

MA226 : Monte-Carlo Simulation
Generating Standard Normal Distribution
Assignment 6

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1 Problem 1

We have to generate sample from standard normal distribution using Box-Muller Transform and Marsaglia-Bray Method.

1.1 Source code of the solution

```
n<-100

string<-"hund"

#Box-Muller
u1<-runif(n)
u2<-runif(n)

z1<-sqrt(-2*log(u1))*cos(2*pi*u2)
z2<-sqrt(-2*log(u1))*sin(2*pi*u2)

sample1<-c(z1,z2)

png(paste("que1_",string,"1.png"))
hist(sample1,breaks=50,col="cyan",plot=TRUE)

#Masaglia-Bray
i<-1

r1<-vector(length=n)
r2<-vector(length=n)

while(i<=n){
  u1<-runif(1)
  u2<-runif(1)

  u1<-2*u1-1
  u2<-2*u2-1

  x<-u1*u1+u2*u2
  if(x < 1){
    temp<-sqrt(-2*log(x)/x)
    r1[i]<-u1*temp
    r2[i]<-u2*temp
    i<-i+1
  }
}

sample2<-c(r1,r2)

png(paste("que1_",string,"2.png"))
hist(sample2,breaks=50,col="cyan",plot=TRUE)
```

1.2 Generated Histograms

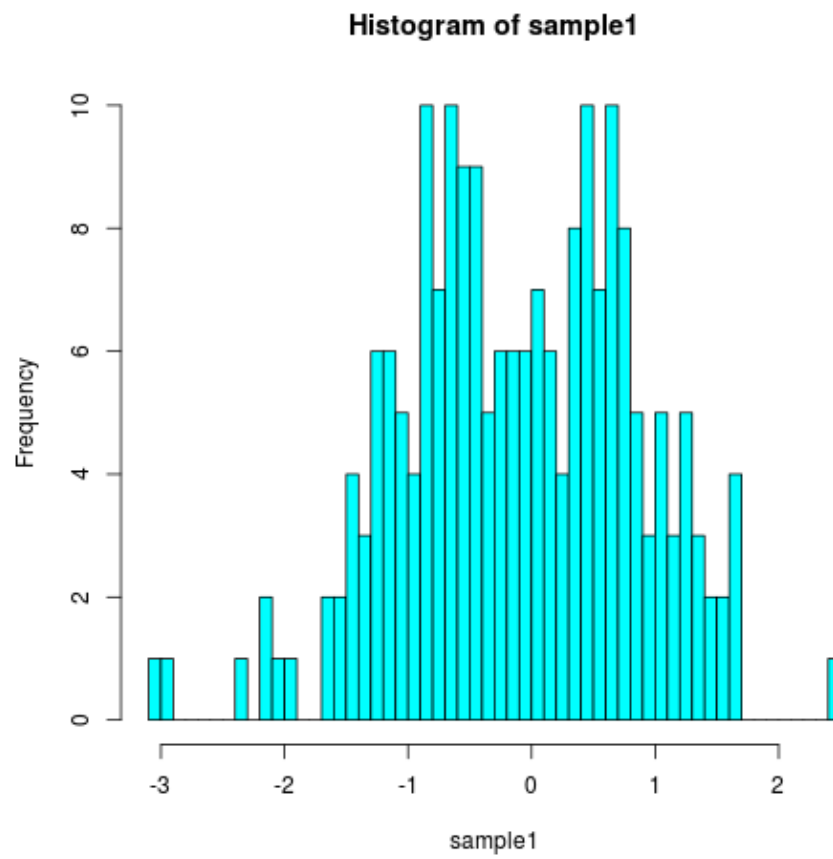


Abbildung 1: Using Box-Muller for 100 sample

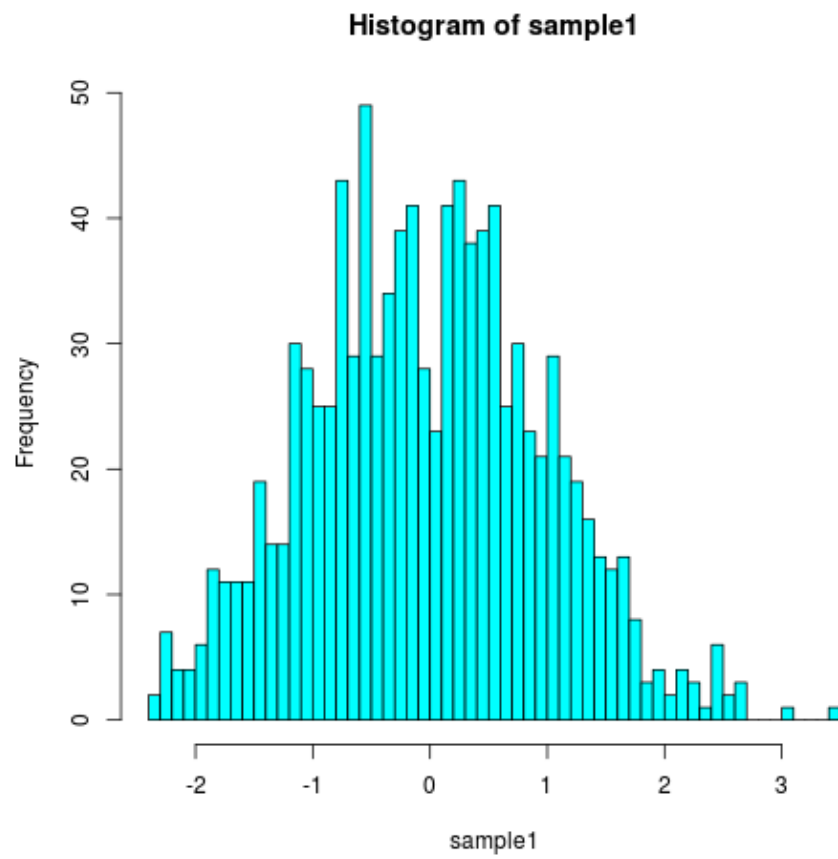


Abbildung 2: Using Box-Muller for 500 sample

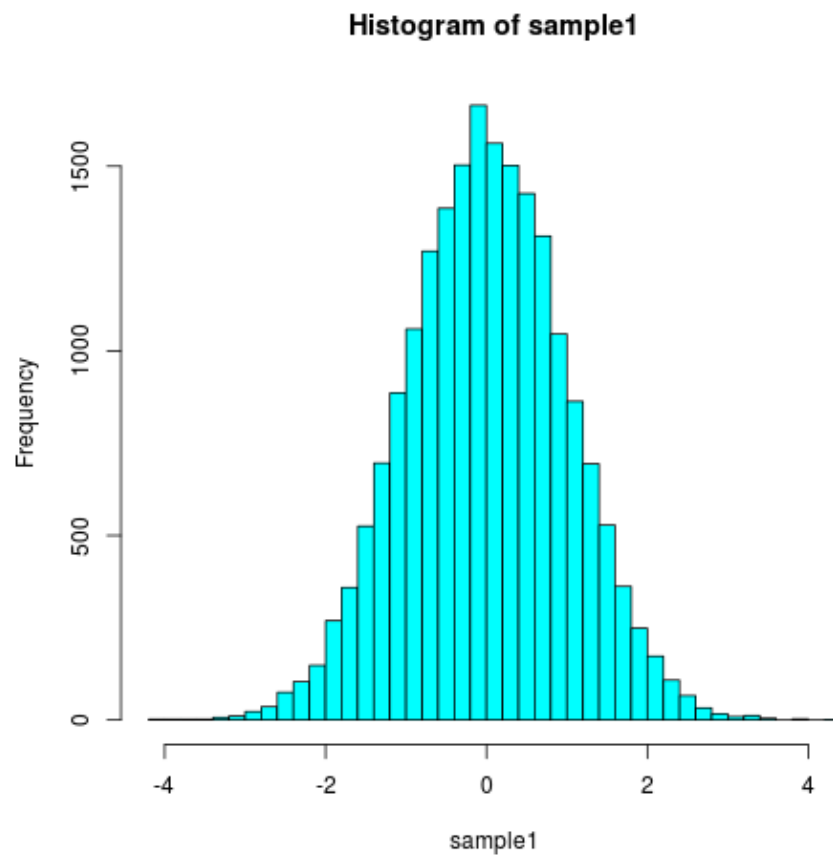


Abbildung 3: Using Box-Muller for 10000 sample

