More general Gaussian likelihoods: GGaussian and GGaussianS

Parameterisation

The Gaussian distribution is

$$f(y) = \frac{\sqrt{\tau}}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\tau (y - \mu)^2\right)$$

for continuously responses y where

 μ : is the mean

 τ : is the precision

These likelihood families generalise the normal procedure. We allow for two linear predictors in the model. One needs to be "simple", only consists of fixed effects, while the other one is general and defined via the formula.

For family="ggaussian", then

$$link(\mu) = formula$$
 $link.simple(\tau) = simple$

while for family="ggaussianS", then

$$link(\tau) = formula$$
 $link.simple(\mu) = simple$

The default link is the *identity* for the mean and the *log* for the precision. We will describe each model separately, as the specifications are a little different.

Family ggaussian

Link-function

The mean is given by the linear predictor η from the formula

$$link(\mu) = \eta$$

where the default is the *identity*-link. The precision is given as

link.simple
$$(\frac{1}{s}\tau) = \beta_1 z_1 + \beta_2 z_2 + \ldots + \beta_m z_m, \quad m \le 10$$

where s is a **fixed** scaling (or often log-offset) and similar to the argument scale for family="gaussian". For the log-link, the precision is expressed as

$$\tau = s \times \exp\left(\beta_1 z_1 + \beta_2 z_2 + \ldots + \beta_m z_m\right)$$

Note: there is no default intercept in the simple model, so it is common to set $z_1 = 1$.

Hyperparameters

The hyperparameters in the model are $\theta_1 = \beta_1, \dots, \theta_m = \beta_m$, and the priors are defined on $\theta_1, \dots, \theta_m$. **Note:** The default prior for θ_1 is similar to the default prior for the precision for the Gaussian family, please change if you do not want this feature.

Specification

- family="ggaussian"
- Required arguments: **inla.mdata**-object that defines the response, s and covariates. The **inla.mdata** object is defined as

inla.mdata
$$(y, s, z_1, \ldots, z_m)$$

where each argument are vectors of the same length.

- Note: The scaling argument s MUST be given as it has no default value. Often we can just use s = 1 as it will auto-expand.
- Note: m=0 is allowed, which means that $\tau=s$ with with log link.simple.

Hyperparameter spesification and default values

```
family="ggaussian"
\operatorname{doc} Generalized Gaussian
hyper
    theta1
         hyperid 66501
         name beta1
         short.name beta1
         output.name beta1 for ggaussian observations
         output.name.intern beta1 for ggaussian observations
         initial 4
         fixed FALSE
         prior normal
         param 9.33 0.61
         to.theta function(x) x
         from.theta function(x) x
    theta2
         hyperid 66502
         name beta2
         short.name beta2
         output.name beta2 for ggaussian observations
         output.name.intern beta2 for ggaussian observations
         initial 0
         fixed FALSE
         prior normal
         param 0 10
         to.theta function(x) x
         from.theta function(x) x
    theta3
         hyperid 66503
```

```
name beta3
    short.name beta3
    output.name beta3 for ggaussian observations
    output.name.intern beta3 for ggaussian observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 66504
    name beta4
    short.name beta4
    output.name beta4 for ggaussian observations
    output.name.intern beta4 for ggaussian observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 66505
    name beta5
    short.name beta5
    output.name beta5 for ggaussian observations
    output.name.intern beta5 for ggaussian observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 66506
    name beta6
    short.name beta6
    output.name beta6 for ggaussian observations
    output.name.intern beta6 for ggaussian observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
```

```
to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 66507
    name beta7
    short.name beta7
    output.name beta7 for ggaussian observations
    output.name.intern beta7 for ggaussian observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 66508
    name beta8
    short.name beta8
    output.name beta8 for ggaussian observations
    output.name.intern beta8 for ggaussian observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 66509
    name beta9
    short.name beta9
    output.name beta9 for ggaussian observations
    output.name.intern beta9 for ggaussian observations
    initial 0
    fixed FALSE
    prior normal
    param 0 10
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 66510
    name beta10
    short.name beta10
    output.name beta10 for ggaussian observations
    output.name.intern beta10 for ggaussian observations
```

```
initial 0
          fixed FALSE
          prior normal
          param 0 10
          to.theta function(x) x
          from.theta function(x) x
survival FALSE
discrete FALSE
link default identity
link.simple default log
\operatorname{pdf} ggaussian
Example
n <- 1000
x <- rnorm(n)
xx <- rnorm(n)
eta \leftarrow 0.1 + 1.1 * x + 2.2 * xx
s <- runif(n)
z \leftarrow rnorm(n)
zz <- rnorm(n)</pre>
eta.prec \leftarrow 1 + 0.55 * z + 1.1 * zz
y <- eta + 1/sqrt(s * exp(eta.prec)) * rnorm(n)
Y <- inla.mdata(y, s, 1, z, zz)
r \leftarrow inla(Y ~1 + x + xx,
           data = list(Y = Y, x = x, xx = xx, z = z, zz = zz, s = s),
           family = "ggaussian")
summary(r)
```

