To show how income and substitution effects can lead to our results, we write down a simple model of fertility choices. h is an individual's level of human capital, which (for now) we identify with his or her wage W. Raising a child takes time b. People maximize utility from the number of children N, and income $Y \equiv (1 - bN)W$. An individual's payoff is

$$U(N) = u(Y) + aN.$$

Here a captures the strength of preference for children. $u(\cdot)$ is concave and increasing. We treat N as continuous, in line with the literature: this can be thought of as the expected number of children among similar people. The marginal benefit of an extra child is $\frac{dU}{dN} = -bWu'(Y) + a$. The effect of an increase in wages (or human capital) on this marginal benefit is

$$\frac{d^2U}{dNdh} = \underbrace{-bu'(Y)}_{\text{Substitution effect}} \underbrace{-bYu''(Y)}_{\text{Income effect}}.$$

The *substitution effect* is negative and reflects that when wages increase, time devoted to childcare costs more in foregone income. The positive *income effect* reflects that when wage income is higher, the marginal loss of wages from children is less painful. Letting $u(\cdot)$ take the constant relative risk aversion (CRRA) form $u(y) = \frac{y^{1-\sigma}-1}{1-\sigma}$, where $\sigma > 0$ captures risk aversion in income, we solve for optimal fertility N^*, and examine the *fertility-human capital relationship*, $\frac{dN^*}{dh}$. For moderate levels of risk aversion ($0.5 < \sigma < 1$):

1. The fertility-human capital relationship is negative: $\frac{dN^*}{dh} < 0$.
2. The relationship is weaker (closer to zero) at higher wages and/or levels of human capital.
3. The relationship is more negative when the time burden of children, b, is larger.

To examine education and fertility timing, we extend the model to two periods. For convenience we ignore time discounting; we also assume that credit markets are imperfect so that agents cannot borrow. Write

$$U(N_1, N_2) = u(Y_1) + u(Y_2) + a(N_1 + N_2) \tag{1}$$

Instead of identifying human capital h with wages, we now allow individuals to choose a level of education $s \in [0, 1]$, which has a time cost in period 1. Education increases period 2 wages, and is complementary to human capital $h > 0$. For period 2 wages we assume the simple functional form $w(s, h) = sh$. We normalize period 1 wages to 1. Maintaining the assumption of CRRA utility, we examine total fertility $N^* \equiv N_1^* + N_2^*$. For $0.5 < \sigma < 1$, facts 1-3 continue to hold (fact 2 in a neighborhood of $\sigma = 1$). In addition, for σ in a neighborhood of 1:

4. The fertility-human capital relationship is weaker at higher levels of education s.

5. The relationship is weaker among those who start fertility in period 2 ($N_1^* = 0$) than among those who start fertility in period 1 ($N_1^* > 0$).

Facts 1-5 match our empirical results. In this model, polygenic scores which correlate with human capital would have a negative correlation with fertility at low income and education levels, and a weaker or zero correlation at higher levels. For scores which are highly correlated with human capital, the correlation would also be weaker at higher levels of the score itself. The correlation would be more negative when the time burden of children b is high, e.g. for single parents. Lastly, the correlation would be weaker among those starting fertility later.

An alternative theory is that polygenic scores correlate with the motivation to have children (parameter a in the model). But this would not explain the pattern of our results across income and education. Indeed, in the one-period model, while $\frac{dN^*}{da} > 0$, this relationship actually becomes stronger at higher wages.

The above does not provide a complete theory of fertility. Rather, it shows that a relatively simple economic model can explain many of our results. In the appendix, we discuss other models of fertility from the economic literature, as well as causal evidence for the relationship between human capital, income and fertility.

