Math 130 04 – A Survey of Calculus

Homework assignment 2

Due: Tuesday, September 13, 2022

Write your answers on a separate sheet of paper. Write your name at the top of each page you use, and number each page. Remember to number your answers correctly.

Sets and functions

1. Consider the sets

$$A = \{1, 3, 5, 7, 9\} \quad B = \{2, 4, 6, 8\} \quad C = \{1, 2, 3, 7, 8, 9\}.$$

What are the following sets? Justify your answers.

(a) $A \cap B$ (The intersection of A and B.)

(b) $A \cap B \cap C$

(c) $A \cup (B \cap C)$

(d) $(A \cup B) \cap C$

Solution:

(a) $A \cap B = \emptyset$

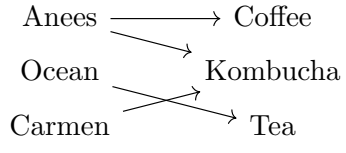
(b) $A \cap B \cap C = \emptyset$

(c) $A \cup (B \cap C) = A \cup \{2, 8\} = \{1, 2, 3, 5, 7, 8, 9\}$

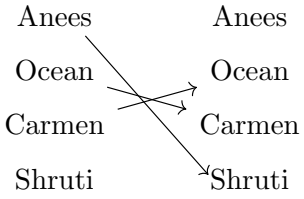
(d) $(A \cup B) \cap C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \cap C = \{1, 2, 3, 7, 8, 9\}$

2. Write the domain and the range of each of the following correspondences. Which of the following correspondences are functions? Justify your answers.

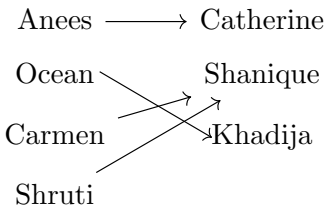
(a)



(b)



(c)



Solution:

(a)

Domain : {Anees, Ocean, Carmen} *Range* : {Coffee, Kombucha, Tea}

The correspondence **is not** a function, because the element "Anees" of the domain maps to more than one element of the range ("Coffee" and "Kombucha").

(b)

Domain : {Anees, Ocean, Carmen, Shruti} *Range* : {Anees, Ocean, Carmen, Shruti}

The correspondence **is not** a function, because the element "Shruti" of the domain does not map to any element of the range.

(c)

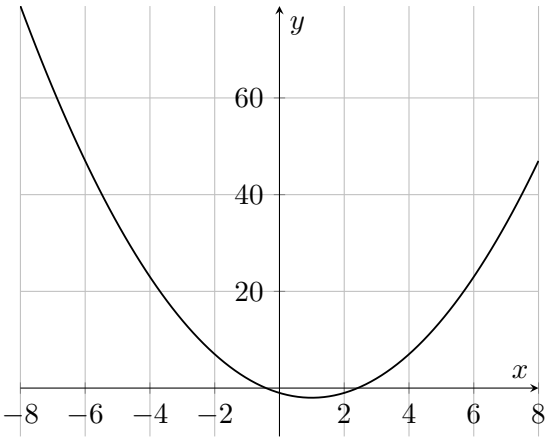
Domain : {Anees, Ocean, Carmen, Shruti} *Range* : {Catherine, Shanique, Khadija}

The correspondence **is** a function, because every element of the domain maps to exactly one element of the range.

