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Profiles of Fertility, Labour Supply and Wages of Married Women: A Complete Life-Cycle Model

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This paper is an econometric examination of female fertility and labour-supply decisions. Based upon utility-maximizing choice, fertility and labour-supply demand functions are specified and estimated jointly with a wage-accumulation equation. A main contribution of the paper is the demonstration of a maximum-likelihood estimation method which avoids the main selectivity-bias problems in this area.

1. INTRODUCTION

Of all the major economic and demographic events in the postwar United States period, the trends in fertility and female labour supply stand out as among the most dramatic. Fertility rates, after their Depression lows, rose unexpectedly after World War II far more than was anticipated as an adjustment to the War, and continued to rise until the late 1950's and early 1960's, when they again took an unexpected turn downward all the way to today's historical lows. Female labour-force-participation rates, continuing their historical climb, grew throughout the postwar period at an increasing pace, by 4 percentage points during the 1950's, by 5 percentage points during the 1960's, and by 8 percentage points during the 1970's. These two trends are of course closely related, for most married women have traditionally spent at least some minimal time out of the labour force while caring for their own children. Thus, for example, the slower growth rate of labour-force-participation rates in the 1950's is probably connected with the higher fertility rates in that period.

In this paper a complete model of female labour supply and fertility choice is constructed and estimated. The model is more complete than previous models in several respects. First, labour supply and fertility decisions are modelled as completely joint in the same sense as the consumption of two goods is joint. Second, both are modelled as life-cycle decisions, that is, as decisions regarding sequences of labour-supply and fertility events rather than levels of the same. Hence the timing of both is important. Third, the life-cycle path of wage rates is made endogenous to the model—given a wage function showing the effect of work experience on future wages, the path of wages is determined automatically by the chosen path of labour supply, which in turn is related to fertility decisions that require some period of non-work. Econometrically, the three profiles for labour supply, fertility, and wages are estimated with a full-information maximum likelihood technique that solves several econometric problems, such as the problem of heterogeneity of tastes; the selectivity problem of missing wage rates for non-workers; and the problem of simultaneous-equations bias.

The value of constructing a complete model is best seen by briefly reviewing past work in this area. The modern literature in the area is based upon the work of Becker fertility decision could be viewed as an economic one, and that one of the costs of having a child is the foregone earnings of the person caring for the child in the home, usually the mother. Consequently the wage rate was assigned a central role. An additional step was taken by Willis (1974), who formally modelled this joint fertility-labour-supply decision in great detail. However, the major weakness of these models has always been their static nature, for they generally attempt to describe the choice of completed fertility, rather than its time path. The definition of labour supply in such a model is unclear, for having children implies a certain pattern of labour supply rather than any particular average labour supply in all periods.

Nevertheless, few empirical analyses have recognized the interrelatedness of fertility and labour supply in even a static sense, much less a dynamic one. The labour-supply literature is particularly deficient in this regard, for the outpouring of studies over the past ten or fifteen years has been based upon a simple static model in which current labour supply is a function of the current stock of children and the current wage rate. The endogeneity of the latter variables from a life-cycle perspective makes the results of this literature suspect. For example, one result of this literature is that female labour supply is much more sensitive than is male labour supply to changes in wage rates and changes in the number of children in the home. But endogenous variables are typically statistically significant (though spuriously) anyway. The fertility literature, though much smaller in volume than the labour-supply literature, is similarly deficient. Typically the completed fertility of women over 40 or 45 years of age is regressed upon a set of exogenous variables excluding the wage rate or any labour-supply variables, giving at best an extreme reduced form equation. However, more recently, instrumental variables for wage rates and labour supply have been included in age-specific fertility equations (Butz and Ward (1979), Ward and Butz (1980)).[1]