Appendix

Solution for the one-period model

Differentiating and setting $\frac{dU}{dN} = 0$ gives the first order condition for an optimal choice of children $N^* > 0$:

$$\frac{bW}{(W(1-bN^*))^\sigma} \geq a, \text{ with equality if } N^* > 0.$$

Rearranging gives

$$N^* = \max\left\{\frac{1}{b}\left(1 - \left(\frac{b}{a}\right)^{1/\sigma} W^{(1-\sigma)/\sigma}\right), 0\right\}. \tag{2}$$

Note that when $\sigma < 1$, for high enough W, $N^* = 0$. Differentiating gives the effect of wages on fertility for $N^* > 0$. This is also the fertility-human capital relationship:

$$\frac{dN^*}{dh} = \frac{dN^*}{dW} = -\frac{1}{b}\left(\frac{b}{a}\right)^{1/\sigma} \frac{1-\sigma}{\sigma} W^{(1-2\sigma)/\sigma}. \tag{3}$$

This is negative if $\sigma < 1$. Also,

$$\frac{d^2N^*}{dW^2} = -\frac{1}{b}\left(\frac{b}{a}\right)^{1/\sigma} \frac{1-\sigma}{\sigma}\frac{1-2\sigma}{\sigma} W^{(1-3\sigma)/\sigma}$$

For $0.5 < \sigma < 1$, this is positive, so the effect of fertility on wages shrinks towards zero as wages increase (and becomes 0 when $N^* = 0$). Next, we consider the time cost of children b:

$$\frac{d^2N^*}{dWdb} = -\left(\frac{1}{a}\right)^{1/\sigma}\left(\frac{1-\sigma}{\sigma}\right)^2 (Wb)^{(1-2\sigma)/\sigma} < 0.$$

Lastly we consider the effect of a. From (2), N^* is increasing in a. Differentiating (3) by a gives

$$\frac{d^2N^*}{dadW} = b^{1/\sigma - 1}\frac{1-\sigma}{\sigma^2} W^{(1-2\sigma)/\sigma} a^{-1/\sigma - 1}$$

which is positive for $\sigma < 1$.

Solution for the two-period model

Period 1 and period 2 income are:

$$Y_1 = 1 - s - bN_1 \tag{4}$$

$$Y_2 = w(s,h)(1 - bN_2) \tag{5}$$

Write the Lagrangian of utility U (1) as

$$\mathcal{L}(N_1, N_2, s) = u(Y_1) + u(Y_2) + a(N_1 + N_2) + \lambda_1 N_1 + \lambda_2 N_2 + \lambda_3(\frac{1}{b} - N_2) + \mu s$$

Lemma 4 below shows that if $\sigma > 0.5$, this problem is globally concave, guaranteeing that the first order conditions identify a unique solution. We assume $\sigma > 0.5$ from here on.

Plugging (4) and (5) into the above, we can derive the Karush-Kuhn-Tucker conditions for an optimum (N_1^*, N_2^*, s^*) as:

$$\frac{d\mathcal{L}}{dN_1} = -bY_1^{-\sigma} + a + \lambda_1 = 0, \text{ with } \lambda_1 = 0 \text{ if } N_1^* > 0; \tag{6}$$

$$\frac{d\mathcal{L}}{dN_2} = -bs^*hY_2^{-\sigma} + a + \lambda_2 - \lambda_3 = 0, \text{ with } \lambda_2 = 0 \text{ if } N_2^* > 0, \lambda_3 = 0 \text{ if } N_2^* < \frac{1}{b}; \tag{7}$$

$$\frac{d\mathcal{L}}{ds} = -Y_1^{-\sigma} + h(1 - bN_2^*)Y_2^{-\sigma} + \mu = 0; \tag{8}$$

$$N_1^*, N_2^*, s^*, \lambda_1, \lambda_2, \lambda_3, \mu \geq 0; N_2^* \leq \frac{1}{b}. \tag{9}$$

Note that the Inada condition ($\lim_{x\to 0} u'(x) = \infty$) for period 1 rules out $s^* = 1$ and $N_1 = 1/b$, so we need not impose these constraints explicitly. Also, so long as $N_2^* < 1/b$, the same condition rules out $s^* = 0$. We consider four cases, of which only three can occur.

