

# Natural selection in contemporary humans is linked to income and substitution effects

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## Abstract

Natural selection has been documented in contemporary humans, but little is known about the mechanisms behind it. We test for natural selection through the association between 33 polygenic scores and fertility, across two generations, using data from UK Biobank (N = 409,629 British subjects with European ancestry). Consistently over time, polygenic scores associated with lower (higher) earnings, education and health are selected for (against). Selection effects are concentrated among lower SES groups, younger parents, people with more lifetime sexual partners, and people not living with a partner. The direction of natural selection is reversed among older parents (22+), or after controlling for age at first live birth. These patterns are in line with economic theories of fertility, in which higher earnings may either increase or decrease fertility via income and substitution effects in the labour market. Studying natural selection can help us understand the genetic architecture of health outcomes: we find evidence in modern day Great Britain for multiple natural selection pressures that vary between subgroups in the direction and strength of their effects, that are strongly related to the socio-economic system, and that may contribute to health inequalities across income groups.

Living organisms evolve through natural selection, in which allele frequencies change in the population through differential reproduction rates. Studying the mechanisms behind natural selection can help us better understand how individual differences in complex traits and disease risk arise (Benton et al. 2021). Recent work confirms that natural selection is taking place in modern human populations, using genome-wide analysis (Barban et al. 2016; Beauchamp 2016; Kong et al. 2017; Sanjak et al. 2018). In particular, genetic variants associated with higher educational attainment are being selected against, although effect sizes appear small.

As yet we know little about the social mechanisms behind these effects. This study uses data from UK Biobank (Bycroft et al. 2018) to learn more. We test for natural selection on 33 different polygenic scores by estimating their correlation

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with fertility. We extend the analysis over two generations, using data on respondents’ number of siblings as well as their number of children. This is interesting because consistent natural selection over multiple generations could lead to substantive effects in the long run. Most importantly, we examine reproductive rates in different subgroups of the population, in order to uncover patterns that can help illuminate the mechanisms behind modern natural selection.

We find selection effects on many polygenic scores. Effects are largely consistent across generations. The strength of natural selection on a polygenic score is associated with that score’s correlation with education and earnings: scores that predict lower education and earnings are being selected for. Also, across the board, polygenic scores have stronger relationships with fertility among specific subgroups. Selection effects are stronger in groups with lower income and less education, among younger parents, people not living with a partner, and people with more lifetime sexual partners. Outside these groups, effects are weaker and often statistically insignificant. In some subgroups, the direction of selection is even reversed: polygenic scores predicting higher education and earnings are associated with *higher* fertility.

These patterns are in line with economic theories of fertility (Becker 1960). In these, higher potential earnings have two opposite effects on fertility: a fertility-increasing *income effect* (higher income makes children more affordable), and a fertility-lowering *substitution effect* (time spent on childrearing has a higher cost in foregone earnings). Our results suggest that the substitution effect dominates for single parents and younger parents, while among couples and older parents (22+) effects are more evenly balanced.

## Results

We created polygenic scores for 33 traits in 409,629 individuals, corrected for ancestry using 100 genetic principal components (see Materials and Methods). Figure 1 plots mean polygenic scores in the sample by 5-year birth intervals. Several scores show consistent increases or declines over this 30-year period, of the order of 5% of a standard deviation. These changes could reflect natural selection within the UK population, but also emigration, or ascertainment bias within the sample. Respondents have higher income and are better educated than the UK population, and they may also differ on other unobserved characteristics (Fry et al. 2017). Since richer and more educated people also live longer, this bias could also increase with age.

To test directly for natural selection, we regress respondents’ number of children ( $y_i$ ) on their polygenic scores (PGS):

$$y_i = \alpha + \beta \text{PGS}_i + \varepsilon_i \quad (1)$$

The “selection effect,”  $\beta$ , reflects the strength of natural selection within the sample. In fact, since polygenic scores are



**Figure 1:** Mean polygenic scores (PGS) by birth year in UK Biobank. Points are means for 5-year intervals. Lines are 95% confidence intervals. Green triangles show a significant linear increase over time ( $p < 0.05/33$ ). Red squares show a significant decrease.

normalized,  $\beta$  is the expected polygenic score among children of the sample (Beauchamp 2016).<sup>1</sup> To learn more about the underlying mechanisms, we split the sample, starting with basic demographic variables including education, income and sex. These are all potential sources of ascertainment bias: as well as the ascertainment for income and education, mentioned above, the sample sex ratio skews 54.05% female.

Figure 2 plots selection effects for each polygenic score, grouping respondents by age of completing full-time education, and by household income. Effects are larger and more significant for the lowest income category, and for the lowest education category. Note that the overall effect is not a simple average of the effect among the different subgroups, because polygenic scores may also shift respondents between the subgroups. For example, a high polygenic score for educational attainment may predict having fewer children among early school leavers, but it may also increase the age at which a respondent leaves school.

These results could be explained by age, if older respondents have lower income and are less educated, and also show more natural selection on polygenic scores. However, if we rerun the regressions, interacting the polygenic score with income category and also with a quadratic in age, the interaction with income remains significant at 0.05/33 for 16 out of 33 regressions. Similarly if we interact the PGS with age of leaving full time education and a quadratic in age, the interaction with age leaving full time education remains significant at 0.05/33 for 12 out of 33 regressions.

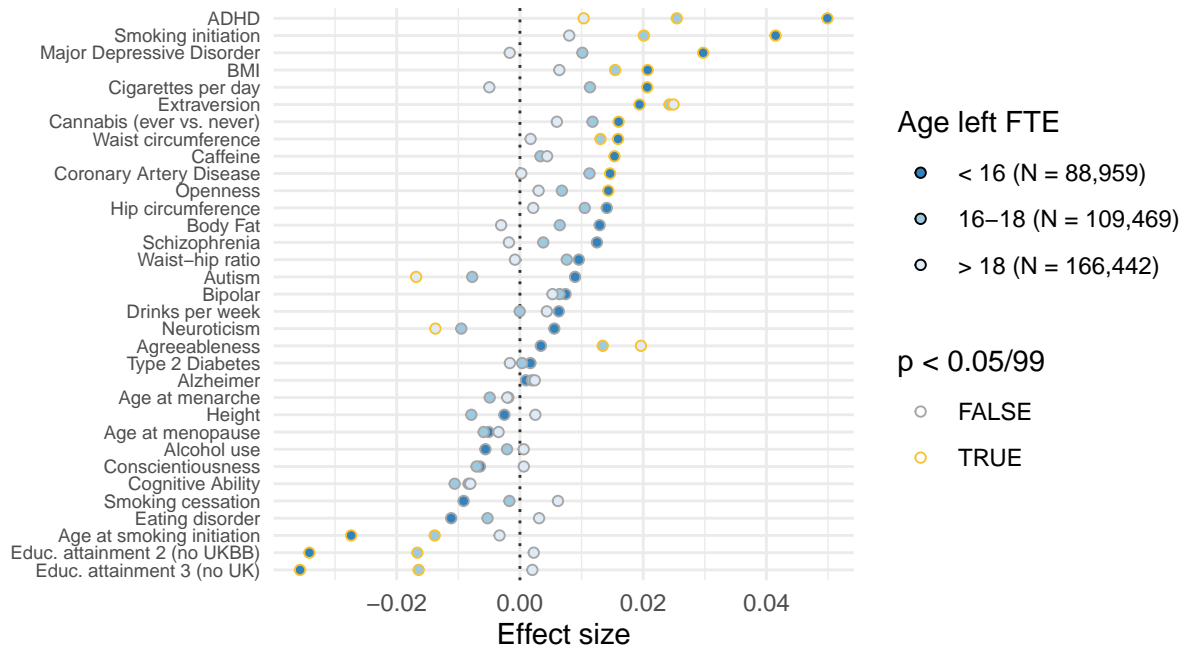
Selection effects are also different between men and women (Appendix Figure 8). Differences are particularly large for educational attainment, height and MDD. Several polygenic scores for mental illness and personality traits are more selected for (or less against) among women, including major depressive disorder (MDD), schizophrenia and neuroticism, while extraversion is more selected for among men. Scores for waist circumference and waist-hip ratio are less selected for among women, and scores for educational attainment are more selected against, though the difference is only significant for the (“EA3”) educational attainment score of Lee et al. (2018). One possible reason for these sex differences is that polygenic scores may affect fertility via success in marriage markets, and men and women may value different characteristics in these.

We next focus on variables related to household type and reproductive strategy. We split males and females by lifetime number of sexual partners, at the median value of 3 (Figure 3a). For both sexes, selection effects are larger and more significant among those with more than 3 partners. Next we split respondents by whether they were living with a spouse or partner at the time of interview. Effects are larger among those not living with a spouse or partner (Figure 3b).

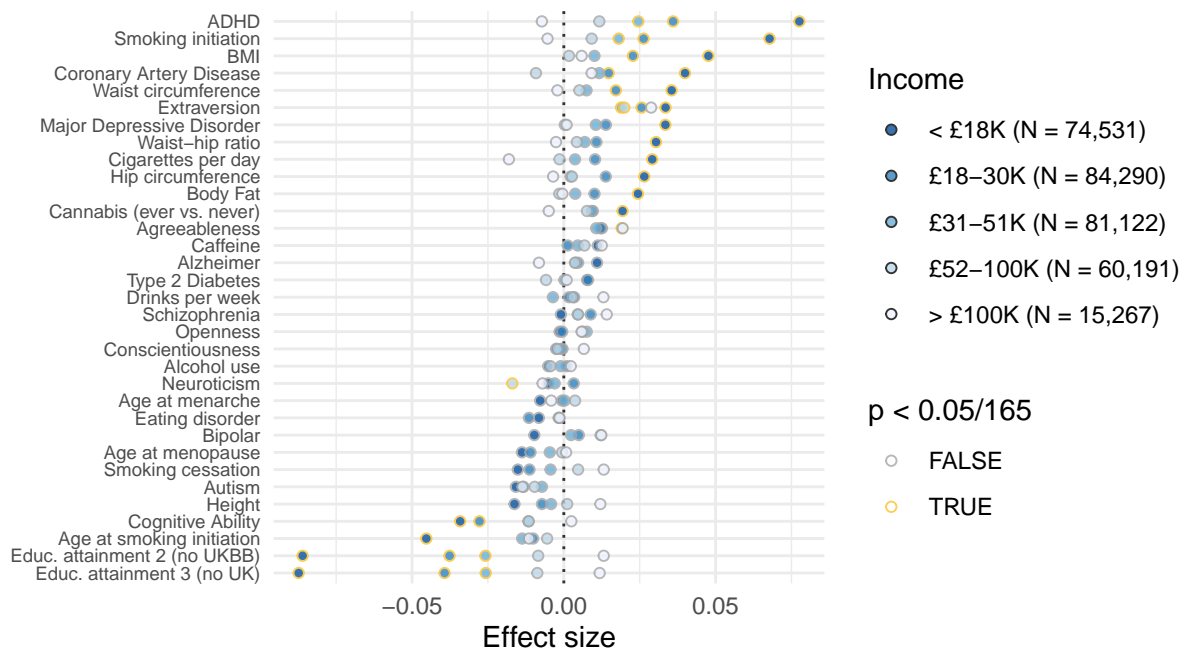
Lastly, we split female respondents by age at first live birth (AFLB).<sup>2</sup> There is evidence for genetic effects on AFLB (Barban et al. 2016), and there is a close link between this variable and number of children born. Figure 4a shows

<sup>1</sup>The selection effect  $\beta$  equals  $Cov(Y, PGS)/Var(PGS)$  where  $Y$  is the number of children. Since PGS are normalized to variance 1, this reduces to  $Cov(Y, PGS) \equiv E(Y PGS) - E(Y)E(PGS)$ , which in turn reduces to  $E(Y PGS)$  as  $E(PGS) = 0$ . This is the polygenic score weighted by the number of children, which is the average polygenic score in the next generation.

<sup>2</sup>This information is unavailable for men.



