MA226 : Monte-Carlo Simulation
Geometric Brownian Motion

Assignment 11

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1 Problem 1

We have to generate 10 sample paths from geometric brownian motion and plot them for various values of μ and σ.

$$S(t_{i+1}) = S(t_i)e^{((t_{i+1}-t_i)[\mu-\frac{\sigma^2}{2}]+\sigma\sqrt{t_{i+1}-t_i}.Z_{i+1})}$$

Values of μ used are 0.06 and -0.06. Values of σ used are 0.3 and 0.4.

1.1 Source code of the solution

```
mean<-c(0.06,-0.06)
sigma<-c(0.3,0.4)

interval<-c(0,5)
n<-10
k<-5000

division<-(interval[2]-interval[1])/k
sqr_division<-division^(1/2)

for(mu in mean){
        for(sd in sigma){

                paths<-matrix(nrow=k,ncol=n)

                for(i in 1:n){

                        paths[1,i]<-5

                        for(j in 2:k){
                                paths[j,i]<-paths[j-1,i]*
                                exp((mu-(sd^2)/2)*division+
                                        sd*sqr_division*rnorm(1) )
                        }
                }

                filename<-paste("que1_",mu,"_",sd,".png")
                png(filename)
                matplot(paths,col=1:10,type='l')
        }
}
```

1.2 Plots

In the following plots, each color represents unique brownian path.

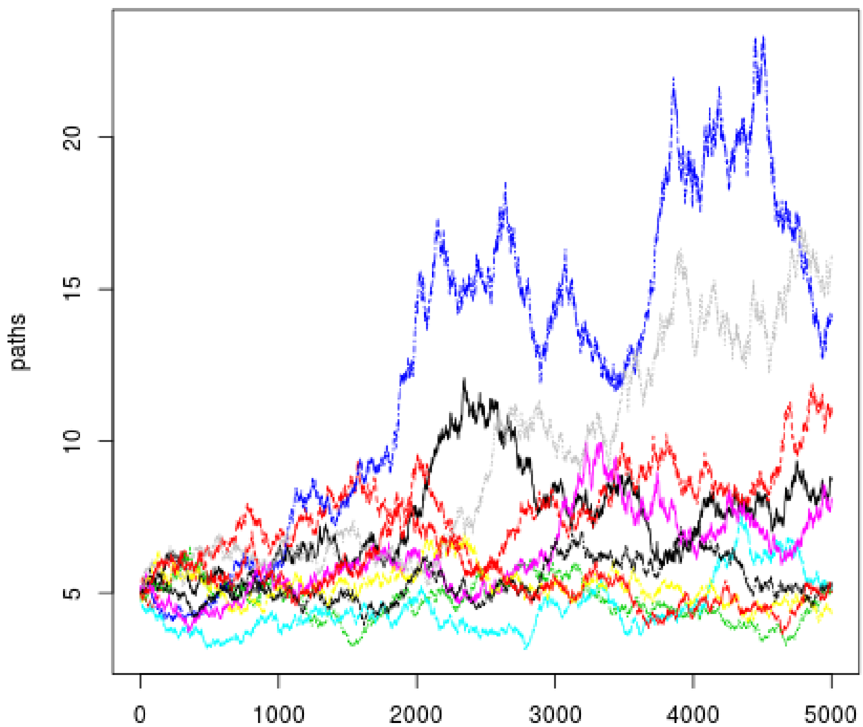

Abbildung 1: Paths for Geometric Brownian Motion for $\mu = 0.06$ and $\sigma = 0.3$.