There has been recently a movement toward estimating simultaneous fertility-labour-supply models (Cain and Dooley (1976), Fleisher and Rhodes (1979), Link and Settle (1981) and Schultz (1978, 1980)). However, these models are often difficult to interpret because they are usually static in conception and specification while the underlying process is sequential. Also, the formulation of these models as simultaneous (in the econometric sense of the term) raises identification problems that can be rather thorny (Rosenzweig and Wolpin (1980)). However, as Mincer (1963, p. 78) pointed out in his original article, such problems do not arise when fertility and labour supply decisions are viewed as joint consumer-demand choices. In any case, recent work by Heckman and MaCurdy (1980) and MaCurdy (1980) has used panel data to model the labour-supply decisions of married women within a life-cycle context. However, as a simplifying assumption, these models assume fertility profiles to be exogenous, and the Heckman–MaCurdy model assumes the wage profile to be exogenous.[2]

The need for studies of more complete models is therefore apparent. The model presented here represents only a beginning in the necessary modelling effort. In the following Section, a more complete econometric model is developed. Its estimates are presented in Section 3 along with a discussion of their implications for various topics of interest in the fertility and labour-supply literatures. The last section is a brief summary that suggests future avenues of research.

2. AN ECONOMETRIC MODEL OF FERTILITY, LABOUR SUPPLY AND WAGES

The econometric equations for fertility and labour supply presented here are assumed to be reduced-form representations of demand functions that are the solutions to an intertemporal optimization problem.[3] A couple at the beginning of marriage is assumed to plan its lifetime pattern of consumption, labour supply of the wife (the labour supply of the husband is taken as exogenous), and births. These patterns are chosen by maximizing some intertemporal preference function defined over consumption, leisure of the wife, and the number of children at each point in time, subject to several constraints. First, there is a standard budget constraint stating that consumption and other forms of money expenditure cannot exceed total lifetime resources, equal to the present value of the earnings of the husband and wife plus the initial level of assets. Second, there is a constraint on the rate at which children can be had, constraining the couple from building up the stock to some desired level immediately. Third, there is an "inverse production function" for children indicating the amount of time of the wife required to care for children of different ages. Fourth, there is a wage-accumulation equation (say, of the learning-by-doing type of Weiss (1972)) in which work at one point in time increases the stock of human capital and hence the potential wage at all later points in time. The couple in choosing its optimal profile thus faces a number of tradeoffs. For example, having children increases the home-time of the wife, either reducing her leisure time or her market earnings, or both; if the latter, her future earning power is also reduced. But the time path of child-care time required is also important, for as children age, less time is required and more leisure and/or market work becomes possible.

The implication of these considerations for econometric estimation is primarily that only exogenous variables should be included in the fertility and female labour-supply equations. These include, most importantly, the level of exogenous wealth available to the couple, plus the level of the wage at the beginning of marriage. Subsequent wages are endogenous since they are a result of labour-supply decisions themselves, through the human-capital accumulation process. The wage equation itself must also be econometrically estimated, since it is unobserved. However, it is of course to be considered a budget constraint rather than a behavioural equation.

The following three-equation model will be estimated:

$$\ln W_t^* = \ln W_{t-1}^* + \gamma S_{t-1} - \delta \tag{1}$$

$$\ln W_t = \ln W_1^* + \gamma \textstyle\sum_{\tau=1}^{t-1} S_\tau - \delta(t-1) + e_t \tag{1'}$$

$$B_t^* = a_0 + a_1 f(t) + a_2 \ln W_1^* + a_3 Y + V a_4 + a_5 B_{t-1} + u_t \tag{2}$$

$$\begin{aligned} B_t &= 1 \quad \text{if } B_t^* \leqq 0 \\ &= 0 \quad \text{if } B_t^* < 0 \end{aligned}$$

$$S_t^* = b_0 + b_1 f(t) + b_2 \ln W_1^* + b_3 Y + V b_4 + b_5 B_{t-1} + v_t \tag{3}$$

$$\begin{aligned} S_t &= 1 \quad \text{if } S_t^* \geqq 0 \\ &= 0 \quad \text{if } S_t^* < 0. \end{aligned}$$