(a) Age left full-time education

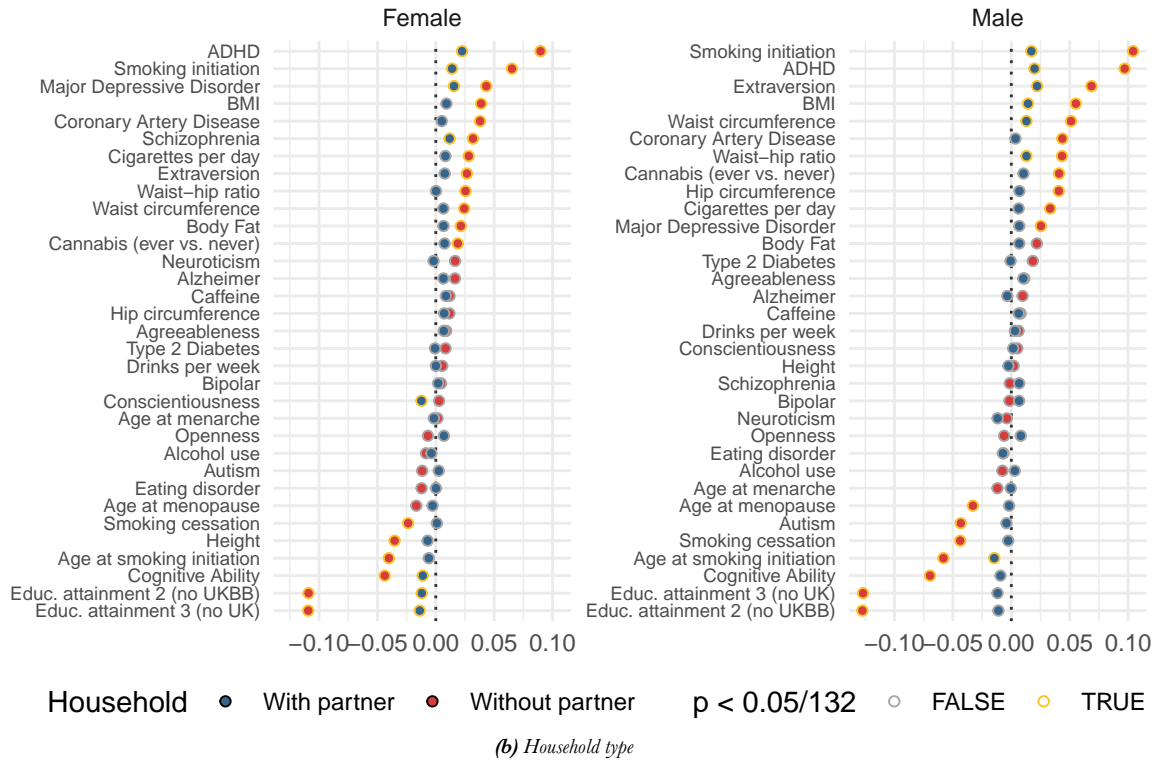
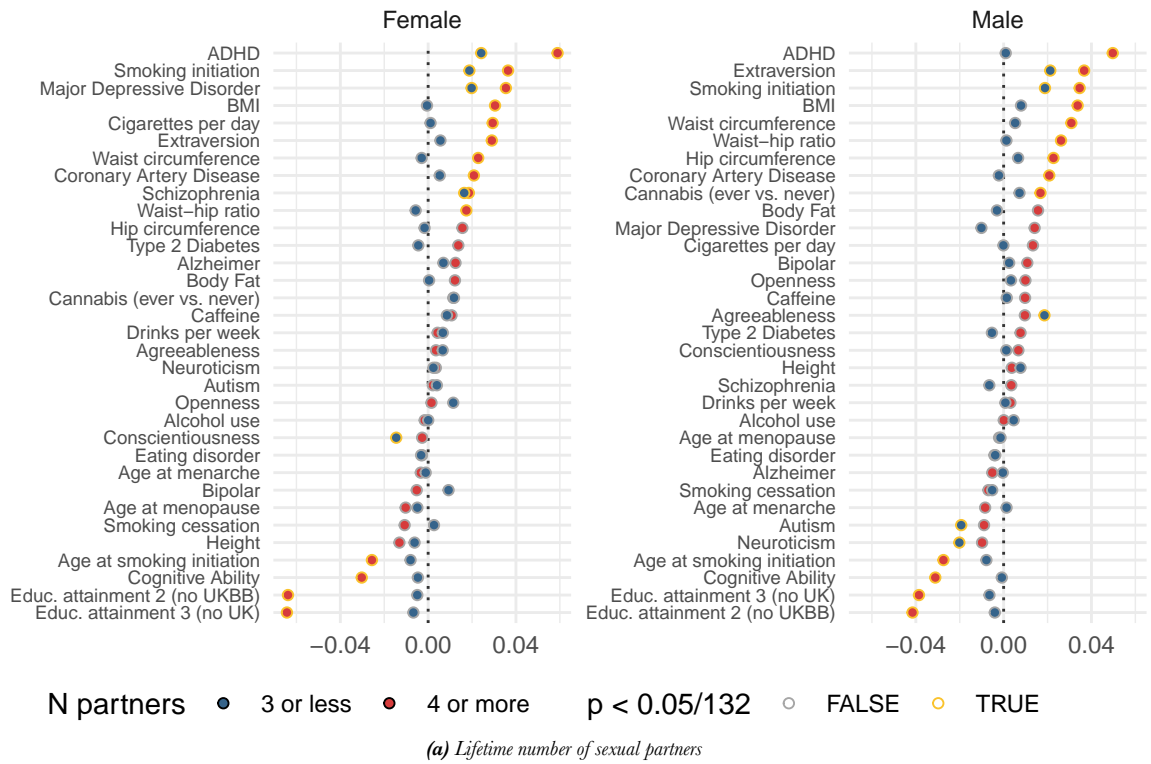


(b) Household income

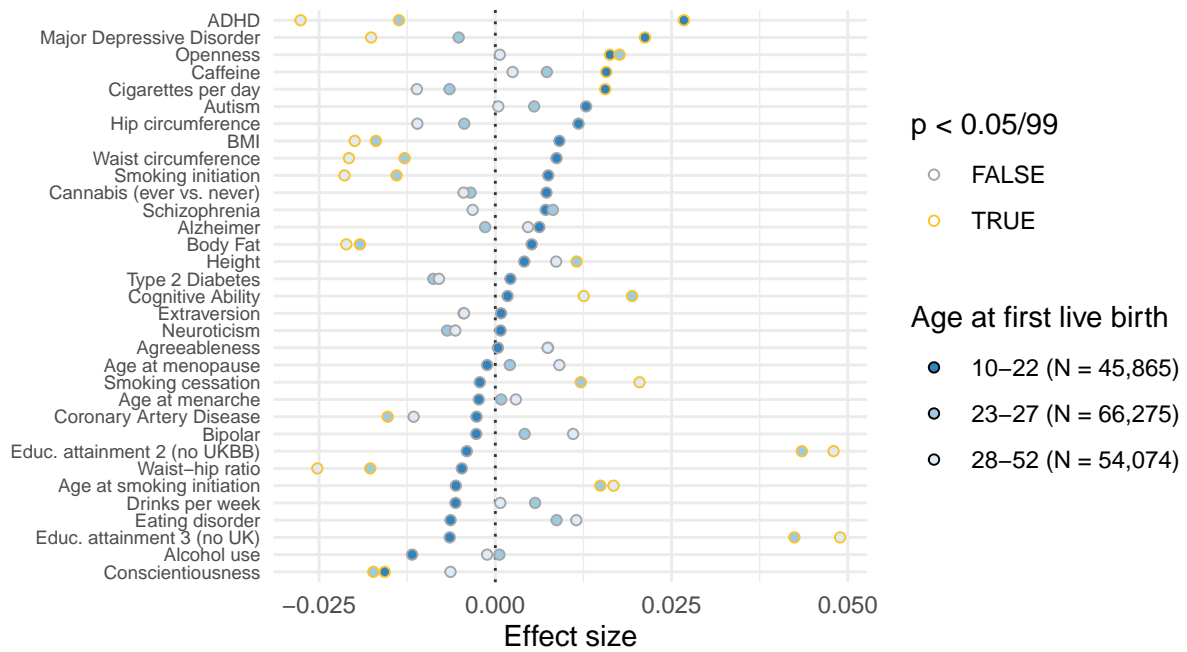
**Figure 2:** Selection effects by education and income. Each point represents a single bivariate regression of number of children on a polygenic score.

effect sizes estimated separately for each tercile of AFLB. Several effects are strikingly different across terciles. ADHD and MDD are selected for amongst the youngest third of mothers, but selected against among the oldest two-thirds. Educational attainment is selected for among the oldest two-thirds of mothers, but is not significantly selected among the youngest third. Similarly, several polygenic scores for body measurements are selected against only among older mothers. The correlation between effect sizes for the youngest and oldest terciles is -0.46.

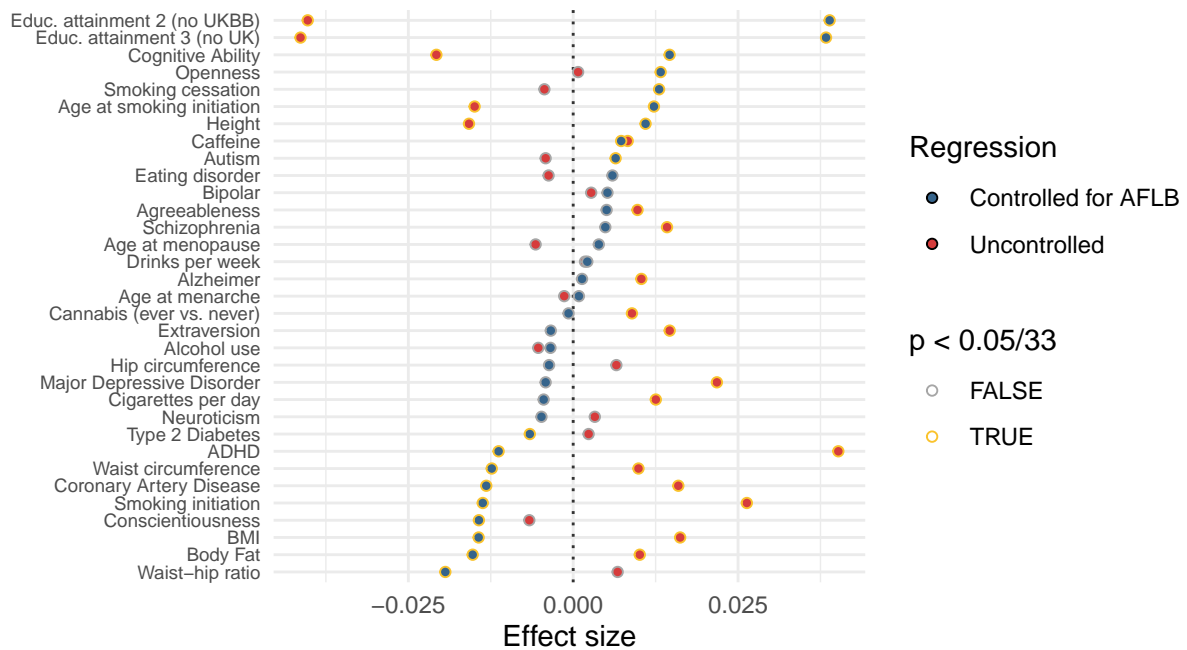
To investigate this further, we regress *number of children* on polygenic scores *controlling* for AFLB, again among females (Figure 4b). In 24 out of 33 cases, effects change sign when controls are added. The correlation between effect sizes controlling for AFLB, and raw effect sizes, is -0.79. Thus, selection effects seem to come through two opposing channels: an effect on AFLB, and an opposite-signed effect on number of children controlling for AFLB. Note that we do not claim that AFLB is exogenous to an individual's polygenic scores – indeed, our analyses show that it is not (Appendix Figure 15). Rather, we argue that AFLB mediates part of the relationship between polygenic scores and fertility, and that once this is controlled for, the remaining part of the relationship has the opposite sign.



**Figure 3:** Selection effects by number of sexual partners and household type.



(a) Effect sizes by age at first live birth terciles (women only)



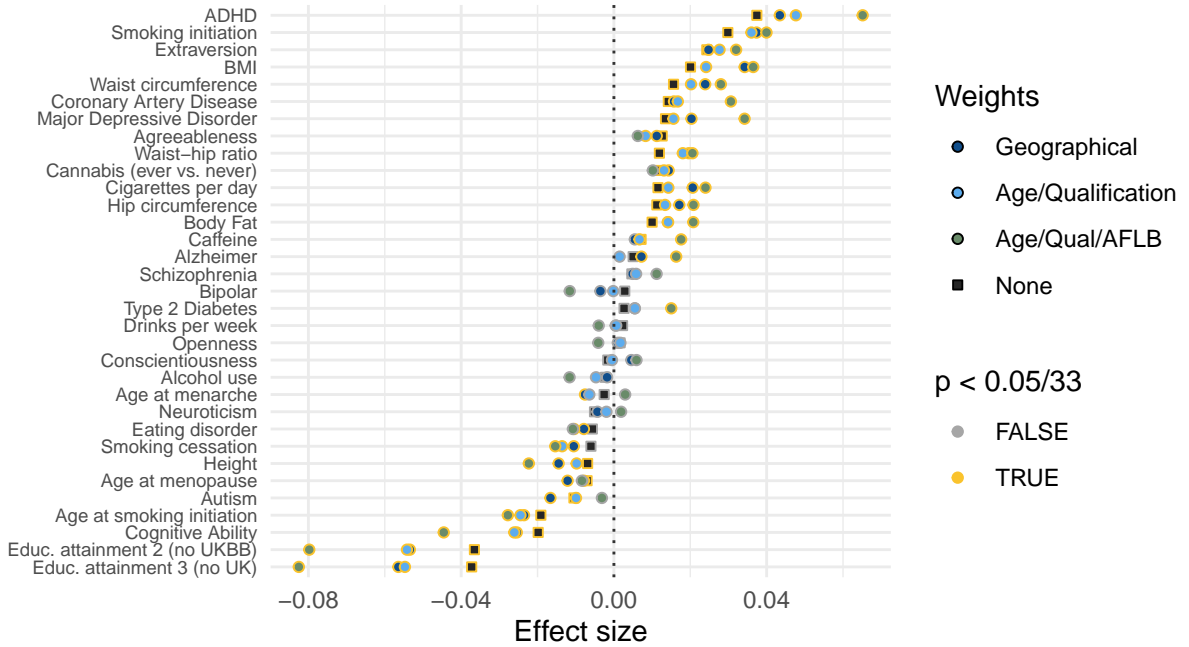
(b) Effect sizes controlling for age at first live birth. Effect sizes without controls (for women only) shown for comparison.

**Figure 4:** Selection effects by age at first live birth, and controlling for age at first live birth (women only).



## Correcting for sampling bias

Since UK Biobank participants are not representative of the wider population, our results suggest that naïve estimates of natural selection are at risk of ascertainment bias. To correct for this, we weight participants using population data. We try three alternative weighting schemes: weighting by geography, age and presence/absence of a partner; weighting by age and highest educational qualification; and age, highest qualification, and age at first live birth, for women only. Figure 5 plots selection effects among the entire sample, estimated with the three weighting schemes. Mean effect sizes across all polygenic scores are increased by a factor of 1.4 (geographical weighting), 1.22 (age/qualification) or 1.89 (age/qualification/AFLB). Estimates might be further affected by weighting on other demographic variables. Since the UK Biobank sample seems to be ascertained in ways that shrink estimates of selection effects, we suspect that our weighted estimates are still conservative.<sup>3</sup>



**Figure 5:** Selection effects: weighted regressions. Unweighted estimates are shown for comparison.

## Selection in the parents' generation

The UK Biobank data contains information on respondents' number of siblings. Since respondents' polygenic scores are equal in expectation to the mean scores of their parents, we can use this to look at selection effects in the parents' generation. We estimate equation (1) using *number of siblings* (including the respondent) as the dependent variable. The

<sup>3</sup>We also check for purifying selection by estimating (1) with a quadratic term. See Appendix Figure 9.

parents’ generation has an additional source of ascertainment bias: sampling parents of respondents overweights parents who have many children. For instance, parents of three children will have, on average, three times more children represented in UK Biobank than parents of one child. Parents of no children will by definition not be represented. To compensate, we reweight our preferred weightings (Age/Qualification) by the inverse of *number of siblings*.