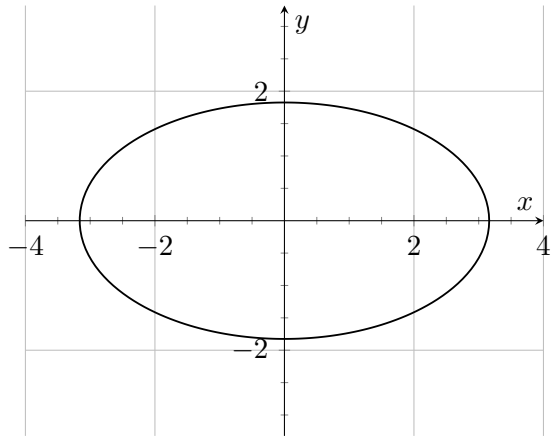
Graphs

3. Which of the following graphs are functions? Justify your answers.

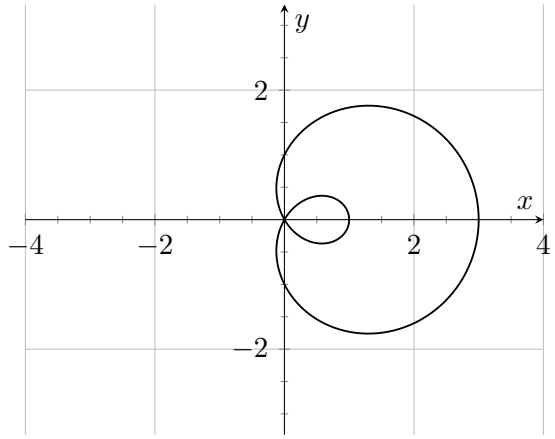
(a)



(b)



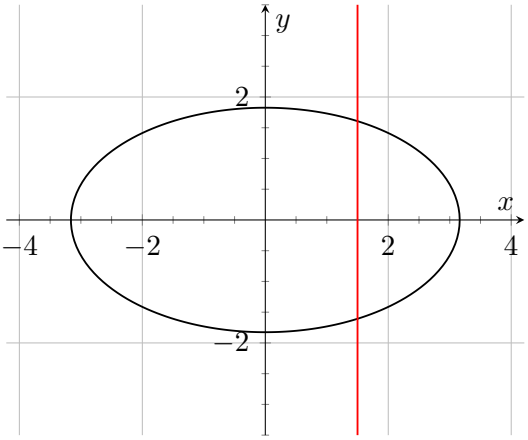
(c)



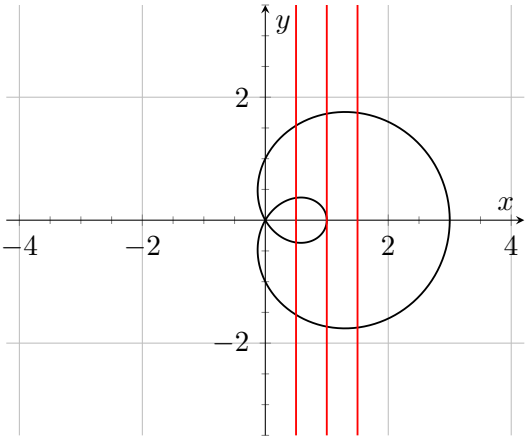
Solution:

(a) The graph is a function, since it satisfies the vertical line test. That is, every vertical line passes through exactly one point of the graph.

(b) The graph is not a function, since it fails the vertical line test. There are vertical lines that pass through two points of the graph, like the red line below.



(c) The graph is not a function, since it fails the vertical line test. There are vertical lines that pass through two, three, and four points of the graph, like the red lines below.



4. A new tech manufacturer, Lemon Inc., released a phone (the piePhone) in 2022 at a price point of $500. The piePhone was a success, and by September, the 2 million units of the piePhone produced in 2022 had sold out. In 2023, Lemon plans to release the piePhone2 (a minor upgrade to the piePhone). To decide how to price the piePhone2, Lemon has studied the market.

