`

short.name `size2`

output.name `size2 for zero-inflated nbinomial_1_strata3`

output.name.intern `log_size2 for zero-inflated nbinomial_1_strata3`

initial `2.30258509299405`

fixed `FALSE`

prior `pc.mgamma`

param `7`

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta4

hyperid `98004`

name `log size 3`

short.name `size3`

output.name `size3 for zero-inflated nbinomial_1_strata3`

output.name.intern `log_size3 for zero-inflated nbinomial_1_strata3`

initial `2.30258509299405`

fixed `TRUE`

prior `pc.mgamma`

param `7`

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta5

hyperid `98005`

name `log size 4`

short.name `size4`

output.name `size4 for zero-inflated nbinomial_1_strata3`

output.name.intern `log_size4 for zero-inflated nbinomial_1_strata3`

initial `2.30258509299405`

fixed `TRUE`

prior `pc.mgamma`

param `7`

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta6

    hyperid `98006`

    name `log size 5`

    short.name `size5`

    output.name `size5 for zero-inflated nbinomial_1_strata3`

    output.name.intern `log_size5 for zero-inflated nbinomial_1_strata3`

    initial `2.30258509299405`

    fixed `TRUE`

    prior `pc.mgamma`

    param `7`

    to.theta `function(x) log(x)`

    from.theta `function(x) exp(x)`

theta7

    hyperid `98007`

    name `log size 6`

    short.name `size6`

    output.name `size6 for zero-inflated nbinomial_1_strata3`

    output.name.intern `log_size6 for zero-inflated nbinomial_1_strata3`

    initial `2.30258509299405`

    fixed `TRUE`

    prior `pc.mgamma`

    param `7`

    to.theta `function(x) log(x)`

    from.theta `function(x) exp(x)`

theta8

    hyperid `98008`

    name `log size 7`

    short.name `size7`

    output.name `size7 for zero-inflated nbinomial_1_strata3`

    output.name.intern `log_size7 for zero-inflated nbinomial_1_strata3`

    initial `2.30258509299405`

    fixed `TRUE`

    prior `pc.mgamma`

    param `7`

    to.theta `function(x) log(x)`

    from.theta `function(x) exp(x)`

theta9

    hyperid `98009`

    name `log size 8`

    short.name `size8`

    output.name `size8 for zero-inflated nbinomial_1_strata3`

    output.name.intern `log_size8 for zero-inflated nbinomial_1_strata3`

    initial `2.30258509299405`

        fixed `TRUE`

        prior `pc.mgamma`

        param 7

        to.theta `function(x) log(x)`

        from.theta `function(x) exp(x)`

    theta10

        hyperid 98010

        name `log size 9`

        short.name `size9`

        output.name `size9 for zero-inflated nbinomial_1_strata3`

        output.name.intern `log_size9 for zero-inflated nbinomial_1_strata3`

        initial 2.30258509299405

        fixed `TRUE`

        prior `pc.mgamma`

        param 7

        to.theta `function(x) log(x)`

        from.theta `function(x) exp(x)`

    theta11

        hyperid 98011

        name `log size 10`

        short.name `size10`

        output.name `size10 for zero-inflated nbinomial_1_strata3`

        output.name.intern `log_size10 for zero-inflated nbinomial_1_strata3`

        initial 2.30258509299405

        fixed `TRUE`

        prior `pc.mgamma`

        param 7

        to.theta `function(x) log(x)`

        from.theta `function(x) exp(x)`

survival `FALSE`

discrete `FALSE`

link `default log`

pdf `zeroinflated`

0.0.1   Zero and N-inflated Binomial likelihood: type 3

This is the case where

$$
\begin{aligned}
\mathrm{Prob}(y\,|\ldots) \;=\; & p_0 \times 1_{[y=0]} + \\
& p_N \times 1_{[y=N]} + \\
& (1 - p_0 - p_N) \times \mathrm{binomial}(y, N, p)
\end{aligned}
$$

where:

$$
p = \frac{\exp(\eta)}{1 + \exp(\eta)} \qquad p_0 = \frac{p^{\alpha_0}}{1 + p^{\alpha_0} + (1-p)^{\alpha_N}} \qquad p_N = \frac{(1-p)^{\alpha_N}}{1 + p^{\alpha_0} + (1-p)^{\alpha_N}}
$$

There are 2 hyperparameters, α_0 and α_N, governing zero-inflation where: The zero-inflation parameters α_0 and α_N are represented as $\theta_0 = \log(\alpha_0)$; $\theta_N = \log(\alpha_N)$ and the prior and initial value is given for θ_0 and θ_N respectively.

Here is an example

```
nsim<-10000
x<-rnorm(nsim)
alpha0<-1.5
alphaN<-2.0
p = exp(x)/(1+exp(x))
p0 = p^alpha0 / (1 + p^alpha0 + (1-p)^alphaN)
pN = (1-p)^alphaN / (1 + p^alpha0 + (1-p)^alphaN)
P<-cbind(p0, pN, (1-p0 -pN))
N<-rpois(nsim,20)
y<-rep(0,nsim)
for(i in 1:nsim)
    y[i]<-sum(rmultinom(1,size = 1,P[i,])*c(0,N[i],rbinom(1,N[i],p[i])))
formula = y ~1 + x
r = inla(formula, family = "zeroninflatedbinomial3",  Ntrials = N, verbose = TRUE,
         data = data.frame(y, x))
```

and the default settings

doc `Zero and N inflated binomial, type 3`

hyper

    theta1

        hyperid `93101`

        name `alpha0`

        short.name `alpha0`

        output.name `alpha0 parameter for zero-n-inflated binomial_3`

        output.name.intern `intern alpha0 parameter for zero-n-inflated binomial_3`

        initial `1`

        fixed `FALSE`

        prior `loggamma`

        param `1 1`

        to.theta `function(x) log(x)`

        from.theta `function(x) exp(x)`

    theta2

        hyperid `93102`

        name `alphaN`

        short.name `alphaN`

        output.name.intern `intern alphaN parameter for zero-n-inflated binomial_3`

        output.name `alphaN parameter for zero-n-inflated binomial_3`

        initial `1`

        fixed `FALSE`

        prior `loggamma`

        param `1 1`

          to.theta `function(x) log(x)`

          from.theta `function(x) exp(x)`

survival `FALSE`

discrete `FALSE`

link `default logit loga cauchit probit cloglog ccloglog loglog robit sn`

pdf `zeroinflated`