Math 130 04 – A Survey of Calculus

Practice Exam

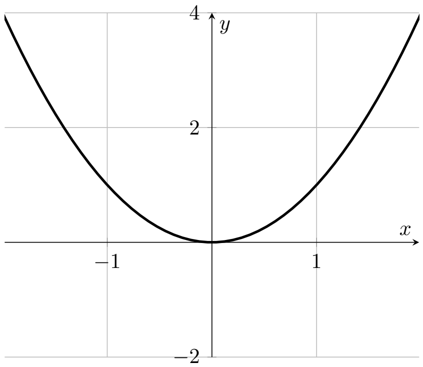
December 13, 2022 **Time: 2 hours**

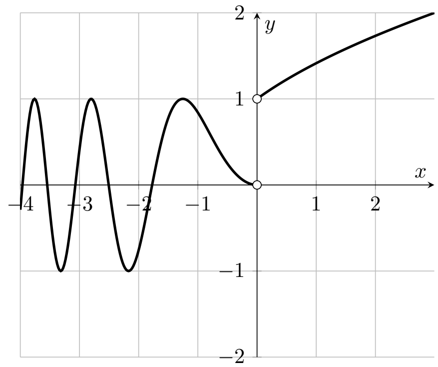
Instructions:

- You have exactly 2 hours to finish the exam.
- You are allowed to use your personal notes (paper only) and a graphing calculator. No other devices (computers, cell phones, tablets) may be used.
- You **must** write your name and student ID at the top of the first page, and you **must** initial every page that you use.
- This exam has **five** questions, each worth **five** points. Your goal is to get **18** points in total.
- Any extra points (> 18) will eventually count towards increasing your grade ($A \to A^+$, $B^+ \to A$, $B^- \to B$, and so on) at the end of the semester.
- Each question is divided into subquestions. The points that each subquestion is worth are indicated next to it.
- Write your answers clearly and neatly in the space provided after each question.
- Ask for extra sheets of paper if you need them.
- Number your answers correctly (especially if you're using extra sheets of paper).
- Justify your answers **fully and clearly.** Answers with no explanation (*even if the final calculation is correct*) are worth **zero** points. Answers with a full and correct explanation but a calculation error are worth more than 90% of the points.

Your Name:
Your Student ID:

1. (a) (2 points) Which of the following graphs represent real functions? Which of the functions is continuous over the interval $[-1, 1]$?

i. 

ii. 

(b) (3 points) Calculate the following limits.

i. $\lim_{x \to 2} \dfrac{3x^2 - 6}{x^2 - 3}$ ii. $\lim_{x \to 2} \dfrac{x^4 - 3x^2 - 4}{x - 2}$ iii. $\lim_{x \to \infty} \dfrac{x^2 + 4x - 3}{x^3 - 1}$

2. (a) (3 points) Calculate the derivatives of the following functions.

   i. $f(x) = 6x^{1/3} + 2x^{-3/4}$ ii. $f(x) = \ln(x^2 + 3)$ iii. $f(x) = e^{(3x^3 - \ln(x))}$

   (b) (2 points) Calculate the Riemann integrals of the following functions.

   i. $f(x) = 4x^3 + 2x^{-2/3}$ ii. $f(x) = e^{4x} + x^{-3/2}$

3. Consider the following function.

$$f(x) = \begin{cases} x^3 - 6(x^2+4)^{1/2} & \text{if } x \geq 0 \\ \\ 4x^2 - 12 & \text{if } x < 0 \end{cases}$$

(a) (1 point) Is f continuous at 0?

(b) (2 points) Is f differentiable at 0?

(c) (2 points) Does f have a local maximum or a local minimum at 0?

4. A company estimates their total cost function to produce x units to be

$$C(x) = 4000 + 0.25x^2 \qquad \text{thousand dollars.}$$

The company also estimates that in order to sell x units, each unit must be priced at

$$f(x) = 150 - 0.5x \qquad \text{thousand dollars.}$$

(a) (2 points) Assuming x units are produced and sold, calculate the total revenue function $R(x)$ and the total profit function $P(x)$.

(b) (2 points) How many units must be produced and sold to maximize profit? What is the maximum profit?

(c) (1 point) What price per unit must be charged to maximize profit?

5. Like all mammals, humans' bodies are maintained at a fixed temperature (98.6 degrees Fahrenheit) while they are alive. When a person dies, their corpse's temperature decreases as follows: At x hours after death, the corpse's temperature is

$$T(x) = T_0 + (98.6 - T_0)e^{-kx} \qquad \text{degrees Fahrenheit,}$$

where T_0 is the ambient temperature (of the room or environment) and k is a positive constant real number.

Upon arrival, a coroner finds the temperature of a corpse to be 61.6 degrees Fahrenheit. After 1 hour, the coroner measures the corpse's temperature to be 57.2 degrees Fahrenheit. The corpse is in a location whose ambient temperature is 10 degrees Fahrenheit.

(a) (2 points) If the coroner arrived x hours after the person died, then use the equation $\dfrac{T(x)}{T(x+1)} = \dfrac{61.6}{57.2}$ to find the constant k.

(b) (1½ points) If the coroner arrived at 11 PM, when did the person die?

(c) (1½ points) What was the rate of change of the corpse's temperature (in degrees Fahrenheit per hour) when the coroner arrived?