# MA226 : Monte-Carlo Simulation
# Brownian Motion
Assignment 10

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# 1 Problem 1

We have to generate 10 sample paths from standard brownian motion.

$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i}.Z_{i+1}$$

## 1.1 Source code of the solution

```
interval<-c(0,5)
n<-10
paths<-matrix(nrow=5000,ncol=10)
k<-5000

division<-(interval[2]-interval[1])/5000

for(i in 1:n){
        path<-vector(length=k)

        paths[1,i]=0

        for(j in 2:k){
                paths[j,i]=paths[j-1,i]+sqrt(division)*rnorm(1)
        }
}

cat("Expected Value of w(2) : ",mean(paths[2000,]),"\n")
cat("Expected Value of w(5) : ",mean(paths[5000,]),"\n")

png("que1_single.png")
matplot (paths,col=1:10,type='l')
```

## 1.2 Observation

Expected Value of w(2) : -0.06535669
Expected Value of w(5) : 1.021281

## 1.3 Plots

In the following plots, each color represents unique brownian path.

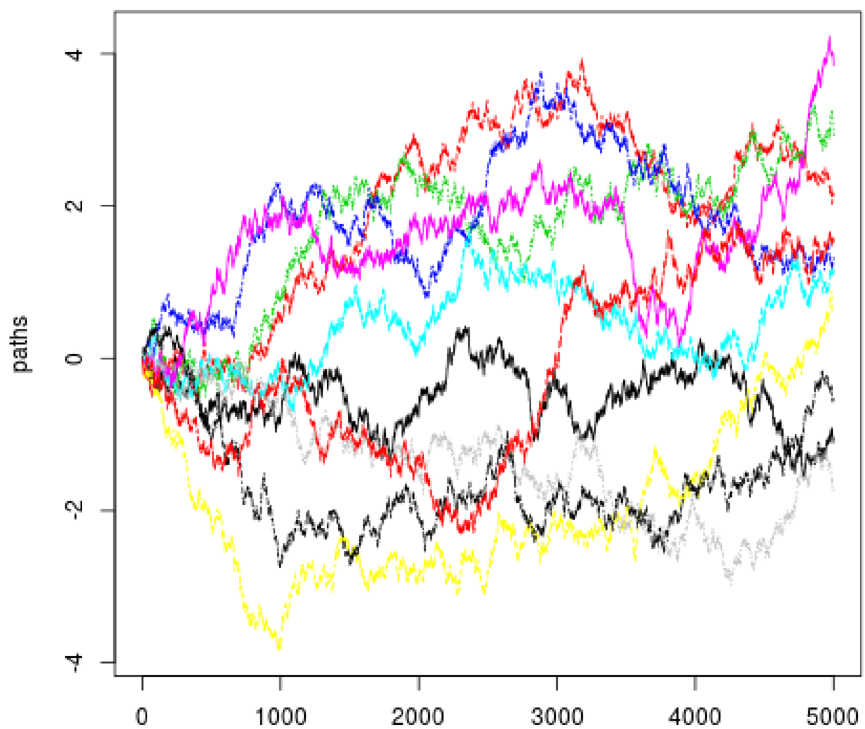


Abbildung 1: Paths for standard Brownian Motion.

# 2 Problem 2

We have to generate 10 sample paths from brownian motion with $mean = 0.06$ and $sigma = 0.3$.

$$X(t_{i+1}) = X(t_i) + \mu(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i}.Z_{i+1}$$

## 2.1 Source code of the solution

```
interval<-c(0,5)
n<-10
paths<-matrix(nrow=5000,ncol=10)
k<-5000

division<-(interval[2]-interval[1])/5000
mean<-0.06
sigma<-0.3

for(i in 1:n){
        path<-vector(length=k)

        paths[1,i]=5

        for(j in 2:k){
                paths[j,i]=paths[j-1,i]+ mean*division +sigma*sqrt(division)*rnorm(1)
        }
}

cat("Expected Value of w(2) : ",mean(paths[2000,]),"\n")
cat("Expected Value of w(5) : ",mean(paths[5000,]),"\n")

png("que2.png")
matplot(paths,col=1:10,type='l')
```

## 2.2 Observation

Expected Value of w(2) : 4.818397
Expected Value of w(5) : 4.986213

## 2.3 Plots

In the following plots, each color represents unique brownian path.