Selection effects are highly correlated across the two generations, and most share the same sign (Appendix Figure 10). Absolute effect size estimates are larger for the parents’ generation. We treat this result cautiously, because when we split respondents up by year of birth, we find few differences in effect sizes between early- and late-born respondents, for either generation. In other words, since estimated effect sizes change across “generations,” but do not change over time within either generation, the change may be due to remaining ascertainment bias within the sample or other effects. In particular, effect sizes in both generations may depend on polygenic scores’ correlation with childlessness, and we cannot estimate this for the parents’ generation.

Although the direction of selection effects does not change between the generations, there are other differences. Compared to our standard polygenic scores (residualized on 100 principal components of genetic data), selection effects on unresidualized scores are about ten percent higher on average for *number of siblings*, whereas effects for *number of children* barely change (Appendix Figure 12). This could be because earlier fertility is driven more by geographically clustered deprivation (e.g. via an insurance motive, Rendall and Bahchieva 1998), which may correlate with the broad-scale genetic variation captured by principal components, such as ancestry.

We also check whether selection effects differ by socio-economic status in the parents’ generation. We have no information about parents’ income, so we use the 1971 Townsend deprivation score of respondents’ birthplace as a proxy (Townsend 1987). Results (Appendix Figure 11) show the same pattern as for respondents: effect sizes are larger and more often significant in the most deprived areas.

Lastly, the siblings data lets us check for a “quantity-quality tradeoff” between number of children and number of grandchildren (for the parents’ generation). We do not find any: in fact, the correlation between *number of siblings* and *number of children* is positive ( $\rho = 0.1$ ,  $p < 2 \times 10^{-17}$ ).

### ***Economic fertility theory and natural selection***

These results show that selection effects are weaker, absent, or even reversed among some subgroups of the population. One possible explanation for this is given by the economic theory of fertility (Becker 1960; Willis 1973; Becker and Tomes 1976). According to this theory, increases in a person’s wage affect their fertility via two opposing channels. There is an *income effect* by which children become more affordable, like any other good. There is also a *substitution effect*: since childrearing has a cost in time, the opportunity cost of childrearing increases if one’s market wage is higher.

The income effect would lead higher earners to have more children. The substitution effect would lead higher earners to have fewer children. If so, then genetic variants which affect earnings potential in the labour market may cause opposing effects on fertility. The income effect will lead to natural selection in favour of earnings-increasing variants. The substitution effect will do the reverse.

To demonstrate this, we write down a simple model of fertility choices.  $h$  is an individual's level of human capital, which (for now) we identify with his or her wage  $W$ . Raising a child takes time  $b$ . People maximize utility from the number of children  $N$ , and income  $Y \equiv (1 - bN)W$ . An individual's payoff is

$$U(N) = u(Y) + aN.$$

Here  $a$  captures the strength of preference for children.  $u(\cdot)$  is concave and increasing. We treat  $N$  as continuous, in line with the literature: this can be thought of as the expected number of children among similar people. The marginal benefit of an extra child is  $\frac{dU}{dN} = -bWu'(Y) + a$ . The effect of an increase in wages (or human capital) on this marginal benefit is

$$\frac{d^2U}{dNdh} = \underbrace{-bu'(Y)}_{\text{Substitution effect}} \underbrace{-bYu''(Y)}_{\text{Income effect}}.$$

The *substitution effect* is negative and reflects that when wages increase, time devoted to childcare costs more in foregone income. The positive *income effect* reflects that when wage income is higher, the marginal loss of wages from children is less painful. Letting  $u(\cdot)$  take the constant relative risk aversion (CRRA) form  $u(y) = \frac{y^{1-\sigma}-1}{1-\sigma}$ , where  $\sigma > 0$  captures risk aversion in income, we solve for optimal fertility  $N^*$ , and examine the *fertility-human capital relationship*,  $\frac{dN^*}{dh}$ . For moderate levels of risk aversion ( $0.5 < \sigma < 1$ ):

1. The fertility-human capital relationship is negative:  $\frac{dN^*}{dh} < 0$ .
2. The relationship is weaker (closer to zero) at higher wages and/or levels of human capital.
3. The relationship is more negative when the time burden of children,  $b$ , is larger.

To examine education and fertility timing, we extend the model to two periods. For convenience we ignore time discounting, and assume that credit markets are imperfect so that agents cannot borrow. Write

$$U(N_1, N_2) = u(Y_1) + u(Y_2) + a(N_1 + N_2) \tag{2}$$

Instead of identifying human capital  $h$  with wages, we now allow individuals to choose a level of education  $s \in [0, 1]$ , which has a time cost in period 1. Education increases period 2 wages, and is complementary to human capital  $h > 0$ . For period 2 wages we assume the simple functional form  $w(s, h) = sh$ . We normalize period 1 wages to 1. Maintaining the assumption of CRRA utility, we examine total fertility  $N^* \equiv N_1^* + N_2^*$ . For  $0.5 < \sigma < 1$ , facts 1-3 continue to hold (fact 2 in a neighborhood of  $\sigma = 1$ ). In addition, for  $\sigma$  in a neighborhood of 1:

4. The fertility-human capital relationship is weaker at higher levels of education  $s$ .
5. The relationship is weaker among those who start fertility in period 2 ( $N_1^* = 0$ ) than among those who start fertility in period 1 ( $N_1^* > 0$ ).

Facts 1-5 match our empirical results. In this model, polygenic scores which correlate with human capital would have a negative correlation with fertility at low income and education levels, and a weaker or zero correlation at higher levels (Figure 2). For scores which are highly correlated with human capital, the correlation would also be weaker at higher levels of the score itself, giving rise to diversifying selection (Appendix Figure 9). The correlation would be more negative when the time burden of children  $b$  is high, e.g. for single parents (Figure 3). Lastly, the correlation would be weaker among those starting fertility later (Figure 4).

An alternative theory is that polygenic scores correlate with the motivation to have children (parameter  $a$  in the model). However, this theory would not explain why our results are weaker at higher incomes and education levels. Indeed, in the one-period model, while  $\frac{dN^*}{da} > 0$ , this relationship actually becomes stronger at higher wages.

The above does not provide a complete theory of fertility. Rather, it shows that a relatively simple economic model can explain many of our results. In the appendix, we discuss other models of fertility from the economic literature, as well as causal evidence for the relationship between human capital, income and fertility.

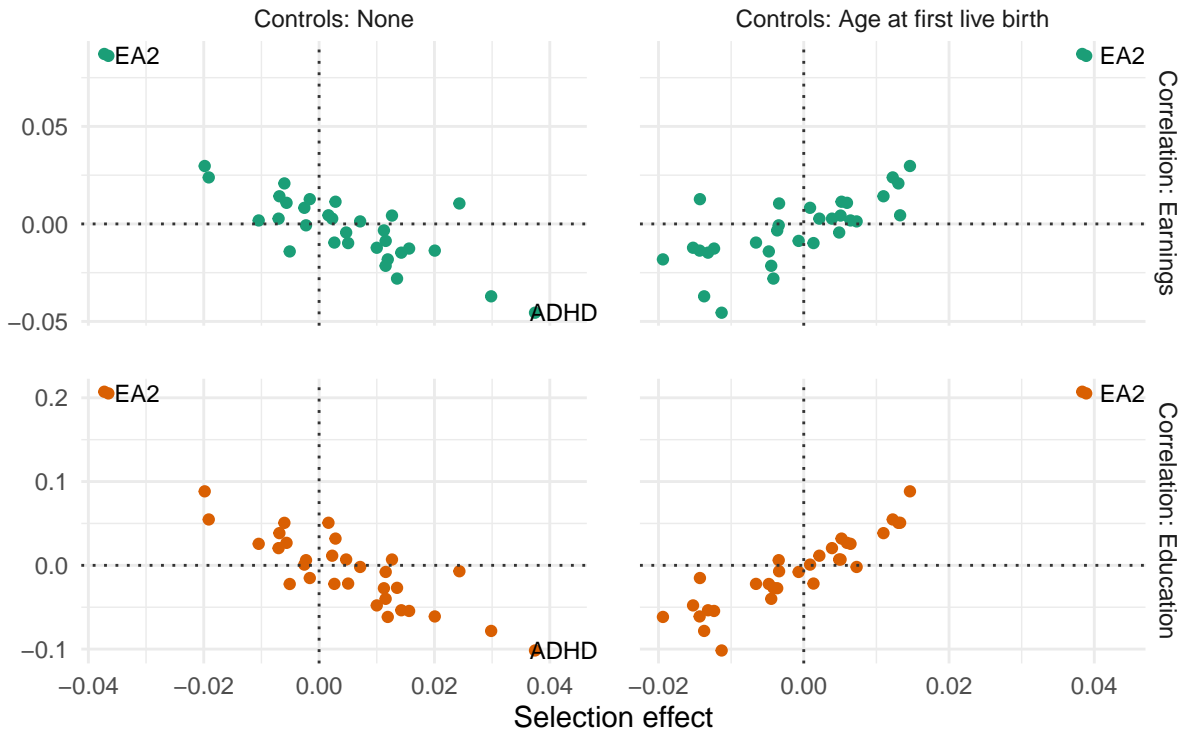
This theory can explain why selection effects have opposite signs when AFLB is controlled for. Suppose that people are aware of their endowments of human capital, i.e. skills and characteristics that are valuable in the labour market. Suppose also that education and innate human capital are complements. Individuals choose how long to stay in education; after leaving education they enter the labour market and form families. If so, then people with less human capital will leave education earlier and have their first child earlier (cf. Caucutt, Guner, and Knowles 2002; Monstad, Proper, and Salvanes 2008). This is a pure substitution effect: the income effect plays no role in the decision to leave education, since people are not yet earning wages.<sup>4</sup> Whilst in the labour market, higher earners will have more children (i.e. income effects dominate). These assumptions will lead to the pattern we observe: earnings-increasing variants will predict later first births (and fewer children overall), but they will also predict more children within any given age at first live birth.

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<sup>4</sup>This argument assumes that capital market imperfections hinder people from borrowing against future income.

The theory can also explain why the natural selection pressure is weaker among higher-income parents. Becker and Tomes (1976) show that if parents care about child quality as well as quantity, under certain assumptions the income effect will be stronger at higher income levels; this can lead to a U-shaped relationship between income and fertility. Empirically, Cohen, Dehejia, and Romanov (2013) find that income decreases fertility at low income levels but increases it at higher income levels. If, in our data, at high income levels income and substitution effects are roughly balanced, then income-linked polygenic scores will be neither selected for nor against.

Lastly, it is often assumed that the substitution effect will dominate for lone parents, or those in unstable relationships, since they have less opportunity to share childcare responsibilities, while the income effect will dominate for couples who are able to reap the gains from specialization (Becker and others 1981). Indeed, US fertility decreases faster with education among single mothers than married mothers (Baudin, De La Croix, and Gobbi 2015). This can explain why earnings-increasing genetic variants decrease fertility more among single parents than among couples. The same logic could explain our results for lifetime number of sexual partners, if this is associated with relationship stability.



**Figure 6:** Selection effects, with and without controls for age at first live birth, by correlations with earnings and educational attainment. Each point represents one polygenic score.

## Testing the theory

We test this explanation in two ways. First, the economic theory predicts that genetic variants will be selected for (or against) in proportion to their effect on earnings. Figure 6 plots selection effects on each polygenic score against that score’s correlation with earnings in a respondent’s first job, and against its correlation with educational attainment, a predictor of lifetime earnings. (See also the Supplementary Animations for the uncorrected effects per AFLB group, which show the correlation reversing after age 22.) The raw relationships (left column) are strongly negative. If we plot selection effects controlling for age at first live birth (right column), the effect reverses and becomes positive. Thus, the labour market appears to play an important role in natural selection. Substitution effects dominate income effects overall, which fits the known association between income and lower fertility (Becker 1960; Jones and Tertilt 2006).

Second, we re-estimate equation (1) for each polygenic score, controlling for earnings in first job and education levels. Effect sizes are generally reduced (Appendix Figure 17), and only 4 out of 33 scores are significant at  $p = 0.05/33$ . This suggests that earnings and education indeed mediate the effect of polygenic scores on fertility.

An alternative theory is that traits selected for are linked to externalizing behaviour, risk-seeking and low time discount rates, via the channel of early sexual behaviour (Mills et al. 2020). The data here provide some support for this: scores which might plausibly be linked to externalizing behaviour, like ADHD and (younger) age at smoking initiation, are selected for. However, this theory is less good at explaining variation in selection across the full range of scores, including physical measures, e.g. waist-hip ratio and BMI. Externalizing behaviour also does not explain why selection effects should work in the opposite direction among older mothers, whereas the economic theory can explain this via income effects, as described above. We test the alternative theory directly by re-estimating equation (1) controlling for a measure of risk attitude (UK Biobank field 2040). While risk attitude is always a highly significant predictor of number of children, it has little impact on the effect size or significance of the polygenic scores. The median ratio of effect sizes between regressions with and without controls is 0.97; all scores which are significant at  $p \leq 0.05/33$  in uncontrolled regressions remain so when controlling for risk attitude. Thus, although risk attitude does predict number of children, it appears not to mediate selection effects. Overall, we believe that the economic theory is the most likely explanation.

