最优化3理论部分 18300290007 加兴华

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For each of the following optimization problems (i) show that it is **convex**, (ii) write a CVX **code** that solves it, and (iii) write down the **optimal solution and optimal value** (by running CVX). You need to write all the above step and publish your codes (using "publish" in MATLAB) in a PDF file. You should upload an electronic version on elearning.

1.

$$\begin{aligned} \min\quad & x_1^2 + 2x_1x_2 + 2x_2^2 + x_3^2 + 3x_1 - 4x_2 \\ \text{s.t.}\quad & \sqrt{2x_1^2 + x_1x_2 + 4x_2^2 + 4} + \frac{(x_1 - x_2 + x_3 + 1)^2}{x_1 + x_2} \le 6 \\ & x \ge 1 \end{aligned}$$

first:

$f(x) = x_1^2 + 2x_1x_2 + 2x_2^2 + x_3^2 + 3x_1 - 4x_2$

$\nabla f(x) = [2x_1 + 2x_2 + 3, 2x_1 + 4x_2 - 4, 3]'$

$$H = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\det(H_1) = 2; \det(H_2) = 4, \det(H_3) = 0$

$\therefore H \succcurlyeq 0 \,.\, f(x)\ is\ convex$

second:

$$g(x) = \sqrt{2x_1^2 + x_1x_2 + 4x_2^2 + 4} = \sqrt{\left\| \begin{pmatrix} \sqrt{1.5}x_1 \\ \frac{x_1 + x_2}{\sqrt{2}} \\ 0 \\ 2 \end{pmatrix} \right\|_2^2} = \left\| \begin{pmatrix} \sqrt{1.5}x_1 \\ \frac{x_1 + x_2}{\sqrt{2}} \\ 0 \\ 2 \end{pmatrix} \right\|_2 \quad is\ convex\ because\ it\ is\ norm\ function$$

$h(x) = \frac{(x_1 - x_2 + x_3 + 1)^2}{x_1 + x_2} = \frac{y^T y}{t},$

$y = [1, -1, 1]x + 1;\ t = x_1 + x_2 > 0\ ; x \ge 1$ is convex owning to y is an affine transformation &h(x) is perspective function

so it's a convex optimization problem

2.

$$\begin{aligned} \min\quad & x_1 + x_2 + x_3 + x_4 \\ \text{s.t.}\quad & (x_1 - x_2)^2 + (x_3 + 2x_4)^4 \le 5 \\ & x_1 + 2x_2 + 3x_3 + 4x_4 \le 6 \\ & x \ge 0 \end{aligned}$$

first:

$f(x) = x_1 + x_2 + x_3 + x_4 = [1,1,1,1]x, \quad x \ge 0$ is convex owning to it is an affine transformation & $x \in$ convex

second:

$g(x) = (x_1 - x_2)^2 = y^2 \quad , y = [1, -1, 0, 0]x$ is convex owning to $x \in$ convex & y^2 is convex

$h(x) = (x_3 + 2x_4)^2 = y^4, \quad y = [0, 0, 1, 2]x$ is convex owning to $x \in$ convex & y^2 is convex

$I(x) = x_1 + 2x_2 + 3x_3 + 4x_4 = [1,2,3,4]x, \quad x \ge 0$ is convex owning to it is an affine transformation & $x \in$ convex

so it's a convex optimization problem

3.

$$\begin{aligned} \min \quad & |2x_1+3x_2+x_3| + \|x\|^2 + \sqrt{2x_1^2+4x_1x_2+7x_2^2+10x_2+6} \\ \text{s.t.} \quad & \frac{x_1^2+1}{x_2} + 2x_1^2+5x_2^2+10x_3^2+4x_1x_2+2x_1x_3+2x_2x_3 \le 7 \\ & \max\{x_1+x_2, x_3, x_1-x_3\} \le 19 \\ & x_1 \ge 0 \\ & x_2 \ge 1. \end{aligned}$$

first:

$f_1(x) = |2x_1+3x_2+x_3| = |y|$, $y = [2,3,1,]x$ is convex owning to it is abs fucntion & y is an affine transformation

$f_2(x) = \|x\|^2$ is convex beacause it is norm function

$f_3(x) = \sqrt{2x_1^2+4x_1x_2+7x_2^2+10x_2+6} = \left\|\begin{bmatrix}\sqrt{2}(x_1+x_2)\\ \sqrt{5}(x_2+1)\\ 1\end{bmatrix}\right\|_2$ *is convex because it is norm function*

then $f(x)=f_1(x)+f_2(x)+f_3(x)$ is convex

second:

$g_1(x) = \frac{x_1^2+1}{x_2} = \frac{y^Ty}{t}$, $y = \begin{bmatrix}x_1\\1\end{bmatrix}$ $\& t = x_2$ *is convex owning to* y is an affine transformation &h(x) is perspective function

$g_2(x) = 2x_1^2+5x_2^2+10x_3^2+4x_1x_2+2x_1x_3+2x_2x_3 = x'Ax$, $A = \begin{bmatrix}2&2&1\\2&5&1\\1&1&10\end{bmatrix}$ *is convex because* $\det(A_{i=1,2,3}) \ge 0$, meaning $A \succcurlyeq 0$

$h(x)=\max\{x_1+x_2, x_3, x_1-x_3\}$ is convex because the members of max are affine transformation and thus convex

so it's a convex optimization problem

4.

$$\begin{aligned} \min \quad & \sqrt{2x_1^2+3x_2^2+x_3^2+4x_1x_2+7} + (x_1^2+x_2^2+x_3^2+1)^2 \\ \text{s.t.} \quad & \frac{(x_1+x_2)^2}{x_3+1} + x_1^8 \le 7 \\ & x_1^2+x_2^2+4x_3^2+2x_1x_2+2x_1x_3+2x_2x_3 \le 10 \\ & |x_1+x_2-x_3|^2 \le 20 \\ & x \ge 0. \end{aligned}$$

first:

$f_1(x) = \sqrt{2x_1^2+3x_2^2+x_3^2+4x_1x_2+7} = \left\|\begin{bmatrix}\sqrt{2}(x_1+x_2)\\ x_2\\ x_3\\ \sqrt{7}\end{bmatrix}\right\|_2$ *is convex because it is norm function*

$f_2(x) = (x_1^2+x_2^2+x_3^2+1)^2 = \left\|\begin{bmatrix}x_1\\x_2\\x_3\\1\end{bmatrix}\right\|_2^4$ *is convex because* $x^4 (x \ge 0) \uparrow\uparrow$ *& inside is norm function*

so $f(x) = f_1(x) + f_2(x)$ is convex.

second:

$g(x)=\frac{(x_1+x_2)^2}{x_3+1} + x_1{}^8$ *is convex because* $\frac{(x_1+x_2)^2}{x_3+1}$ is affine transformation inside an perspective function & $x_1{}^8 (x_1 \ge 0)$ *is convex*

$h(x)=x_1^2+x_2^2+4x_3^2+2x_1x_2+2x_1x_3+2x_2x_3 = x'\begin{bmatrix}1&1&1\\1&1&1\\1&1&4\end{bmatrix}x$ *is convex because* $\begin{bmatrix}1&1&1\\1&1&1\\1&1&4\end{bmatrix} \succcurlyeq 0$

$i_3(x) = |x_1+x_2-x_3|^2$ is convex because it is affine transformation inside an abs function inside an $x^2 (x > 0)$

so it's a convex optimization problem

5.

$$\begin{aligned} \min \quad & \frac{x_1^4}{x_2^2} + \frac{x_2^4}{x_1^2} + 2x_1x_2 + |x_1+5| + |x_2+5| + |x_3+5| \\ \text{s.t.} \quad & ((x_1^2+x_2^2+x_3^2+1)^2+1)^2 + x_1^4 + x_2^4 + x_3^4 \le 200 \\ & \max\{x_1^2+4x_1x_2+9x_2^2, x_1, x_2\} \le 40 \\ & x_1 \ge 1 \\ & x_2 \ge 1. \end{aligned}$$

first:

$f_1(x) = \frac{x_1^4}{x_2^2} + \frac{x_2^4}{x_1^2} + 2x_1x_2 = \left(\frac{x_1^2}{x_2} + \frac{x_2^2}{x_1}\right)^2, (x_1, x_2 \ge 1)$ *is convex because it is two* perspective functions together inside $x^2 (x \ge 0)$

$f_2(x) = |x_1+5| + |x_2+5| + |x_3+5|$ *is convex because it is three abs functions together*

so $f(x) = f_1(x) + f_2(x)$ is convex

second:

$g(x) = \left((x_1^2+x_2^2+x_3^2+1)^2+1\right)^2 = \left(\left\|\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{bmatrix}\right\|_2^4 + 1\right)^2$ is convex because it is norm function inside another norm function.

