DATA130026 Optimization
Assignment 11
Due Time: at the beginning of the class, Jun. 8, 2021

1. For each of the following functions on $\mathbb{R}^n$, explain how to calculate a subgradient at a given x.

   (a) $f(x) = \sup_{0\le t\le 1} p(t)$, where $p(t) = x_1 + x_2 t + \ldots + x_n t^{n-1}$.

   (b) $f(x) = x_{[1]} + x_{[2]} + \ldots + x_{[k]}$, where $x_{[i]}$ denotes the ith largest elements of x.

   (c) $f(x) = \|Ax - b\|_2 + \|x\|_2$ where $A \in \mathbb{R}^{m\times n}$.

2. Under the same notations in lecture slides, prove that with diminishing but non-summable step size $\alpha_i = \frac{R}{G\sqrt{i}}$, we have

$$f_{bs}^k - f^* \le O(\frac{RG}{\sqrt{k}}).$$

   (Hint: Use $f_{bs}^k \le \bar{f}_{bs}^k$, where $\bar{f}_{bs}^k = \min_{i=k/2,\ldots,k} f(x_k)$.)

3. Write a MATLAB code for solving the Lasso problem using subgradent method:

$$\min_{x\in\mathbb{R}^n} \frac{1}{2}\|Ax - b\|_2^2 + \tau\|x\|_1$$

   where $\tau > 0$ is a weighting parameter, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^n$ are given data. Choose $x = 0$ as the starting point. Terminate your code after 10000 iterations. Use the following Matlab code to generate the data:

```
m = 100; n = 500; s = 50;
A = randn(m,n);
xs = zeros(n,1); picks = randperm(n); xs(picks(1:s)) = randn(s,1);
b = A*xs;tau=0.001;
```

   Use constant step size, constant step length, dimnishing step size and Polyak's step size. Try three different constants or parameter for constant step size, constant step length and dimnishing step size. Plot fours figures to show the evolutions for `f(xk)-f*` (the optimal value can be computed by CVX) for the four step size rules.