## Discussion

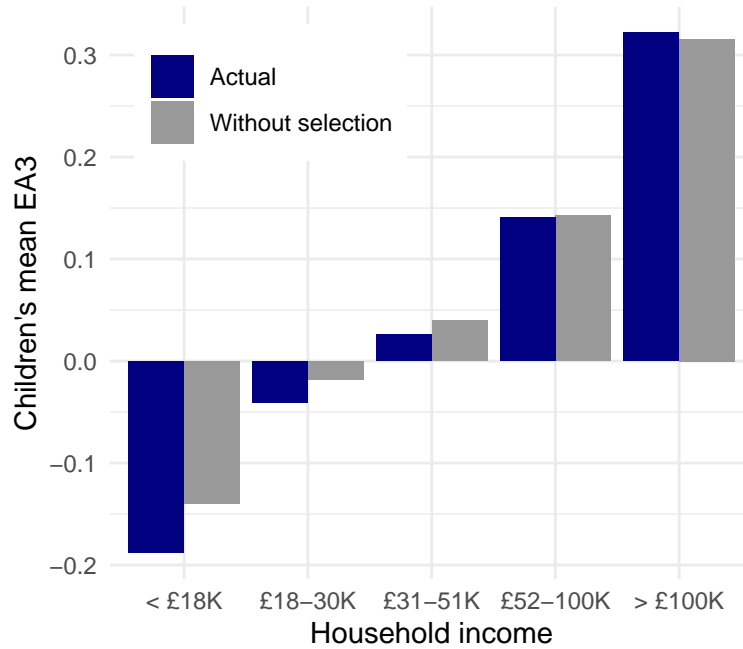
Previous work has documented natural selection in modern populations on variants underlying polygenic traits (Beauchamp 2016; Kong et al. 2017; Sanjak et al. 2018). We show that correlations between polygenic scores and fertility are highly concentrated among specific subgroups of the population, including people with lower income,

lower education, younger first parenthood, and more lifetime sexual partners. Indeed, among older mothers (22+), selection effects are actually reversed. Furthermore, the size of selection effects on a polygenic score correlates with that score’s association with labour market earnings. The economic theory of fertility provides a parsimonious explanation for this.

Polygenic scores which correlate with high (low) earnings and more (less) education are being selected against (selected for). In addition, many of the phenotypes under positive selection are linked to disease risk, or are what many people would see as undesirable to have. For example, most people would probably prefer to have high educational attainment, a low risk of ADHD and major depressive disorder, and a low risk of coronary artery disease, but natural selection is pushing against genes associated with these traits. Potentially, this could increase the health burden on modern populations, but that depends on effect sizes. Our results suggest that naïve estimates can be affected by sample ascertainment bias. This problem may be less serious in surveys which aim to be representative (as the UK Biobank does not). However, there is still scope for bias, since not all respondents consent to the collection of genetic data. For instance, completion rates for genotype data in the US Health and Retirement Study were around 80-85% (HRS 2020). Researchers should be aware of the risks of ascertainment when studying modern natural selection.

We also do not know how estimated effect sizes of natural selection will change as more accurate measures of genetic variation are produced. And we are unsure whether genetic variants underlying other phenotypes will show a similar pattern of natural selection to those studied here. In addition, genetic effects on educational attainment have been shown to be inflated in population-based samples as compared to within-family designs, likely because of indirect genetic effects, gene-environment correlations, and/or assortative mating (Lee et al. 2018; Selzam et al. 2019; Kong et al. 2018). In short, it is probably too early to tell whether modern natural selection has a substantively important effect on the genetic make-up of the population. Nevertheless, we note that selection effects on our measured polygenic scores are still relatively small, even after reweighting to account for ascertainment bias.

Because selection effects are concentrated in lower-income groups, they may also increase inequality with respect to polygenic scores. For example, Figure 7 graphs mean polygenic scores for educational attainment (EA3) among children from households of different income groups. The blue bars show the actual means, i.e. parents’ mean polygenic score weighted by number of children. The grey bars show the hypothetical means if all households had equal numbers of children. Natural selection against genes associated with educational attainment, which lowers the mean, is stronger at the bottom of the income distribution, and this increases the differences between groups. Overall, natural selection increases inequality for 29 out of 33 polygenic scores (Appendix Table 4). Since many polygenic scores are predictive of disease risk, this increase in inequality could potentially increase health inequalities. In general, the evolutionary history of anatomically modern humans is related to disease risk (Benton et al. 2021); understanding the role of contemporary natural selection may aid in mapping the genetic architecture of current health disparities.



**Figure 7:** Mean polygenic score for educational attainment (EA3) of children by household income group. Blue is actual. Grey is hypothetical in the absence of selection effects.

Existing evidence on human natural selection has led some to “biocosmic pessimism,” including speculation that the Fermi paradox may be resolved by dysgenic selection among alien species (Sarraf, Feltham, and others 2019). Others are more sanguine, and argue that natural selection’s effects are outweighed by environmental improvements, such as those underlying (e.g.) the Flynn effect (Flynn 1987). The evidence here may add some nuance to this debate. Patterns of natural selection have been relatively consistent across the past two generations, but they are not the outcome of a universal, society-wide phenomenon. Instead they result from a sum of opposing forces, operating in different parts of society and pulling in different directions.

Any model of fertility is implicitly a model of natural selection, but so far, the economic and human genetics literatures have developed in parallel. Integrating the two could deepen our understanding of natural selection in modern societies. Economics possesses a range of theoretical models on the effects of skills, education and income (see Hotz, Klerman, and Willis 1997; Lundberg and Pollak 2007). One perennial problem is how to test these theories in a world where education, labour and marriage markets all interact. Genetic data, such as polygenic scores, could help to pin down the direction of causality, for example via Mendelian randomization (Davey Smith and Ebrahim 2003). Conversely, theory and empirical results from economics can shine a light on the mechanisms behind natural selection, and thereby on the nature of individual differences in complex traits and disease risk.



## Materials and methods

We use participant data from UK Biobank (Bycroft et al. 2018), which has received ethical approval from the National Health Service North West Centre for Research Ethics Committee (reference: 11/NW/0382). We limit the sample to white British participants of European descent, as defined by genetic estimated ancestry and self-identified ethnic group (Abdellaoui et al. 2019), giving a sample size of 409,629. For regressions on number of children we use participants over 45, since most fertility is completed by this age. This gives a sample size of 371,088.

Polygenic scores were computed by summing the alleles across  $\sim 1.3$  million genetic variants weighted by their effect sizes as estimated in 33 genome-wide association studies (GWASs) that excluded UK Biobank. To control for population stratification, we corrected the polygenic scores for 100 principal components (PCs). To compute polygenic scores and PCs, the same procedures were followed as described in Abdellaoui et al. (2019).

Earnings in first job are estimated from mean earnings in the 2007 Annual Survey of Hours and Earnings, using the SOC 2000 job code (Biobank field 22617).

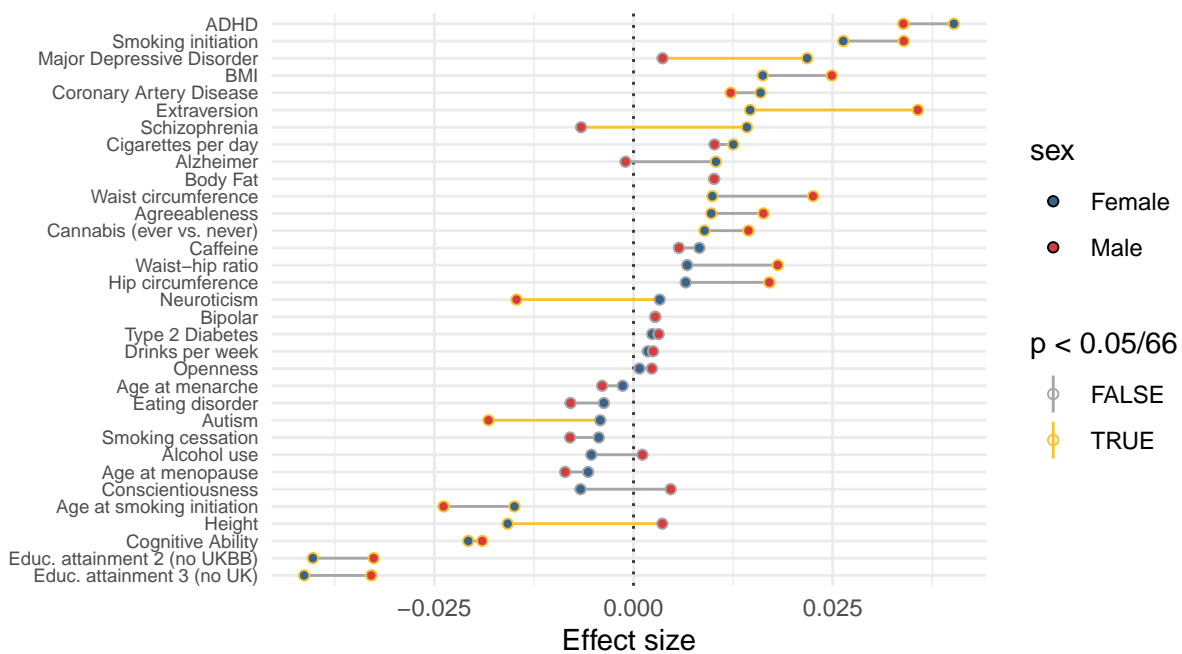
Population data for weighting is taken from the 2011 UK Census and the 2006 General Household Survey (GHS). Weighting for Age/Qualification and Age/Qualification/AFLB weights was done using marginal totals from a linear model, using the `calibrate()` function in the R “survey” package (Lumley 2020). Geographical weighting was done with iterative post-stratification using the `rake()` function, on Census Middle Layer Super Output Areas, sex and presence/absence of a partner.

## Acknowledgements

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# Appendix

## Natural selection by sex



**Figure 8:** Selection effects by sex. Highlighted lines are significant differences. Highlighted points are significantly different from 0.

## Weighted regressions

Table 1 gives effect sizes as a proportion of the unweighted effect size, for all polygenic scores which are consistently signed and which are significantly different from zero in unweighted regressions.

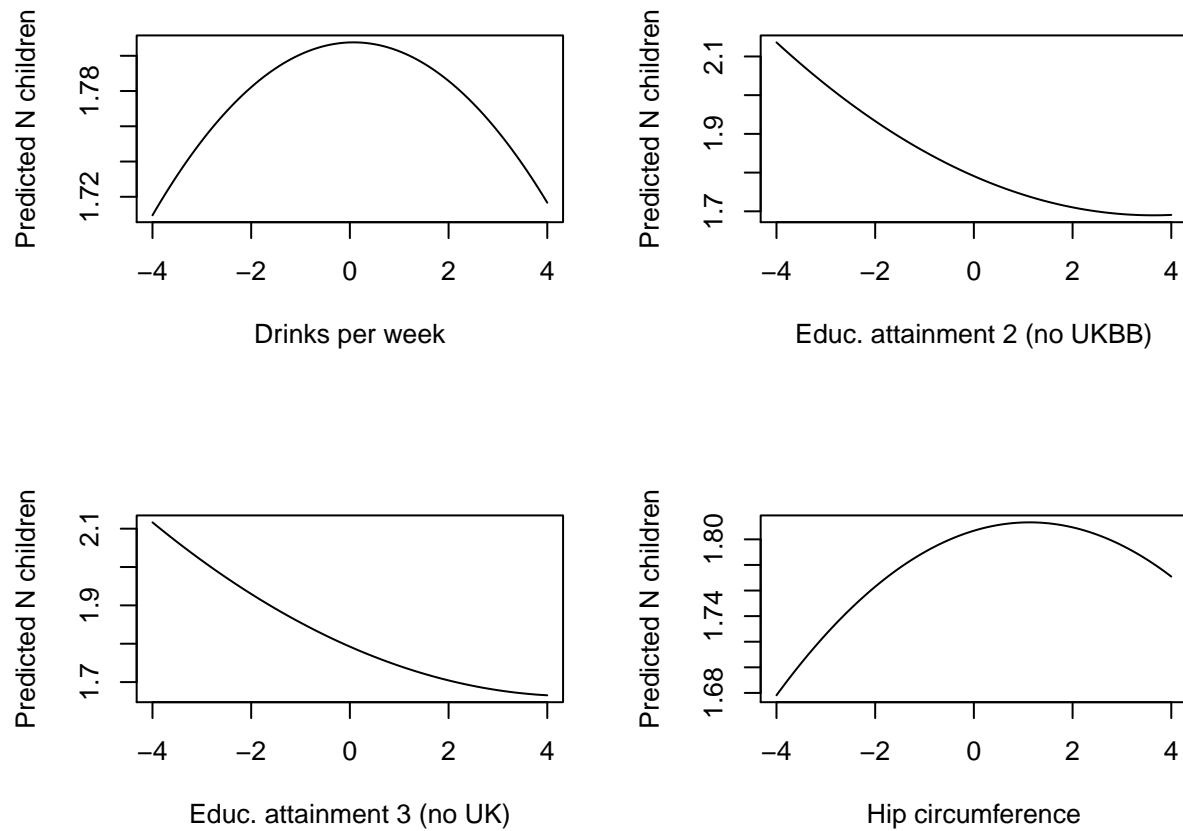
**Table 1:** *Weighted effect sizes as a proportion of unweighted effect sizes.*

| PGS                          | Weighting    |                   |               |
|------------------------------|--------------|-------------------|---------------|
|                              | Geographical | Age/Qualification | Age/Qual/AFLB |
| Height                       | 2.10         | 1.42              | 1.41          |
| Cigarettes per day           | 1.79         | 1.24              | 1.91          |
| Age at menopause             | 1.73         | 1.12              | 1.47          |
| BMI                          | 1.71         | 1.20              | 2.25          |
| Waist-hip ratio              | 1.65         | 1.52              | 3.05          |
| Autism                       | 1.59         | 0.95              | 0.75          |
| Waist circumference          | 1.53         | 1.29              | 2.83          |
| Hip circumference            | 1.52         | 1.19              | 3.18          |
| Educ. attainment 3 (no UK)   | 1.52         | 1.47              | 2.00          |
| Major Depressive Disorder    | 1.50         | 1.15              | 1.57          |
| Educ. attainment 2 (no UKBB) | 1.46         | 1.48              | 1.98          |
| Body Fat                     | 1.41         | 1.42              | 2.06          |
| Cognitive Ability            | 1.28         | 1.32              | 2.15          |
| Smoking initiation           | 1.25         | 1.21              | 1.52          |
| Cannabis (ever vs. never)    | 1.24         | 1.14              | 1.14          |
| Age at smoking initiation    | 1.23         | 1.28              | 1.86          |
| ADHD                         | 1.16         | 1.27              | 1.62          |
| Coronary Artery Disease      | 1.10         | 1.17              | 1.92          |
| Extraversion                 | 1.02         | 1.14              | 2.19          |
| Agreeableness                | 0.89         | 0.66              | 0.64          |
| Caffeine                     | 0.77         | 0.94              | 2.13          |
| <i>Mean</i>                  | 1.40         | 1.22              | 1.89          |
| <i>Median</i>                | 1.46         | 1.21              | 1.92          |

Only consistently-signed and significant (when unweighted) estimates are shown.  
Age/Qual/AFLB as a proportion of unweighted regressions including females only.