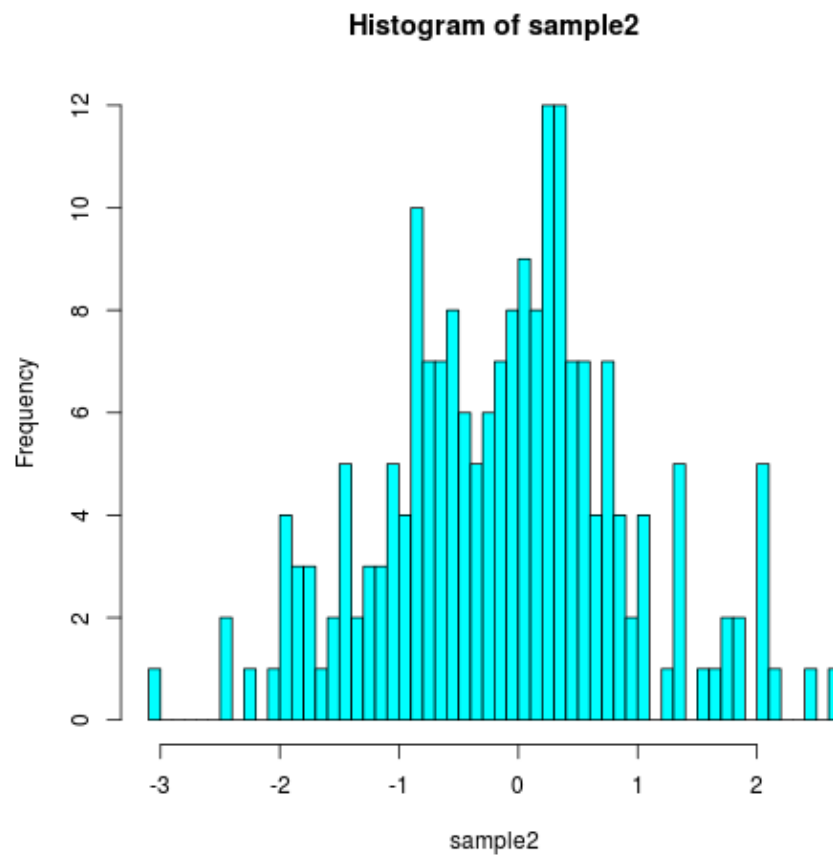


Abbildung 4: Using Marsaglia-Bray for 100 sample

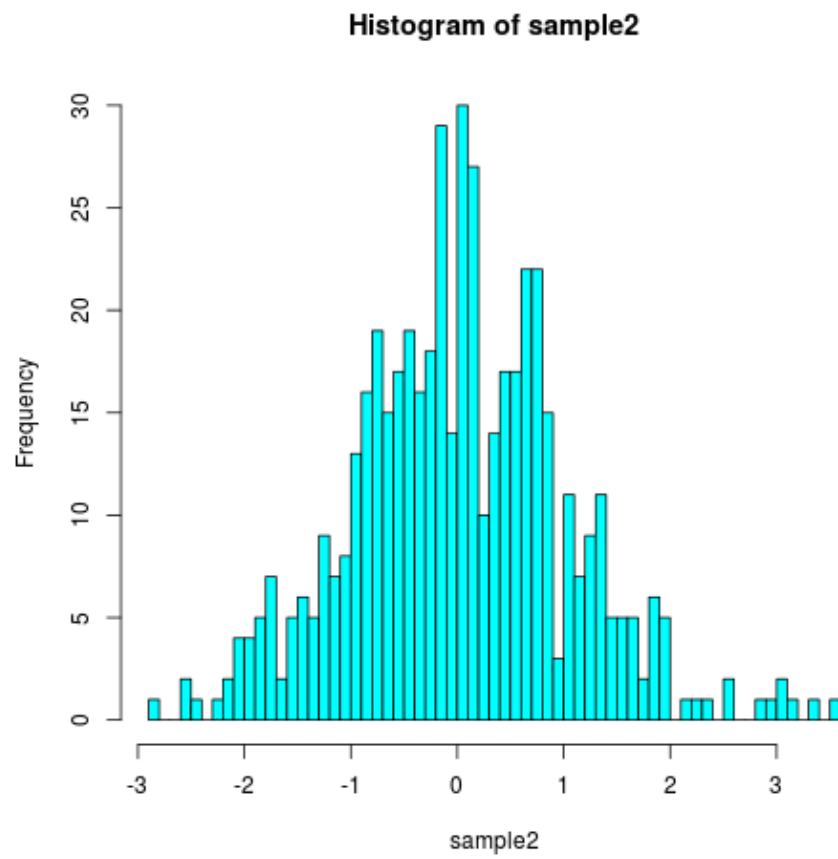


Abbildung 5: Using Marsaglia-Bray for 500 sample

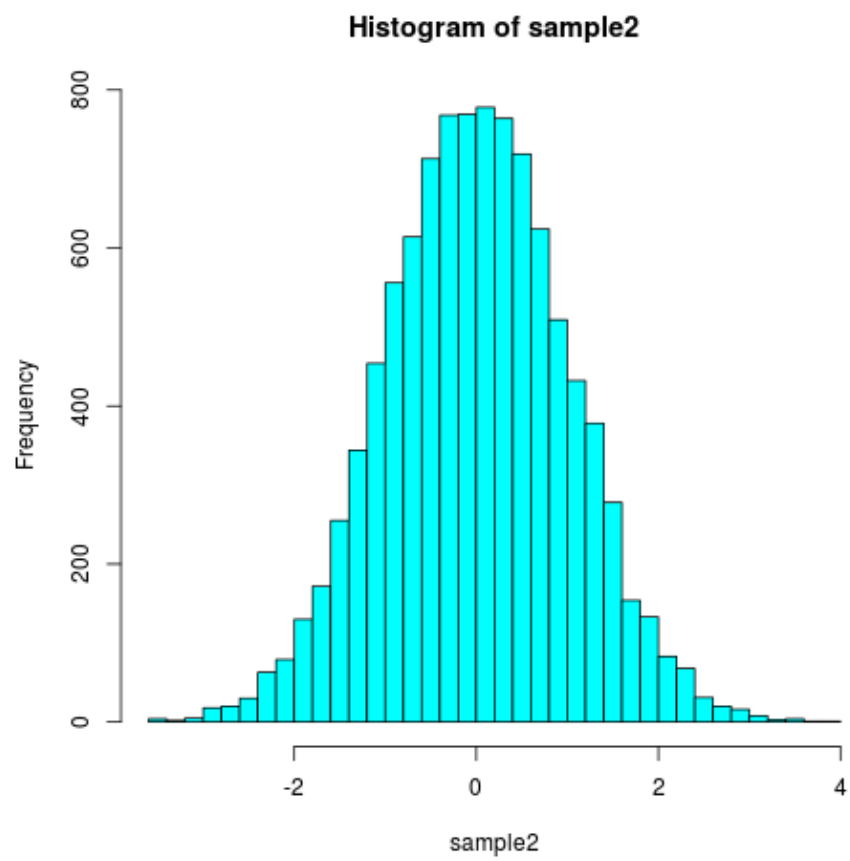


Abbildung 6: Using Marsaglia-Bray for 10000 sample

2 Problem 2

In this problem we have to plot emirical and theoritical distribution of $N(0, 5)$ and $N(5, 5)$.

