MA226 : Monte-Carlo Simulation
Variation Reduction Techniques
Assignment 8

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1 Problem 1

We have to use Monte Carlo Estimator for random variable $I = E[e^{U^{1/2}}]$

1.1 Source code of the solution

```
e<-c(2,3,4,5)
for(x in e){
        cat("\nSample Size: ",x,"\n")
        M<-10^x

        u1<-runif(M)
        u2<-runif(M)

        sample<-(exp(u1^(1/2))+exp(u1^(1/2)))/2

        cat("Mean without-antithetic : ",mean(sample),"\n")
        cat("Variance without-antithetic : ",var(sample),"\n")

        var_original<-var(sample)


        u1<-runif(M)
        u2<-1-u1

        sample1<-exp(u1^(1/2))
        sample2<-exp(u2^(1/2))
        sample<-(sample1+sample2)/2

        mean_sample<-mean(sample)
        var_sample<-var(sample)

        cat("Mean of Y : ",mean(sample),"\n")
        cat("Variance of Y : ",var(sample),"\n")

        cat("Percentage reduction in Variance: ",
                (var_original-var(sample))/(var_original)*100," %\n")
        alpha<-0.05
        x<-mean_sample-qnorm(1-alpha/2)*var_sample/sqrt(M)
        y<-mean_sample+qnorm(1-alpha/2)*var_sample/sqrt(M)
        cat("Confidence Interval: (",x," , ",y,")","\n")
}
```

1.2 Observation

Sample Size: 2
Mean: 1.967482
Variance: 0.1850384
Confidence Interval: (1.931215 , 2.003749)

Sample Size: 3
Mean: 1.995675
Variance: 0.2132864

Confidence Interval: (1.982456 , 2.008894)

Sample Size: 4
Mean: 1.994284
Variance: 0.1934042
Confidence Interval: (1.990494 , 1.998075)

Sample Size: 5
Mean: 2.00165
Variance: 0.1938415
Confidence Interval: (2.000448 , 2.002851)

2 Problem 2

We have to to estimate following random variable using antithetic variates.

. The given random variable $Y_i = \frac{e^{{U_i}^{1/2}} + e^{1-U_i^{1/2}}}{2}$

2.1 Source code of the solution

```
e<-c(2,3,4,5)
for(x in e){
        cat("\nSample Size: ",x,"\n")
        M<-10^x

        u1<-runif(M)
        u2<-runif(M)

        sample<-(exp(u1^(1/2))+exp(u1^(1/2)))/2

        cat("Mean without-antithetic : ",mean(sample),"\n")
        cat("Variance without-antithetic : ",var(sample),"\n")

        var_original<-var(sample)


        u1<-runif(M)
        u2<-1-u1

        sample1<-exp(u1^(1/2))
        sample2<-exp(u2^(1/2))
        sample<-(sample1+sample2)/2

        mean_sample<-mean(sample)
        var_sample<-var(sample)

        cat("Mean of Y : ",mean(sample),"\n")
        cat("Variance of Y : ",var(sample),"\n")

        cat("Percentage reduction in Variance: ",
                (var_original-var(sample))/(var_original)*100," %\n")
        alpha<-0.05
        x<-mean_sample-qnorm(1-alpha/2)*var_sample/sqrt(M)
        y<-mean_sample+qnorm(1-alpha/2)*var_sample/sqrt(M)
        cat("Confidence Interval: (",x," , ",y,")","\n")
}
```

2.2 Observation

Sample Size: 2
Mean without-antithetic : 2.022732
Variance without-antithetic : 0.1862374
Mean of Y : 1.996541
Variance of Y : 0.001562767
Percentage reduction in Variance: 99.16087
Confidence Interval: (1.996235 , 1.996847)

Sample Size: 3
Mean without-antithetic : 2.014986
Variance without-antithetic : 0.1968573
Mean of Y : 2.000812
Variance of Y : 0.001030568
Percentage reduction in Variance: 99.47649
Confidence Interval: (2.000748 , 2.000876)

Sample Size: 4
Mean without-antithetic : 1.992309
Variance without-antithetic : 0.1980112
Mean of Y : 2.000028
Variance of Y : 0.001067097
Percentage reduction in Variance: 99.46109
Confidence Interval: (2.000007 , 2.000049)

Sample Size: 5
Mean without-antithetic : 2.000763
Variance without-antithetic : 0.1935972
Mean of Y : 2.000117
Variance of Y : 0.00106421
Percentage reduction in Variance: 99.4503
Confidence Interval: (2.000111 , 2.000124)

3 Problem 3

We have to to estimate above given random variable using control variate estimate. We take control variate estimate as $U^{1/2}$.

3.1 Source code of the solution

```
#control variate

e<-c(2,3,4,5)
for(x in e){
        cat("\nSample Size: ",x,"\n")

        M<-10^x
        #simulation to estimate correlation
        u1<-sqrt(runif(M))
        u2<-exp(u1)
        c<-(cov(u1,u2))/var(u1)

        #variance reduction
        #sam<-exp(sqrt(u1))
        w<-u2 + c*(0.66 - u1)

        cat("Mean of sample : ",mean(w),"\n")
        cat("Variance of sample : ",var(w),"\n")

        cat("Percentage reduction in Variance: ",
                (var(u2)-var(w))/(var(u2))*100," %\n")

        mean_sample<-mean(w)
        var_sample<-var(w)
        alpha<-0.05
        x<-mean_sample-qnorm(1-alpha/2)*var_sample/sqrt(M)
        y<-mean_sample+qnorm(1-alpha/2)*var_sample/sqrt(M)
        cat("Confidence Interval: (",x," , ",y,")","\n")
}
```

3.2 Observation

Sample Size: 2
Mean of sample : 1.990714
Variance of sample : 0.003255635
Percentage reduction in Variance: 98.41249
Confidence Interval: (1.990076 , 1.991352)

Sample Size: 3
Mean of sample : 1.988595
Variance of sample : 0.002865602
Percentage reduction in Variance: 98.5398
Confidence Interval: (1.988417 , 1.988772)

Sample Size: 4
Mean of sample : 1.986998
Variance of sample : 0.002730015
Percentage reduction in Variance: 98.58755
Confidence Interval: (1.986945 , 1.987052)

Sample Size: 5
Mean of sample : 1.98782
Variance of sample : 0.002717987
Percentage reduction in Variance: 98.60639
Confidence Interval: (1.987803 , 1.987837)