### Purifying and diversifying selection

We rerun equation (1), adding a quadratic term in  $PGS_i$ , and using our age/qualification weights. Scores for hip circumference and drinks per week show significant purifying selection ( $p < 0.05/33$ , negative coefficient). Scores for educational attainment (EA2 and EA3) show significant diversifying selection ( $p < 0.05/33$ , positive coefficient), which reduces the strength of selection against educational attainment at very high levels of the PGS. Figure 9 plots predicted number of children against polygenic score from these regressions.

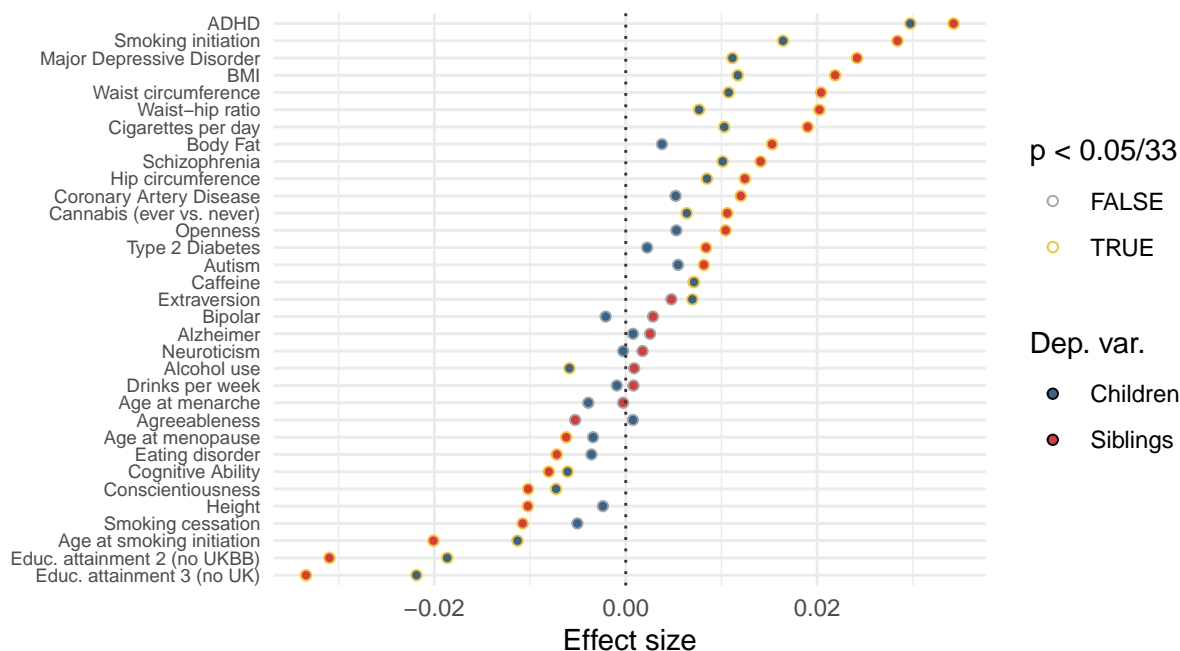


**Figure 9:** Purifying/diversifying selection: predicted number of children by polygenic score.

## Parents' generation

Figure 10 shows regressions of *number of siblings*, i.e. parents' number of children, on polygenic scores. By definition, members of the parents' generation who had no children cannot be included in this data. For a clean comparison with the respondents' generation, we rerun regressions on *number of children* excluding those with no children, and show results in the figure.

Excluding childless people in the parents' generation could bias our estimates. To learn about this, we compare effect sizes excluding and including childless people in the *current* generation. The correlation between the two sets of effect sizes is 0.92. So, patterns across different scores are broadly similar whether the childless are counted or not. However, absolute effect sizes are smaller when the childless are excluded, for 26 out of 33 scores; the median percentage change is -51.49%. The fact that childless people have such a strong effect on estimates makes it hard to compare total effect sizes across generations. In particular, since the parents' generation has a different distribution of numbers of children, childless people may have had more or less effect in that generation.



**Figure 10:** Selection effects, respondents' parents vs. respondents.

As an alternative approach, we run regressions interacting polygenic scores with birth year, median split at 1950 ("early born" versus "late born"). We use both *number of children* and *number of siblings* as a dependent variable. We weight using age/qualification cells, and further adjust for selection in the parents' generation (see above).

Tables 2 and 3 summarize the results. There is no evidence for changes in selection effects within the parents' gen-

eration. In the respondents' generation, effect sizes were significantly larger in absolute size among the later-born for four polygenic scores: cognitive ability, EA2, EA3 and extraversion. These changes are inconsistent with the intergenerational change, where estimated effect sizes were larger among the earlier, parents' generation. One score, conscientiousness, showed a significant change in sign, from negative to positive effects on fertility.

Overall, there is weak evidence for change over time. The clearest results are that (a) the direction of selection, and (b) the pattern of relative effect sizes across scores, are broadly consistent over time.

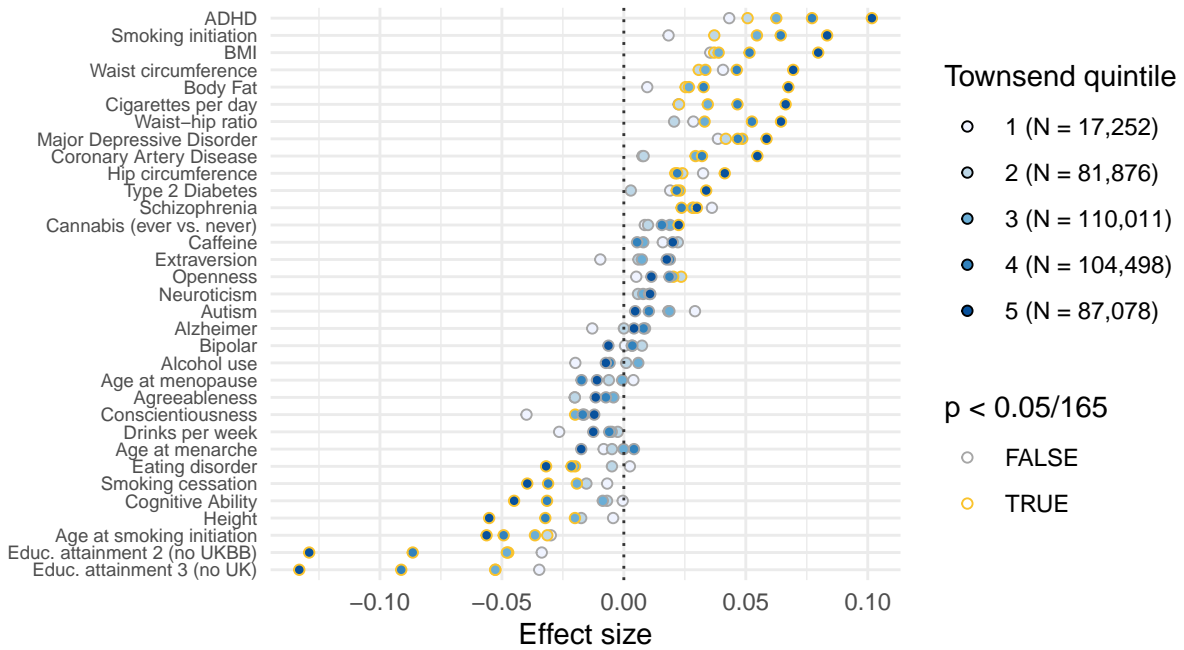
**Table 2:** *Change in selection effects between parents of early and late born respondents (regressions on number of siblings).*

| Change                                    | Number of scores |
|---|------------------|
| Insignificant                             | 33               |
| Significance is measured at $p < 0.05/66$ |                  |

**Table 3:** *Change in selection effects between early and late born respondents (regressions on number of children).*

| Change                                    | Number of scores |
|---|------------------|
| Change sign                               | 1                |
| Insignificant                             | 28               |
| Size increasing                           | 4                |
| Significance is measured at $p < 0.05/66$ |                  |

Figure 11 plots effects on *number of siblings* by Townsend deprivation quintile of birth area.

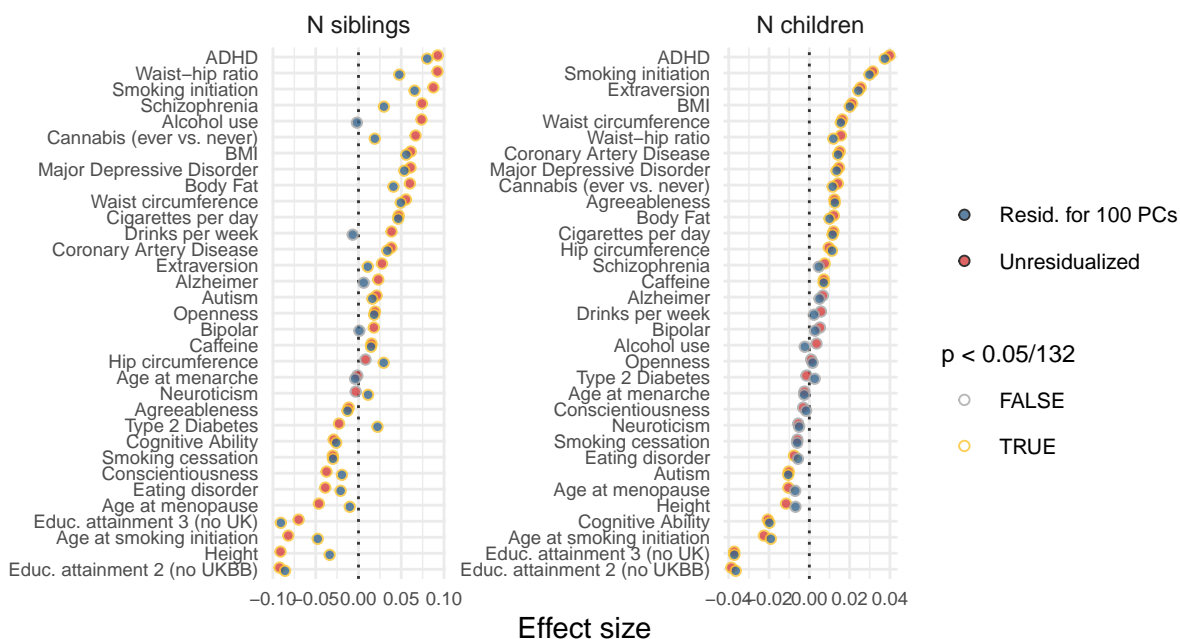


**Figure 11:** *Selection effects in the parents' generation by Townsend deprivation quintile of birth area.*

## Selection effects on raw polygenic scores

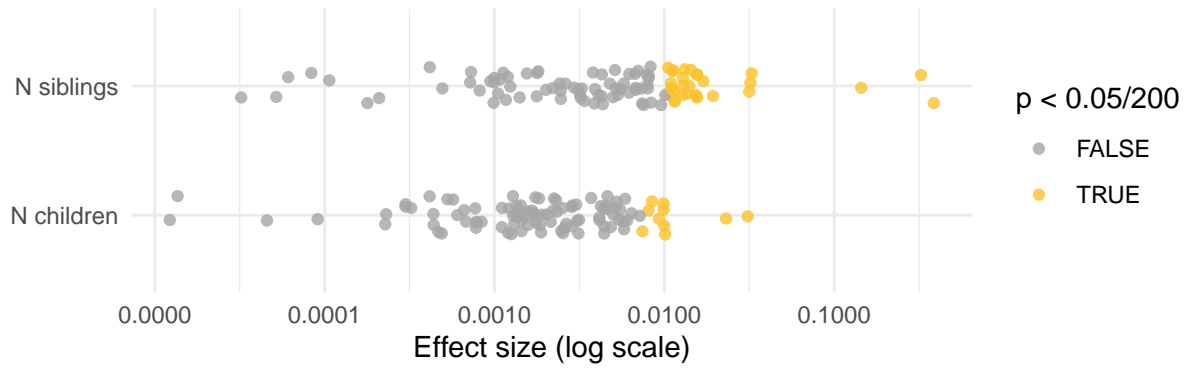
Figure 12 compares selection effects on polygenic scores residualized for the top 100 principal components of the genetic data, to selection effects on raw, unresidualized polygenic scores. In siblings regressions, effect sizes are larger for raw scores – sometimes much larger, as in the case of height. 28 out of 33 “raw” effect sizes have a larger absolute value than the corresponding “residualized” effect size. The median proportion between raw and controlled effect sizes is 0.87. Among the children regressions, this no longer holds. Effect sizes are barely affected by controlling for principal components.

Overall, 81.82 per cent of effect sizes are consistently signed across all four regressions (on children and siblings, and with and without residualization).



**Figure 12:** Selection effects using unresidualized polygenic scores on number of siblings/children.

To get a further insight into this we regress  $n$  siblings and  $n$  children on individual principal components. As Figure 13 shows, effects are larger and more significant in siblings regressions. 29 principal components significantly predicted number of siblings, while only 10 significantly predicted number of children.

