Math 150 03 – Calculus I

Homework assignment 6

Due: Wednesday, November 8, 2023

1. Find the antiderivatives of the following functions using the Anti-Sum and Anti-Constant Multiple rules.

    (a) $\frac{1}{3}x^5 + x^3 - 4$

    (b) $\frac{x^2 - 3x + 2}{x^2}$

    Recall that the Anti-Chain rule can be stated as follows.

    Anti-Chain Rule: "If $h(x) = g'(f(x)) \cdot f'(x)$, then the antiderivative of h is $\left(\int h\right)(x) = g(f(x)) + C$, where C is any constant real number."

2. (a) Recall that $\tan(x) = \frac{\sin(x)}{\cos(x)}$. Use the Anti-Chain rule to calculate the antiderivative of $h(x) = \tan(x)$.

    (b) Calculate $\frac{d}{dx}[\tan(x)]$.

    (c) Recall that $\sec(x)^2 = \tan(x)^2 + 1$. Use this and the previous answer to calculate the antiderivative of $h(x) = \tan(x)^2$.

    Recall that the Anti-Chain rule can also be stated as follows.

    Anti-Chain Rule: "If $h(x) = g(f(x)) \cdot f'(x)$, then the antiderivative of h is $\left(\int h\right)(x) = \left(\int g\right)(f(x))$, where $\left(\int g\right)$ is the antiderivative of g."

3. Write each of the following functions as $g(f(x)) \cdot f'(x)$ for an appropriate choice of the functions g and f. Use the Anti-Chain rule to evaluate their antiderivatives.

    (a) $h(x) = \frac{(\ln(x))^8}{x}$

    (b) $h(x) = \frac{e^{2x}}{3 + e^{2x}}$

    (c) $h(x) = \frac{e^3\sqrt{x}}{\sqrt{x}}$

4. Let $h(x) = x\sqrt{x - 1}$. We would like to find the antiderivative of h.

    (a) Let $f(x) = x - 1$. Show that we can write h as $h(x) = \left((f(x) + 1) \cdot f(x)^{1/2}\right) \cdot f'(x)$.

    (b) Expand the previous expression to show that we can write h as

    $$h(x) = i(f(x)) \cdot f'(x) + j(f(x)) \cdot f'(x)$$

    (c) Use the Anti-Sum and Anti-Chain rules to calculate the antiderivative of h.