Here S_t is an employment-status variable equal to one if employed and zero if not. Equation (1) is a standard human-capital accumulation equation in which the "permanent" wage (i.e. the rental rate on human capital times the stock of human capital) is increased by past work experience at the rate γ and which depreciates at the rate δ during each period of non-work. The actual wage contains an error term: $\ln W_t = \ln W_t^* + e_t$. Applying the wage equation recursively to previous periods yields the wage equation (1′), showing the wage to be a function of the initial permanent wage, $\ln W_1^*$, and cumulative work experience ($S_0 = 0$).[4]

Equations (2) and (3) are the fertility and labour-supply equations, respectively, each formulated as a probit equation with latent variables B_t^* and S_t^*, and with error terms u_t and v_t, respectively. The fertility equation is specified in terms of the birth rate, $B_t^* = \dot{K}(t, X)$, rather than K_t itself.[5] The variables B_t and S_t are determined by $f(t)$, some function of marital duration (e.g. time dummies); ln W_1^*, the level of the permanent initial wage; Y, the level of exogenous wealth, consisting of the value of family assets plus the present value of the husband's earnings stream; V, a vector of exogenous taste variables; and the lagged fertility rate (discussed below). It is assumed in the model that fertility and labour-supply profiles are planned on the basis of the permanent wage (or the "expected" wage, alternatively), rather than the actual wage.[6] It is also assumed that the rate of return to work experience and all relevant goods prices are constant over the relatively short time period of the data (see below). On the basis of theoretical considerations discussed in Moffitt (1980*a*), it is hypothesized that $a_2 < 0$ and $b_2 > 0$—that is, that upward shifts in the wage profile lower the profile of births and raise the profile of employment rates—and that $a_3 > 0$ and $b_3 < 0$, which will occur if children and leisure are both normal goods.[7]

One obvious problem with estimation is the fact that wage rates are not available for non-workers. In addition, the permanent initial wage, ln W_1^*, is not even observed for workers; hence equation (1′) cannot be estimated as it is. Nevertheless, the solution is standard, even though in a somewhat different time series context.

A fourth equation is required for ln W_1^*:

$$\ln W_1^* = Z\eta + \mu_w,$$

where Z is a vector of human capital variables (e.g. years of education) and μ_w is a random error term specific to the individual (e.g. "ability"). Inserting this equation into the structural system we obtain the reduced-form system:

$$\ln W_t = Z\eta + \gamma \textstyle\sum_{\tau=1}^{t-1} S_\tau - \delta(t-1) + \mu_w + e_t \tag{4}$$

$$B_t^* = a_0 + a_1 f(t) + a_2 Z\eta + a_3 Y + Va_4 + a_5 B_{t-1} + u_t' \tag{5}$$

$$S_t^* = b_0 + b_1 f(t) + b_2 Z\eta + b_3 Y + Vb_4 + b_5 B_{t-1} + v_t', \tag{6}$$

where $u_t' = a_2\mu_w + u_t$ and $v_t' = b_2\mu_w + v_t$.

The data that will be used for estimation consist of a panel of up to 11 annual observations on individual fertility, employment, and wages over several years of marriage. To estimate the simultaneous system a full-information maximum-likelihood (FIML) procedure will be used. The log-likelihood function for the problem is:

$$L = \textstyle\sum_i \log(P_i)$$

where P_i is the probability of observing T_i^b periods of fertility events, T_i^s periods of employment events, and T_i^w wages for the i-th individual:

$$P_i = \text{Prob}\,(B_1, B_2, \ldots, B_{T_i^b}, S_1, S_2, \ldots, S_{T_i^s}, \ln W_1, \ln W_2, \ldots, \ln W_{T_i^w}).$$

The maximum number of periods observed for each individual is eleven, but for data reasons (see below) a full set of birth and employment variables is not always observed. In addition, wage rates are not observed during periods of non-work.