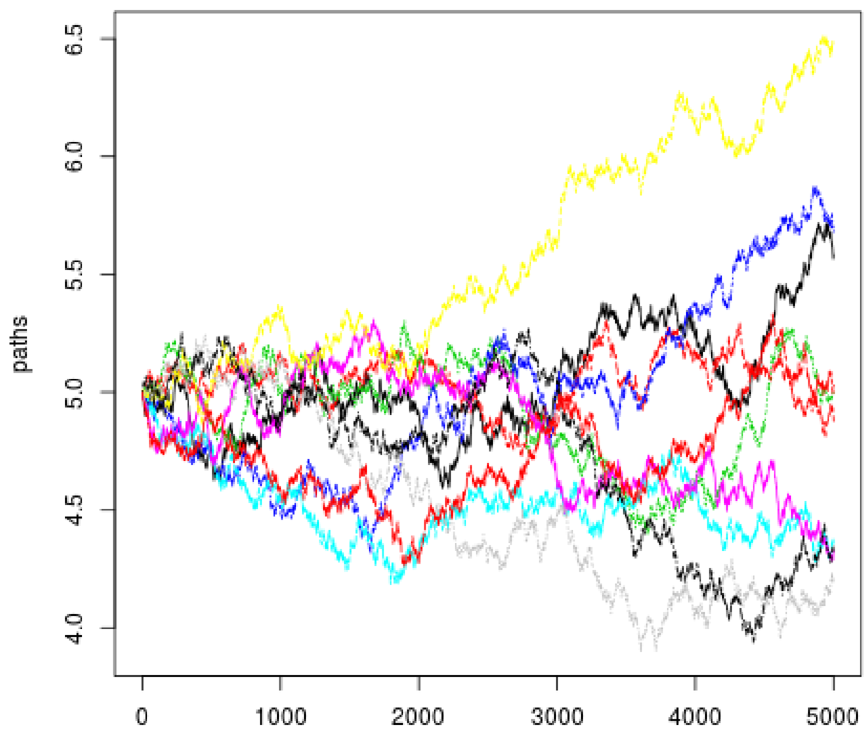


Abbildung 2: Paths for standard Brownian Motion.

# 3 Problem 3

We have to generate 10 sample paths from brownian motion with $mu(t) = 0.0325 - 0.05t$ and $sigma(t) = 0.012 + 0.0138t + 0.00125t^2$.

$$Y(t_{i+1}) = Y(t_i) + \mu(t_i)(t_{i+1} - t_i) + \sigma(t_i)\sqrt{t_{i+1} - t_i}.Z_{i+1}$$

## 3.1 Source code of the solution

```
interval<-c(0,5)
n<-10
paths<-matrix(nrow=5000,ncol=10)
k<-5000

division<-(interval[2]-interval[1])/5000


mu<-function(x){
        return(0.0325 - 0.05*x)
}

sig<-function(x){
        return(0.012+0.0138*x+0.00125*(x^2))
}

for(i in 1:n){

        paths[1,i]=5

        for(j in 1:(k-1)){
                paths[j+1,i]<-paths[j,i]+ mu((j-1)*division)*division +
                        sig((j-1)*division)*sqrt(division)*rnorm(1)
        }
}

cat("Expected Value of w(2) : ",mean(paths[2000,]),"\n")
cat("Expected Value of w(5) : ",mean(paths[5000,]),"\n")

png("que3.png")
matplot(paths,col=1:10,type='l')
```

## 3.2 Observation

Expected Value of w(2) : 4.97063
Expected Value of w(5) : 4.586572

## 3.3 Plots

In the following plots, each color represents unique brownian path.

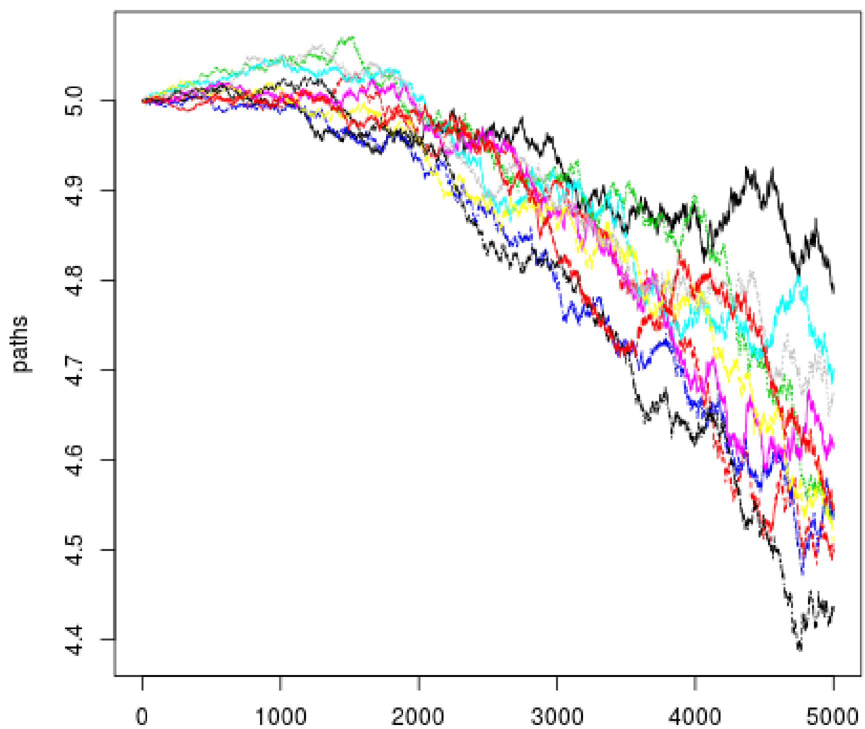


Abbildung 3: Paths for given Brownian Motion.