

# The Classical Measurement Error (MEC) model

## Parametrization

This is an implementation of the classical ME model for a fixed effect. It is best described by an example, let the model be

$$\eta = \beta x + \epsilon$$

where  $\eta$  is the linear predictor,  $\beta$  the effect of the true covariate  $x$  with zero mean Gaussian noise  $\epsilon$ . The issue is that  $x$  is not observed directly, but only through  $w$ , where

$$w = x + u$$

where  $u$  is zero mean Gaussian noise. Even though this setup is possible to implement using basic features ("copy" and multiple likelihoods), we provide the following model which reparametrizes the above,

$$\eta = \nu + \epsilon$$

where  $\nu = \beta x$  has the correct distribution depending on various parameters:  $\beta$  has prior  $\pi(\beta)$ , and  $x$  is apriori  $\mathcal{N}(\mu_x I, \tau_x I)$ <sup>1</sup>. The error is apriori  $u \sim \mathcal{N}(0, \tau_u \mathbf{D})$ , where  $\tau_u$  is the observational precision of the error  $\text{Prec}(u|x)$  with possible heteroscedasticity, encoded in the entries  $d_i$  of the diagonal matrix  $\mathbf{D}$ . The vector  $s$  contains the fixed scalings  $s = (d_1, \dots, d_n)$  (with  $n$  the number of data points).

## Hyperparameters

This model has 4 hyperparameters,  $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$  where  $\theta_2$ ,  $\theta_3$  and  $\theta_4$  are default set to be fixed (ie defined to be known). The values of  $\theta_2$ ,  $\theta_3$  and  $\theta_4$  are set to mimic a classical fixed effect, so they will always make sense. To achieve the ME model, please use the appropriate choices for (some of) these parameters!

The hyperparameter specification is as follows:

$$\theta_1 = \beta$$

and the prior is defined on  $\theta_1$ ,

$$\theta_2 = \log(\tau_u)$$

and the prior is defined on  $\theta_2$ ,

$$\theta_3 = \mu_x$$

and the prior is defined on  $\theta_3$ ,

$$\theta_4 = \log(\tau_x)$$

and the prior is defined on  $\theta_4$ .

## Specification

The MEC is specified inside the `f()` function as

```
f(w, [<weights>], model="mec", scale = <s>, values= <w>, hyper = <hyper>)
```

The `w` are the observed values of the true but unknown covariates  $x$ , with the *assumption*, that if two or more elements of `w` are *identical*, then they refer to the *same* element in the true covariate  $x$ . If data points with identical  $w$  values belong to different  $x$  values (e.g., different individuals), please add a *tiny* random value to  $w$  to make this difference obvious to the model.

The fixed scaling of the observational precision is given in argument `scale`. If the argument `scale` is not given, then  $s$  is set to 1.

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<sup>1</sup>Note: The second argument in  $\mathcal{N}(,)$  is the precision not the variance.

## Hyperparameter specification and default values

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hyper

theta1

hyperid 2001  
name beta  
short.name b  
prior gaussian  
param 1 0.001  
initial 1  
fixed FALSE  
to.theta function(x) x  
from.theta function(x) x

theta2

hyperid 2002  
name prec.u  
short.name prec  
prior loggamma  
param 1 1e-04  
initial 9.21034037197618  
fixed TRUE  
to.theta function(x) log(x)  
from.theta function(x) exp(x)

theta3

hyperid 2003  
name mean.x  
short.name mu.x  
prior gaussian  
param 0 1e-04  
initial 0  
fixed TRUE  
to.theta function(x) x  
from.theta function(x) x

theta4

hyperid 2004  
name prec.x  
short.name prec.x  
prior loggamma  
param 1 10000  
initial -9.21034037197618  
fixed TRUE  
to.theta function(x) log(x)  
from.theta function(x) exp(x)

```

constr FALSE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values FALSE

pdf mec

```

## Example

```

n = 100
beta = 4
prec.y = 1
prec.u = 1
prec.x = 1
## true unobserved covariate
x = rnorm(n, sd = 1/sqrt(prec.x))
## the observed covariate with heteroscedastic scaling
s = runif(n,min=0.5,max=2)
w = x + rnorm(n, sd = 1/sqrt(prec.u*s))
## regression model using the unobserved 'x'
y = 1 + beta*x + rnorm(n, sd = 1/sqrt(prec.y))

## prior parameters
prior.beta = c(0, 0.0001)
prior.prec.u = c(10, 9)
prior.prec.x = c(10, 9)
prior.prec.y = c(10, 9)

formula = y ~ 1 +
f(w, model="mec", scale=s, values=w,
  hyper = list(
    beta = list(
      prior = "gaussian",
      param = prior.beta,
      fixed = FALSE
    ),
    prec.u = list(
      prior = "loggamma",
      param = prior.prec.u,
      initial = log(prec.u),
      fixed = FALSE
    ),
  ),

```

```

    prec.x = list(
      prior = "loggamma",
      param = prior.prec.x,
      initial = log(prec.x),
      fixed = FALSE
    ),
    mean.x = list(
      prior = "gaussian",
      initial = 0,
      fixed=TRUE
    )
  )
)

r = inla(formula,
  data = data.frame(y, w, s),
  family = "gaussian",
  control.family = list(
    hyper = list(
      prec = list(param = prior.prec.y,
        initial = log(prec.y),
        fixed=FALSE
      )
    )
  )
)

summary(r)

```

## Notes

- INLA provides the posteriors of  $\nu_i = \beta x_i$  and NOT  $x_i$ .
- The posteriors of  $\nu_i$  come (default) in the order given by the sorted (from low to high) values of **w**. The entry **\$ID** gives the mapping.
- The option **scale** defines the scaling in the same order as argument **values**. It is therefore adviced to also give argument **values** when **scale** is used to be sure that they are consistent.