Case 1: $N_1^* > 0, N_2^* > 0$

Rearranging (6), (7) and (8) gives:

$$N_1^* = \frac{1}{b}\left(1 - s^* - \left(\frac{b}{a}\right)^{1/\sigma}\right); \tag{10}$$

$$N_2^* = \frac{1}{b}\left(1 - \left(\frac{b}{a}\right)^{1/\sigma} (s^* h)^{(1-\sigma)/\sigma}\right); \tag{11}$$

$$s^* = \frac{1 - bN_1^*}{1 + ((1 - bN_2^*)h)^{1-1/\sigma}}. \tag{12}$$

Plugging the expressions for N_1^* and N_2^* into s^*gives

$$s^* = \frac{s^* + \left(\frac{b}{a}\right)^{1/\sigma}}{1 + \left(\left(\frac{b}{a}\right)^{1/\sigma} s^{*(1-\sigma)/\sigma} h^{1/\sigma}\right)^{1-1/\sigma}}$$

which simplifies to

$$s^* = \left(\frac{b}{a}\right)^{1/(2\sigma-1)} h^{(1-\sigma)/(2\sigma-1)}. \tag{13}$$

Plugging the above into (10) and (11) gives:

$$N_1^* = \frac{1}{b}\left(1 - \left(\frac{b}{a}\right)^{1/(2\sigma-1)} h^{(1-\sigma)/(2\sigma-1)} - \left(\frac{b}{a}\right)^{1/\sigma}\right);$$

$$N_2^* = \frac{1}{b}\left(1 - \left(\frac{b}{a}\right)^{1/(2\sigma-1)} h^{(1-\sigma)/(2\sigma-1)}\right).$$

Note that that $N_1^* < N_2^*$. For these both to be positive requires low values of h if $\sigma < 1$ and high values of h if $\sigma > 1$. Also:

$$w(s^*, h) \equiv s^* h = \left(\frac{b}{a}\right)^{1/(2\sigma-1)} h^{\sigma/(2\sigma-1)}.$$

Observe that $w(s^*, h)$ is increasing in h for $\sigma > 0.5$, and convex iff $0.5 < \sigma < 1$.

While N_1^* and N_2^* are positive, they have the same derivative with respect to h:

$$\frac{dN_t^*}{dh} = -\frac{1}{b}\left(\frac{b}{a}\right)^{1/(2\sigma-1)} \frac{1-\sigma}{2\sigma-1} h^{(1-\sigma)/(2\sigma-1)-1} \tag{14}$$

Examining this and expression (13) gives:

Lemma 1. *For $\sigma < 1$, case 1 holds for h low enough, and in case 1, N_1^* and N_2^* decrease in h, while s^* increases in h.*

For $\sigma > 1$, case 1 holds for h high enough, and in case 1 N_1^* and N_2^* increase in h, while s^* decreases in h.

N_t^* is convex in h for $\sigma > 2/3$, and concave otherwise. s^* is convex in h if $\sigma < 2/3$, and concave otherwise.

Case 2: $N_1^* = 0, N_2^* > 0$

Replace $N_1^* = 0$ into the first order condition for s^* from (8), and rearrange to give:

$$s^* = \frac{1}{1 + ((1 - bN_2)h)^{1-1/\sigma}}.$$

Now since $N_2^* > 0$, we can rearrange (7) to give

$$N_2^* = \frac{1}{b}\left(1 - \left(\frac{b}{a}\right)^{1/\sigma} (s^* h)^{(1-\sigma)/\sigma}\right). \tag{15}$$

Plugging this into s^* gives

$$s^* = \frac{1}{1 + \left(\frac{bh}{a}\right)^{(\sigma-1)/\sigma^2} (s^*)^{-(1-\sigma)^2/\sigma^2}}$$

which can be rearranged to

$$(1 - s^*)(s^*)^{(1-2\sigma)/\sigma^2} = \left(\frac{a}{bh}\right)^{(1-\sigma)/\sigma^2}. \tag{16}$$