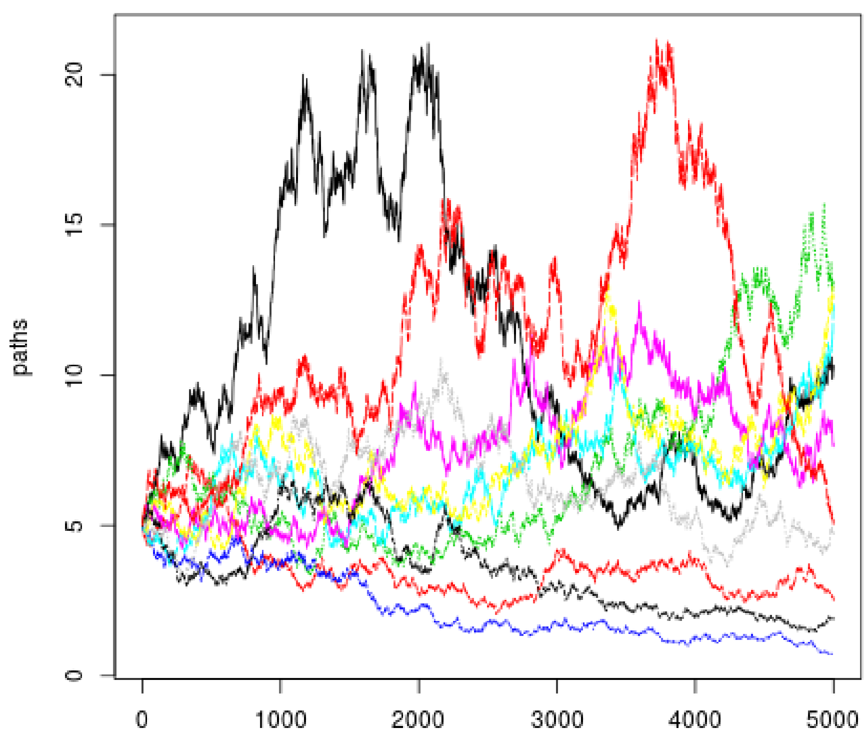

Abbildung 2: Paths for Geometric Brownian Motion for $\mu = 0.06$ and $\sigma = 0.3$.

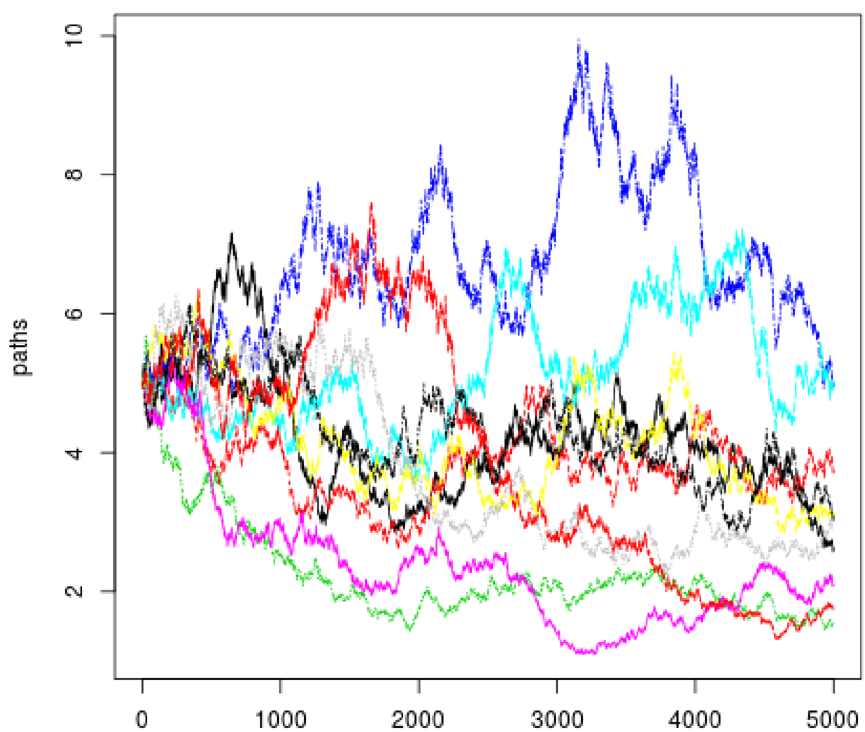

Abbildung 3: Paths for Geometric Brownian Motion for $\mu = -0.06$ and $\sigma = 0.4$.