$h(x) = \max\{x_1^2+4x_1x_2+9x_2^2, x_1, x_2\} = \max\left\{\left\|\begin{bmatrix} x_1+2x_2 \\ \sqrt{5}x_2 \end{bmatrix}\right\|_2, x_1, x_2\right\}$ *is convex because it is* max *function of* 3 *convex functions.*

so it's a convex optimization problem

6. Suppose that we are given 40 points in the plane. Each of these points belongs to one of two classes. Specifically, there are 19 points of class 1 and 21 points of class 2. The points are generated and plotted by the MATLAB commands

```
rand('seed',id);
x=rand(40,1);
y=rand(40,1);
class=[2*x<y+0.5]+1;
A1=[x(find(class==1)),y(find(class==1))];
A2=[x(find(class==2)),y(find(class==2))];
plot(A1(:,1),A1(:,2),'*','MarkerSize',6)
hold on
plot(A2(:,1),A2(:,2),'d','MarkerSize',6)
hold off
```

where `id` should be your student id number. Note that the rows of $A_1 \in \mathbb{R}^{19\times 2}$ are the 19 points of class 1 and the rows of $A_2 \in \mathbb{R}^{21\times 2}$ are the 21 points of class 2. Write a CVX-based code for finding the maximum-margin line separating the two classes of points.

we can get a convex optimization problem:

$\min \|w\|_2$

s.t. $A_1 w + b \le -1$

$A_2 w + b \ge 1$

then just solve it by cvx!

Table of Contents


```
% optimation homework3,coding part
```

1

```
A=[1,1,0;1,2,0;0,0,1];
cvx_begin quiet
    variable x(3)
    minimize(quad_form(x ,A)+3*x(1)-4*x(2))
    subject to
        norm([(x(1)+x(2))/sqrt(2);sqrt(3/2)*x(1);...
sqrt(7/2)*x(2);2])+quad_over_lin((x(1)-x(2)+x(3)+1),(x(1)+x(2)))<=6

        x>=1
cvx_end
fprintf("(1)\nbest x transposed is ")
disp(x')
fprintf("optimal value is %d\n",quad_form(x ,A)+3*x(1)-4*x(2))
```

```
(1)
best x transposed is             1            1            1

optimal value is 5.000000e+00
```

2

```
cvx_begin quiet
variable x(4)
    minimize(sum(x))
    subject to
        square_pos(x(1)-x(2))+square_pos(x(3)+2*x(4))<=5
        x(1)+2*x(2)+3*x(3)+4*x(4)<=6
        x>=0
cvx_end
fprintf("(2)\nbest x transposed is ")
disp(x')
fprintf("optimal value is %d\n",sum(x))
```

```
(2)
```

```
best x transposed is    9.0403e-10   5.6793e-10   9.0093e-10
 9.7886e-10

optimal value is 3.351756e-09
```

3

```
A=[2,2,1;2,5,1;1,1,10];
cvx_begin quiet
variable x(3)

 minimize(abs(2*x(1)+3*x(2)+x(3))+square_pos(norm(x))+norm([sqrt(2)*...
(x(1)+x(2)),sqrt(5)*(x(2)+1),1]))

    subject to
        quad_over_lin([x(1);1],x(2))+quad_form(x,A)<=7
        max([x(1)+x(2),x(3),x(1)-x(3)])<=19
        x(1)>=0
        x(2)>=1
cvx_end
fprintf("(3)\nbest x transposed is ")
disp(x')
fprintf("optimal value is %d\n",abs(2*x(1)+3*x(2)+x(3))+...
square_pos(norm(x))+norm([sqrt(2)*(x(1)+x(2)),sqrt(5)*(x(2)+1),1]))
```

```
(3)
best x transposed is    4.7384e-09            1     -0.43166

optimal value is 8.550502e+00
```

4

```
A=[1,1,1;1,1,1;1,1,4];
cvx_begin quiet
variable x(3)
    minimize(norm([sqrt(2)*(x(1)+x(2)),x(2),x(3),sqrt(7)])+...
square_pos((x(1)^2+x(2)^2+x(3)^2+1)))

    subject to
        quad_over_lin(x(1)+x(2),x(3)+1)+x(1)^8<=7
        quad_form(x,A)<=10
        square_pos(abs(x(1)+x(2)-x(3)))<=20
        x>=0
cvx_end
fprintf("(4)\nbest x transposed is ")
disp(x')
fprintf("optimal value is %d
\n",norm([sqrt(2)*(x(1)+x(2)),x(2),x(3),...
sqrt(7)])+square_pos((x(1)^2+x(2)^2+x(3)^2+1)))
```

```
(4)
best x transposed is    1.8775e-05   1.8383e-05   2.0013e-05
```

```
optimal value is 3.645751e+00
```