Differentiate the left hand side of the above to get

$$\begin{aligned}
&\frac{1-2\sigma}{\sigma^2}(1 - s^*)(s^*)^{(1-2\sigma)/\sigma^2-1} - (s^*)^{(1-2\sigma)/\sigma^2} \\
=&\frac{1-2\sigma}{\sigma^2}(s^*)^{(1-2\sigma)/\sigma^2-1} - \frac{\sigma^2+1-2\sigma}{\sigma^2}(s^*)^{(1-2\sigma)/\sigma^2} \\
=&\frac{1-2\sigma}{\sigma^2}(s^*)^{(1-2\sigma)/\sigma^2-1} - \frac{(1-\sigma)^2}{\sigma^2}(s^*)^{(1-2\sigma)/\sigma^2}.
\end{aligned} \tag{17}$$

This is negative if and only if

$$s^* > \frac{1-2\sigma}{(1-\sigma)^2}$$

which is always true since $\sigma > 0.5$. Note also that since $\sigma > 0.5$, then the left hand side of (16) approaches infinity as $s^* \to 0$ and approaches 0 as $s^* \to 1$. Thus, (16) implicitly defines the unique solution for s^*.

Conditions (6) and (7) give the bounds for this case. When $\sigma < 1$, (6) puts a minimum on h and (7) puts a maximum on h. The situation is reversed for $\sigma > 1$.

To find how s^* changes with h, note that the right hand side of the above decreases in h for $\sigma < 1$, and increases in h for $\sigma > 1$. Putting these facts together: for $\sigma < 1$, when h increases the RHS of (16) decreases, hence the LHS decreases and s^* increases, i.e. s^* is increasing in h. For $\sigma > 1$, s^* is decreasing in h.

To find how N_2^* changes with h, we differentiate (15):

$$\frac{dN_2^*}{dh} = -\frac{1}{b}\left(\frac{b}{a}\right)^{1/\sigma}\frac{1-\sigma}{\sigma}(s^*h)^{(1-2\sigma)/\sigma}(s^* + h\frac{ds^*}{dh}) \tag{18}$$

which is negative for $\sigma < 1$, since $\frac{ds^*}{dh} > 0$ in this case.

Differentiating again:

$$\begin{aligned}\frac{d^2N_2}{dh^2} &= -X[\frac{1-2\sigma}{\sigma}(s^*h)^{(1-3\sigma)/\sigma}(s^* + h\frac{ds^*}{dh})^2 + (s^*h)^{(1-2\sigma)/\sigma}(2\frac{ds^*}{dh} + h\frac{d^2s^*}{dh^2})] \\ &= X(s^*h)^{(1-3\sigma)/\sigma}[\frac{2\sigma-1}{\sigma}(s^* + h\frac{ds^*}{dh})^2 - (s^*h)(2\frac{ds^*}{dh} + h\frac{d^2s^*}{dh^2})]\end{aligned}$$

where $X = \frac{1}{b}\left(\frac{b}{a}\right)^{1/\sigma}\frac{1-\sigma}{\sigma} > 0$. Note that $\frac{d^2N_2}{dh^2}$ is continuous in σ around $\sigma = 1$. Note also from (16) that for $\sigma = 1$, s^* becomes constant in σ. The term in square brackets then reduces to $(s^*)^2 > 0$. Putting these facts together, for σ sufficiently close to 1, $\frac{d^2N_2^*}{dh^2} > 0$, i.e. N_2^* is convex in h.