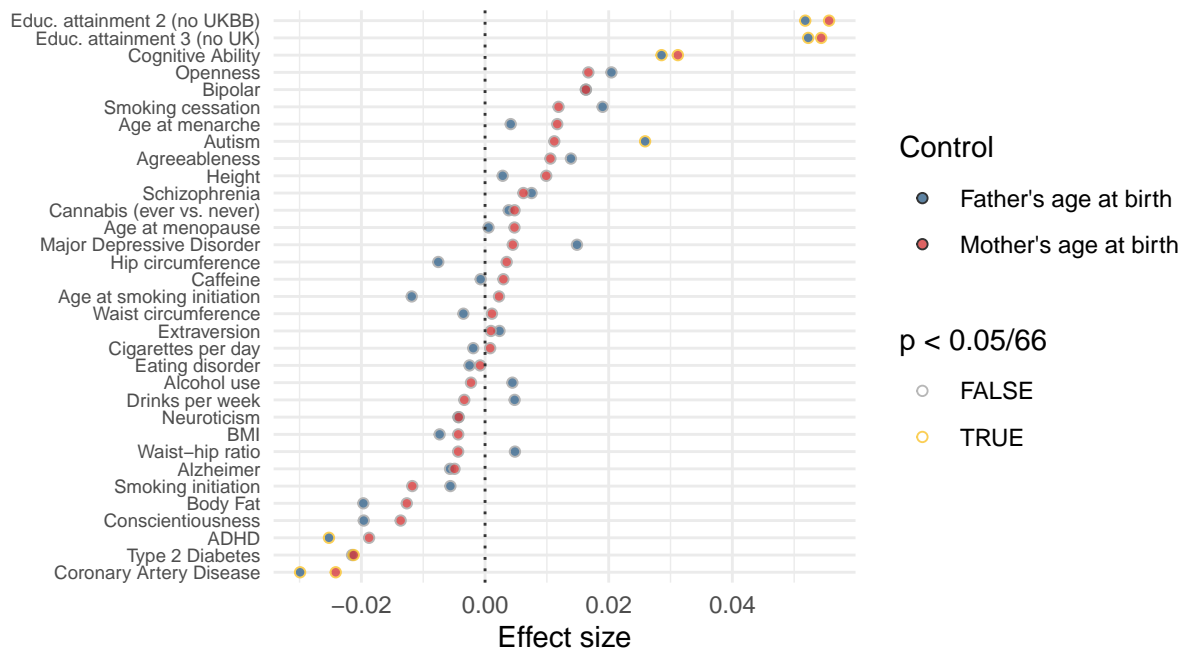


**Figure 13:** Selection effects of 100 principal components of genetic data. Absolute effect sizes are plotted. Each dot represents one bivariate regression. Points are jittered on the Y axis.



## Selection controlling for age at first live birth: respondents' parents

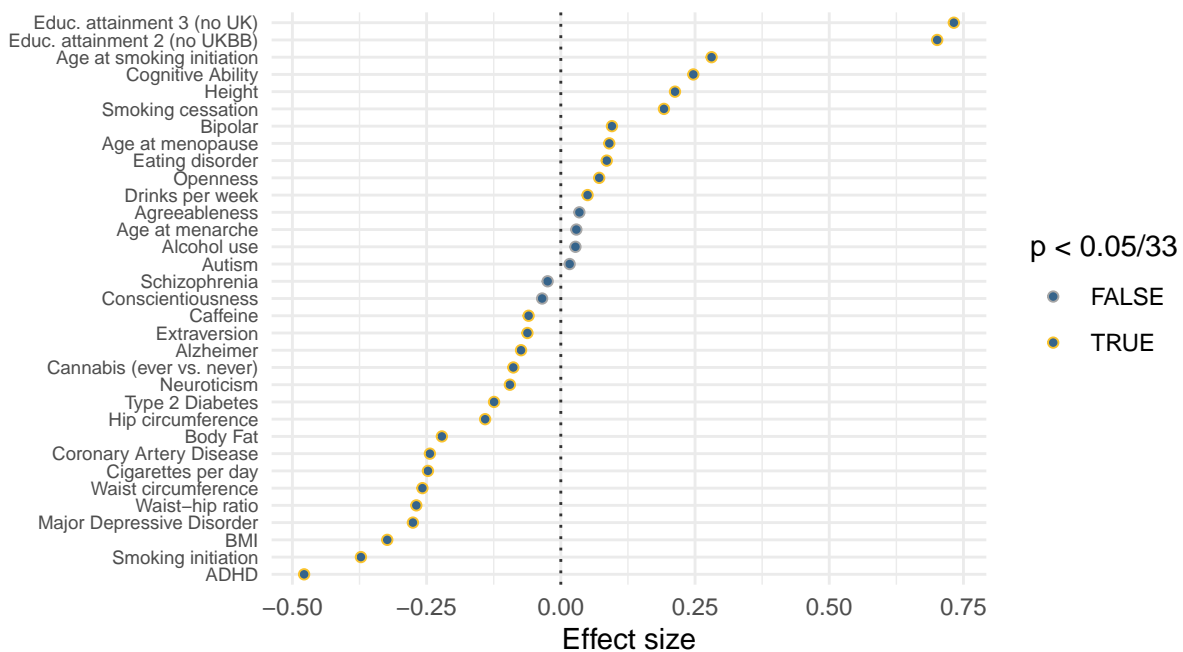
Among the parents' generation, we can control for age at first live birth using the subsets of respondents who reported their mother's or father's age, and who had no elder siblings. We run regressions on *number of siblings* on these subsets, controlling for either parent's age at their birth. Figure 14 shows the results. Effect sizes are very similar, whether controlling for father's or mother's age. As in the respondents' generation, effect sizes are negatively correlated with the effect sizes from bivariate regressions without the age at birth control (father's age at birth:  $\rho$  -0.6; mother's age at birth:  $\rho$  -0.71).



**Figure 14:** Selection effects (parents' generation) among eldest siblings, controlling for parents' age at birth.

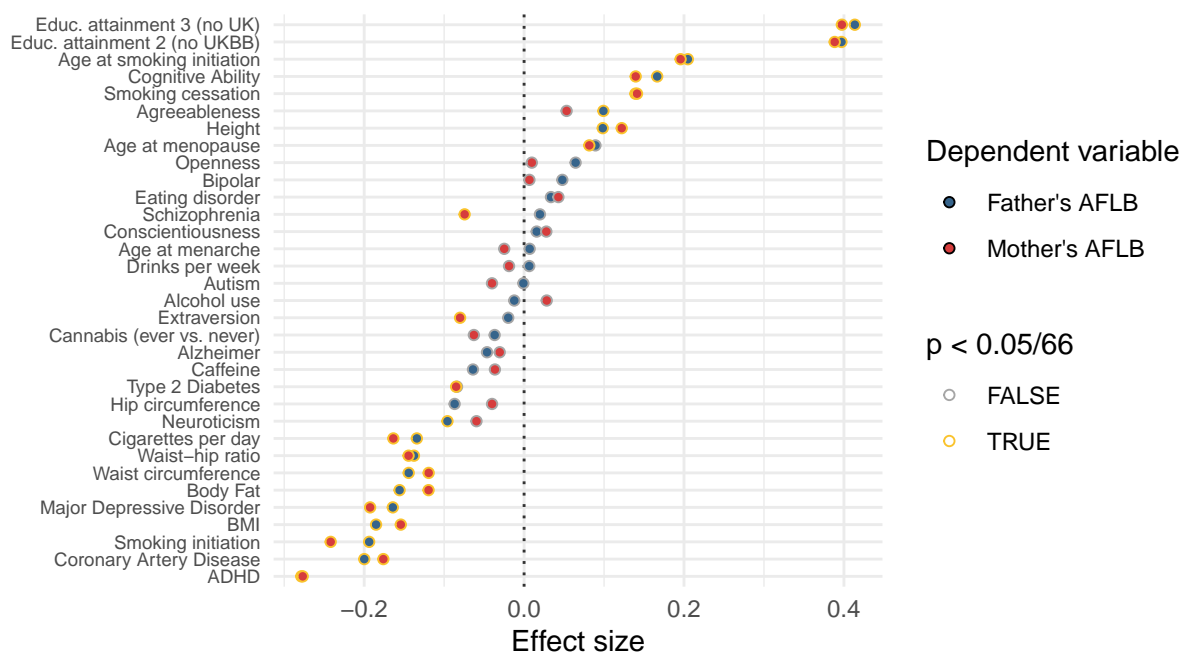
## Effects of polygenic scores on age at first live birth

Our results suggest that polygenic scores may directly correlate with age at first live birth. Figure 15 plots estimated effect sizes from bivariate regressions for respondents. Figure 16 does the same for their parents, using only eldest siblings.<sup>5</sup> Effect sizes are reasonably large. They are also highly correlated across generations. Effect sizes of polygenic scores on father's age at own birth, and on own age at first live birth, have a correlation of 0.98; for mother's age and own age it is 0.98.



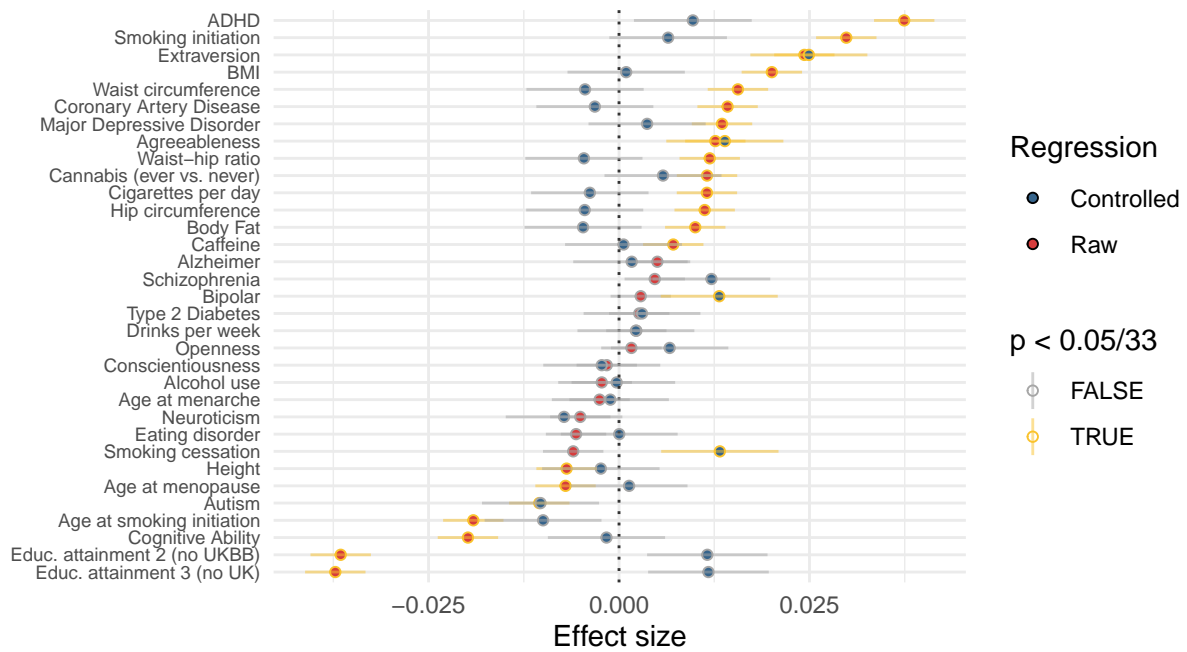
**Figure 15:** Effects of polygenic scores on age at first live birth.

<sup>5</sup>Parental AFLB can only be calculated for this group.



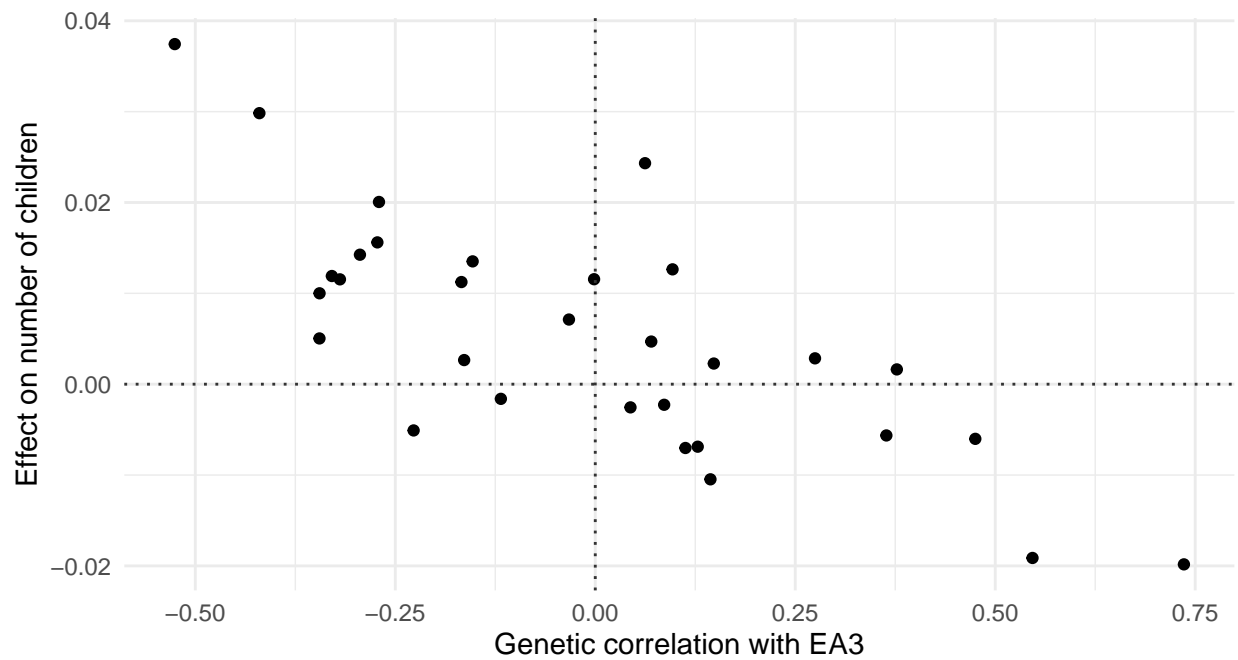
**Figure 16:** Effects of polygenic scores on parents' age at respondent's birth, eldest siblings.