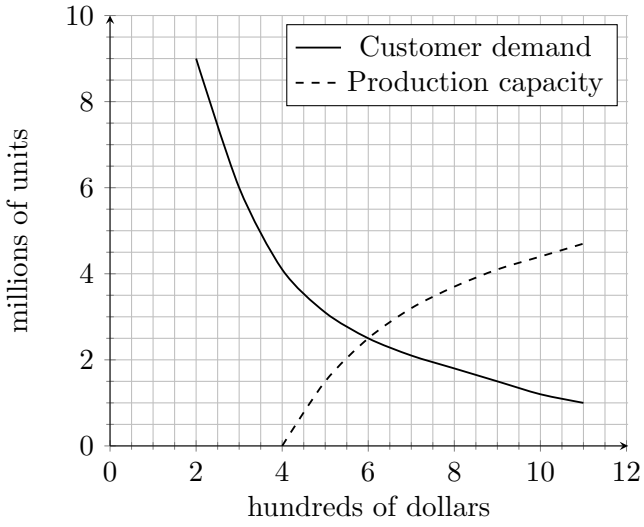


Figure 1: Market study

The result of the study (the "Customer demand" graph above) shows the number of units of the piePhone2 that customers will buy at a given price point. For example, if the piePhone2 is priced at $300, then Lemon can expect to sell 6 million units, but if it is priced at $900, then only 1.5 million units will sell. On top of the market study, Lemon has drawn a graph ("Production capacity") showing how many units they can produce at a given price point per unit. For instance, if the piePhone2 is priced at less than $400, then Lemon can't afford to produce it, but if it is priced at $1000, then Lemon can produce about 4.5 million units in 2023.

(a) Suppose Lemon produces 2 million units of the piePhone2 in 2023.

i. What is the lowest price that Lemon can afford to sell the piePhone2 at?

ii. How many units can Lemon expect to sell?

iii. What will the final price per unit be?

iv. What will Lemon's revenue (number of units sold $\times$ price per unit) be?

(b) Suppose Lemon produces 4 million units of the piePhone2 in 2023.

i. What is the lowest price that Lemon can afford to sell the piePhone2 at?

ii. How many units can Lemon expect to sell?

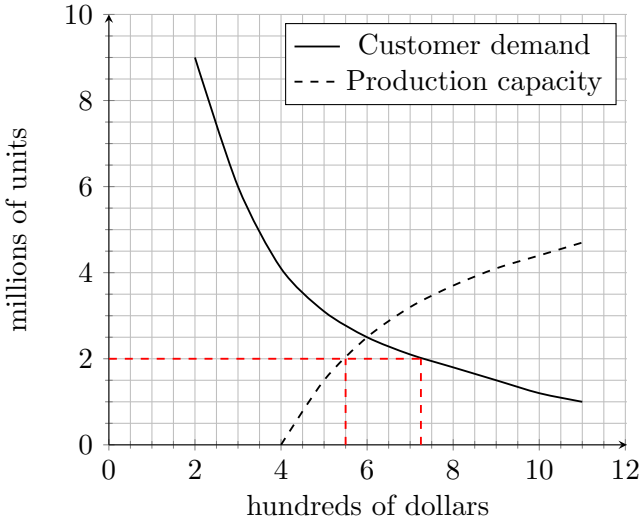
iii. What will the final price per unit be?

iv. What will Lemon's revenue (number of units sold $\times$ price per unit) be?

(c) How many units of the piePhone2 should Lemon produce in 2023 so that every unit will sell? What will the final price per unit be? What will Lemon's revenue be in 2023?

Solution:

(a) We begin by sketching this situation on the graph. Since Lemon produces 2 million units, we draw a horizontal line passing through the y axis at 2.



i. Since the horizontal line passes through the "Production capacity" graph at the point $(x = 5.5, y = 2)$, we see that Lemon can afford to produce 2 million units at a minimum price of \$550 per unit.

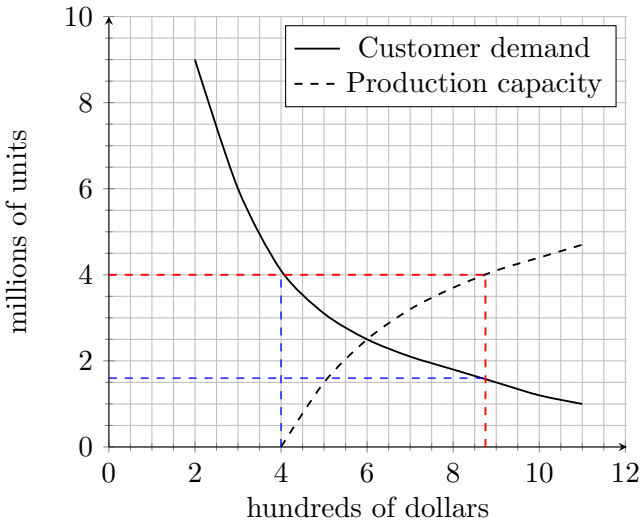
ii. Since the horizontal line passes through the "Customer demand" graph at $(x = 7.25, y = 2)$, we see that customers will buy 2 million units of the piePhone2 at a price point of \$725. Since this is *higher* than the lowest price at which Lemon can afford to produce 2 million units of the piePhone2, Lemon will sell *all* 2 *million units* of the piePhone2.