2.1 Source code generating Empirical and Theoretical Distributions

```
n<-100

string<-"ten-thou"

#Box-Muller
u1<-runif(n)
u2<-runif(n)

z1<-sqrt(-2*log(u1))*cos(2*pi*u2)
z2<-sqrt(-2*log(u1))*sin(2*pi*u2)

sample1<-c(z1, z2)

samplex<-sqrt(5)*sample1
sampley<-sqrt(5)*sample1+5

png("que2_one.png")
plot(ecdf(samplex), col="red")
par(new=TRUE)
samplex<-sort(samplex)
plot(pnorm(samplex, mean=0, sd=sqrt(5)), col="cyan")

png("que2_two.png")
plot(ecdf(sampley), col="red")
par(new=TRUE)
sampley<-sort(sampley)
plot(pnorm(sampley, mean=5, sd=sqrt(5)), col="cyan")
```

2.2 Generated Graphs

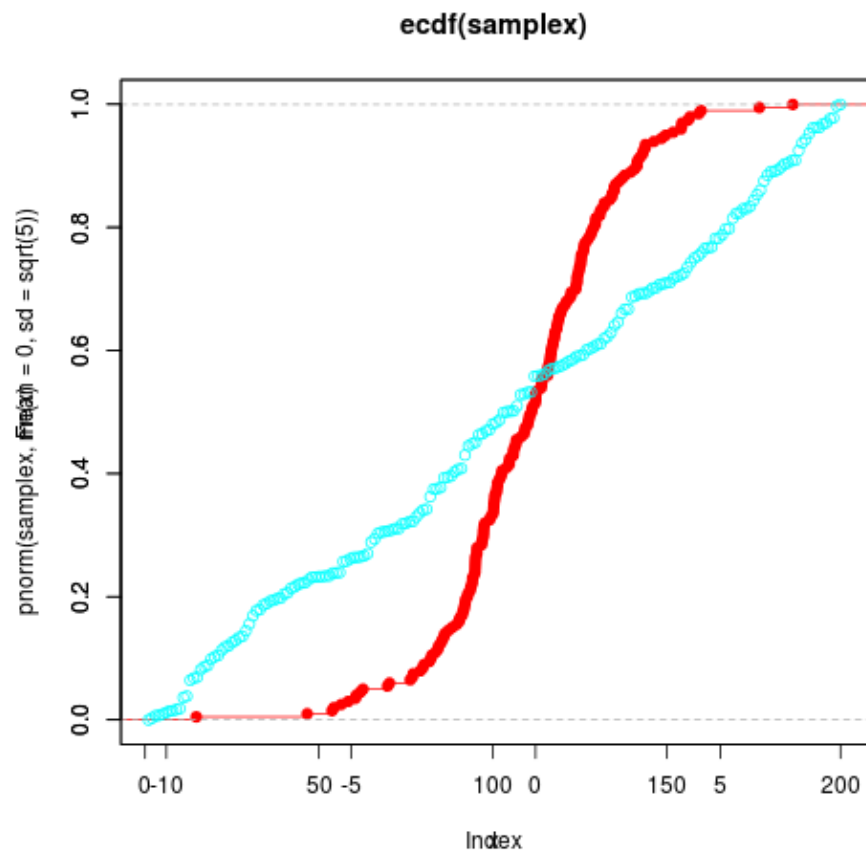


Abbildung 7: Plot of Emirical and Theoritical distribution of $N(0, 5)$

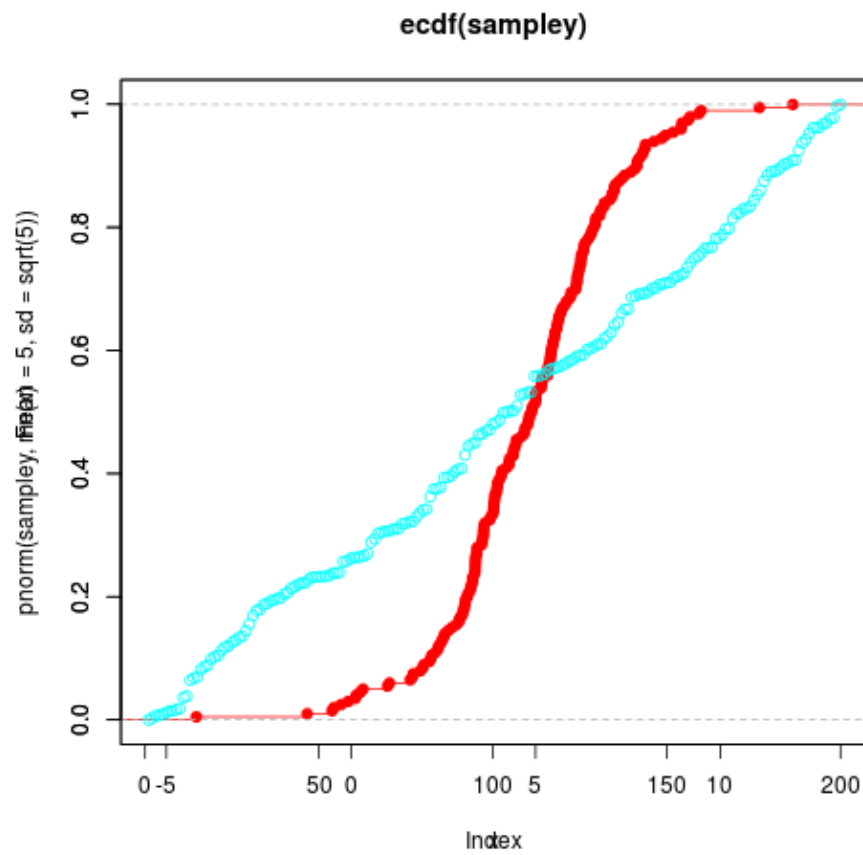


Abbildung 8: Plot of Emirical and Theoritical distribution of $N(5, 5)$

3 Problem 3

In this problem we have to calculate the time taken to calculate Standard Normal distribution using Box-Muller and Marsaglia-Bray Methods.

3.1 Source code for Box-Muller

```
radius<-function(u){
  return (-2*log(u))
}
arg<-function(u){
  return (2*pi*u)
}
n<-10000
stime<-Sys.time();
sample1<-c()
sample2<-c()
for(i in 1:n/2){
  u1<-runif(1)
  u2<-runif(1)
  sample1[i]<-sqrt(radius(u1))*cos(arg(u2))
  sample2[i]<-sqrt(radius(u1))*sin(arg(u2))
}

sample<-c(sample1, sample2)
etime<-Sys.time();
cat("Computation Time of Box-Muller Method: ", etime-stime, "\n");
```

3.2 Source code for Marsaglia-Bray

```