## Controlling for earnings and education



**Figure 17:** Selection effects controlling for earnings in first job (estimated by mean earnings from ASHE 2007 for the SOC2000 job code) and education (left education before 16, 16-18, or after 18). Raw effects are shown for comparison. Lines are 95% confidence intervals uncorrected for multiple testing

### Genetic correlations with EA3



**Figure 18:** Selection effects plotted against genetic correlation with EA3.

Another way to examine the “earnings” theory of natural selection is to compare selection effects of polygenic scores with their genetic correlation with educational attainment (EA3). Since EA3 strongly predicts earnings, if earnings drives differences in fertility, we’d expect a correlation between the two sets of results. Figure 18 shows this is so: the correlation, after excluding EA2, is -0.75. Genetic correlations were calculated using LD score regression from GWAS summary statistics.

## Effects on inequality

Table 4 estimates differences in children’s mean polygenic scores between the highest and lowest income groups. Column “With selection” uses respondents’ scores weighted by Age/Qualification times number of children. Column “Without selection” uses scores weighted by Age/Qualification only, i.e. if all couples had the same number of children.

**Table 4:** Differences in polygenic scores between highest and lowest income group.

| PGS                          | Without<br>selection | With selection | % change |
|------------------------------|----------------------|----------------|----------|
| Age at menarche              | 0.014                | 0.020          | 44.5     |
| Alzheimer                    | −0.036               | −0.047         | 30.1     |
| Caffeine                     | −0.021               | −0.027         | 28.4     |
| Bipolar                      | 0.064                | 0.081          | 27.2     |
| Smoking initiation           | −0.166               | −0.210         | 26.6     |
| Age at smoking initiation    | 0.109                | 0.137          | 25.5     |
| Cigarettes per day           | −0.084               | −0.105         | 25.5     |
| ADHD                         | −0.230               | −0.281         | 22.0     |
| Agreeableness                | 0.045                | 0.055          | 21.6     |
| Hip circumference            | −0.072               | −0.086         | 20.3     |
| Type 2 Diabetes              | −0.039               | −0.046         | 19.5     |
| Smoking cessation            | 0.132                | 0.156          | 18.4     |
| Waist circumference          | −0.137               | −0.160         | 16.9     |
| Openness                     | 0.087                | 0.100          | 15.2     |
| Height                       | 0.140                | 0.161          | 15.1     |
| Coronary Artery Disease      | −0.109               | −0.125         | 14.5     |
| Drinks per week              | 0.059                | 0.068          | 14.4     |
| Waist-hip ratio              | −0.146               | −0.165         | 13.5     |
| BMI                          | −0.156               | −0.176         | 13.4     |
| Educ. attainment 2 (no UKBB) | 0.477                | 0.533          | 11.9     |
| Major Depressive Disorder    | −0.142               | −0.159         | 11.5     |
| Educ. attainment 3 (no UK)   | 0.487                | 0.542          | 11.2     |
| Age at menopause             | 0.036                | 0.039          | 9.2      |
| Extraversion                 | 0.095                | 0.102          | 7.5      |
| Body Fat                     | −0.125               | −0.135         | 7.4      |
| Neuroticism                  | −0.119               | −0.125         | 5.2      |
| Cognitive Ability            | 0.185                | 0.191          | 3.4      |
| Eating disorder              | 0.056                | 0.057          | 1.7      |
| Alcohol use                  | 0.034                | 0.034          | 0.6      |
| Cannabis (ever vs. never)    | −0.043               | −0.042         | −1.6     |
| Autism                       | −0.020               | −0.016         | −19.6    |
| Schizophrenia                | −0.050               | −0.025         | −50.5    |
| Conscientiousness            | −0.013               | 0.001          | −106.7   |

## Proofs

## Appendix

### Solution for the one-period model

Differentiating and setting  $\frac{dU}{dN} = 0$  gives the first order condition for an optimal choice of children  $N^* > 0$ :

$$\frac{bW}{(W(1 - bN^*))^\sigma} \geq a, \text{ with equality if } N^* > 0.$$

Rearranging gives

$$N^* = \max \left\{ \frac{1}{b} \left( 1 - \left( \frac{b}{a} \right)^{1/\sigma} W^{(1-\sigma)/\sigma} \right), 0 \right\}. \quad (3)$$

Note that when  $\sigma < 1$ , for high enough  $W$ ,  $N^* = 0$ . Differentiating gives the effect of wages on fertility for  $N^* > 0$ .

This is also the fertility-human capital relationship:

$$\frac{dN^*}{dh} = \frac{dN^*}{dW} = -\frac{1}{b} \left( \frac{b}{a} \right)^{1/\sigma} \frac{1-\sigma}{\sigma} W^{(1-2\sigma)/\sigma}. \quad (4)$$

This is negative if  $\sigma < 1$ . Also,

$$\frac{d^2 N^*}{dW^2} = -\frac{1}{b} \left( \frac{b}{a} \right)^{1/\sigma} \frac{1-\sigma}{\sigma} \frac{1-2\sigma}{\sigma} W^{(1-3\sigma)/\sigma}$$

For  $0.5 < \sigma < 1$ , this is positive, so the effect of fertility on wages shrinks towards zero as wages increase (and becomes 0 when  $N^* = 0$ ). Next, we consider the time cost of children  $b$ :

$$\frac{d^2 N^*}{dW db} = -\left( \frac{1}{a} \right)^{1/\sigma} \left( \frac{1-\sigma}{\sigma} \right)^2 (Wb)^{(1-2\sigma)/\sigma} < 0.$$

Lastly we consider the effect of  $a$ . From (3),  $N^*$  is increasing in  $a$ . Differentiating (4) by  $a$  gives

$$\frac{d^2 N^*}{da dW} = b^{1/\sigma-1} \frac{1-\sigma}{\sigma^2} W^{(1-2\sigma)/\sigma} a^{-1/\sigma-1}$$

which is positive for  $\sigma < 1$ .

## Solution for the two-period model

Period 1 and period 2 income are:

$$Y_1 = 1 - s - bN_1 \quad (5)$$

$$Y_2 = w(s, h)(1 - bN_2) \quad (6)$$

Write the Lagrangian of utility  $U$  (2) as

$$\mathcal{L}(N_1, N_2, s) = u(Y_1) + u(Y_2) + a(N_1 + N_2) + \lambda_1 N_1 + \lambda_2 N_2 + \lambda_3 \left(\frac{1}{b} - N_2\right) + \mu s$$

Lemma 4 below shows that if  $\sigma > 0.5$ , this problem is globally concave, guaranteeing that the first order conditions identify a unique solution. We assume  $\sigma > 0.5$  from here on.

Plugging (5) and (6) into the above, we can derive the Karush-Kuhn-Tucker conditions for an optimum  $(N_1^*, N_2^*, s^*)$  as:

$$\frac{d\mathcal{L}}{dN_1} = -bY_1^{-\sigma} + a + \lambda_1 = 0, \text{ with } \lambda_1 = 0 \text{ if } N_1^* > 0; \quad (7)$$

$$\frac{d\mathcal{L}}{dN_2} = -bs^*hY_2^{-\sigma} + a + \lambda_2 - \lambda_3 = 0, \text{ with } \lambda_2 = 0 \text{ if } N_2^* > 0, \lambda_3 = 0 \text{ if } N_2^* < \frac{1}{b}; \quad (8)$$

$$\frac{d\mathcal{L}}{ds} = -Y_1^{-\sigma} + h(1 - bN_2^*)Y_2^{-\sigma} + \mu = 0; \quad (9)$$

$$N_1^*, N_2^*, s^*, \lambda_1, \lambda_2, \lambda_3, \mu \geq 0; N_2^* \leq \frac{1}{b}. \quad (10)$$

Note that the Inada condition ( $\lim_{x \rightarrow 0} u'(x) = \infty$ ) for period 1 rules out  $s^* = 1$  and  $N_1 = 1/b$ , so we need not impose these constraints explicitly. Also, so long as  $N_2^* < 1/b$ , the same condition rules out  $s^* = 0$ . We consider four cases, of which only three can occur.



**Case 1:**  $N_1^* > 0, N_2^* > 0$

Rearranging (7), (8) and (9) gives:

$$N_1^* = \frac{1}{b} \left( 1 - s^* - \left( \frac{b}{a} \right)^{1/\sigma} \right); \quad (11)$$

$$N_2^* = \frac{1}{b} \left( 1 - \left( \frac{b}{a} \right)^{1/\sigma} (s^* h)^{(1-\sigma)/\sigma} \right); \quad (12)$$

$$s^* = \frac{1 - bN_1^*}{1 + ((1 - bN_2^*)h)^{1-1/\sigma}}. \quad (13)$$

Plugging the expressions for  $N_1^*$  and  $N_2^*$  into  $s^*$  gives

$$s^* = \frac{s^* + \left( \frac{b}{a} \right)^{1/\sigma}}{1 + \left( \left( \frac{b}{a} \right)^{1/\sigma} s^{*(1-\sigma)/\sigma} h^{1/\sigma} \right)^{1-1/\sigma}}$$

which simplifies to

$$s^* = \left( \frac{b}{a} \right)^{1/(2\sigma-1)} h^{(1-\sigma)/(2\sigma-1)}. \quad (14)$$

Plugging the above into (11) and (12) gives:

$$N_1^* = \frac{1}{b} \left( 1 - \left( \frac{b}{a} \right)^{1/(2\sigma-1)} h^{(1-\sigma)/(2\sigma-1)} - \left( \frac{b}{a} \right)^{1/\sigma} \right);$$

$$N_2^* = \frac{1}{b} \left( 1 - \left( \frac{b}{a} \right)^{1/(2\sigma-1)} h^{(1-\sigma)/(2\sigma-1)} \right).$$

Note that that  $N_1^* < N_2^*$ . For these both to be positive requires low values of  $h$  if  $\sigma < 1$  and high values of  $h$  if  $\sigma > 1$ .

Also:

$$w(s^*, h) \equiv s^* h = \left( \frac{b}{a} \right)^{1/(2\sigma-1)} h^{\sigma/(2\sigma-1)}.$$

Observe that  $w(s^*, h)$  is increasing in  $h$  for  $\sigma > 0.5$ , and convex iff  $0.5 < \sigma < 1$ .

While  $N_1^*$  and  $N_2^*$  are positive, they have the same derivative with respect to  $h$ :

$$\frac{dN_t^*}{dh} = -\frac{1}{b} \left( \frac{b}{a} \right)^{1/(2\sigma-1)} \frac{1-\sigma}{2\sigma-1} h^{(1-\sigma)/(2\sigma-1)-1} \quad (15)$$

Examining this and expression (14) gives:

**Lemma 1.** For  $\sigma < 1$ , case 1 holds for  $h$  low enough, and in case 1,  $N_1^*$  and  $N_2^*$  decrease in  $h$ , while  $s^*$  increases in  $h$ .

For  $\sigma > 1$ , case 1 holds for  $h$  high enough, and in case 1  $N_1^*$  and  $N_2^*$  increase in  $h$ , while  $s^*$  decreases in  $h$ .

$N_t^*$  is convex in  $h$  for  $\sigma > 2/3$ , and concave otherwise.  $s^*$  is convex in  $h$  if  $\sigma < 2/3$ , and concave otherwise.

**Case 2:**  $N_1^* = 0, N_2^* > 0$

Replace  $N_1^* = 0$  into the first order condition for  $s^*$  from (9), and rearrange to give:

$$s^* = \frac{1}{1 + ((1 - bN_2)h)^{1-1/\sigma}}.$$

Now since  $N_2^* > 0$ , we can rearrange (8) to give

$$N_2^* = \frac{1}{b} \left( 1 - \left( \frac{b}{a} \right)^{1/\sigma} (s^*h)^{(1-\sigma)/\sigma} \right). \quad (16)$$

Plugging this into  $s^*$  gives

$$s^* = \frac{1}{1 + \left( \frac{bh}{a} \right)^{(\sigma-1)/\sigma^2} (s^*)^{-(1-\sigma)^2/\sigma^2}}$$

which can be rearranged to

$$(1 - s^*)(s^*)^{(1-2\sigma)/\sigma^2} = \left( \frac{a}{bh} \right)^{(1-\sigma)/\sigma^2}. \quad (17)$$

Differentiate the left hand side of the above to get

$$\begin{aligned} & \frac{1-2\sigma}{\sigma^2} (1 - s^*)(s^*)^{(1-2\sigma)/\sigma^2-1} - (s^*)^{(1-2\sigma)/\sigma^2} \\ &= \frac{1-2\sigma}{\sigma^2} (s^*)^{(1-2\sigma)/\sigma^2-1} - \frac{\sigma^2 + 1 - 2\sigma}{\sigma^2} (s^*)^{(1-2\sigma)/\sigma^2} \\ &= \frac{1-2\sigma}{\sigma^2} (s^*)^{(1-2\sigma)/\sigma^2-1} - \frac{(1-\sigma)^2}{\sigma^2} (s^*)^{(1-2\sigma)/\sigma^2}. \end{aligned} \quad (18)$$

This is negative if and only if

$$s^* > \frac{1-2\sigma}{(1-\sigma)^2}$$

which is always true since  $\sigma > 0.5$ . Note also that since  $\sigma > 0.5$ , then the left hand side of (17) approaches infinity as  $s^* \rightarrow 0$  and approaches 0 as  $s^* \rightarrow 1$ . Thus, (17) implicitly defines the unique solution for  $s^*$ .

Conditions (7) and (8) give the bounds for this case. When  $\sigma < 1$ , (7) puts a minimum on  $h$  and (8) puts a maximum on

$h$ . The situation is reversed for  $\sigma > 1$ .

To find how  $s^*$  changes with  $h$ , note that the right hand side of the above decreases in  $h$  for  $\sigma < 1$ , and increases in  $h$  for  $\sigma > 1$ . Putting these facts together: for  $\sigma < 1$ , when  $h$  increases the RHS of (17) decreases, hence the LHS decreases and  $s^*$  increases, i.e.  $s^*$  is increasing in  $h$ . For  $\sigma > 1$ ,  $s^*$  is decreasing in  $h$ .

To find how  $N_2^*$  changes with  $h$ , we differentiate (16):

$$\frac{dN_2^*}{dh} = -\frac{1}{b} \left(\frac{b}{a}\right)^{1/\sigma} \frac{1-\sigma}{\sigma} (s^*h)^{(1-2\sigma)/\sigma} (s^* + h \frac{ds^*}{dh}) \quad (19)$$

which is negative for  $\sigma < 1$ , since  $\frac{ds^*}{dh} > 0$  in this case.