Family ggaussianS

Link-function

This is the swapped version. The mean is given by a simple linear predictor

link.simple(
$$\mu$$
) = off + $\beta_1 z_1 + \beta_2 z_2 + \ldots + \beta_m z_m$, $m \le 10$

where "off" is a fixed offset and where link.simple is the *identity*-link by default. **Note:** there is no default intercept in the simple model, so it is common to set $z_1 = 1$. The precision is specified in the formula

$$link(\tau) = formula$$

using the log-link as default.

Hyperparameters

The hyperparameters in the model are $\theta_1 = \beta_1, \dots, \theta_m = \beta_m$, and the priors are defined on $\theta_1, \dots, \theta_m$.

Specification

- family="ggaussianS"
- Required arguments: **inla.mdata**-object that defines the response, the offset "off" and covariates. The **inla.mdata** object is defined as

inla.mdata
$$(y, \text{ off}, z_1, \ldots, z_m)$$

where each argument are vectors of the same length.

- Note: The offset argument MUST be given as it has no default value. Often we can just use off= 0 as it will auto-expand.
- Note: m=0 is allowed, which means that $\mu=$ off with with identity link.simple.

Hyperparameter spesification and default values

```
family="ggaussianS"

doc Generalized GaussianS

hyper
```

theta1

```
hyperid 66601
name beta1
short.name beta1
output.name beta1 for ggaussianS observations
output.name.intern beta1 for ggaussianS observations
initial 0
fixed FALSE
prior normal
param 0 0.001
to.theta function(x) x
from.theta function(x) x
```

```
theta2
    hyperid 66602
    name beta2
    short.name beta2
    output.name beta2 for ggaussianS observations
    output.name.intern beta2 for ggaussianS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 0.001
    to.theta function(x) x
    from.theta function(x) x
theta3
    hyperid 66603
    name beta3
    short.name beta3
    {\bf output.} {\bf name} \ {\tt beta3} \ {\tt for} \ {\tt ggaussianS} \ {\tt observations}
    output.name.intern beta3 for ggaussianS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 0.001
    to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 66604
    name beta4
    short.name beta4
    output.name beta4 for ggaussianS observations
    output.name.intern beta4 for ggaussianS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 0.001
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 66605
    name beta5
    short.name beta5
    output.name beta5 for ggaussianS observations
    output.name.intern beta5 for ggaussianS observations
    initial 0
    fixed FALSE
```

```
prior normal
    param 0 0.001
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 66606
    name beta6
    short.name beta6
    output.name beta6 for ggaussianS observations
    output.name.intern beta6 for ggaussianS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 0.001
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 66607
    name beta7
    short.name beta7
    output.name beta7 for ggaussianS observations
    output.name.intern beta7 for ggaussianS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 0.001
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 66608
    name beta8
    short.name beta8
    output.name beta8 for ggaussianS observations
    output.name.intern beta8 for ggaussianS observations
    initial 0
    fixed FALSE
    prior normal
    param 0 0.001
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 66609
    name beta9
    short.name beta9
```

```
output.name.intern beta9 for ggaussianS observations
         initial 0
         fixed FALSE
         prior normal
         param 0 0.001
         to.theta function(x) x
         from.theta function(x) x
     theta10
         hyperid 66610
         name beta10
         short.name beta10
         output.name beta10 for ggaussianS observations
         output.name.intern beta10 for ggaussianS observations
         initial 0
         fixed FALSE
         prior normal
         param 0 0.001
         to.theta function(x) x
         from.theta function(x) x
survival FALSE
discrete FALSE
link default log
link.simple default identity
pdf ggaussian
Example
n <- 1000
x \leftarrow rnorm(n)
xx <- rnorm(n)
off <- runif(n)
z <- rnorm(n)
zz <- rnorm(n)
mean \leftarrow off + 0.1 + 1.1 * z + 2.2 * zz
prec \leftarrow exp(1 + 0.55 * x + 1.1 * xx)
y <- mean + (1/sqrt(prec)) * rnorm(n)</pre>
Y <- inla.mdata(y, off, 1, z, zz)
r \leftarrow inla(Y ~1 + x + xx,
          data = list(Y = Y, off = off, x = x, xx = xx, z = z, zz = zz),
          family = "ggaussianS")
summary(r)
```

output.name beta9 for ggaussianS observations