square<-function(u1, u2){
  return (u1**2+u2**2)
}
temp<-function(x){
  return (sqrt(-2*log(x)/x))
}

n<-10000
stime<-Sys.time();
sample1<-c()
sample2<-c()
i<-1
while(n>=0){
  u1<-runif(1)
  u2<-runif(1)
  u1<-2*u1-1
  u2<-2*u2-1
  x<-square(u1, u2)
  if(x>1){
    next
  }
  y<-temp(x)
  sample1[i]<-u1*y
  sample2[i]<-u2*y
  i=i+1
  n=n-1
}
```

```

        i<-i+1
        n<-n-2
    }
    sample<-c(sample1, sample2)
    etime<-Sys.time();
    cat("Computation Time of Marsaglia-Bray Method: ", etime-stime, "\n");

```

3.3 Analysis

Time taken for Box-Muller: 0.0003612041

Time taken for Masaglia-Bray: 0.002064705

We can clearly see that Box-Muller takes more time than Marsaglia Bray

4 Problem 4

In this section we calculate number of rejections proportional to total sampled uniform random variables.

4.1 Source code for Solution

```

square<-function(u1,u2){
    return (u1**2+u2**2)
}
temp<-function(x){
    return (sqrt(-2*log(x)/x))
}

rejected<-0
n<-10000
ct<-n
stime<-Sys.time();
sample1<-c()
sample2<-c()
i<-1
while(n>=0){
    u1<-runif(1)
    u2<-runif(1)
    u1<-2*u1-1
    u2<-2*u2-1
    x<-square(u1,u2)
    if(x>1){
        rejected<-rejected+1
        next
    }
    y<-temp(x)
    sample1[i]<-u1*y
    sample2[i]<-u2*y
    i<-i+1
    n<-n-2
}

```

```
cat("Rejection_Percentage:", rejected/(rejected+(ct/2)), "\n")
```

4.2 Analysis

Here, rejection probability comes as 0.2092361 which is quite close to $1 - \frac{p^i}{4} = 0.2146$