Summarizing:

Lemma 2. *Case 2 holds for intermediate values of h. In case 2: for $\sigma < 1$, s^* is increasing in h and N_2^* is decreasing in h. For $\sigma > 1$, $s^*$is decreasing in h. For σ close enough to 1, N_2^* is convex in h.*

Case 3: $N_1^* = 0, N_2^* = 0$

We can solve for s^* by substituting values of Y_1 and Y_2 into (8):

$$-(1-s^*)^{-\sigma} + h(s^*h)^{-\sigma} = 0$$

which rearranges to

$$s^* = \frac{1}{1 + h^{(\sigma-1)/\sigma}}. \tag{19}$$

Conditions (6) and (7) become:

$$
\begin{aligned}
-b(1-s^*)^{-\sigma} + a &\le 0 \\
-bs^*h(s^*h)^{-\sigma} + a &\le 0
\end{aligned}
$$

equivalently

$$
\begin{aligned}
\frac{a}{b} &\le (1-s^*)^{-\sigma} \\
\frac{a}{b} &\le s^*h(s^*h)^{-\sigma}
\end{aligned}
$$

which can both be satisfied for a/b close enough to zero. Note from (19) that as $h \to \infty$, s^* increases towards 1 for $\sigma < 1$, and decreases towards 0 for $\sigma > 1$. Note also that the right hand side of the first inequality above approaches infinity as $s^* \to 1$, therefore also as $h \to \infty$ for $\sigma < 1$. Rewrite the second inequality as

$$
\frac{a}{b} < (s^*h)^{1-\sigma} = \left(\frac{h}{1+h^{(\sigma-1)/\sigma}}\right)^{1-\sigma} = \left(h^{-1} + h^{-1/\sigma}\right)^{\sigma-1}
$$

and note that again, as $h \to \infty$, the RHS increases towards infinity for $\sigma < 1$, and decreases towards zero otherwise. Thus, for $\sigma < 1$, both equations will be satisfied for h high enough. For $\sigma > 1$, they will be satisfied for h low enough. Summarizing

Lemma 3. *For $\sigma < 1$, case 3 holds for h high enough, and in case 3, s^* increases in h. For $\sigma > 1$, case 3 holds for h low enough and s^* decreases in h.*

Case 4: $N_1^* > 0, N_2^* = 0$

Rearranging the first order conditions (6) and (7) for N_1^* and N_2^* gives

$$
\begin{aligned}
\frac{a}{b} &= (1-s^*-bN_1^*)^{-\sigma} \\
\frac{a}{b} &\le s^*hY_2^{-\sigma}
\end{aligned}
$$

hence

$$
\begin{aligned}
(1-s^*-bN_1^*)^{-\sigma} &\le s^*hY_2^{-\sigma} = (s^*h)^{1-\sigma} \\
\Leftrightarrow (1-s^*-bN_1^*)^{\sigma} &\ge (s^*h)^{\sigma-1} \\
\Leftrightarrow 1-s^*-bN_1^* &\ge (s^*h)^{1-1/\sigma}
\end{aligned}
$$

Now rearrange the first order condition for s^* from (8), noting that since $N_2^* = 0$, $s^* > 0$ by the Inada condition.

$$
\begin{aligned}
h^{1/\sigma-1}(1-s^*-bN_1^*) &= s^* \\
1-s^*-bN_1^* &= s^*h^{1-1/\sigma}
\end{aligned}
$$

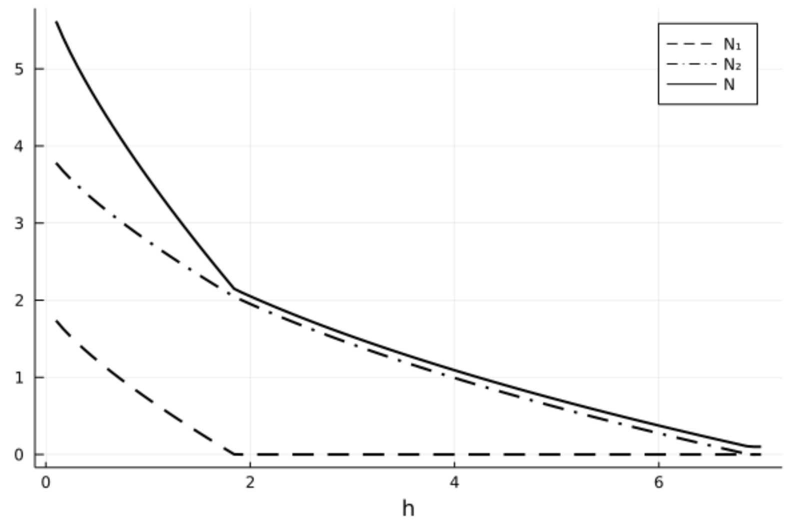

Figure 1: Fertility vs. human capital in the two-period model with $a = 0.4, b = 0.25, \sigma = 0.7$.

