DATA130026 Optimization
Assignment 3
Due Time: at the beginning of the class, Apr. 6, 2021

For each of the following optimization problems (i) show that it is **convex**, (ii) write a CVX **code** that solves it, and (iii) write down the **optimal solution and optimal value** (by running CVX). You need to write all the above step and publish your codes (using "publish" in MATLAB) in a PDF file. You should upload an electronic version on elearning.

1.

$$\begin{aligned} \min \quad & x_1^2 + 2x_1x_2 + 2x_2^2 + x_3^2 + 3x_1 - 4x_2 \\ \text{s.t.} \quad & \sqrt{2x_1^2 + x_1x_2 + 4x_2^2 + 4} + \frac{(x_1 - x_2 + x_3 + 1)^2}{x_1 + x_2} \le 6 \\ & x \ge 1 \end{aligned}$$

2.

$$\begin{aligned} \min \quad & x_1 + x_2 + x_3 + x_4 \\ \text{s.t.} \quad & (x_1 - x_2)^2 + (x_3 + 2x_4)^4 \le 5 \\ & x_1 + 2x_2 + 3x_3 + 4x_4 \le 6 \\ & x \ge 0 \end{aligned}$$

3.

$$\begin{aligned} \min \quad & |2x_1 + 3x_2 + x_3| + \|x\|^2 + \sqrt{2x_1^2 + 4x_1x_2 + 7x_2^2 + 10x_2 + 6} \\ \text{s.t.} \quad & \frac{x_1^2 + 1}{x_2} + 2x_1^2 + 5x_2^2 + 10x_3^2 + 4x_1x_2 + 2x_1x_3 + 2x_2x_3 \le 7 \\ & \max\{x_1 + x_2, x_3, x_1 - x_3\} \le 19 \\ & x_1 \ge 0 \\ & x_2 \ge 1. \end{aligned}$$

4.

$$\begin{aligned} \min \quad & \sqrt{2x_1^2 + 3x_2^2 + x_3^2 + 4x_1x_2 + 7} + (x_1^2 + x_2^2 + x_3^2 + 1)^2 \\ \text{s.t.} \quad & \frac{(x_1 + x_2)^2}{x_3 + 1} + x_1^8 \le 7 \\ & x_1^2 + x_2^2 + 4x_3^2 + 2x_1x_2 + 2x_1x_3 + 2x_2x_3 \le 10 \\ & |x_1 + x_2 - x_3|^2 \le 20 \\ & x \ge 0. \end{aligned}$$

5.

$$\begin{aligned}
\min \quad & \frac{x_1^4}{x_2^2} + \frac{x_2^4}{x_1^2} + 2x_1x_2 + |x_1+5| + |x_2+5| + |x_3+5| \\
\text{s.t.} \quad & ((x_1^2+x_2^2+x_3^2+1)^2+1)^2 + x_1^4 + x_2^4 + x_3^4 \leq 200 \\
& \max\{x_1^2 + 4x_1x_2 + 9x_2^2, x_1, x_2\} \leq 40 \\
& x_1 \geq 1 \\
& x_2 \geq 1.
\end{aligned}$$

6. Suppose that we are given 40 points in the plane. Each of these points belongs to one of two classes. Specifically, there are 19 points of class 1 and 21 points of class 2. The points are generated and plotted by the MATLAB commands

```
rand('seed',id);
x=rand(40,1);
y=rand(40,1);
class=[2*x<y+0.5]+1;
A1=[x(find(class==1)),y(find(class==1))];
A2=[x(find(class==2)),y(find(class==2))];
plot(A1(:,1),A1(:,2),'*','MarkerSize',6)
hold on
plot(A2(:,1),A2(:,2),'d','MarkerSize',6)
hold off
```

where `id` should be your student id number. Note that the rows of $A_1 \in \mathbb{R}^{19\times 2}$ are the 19 points of class 1 and the rows of $A_2 \in \mathbb{R}^{21\times 2}$ are the 21 points of class 2. Write a CVX-based code for finding the maximum-margin line separating the two classes of points.