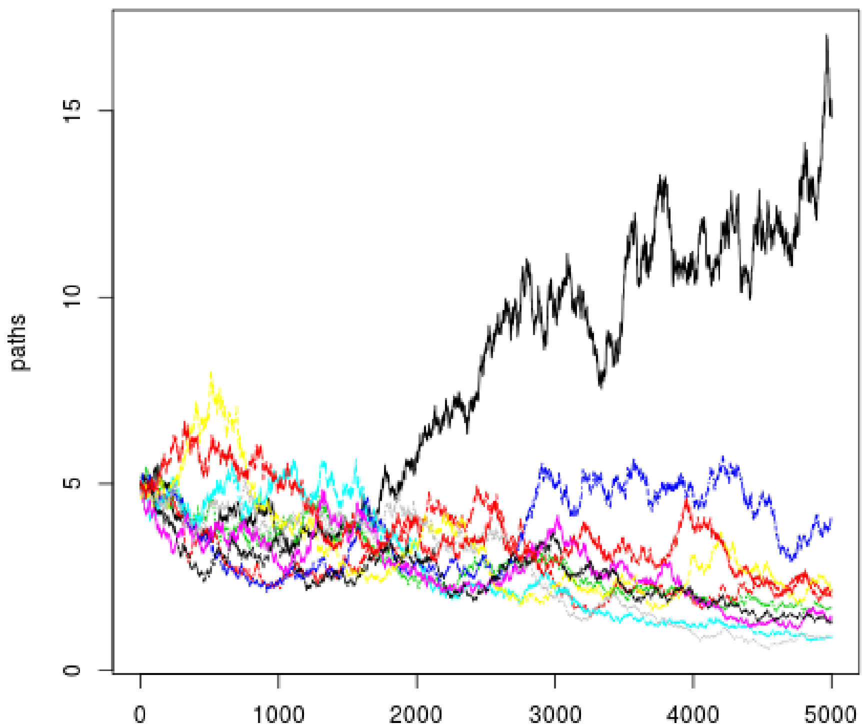

Abbildung 4: Paths for Geometric Brownian Motion for $\mu = -0.06$ and $\sigma = 0.4$.

2 Problem 2

We have to generate 1000 sample paths from geometric brownian motion and verify observed expectation and vaiance and theoritical expection and variance.

2.1 Source code of the solution

```
mean<-c(0.06,-0.06)
sigma<-c(0.3,0.4)

interval<-c(0,5)
n<-1000
k<-5000

division<-(interval[2]-interval[1])/k
sqr_division<-division^(1/2)

theoEx<-function(s0,mu,t){
        return(s0*exp(mu*t))
}

theoVar<-function(s0,mu,sd,t){
        return(s0*s0*exp(2*mu*t)*(exp(sd*sd*t)-1))
}

for(mu in mean){
        for(sd in sigma){

                paths<-matrix(nrow=k,ncol=n)

                for(i in 1:n){

                        paths[1,i]<-5

                        for(j in 2:k){
                                paths[j,i]<-paths[j-1,i]*
                                exp((mu-((sd^2)/2))*division+
                                        sd*sqr_division*rnorm(1) )
                        }
                }
                cat("For mu=",mu,"and sigma=",sd,"\n")
                cat("Observed Expectation = ",mean(paths[5000,]),"\n")
                cat("Observed Variance = ",var(paths[5000,]),"\n")
                cat("Theoritical Expection = ",theoEx(5,mu,5),"\n")
                cat("Theoritical Variance = ",theoVar(5,mu,sd,5),"\n\n")
        }
}
```

2.2 Observation

For mu= 0.06 and sigma= 0.3
Observed Expectation = 6.800481
Observed Variance = 29.58148
Theoritical Expection = 6.749294
Theoritical Variance = 25.88831

For mu= 0.06 and sigma= 0.4
Observed Expectation = 6.822147
Observed Variance = 47.73617
Theoritical Expection = 6.749294
Theoritical Variance = 55.82703

For mu= -0.06 and sigma= 0.3
Observed Expectation = 3.678885
Observed Variance = 7.376168
Theoritical Expection = 3.704091
Theoritical Variance = 7.797409

For mu= -0.06 and sigma= 0.4
Observed Expectation = 3.665376
Observed Variance = 14.21604
Theoritical Expection = 3.704091
Theoritical Variance = 16.81478