5

```
cvx_begin quiet
variable x(3)
    minimize(square_pos(quad_over_lin(x(1),x(2))+...
quad_over_lin(x(2),x(1)))+norm(x+5 ,1))
    subject to
        square_pos(square_pos(x(1)^2+x(2)^2+x(3)^2+1)+1)+...
x(1)^4+x(2)^4+x(3)^4<=200

        max([(x(1)+2*x(2))^2+5*x(2)^2;x(1);x(2)])<=40
        x(1)>=1
        x(2)>=1
cvx_end
fprintf("(5)\nbest x transposed is ")
disp(x')
fprintf("optimal value is %d
\n",square_pos(quad_over_lin(x(1),x(2))+...
quad_over_lin(x(2),x(1)))+norm(x+5 ,1))
```

```
(5)
best x transposed is            1            1      -0.7833

optimal value is 2.021670e+01
```

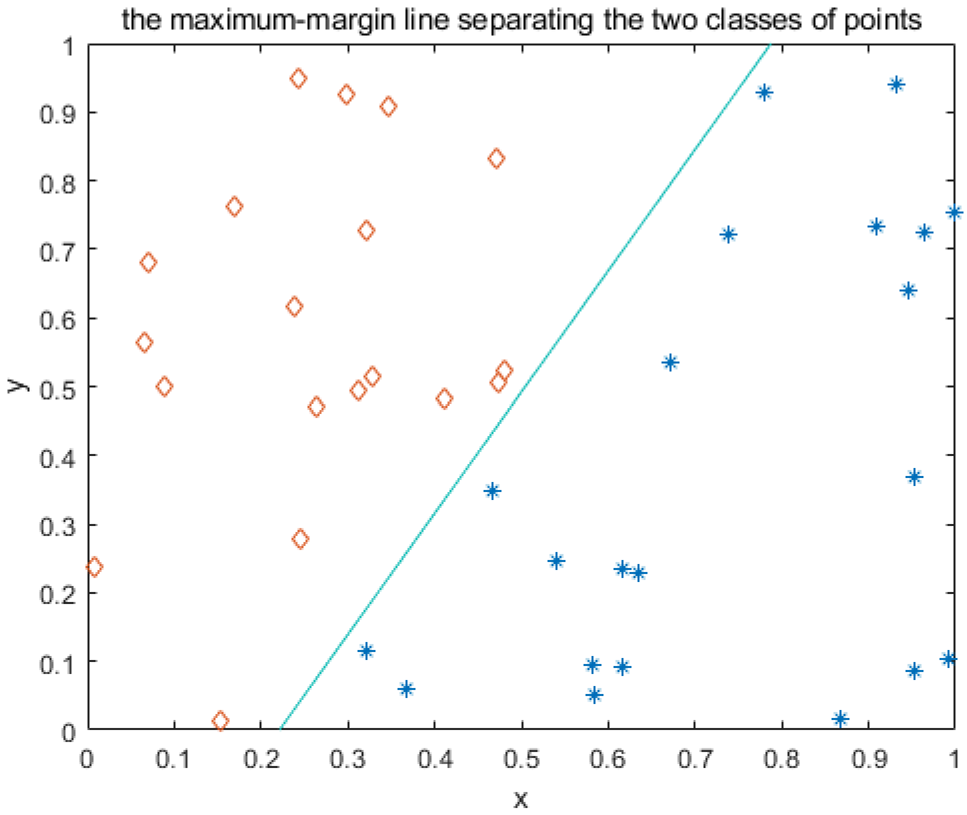
6

```
rand('seed',18300290007);
x=rand(40,1);
y=rand(40,1);
class=[2*x<y+0.5]+1;
A1=[x(find(class==1)),y(find(class==1))];
A2=[x(find(class==2)),y(find(class==2))];
plot(A1(:,1),A1(:,2),'*','MarkerSize',6)
hold on
plot(A2(:,1),A2(:,2),'d','MarkerSize',6)
hold on
cvx_begin quiet
variables w(2) b
    minimize(norm(w))
    subject to
        A1*w+b<=-1
        A2*w+b>=1
cvx_end
fprintf("so the w is\n")
disp(w)
fprintf("b is %d",b)
p1=ezplot(@(x,y) [x,y]*w+b,[0,1]);
title('the maximum-margin line separating the two classes of points')
```

```
so the w is
```

```
-28.834
 16.331
```

```
b is 6.376794e+00
```



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