Remark: This situation is called a **shortage** in economics.

iii. Since (in this situation) customers are willing to pay \$725 per unit of the piePhone2, the final price per unit will be \$725.

iv. Lemon's revenue will be \$725 $\times$ 2 million = 1.45 billion dollars (1450 million dollars).

(b) We begin by sketching this situation on the graph. Since Lemon produces 4 million units, we draw a horizontal line passing through the y axis at 4.



i. Since the horizontal line at $y = 4$ passes through the "Production capacity" graph at the point $(x = 8.75, y = 4)$, we see that Lemon can afford to produce 4 million units at a minimum price of \$875 per unit.

ii. There are two possible outcomes:

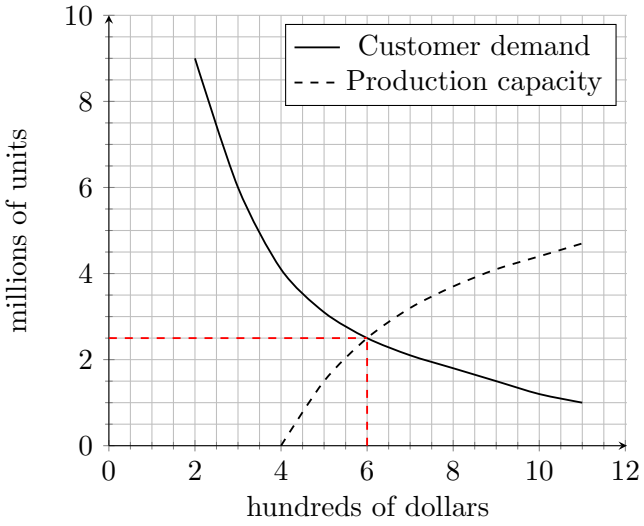
- Since Lemon cannot afford to sell the piePhone2 at less than \$875 per unit, Lemon will sell as many units as customers are willing to buy at this price. Since the vertical line at $x = 8.75$ passes through the "Customer demand" graph at the point $(x = 8.75, y = 1.6)$, in this scenario Lemon will sell 1.6 million units, and hold on to $(4 - 1.6) = 2.4$ million units.
- Alternatively, since the horizontal line at $y = 4$ passes though the "Customer demand" graph at the point $(x = 4, y = 4)$, Lemon can choose to sell all 4 million units at a price of \$400 per unit.

Remark: This situation is called a **surplus** in economics. Lemon will likely choose the first outcome, since it allows them to sell the unsold units (the surplus) later at a discounted price.

iii. If Lemon chooses to sell 1.6 million units, the final price will be \$875 per unit. If they choose to sell all 4 million units, the final price will be \$400 per unit.

iv. If 1.6 million units are sold for \$875 each, Lemon's revenue will be \$875 $\times$ 1.6 million = 1.4 billion dollars (1400 million dollars). If 4 million units are sold for \$400 each, Lemon's revenue will be \$400 $\times$ 4 million = 1.6 billion dollars (1600 million dollars).

(c) In the ideal situation, there will be no surplus and no shortage. This is at the point where the "Customer demand" graph meets the "Production capacity" graph, i.e. the point $(x = 6, y = 2.5)$. This means that Lemon can produce at most 2.5 million units if they want to sell every unit.



In this situation, the price at which customers will buy the piePhone2 is exactly the price at which Lemon can afford to sell it, namely \$600. Therefore the final price will be \$600.

In this situation, Lemon's revenue will be $\$600 \times 2.5$ million $= 1.5$ billion dollars (1500 million dollars).