Differentiating again:

$$\begin{aligned} \frac{d^2N_2}{dh^2} &= -X \left[ \frac{1-2\sigma}{\sigma} (s^*h)^{(1-3\sigma)/\sigma} (s^* + h \frac{ds^*}{dh})^2 + (s^*h)^{(1-2\sigma)/\sigma} (2 \frac{ds^*}{dh} + h \frac{d^2s^*}{dh^2}) \right] \\ &= X (s^*h)^{(1-3\sigma)/\sigma} \left[ \frac{2\sigma-1}{\sigma} (s^* + h \frac{ds^*}{dh})^2 - (s^*h) (2 \frac{ds^*}{dh} + h \frac{d^2s^*}{dh^2}) \right] \end{aligned}$$

where  $X = \frac{1}{b} \left(\frac{b}{a}\right)^{1/\sigma} \frac{1-\sigma}{\sigma} > 0$ . Note that  $\frac{d^2N_2}{dh^2}$  is continuous in  $\sigma$  around  $\sigma = 1$ . Note also from (17) that for  $\sigma = 1$ ,  $s^*$  becomes constant in  $\sigma$ . The term in square brackets then reduces to  $(s^*)^2 > 0$ . Putting these facts together, for  $\sigma$  sufficiently close to 1,  $\frac{d^2N_2^*}{dh^2} > 0$ , i.e.  $N_2^*$  is convex in  $h$ .

Summarizing:

**Lemma 2.** *Case 2 holds for intermediate values of  $h$ . In case 2: for  $\sigma < 1$ ,  $s^*$  is increasing in  $h$  and  $N_2^*$  is decreasing in  $h$ . For  $\sigma > 1$ ,  $s^*$  is decreasing in  $h$ . For  $\sigma$  close enough to 1,  $N_2^*$  is convex in  $h$ .*

**Case 3:**  $N_1^* = 0, N_2^* = 0$

We can solve for  $s^*$  by substituting values of  $Y_1$  and  $Y_2$  into (9):

$$-(1-s^*)^{-\sigma} + h(s^*h)^{-\sigma} = 0$$

which rearranges to

$$s^* = \frac{1}{1 + h^{(\sigma-1)/\sigma}}. \quad (20)$$

Conditions (7) and (8) become:

$$-b(1 - s^*)^{-\sigma} + a \leq 0$$

$$-bs^*h(s^*h)^{-\sigma} + a \leq 0$$

equivalently

$$\frac{a}{b} \leq (1 - s^*)^{-\sigma}$$

$$\frac{a}{b} \leq s^*h(s^*h)^{-\sigma}$$

which can both be satisfied for  $a/b$  close enough to zero. Note from (20) that as  $h \rightarrow \infty$ ,  $s^*$  increases towards 1 for  $\sigma < 1$ , and decreases towards 0 for  $\sigma > 1$ . Note also that the right hand side of the first inequality above approaches infinity as  $s^* \rightarrow 1$ , therefore also as  $h \rightarrow \infty$  for  $\sigma < 1$ . Rewrite the second inequality as

$$\frac{a}{b} < (s^*h)^{1-\sigma} = \left( \frac{h}{1 + h^{(\sigma-1)/\sigma}} \right)^{1-\sigma} = (h^{-1} + h^{-1/\sigma})^{\sigma-1}$$

and note that again, as  $h \rightarrow \infty$ , the RHS increases towards infinity for  $\sigma < 1$ , and decreases towards zero otherwise. Thus, for  $\sigma < 1$ , both equations will be satisfied for  $h$  high enough. For  $\sigma > 1$ , they will be satisfied for  $h$  low enough. Summarizing

**Lemma 3.** *For  $\sigma < 1$ , case 3 holds for  $h$  high enough, and in case 3,  $s^*$  increases in  $h$ . For  $\sigma > 1$ , case 3 holds for  $h$  low enough and  $s^*$  decreases in  $h$ .*

**Case 4:**  $N_1^* > 0, N_2^* = 0$

Rearranging the first order conditions (7) and (8) for  $N_1^*$  and  $N_2^*$  gives

$$\frac{a}{b} = (1 - s^* - bN_1^*)^{-\sigma}$$

$$\frac{a}{b} \leq s^*hY_2^{-\sigma}$$

hence

$$\begin{aligned}
(1 - s^* - bN_1^*)^{-\sigma} &\leq s^* h Y_2^{-\sigma} = (s^* h)^{1-\sigma} \\
\Leftrightarrow (1 - s^* - bN_1^*)^\sigma &\geq (s^* h)^{\sigma-1} \\
\Leftrightarrow 1 - s^* - bN_1^* &\geq (s^* h)^{1-1/\sigma}
\end{aligned}$$

Now rearrange the first order condition for  $s^*$  from (9), noting that since  $N_2^* = 0$ ,  $s^* > 0$  by the Inada condition.

$$\begin{aligned}
h^{1/\sigma-1}(1 - s^* - bN_1^*) &= s^* \\
1 - s^* - bN_1^* &= s^* h^{1-1/\sigma}
\end{aligned}$$

This, combined with the previous inequality, implies

$$\begin{aligned}
(s^* h)^{1-1/\sigma} &\leq s^* h^{1-1/\sigma} \\
\Leftrightarrow (s^*)^{-1/\sigma} &\leq 1
\end{aligned}$$

which cannot hold since  $0 < s^* < 1$ .

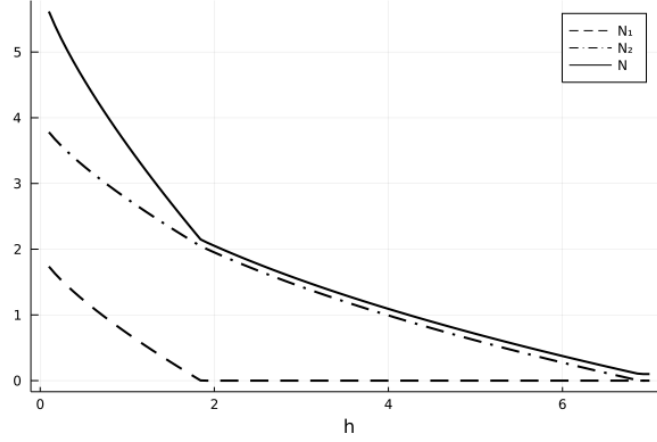
## Comparative statics

We can now examine how the fertility-human capital relationship

$$\frac{dN^*}{dh}, \text{ where } N^* \equiv N_1^* + N_2^*,$$

changes with respect to other parameters. We focus on the case  $\sigma < 1$ , since it gives the closest match to our observations, and since it also generates “reasonable” predictions in other areas, e.g. that education levels increase with human capital. Figure 19 shows how  $N^*$  changes with  $h$  for  $a = 0.4, b = 0.25, \sigma = 0.7$ .

From Lemmas 1, 2 and 3, as  $h$  increases we move from  $N_1^*, N_2^* > 0$  to  $N_1^* = 0, N_2^* > 0$  to  $N_1^* = N_2^* = 0$ . Furthermore, for  $\sigma > 2/3$ ,  $N_1^*$  and  $N_2^*$  are convex in  $h$  when they are both positive, and for  $\sigma$  close enough to 1,  $N_2^*$  is convex in  $h$  when  $N_1^* = 0$ . To show global concavity of  $N^*$  in  $h$  as  $\sigma \nearrow 1$ , all that remains is to check that the derivative is increasing around the points where these 3 regions meet. That is trivially satisfied where  $N_2^*$  becomes 0, since thereafter  $\frac{dN^*}{dh}$  is



**Figure 19:** Fertility vs. human capital in the two-period model with  $a = 0.4, b = 0.25, \sigma = 0.7$ .

zero. The derivative as  $N_1^*$  approaches zero is twice the expression in (15):

$$-\frac{2}{b} \left(\frac{b}{a}\right)^{1/(2\sigma-1)} \frac{1-\sigma}{2\sigma-1} h^{(1-\sigma)/(2\sigma-1)-1} \quad (21)$$

and the derivative to the right of this point is given by (19):

$$-\frac{1}{b} \left(\frac{b}{a}\right)^{1/\sigma} \frac{1-\sigma}{\sigma} (s^*h)^{(1-2\sigma)/\sigma} \left(s^* + h \frac{ds^*}{dh}\right) \quad (22)$$

We want to prove that the former is larger in magnitude (i.e. more negative). Dividing (21) by (22) gives

$$2 \frac{\sigma}{2\sigma-1} \left(\frac{b}{a}\right)^{(1-\sigma)/(\sigma(2\sigma-1))} \frac{h^{(1-\sigma)^2/(\sigma(2\sigma-1))}}{s^*(s^* + h \frac{ds^*}{dh})}$$

Examining (17) shows that as  $\sigma \rightarrow 1$ ,  $s^* \rightarrow 0.5$  and  $\frac{ds^*}{dh} \rightarrow 0$ , and therefore the above approaches

$$2 \frac{1}{(0.5)^2} = 8.$$

Thus, as  $\sigma \nearrow 1$ ,  $N^*$  is guaranteed to be globally convex in  $h$ .

We can now prove the facts stated in the main text.

**Fact 1:** for  $\sigma < 1$ , total fertility  $N^* \equiv N_1^* + N_2^*$  is decreasing in human capital  $h$ .

Furthermore, for  $\sigma$  close enough to 1, fertility is convex in human capital, i.e.

**Fact 2 part 1:** the fertility-human capital relationship is closer to 0 at high levels of  $h$ .

For  $\sigma < 1$ , education levels  $s^*$  increase in  $h$ , and so therefore do equilibrium wages  $w(s^*, h)$ . This, plus fact 1, gives:

**Fact 2 part 2:** for  $\sigma < 1$  and close to 1, the fertility-human capital relationship is weaker among higher earners.

**Fact 4:** for  $\sigma < 1$  and close to 1, the fertility-human capital relationship is weaker at high levels of education.

Next, we compare people who start fertility early ( $N_1^* > 0$ ) versus those who start fertility late ( $N_1^* = 0$ ). Again, for  $\sigma < 1$  the former group have lower  $h$  than the latter group. Thus we have:

**Fact 5:** for  $\sigma < 1$  and close to 1, the fertility-human capital relationship is weaker among those who start fertility late.

Lastly, we prove fact 3. Differentiating  $dN_t^*/dh$  in (15) with respect to  $b$ , for when  $N_1^* > 0$  gives:

$$\frac{d^2 N_t^*}{dhdb} = \frac{2\sigma - 2}{2\sigma - 1} b^{(3-4\sigma)/(2\sigma-1)} \left(\frac{1}{a}\right)^{1/(2\sigma-1)} \frac{1-\sigma}{2\sigma-1} h^{(\sigma-1)^2/(\sigma(2\sigma-1))}$$

which is negative for  $0.5 < \sigma < 1$ . When  $N_1^* = 0$ , differentiating  $dN_2^*/dh$  in (19) gives:

$$\frac{d^2 N_2^*}{dhdb} = -\frac{1-\sigma}{\sigma} b^{(1-2\sigma)/\sigma} \left(\frac{1}{a}\right)^{1/\sigma} \frac{1-\sigma}{\sigma} (s^*h)^{(1-2\sigma)/\sigma} (s^* + h \frac{ds^*}{dh})$$

which again is negative for  $\sigma < 1$ . Therefore:

**Fact 3:** for  $\sigma < 1$ , the fertility-human capital relationship is more negative when the burden of childcare  $b$  is larger.

## Concavity

**Lemma 4.** For  $\sigma > 0.5$ ,  $U$  in equation (2) is concave in  $N_1, N_2$  and  $s$ .

*Proof.* We examine the Hessian matrix of utility in each period. Note that period 1 utility is constant in  $N_2$  and period 2 utility is constant in  $N_1$ . For period 1 the Hessian with respect to  $N_1$  and  $s$  is:

$$\begin{bmatrix} d^2u/dN_1^2 & d^2u/dsdN_1 \\ d^2u/dsdN_1 & d^2u/ds^2 \end{bmatrix} = \begin{bmatrix} -\sigma b^2 & -\sigma b \\ -\sigma b & -\sigma \end{bmatrix} Y_1^{-\sigma-1}$$

with determinant

$$(\sigma^2 b^2 - \sigma^2 b^2) Y_1^{-2\sigma-2} = 0.$$

Thus, first period utility is weakly concave. For period 2 with respect to  $N_2$  and  $s$ , the Hessian is:

$$\begin{bmatrix} d^2u/dN_2^2 & d^2u/dsdN_2 \\ d^2u/dsdN_2 & d^2u/ds^2dN_2 \end{bmatrix} = \begin{bmatrix} -\sigma(bsh)^2Y_2^{-\sigma-1} & -(1-\sigma)bhY_2^{-\sigma} \\ -(1-\sigma)bhY_2^{-\sigma} & -\sigma[h(1-bN_2^*)]^2Y_2^{-\sigma-1} \end{bmatrix}$$

with determinant

$$\begin{aligned} & (-\sigma(bsh)^2Y_2^{-\sigma-1})(-\sigma[h(1-bN_2^*)]^2Y_2^{-\sigma-1}) - (-(1-\sigma)bhY_2^{-\sigma})^2 \\ &= \sigma^2(bsh)^2[h(1-bN_2^*)]^2Y_2^{-2\sigma-2} - (1-\sigma)^2(bh)^2Y_2^{-2\sigma} \\ &= \sigma^2(bh)^2Y_2^{-2\sigma} - (1-\sigma)^2(bh)^2Y_2^{-2\sigma}, \text{ using that } Y_2 = (sh)(1-bN) \\ &= (bh)^2Y_2^{-2\sigma}(\sigma^2 - (1-\sigma)^2) \end{aligned}$$

which is positive if and only if  $\sigma > 0.5$ . Thus, if  $\sigma > 0.5$  then the Hessian is negative definite and thus utility is concave; this combined with weak concavity of period 1, and linearity of  $a(N_1 + N_2)$ , shows that (2) is concave.  $\square$



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