This, combined with the previous inequality, implies

$$\begin{aligned} &(s^* h)^{1-1/\sigma} \leq s^* h^{1-1/\sigma} \\ \Leftrightarrow\ &(s^*)^{-1/\sigma} \leq 1 \end{aligned}$$

which cannot hold since $0 < s^* < 1$.

Comparative statics

We can now examine how the fertility-human capital relationship

$$\frac{dN^*}{dh}, \text{ where } N^* \equiv N_1^* + N_2^*,$$

changes with respect to other parameters. We focus on the case $\sigma < 1$, since it gives the closest match to our observations, and since it also generates "reasonable" predictions in other areas, e.g. that education levels increase with human capital. Figure 1 shows how N^* changes with h for $a = 0.4, b = 0.25, \sigma = 0.7$.

From Lemmas 1, 2 and 3, as h increases we move from $N_1^*, N_2^* > 0$ to $N_1^* = 0, N_2^* > 0$ to $N_1^* = N_2^* = 0$. Furthermore, for $\sigma > 2/3$, N_1^* and N_2^* are convex in h when they are both positive, and for σ close enough to 1, N_2^* is convex in h when $N_1^* = 0$. To show global concavity of N^* in h as $\sigma \nearrow 1$, all that remains is to check that the derivative is increasing around the points where these 3 regions meet. That is trivially satisfied where N_2^* becomes 0, since thereafter $\frac{dN^*}{dh}$ is zero. The derivative as N_1^* approaches zero is twice the expression in (14):

$$-\frac{2}{b}\left(\frac{b}{a}\right)^{1/(2\sigma-1)}\frac{1-\sigma}{2\sigma-1}h^{(1-\sigma)/(2\sigma-1)-1} \tag{20}$$

and the derivative to the right of this point is given by (18):

$$-\frac{1}{b}\left(\frac{b}{a}\right)^{1/\sigma}\frac{1-\sigma}{\sigma}(s^*h)^{(1-2\sigma)/\sigma}(s^*+h\frac{ds^*}{dh}) \tag{21}$$

We want to prove that the former is larger in magnitude (i.e. more negative). Dividing (20) by (21) gives

$$2\frac{\sigma}{2\sigma-1}\left(\frac{b}{a}\right)^{(1-\sigma)/(\sigma(2\sigma-1))}\frac{h^{(1-\sigma)^2/(\sigma(2\sigma-1))}}{s^*(s^*+h\frac{ds^*}{dh})}$$

Examining (16) shows that as $\sigma \to 1$, $s^* \to 0.5$ and $\frac{ds^*}{dh} \to 0$, and therefore the above approaches

$$2\frac{1}{(0.5)^2} = 8.$$

Thus, as $\sigma \nearrow 1$, N^* is guaranteed to be globally concave in h.

We can now prove the facts stated in the main text.

Fact 1: for $\sigma < 1$, total fertility $N^* \equiv N_1^* + N_2^*$ is decreasing in human capital h.

Furthermore, for σ close enough to 1, fertility is convex in human capital, i.e.

Fact 2 part 1: the fertility-human capital relationship is closer to 0 at high levels of h.

For $\sigma < 1$, education levels s^* increase in h, and so therefore do equilibrium wages $w(s^*, h)$. This, plus fact 1, gives:

Fact 2 part 2: for $\sigma < 1$ and close to 1, the fertility-human capital relationship is weaker among higher earners.

Fact 4: for $\sigma < 1$ and close to 1, the fertility-human capital relationship is weaker at high levels of education.

Next, we compare people who start fertility early ($N_1^* > 0$) versus those who start fertility late ($N_1^* = 0$). Again, for $\sigma < 1$ the former group have lower h than the latter group. Thus we have:

Fact 5: for $\sigma < 1$ and close to 1, the fertility-human capital relationship is weaker among those who start fertility late.

Lastly, we prove fact 3. Differentiating dN_t^*/dh in (14) with respect to b, for when $N_1^* > 0$ gives:

$$\frac{d^2N_t^*}{dhdb} = \frac{2\sigma-2}{2\sigma-1}b^{(3-4\sigma)/(2\sigma-1)}\left(\frac{1}{a}\right)^{1/(2\sigma-1)}\frac{1-\sigma}{2\sigma-1}h^{(\sigma-1)^2/(\sigma(2\sigma-1))}$$

which is negative for $0.5 < \sigma < 1$. When $N_1^* = 0$, differentiating dN_2^*/dh in (18) gives:

$$\frac{d^2N_2^*}{dhdb} = -\frac{1-\sigma}{\sigma}b^{(1-2\sigma)/\sigma}\left(\frac{1}{a}\right)^{1/\sigma}\frac{1-\sigma}{\sigma}(s^*h)^{(1-2\sigma)/\sigma}(s^* + h\frac{ds^*}{dh})$$

which again is negative for $\sigma < 1$. Therefore:

Fact 3: for $\sigma < 1$, the fertility-human capital relationship is more negative when the burden of childcare b is larger.

Concavity

Lemma 4. *For $\sigma > 0.5$, U in equation (1) is concave in N_1, N_2 and s.*

Proof. We examine the Hessian matrix of utility in each period. Note that period 1 utility is constant in N_2 and period 2 utility is constant in N_1. For period 1 the Hessian with respect to $N_1$and s is:

$$\begin{bmatrix} d^2u/dN_1^2 & d^2u/dsdN_1 \\ d^2u/dsdN_1 & d^2u/ds^2 \end{bmatrix} = \begin{bmatrix} -\sigma b^2 & -\sigma b \\ -\sigma b & -\sigma \end{bmatrix} Y_1^{-\sigma-1}$$

with determinant

$$(\sigma^2b^2 - \sigma^2b^2)Y_1^{-2\sigma-2} = 0.$$

Thus, first period utility is weakly concave. For period 2 with respect to N_2 and s, the Hessian is:

$$\begin{bmatrix} d^2u/dN_2^2 & d^2u/dsdN_2 \\ d^2u/dsdN_2 & d^2u/ds^2dN_2 \end{bmatrix} = \begin{bmatrix} -\sigma(bsh)^2Y_2^{-\sigma-1} & -(1-\sigma)bhY_2^{-\sigma} \\ -(1-\sigma)bhY_2^{-\sigma} & -\sigma[h(1-bN_2^*)]^2Y_2^{-\sigma-1} \end{bmatrix}$$

with determinant

$$\begin{aligned} &(-\sigma(bsh)^2Y_2^{-\sigma-1})(-\sigma[h(1-bN_2^*)]^2Y_2^{-\sigma-1}) - (-(1-\sigma)bhY_2^{-\sigma})^2 \\ =&\sigma^2(bsh)^2[h(1-bN_2^*)]^2Y_2^{-2\sigma-2} - (1-\sigma)^2(bh)^2Y_2^{-2\sigma} \\ =&\sigma^2(bh)^2Y_2^{-2\sigma} - (1-\sigma)^2(bh)^2Y_2^{-2\sigma}, \text{ using that } Y_2 = (sh)(1-bN) \\ =&(bh)^2Y_2^{-2\sigma}(\sigma^2 - (1-\sigma)^2) \end{aligned}$$

which is positive if and only if $\sigma > 0.5$. Thus, if $\sigma > 0.5$ then the Hessian is negative definite and thus utility is concave; this combined with weak concavity of period 1, and linearity of $a(N_1 + N_2)$, shows that (1) is concave. $\square$