As can be seen from equations (4)–(6), the three variables in the probability statement are functions of several error terms. The error term $e_t' = \mu_w + e_t$ in the wage equation is equivalent to a standard permanent-transitory process assuming that e_t is uncorrelated both over time and with the permanent component, μ_w. The errors in the fertility and employment equations are functions of the permanent component of the wage equation as well as u_t and v_t. The latter two errors are also assumed to follow a permanent-transitory process over time, and the permanent and transitory errors are assumed to be correlated across the equations as well. Thus the model just consists of three permanent-transitory errors. Formally, the assumptions are:

$$\begin{array}{lll} e_t \sim N(0, \sigma_e^2) & u_t \sim N(0,1) & v_t \sim N(0,1) \\ u_t = \mu_B + \eta_t & \mu_B \sim N(0, \sigma_\mu^2) & \eta_t \sim N(0, 1-\sigma_{\mu_B}^2) \\ v_t = \mu_s + \nu_t & \mu_s \sim N(0, \sigma_{\mu_s}^2) & \nu_t \sim N(0, 1-\sigma_{\mu_s}^2) \\ e_t' = \mu_w + e_t & \mu_w \sim N(0, \sigma_{\mu_w}^2). & \end{array}$$

All elements of the contemporaneous error vector $(\mu_B \eta_t \mu_s \nu_t \mu_w e_t)$ are assumed to be uncorrelated, with the exceptions: Corr $(\mu_B, \mu_s) = \rho_1$ and Corr $(\eta_t, \nu_t) = \rho_2$. All time-varying errors (η_t, ν_t, e_t) are assumed to be uncorrelated across time periods. The percentages of variance explained by the three permanent components are:

$$\rho_u = \sigma_{\mu_B}^2$$
$$\rho_v = \sigma_{\mu_s}^2$$
$$\rho_e = \sigma_{\mu_w}^2 / (\sigma_e^2 + \sigma_{\mu_w}^2)$$

The various retrictions imposed on this structure are entirely for purposes of tractability. For example, transitory errors in the wage are assumed uncorrelated with fertility and employment decisions; only the permanent wage component is so assumed. It seems likely, to be sure, that the permanent component of the wage should have the larger effect, but the transitory component probably has a non-zero effect. Also, the components of the wage are assumed uncorrelated with the fertility and labour supply components. This can be partly rationalized on the basis that the wage equation is a demand equation determined by employers whereas the other two are supply equations jointly set by the individual. In principle the errors of different agents should be uncorrelated. But if there are common unobserved variables affecting both agents' decisions, some correlation will arise. In any case, even with the restrictions, the model is extremely burdensome computationally, for the evaluation of the multi-normal probability P_i requires a three-fold numerical integration. The procedure for numerically evaluating the probability is given in the Appendix.[8]

The use of the full-information maximum-likelihood technique avoids the major selection biases that would arise from simpler techniques. For example, estimating the wage equation with ordinary least squares would generate inconsistent parameter estimates because of the selectivity bias attendant upon the fact that only workers (those with $S_t = 1$) have wages, implying above-average values for μ_w, which is in the error term for S_t^*. That is, the values of the error terms in the S_t and W_t equations are correlated. The FIML technique already outlined fully incorporates this correlation between the error terms in a dynamic context just as the original Heckman (1974) model did in a cross-section context. Nevertheless, a disadvantage of maximum likelihood is that it may not be robust to misspecification. Since the model is in many ways a reduced-form approximation to the "true" model, misspecification could be a problem.

A more intrinsically dynamic econometric problem arises because of the presence of cumulative work experience in the wage equation. This variable, whose presence implies a form of state dependence, is endogenous to the system and is correlated with the error term in the wage equation. The heterogeneity in wage levels implies that those with greater cumulative work experience will have greater wages, independent of any true causal effect of work experience on wages. In past single-equation wage models, this potential endogeneity has been addressed by various forms of instrumental-variable techniques (Mincer and Polacheck (1974), Sedlacek (1979)). In the complete model here, the FIML procedure takes account of this correlation, inasmuch as the reduced form of the system is estimated. The correlation between the values of S_t and W_t in *different* time periods, as well as the correlation between values in the same time period discussed in the last paragraph, are both accounted for in the probability expression for P_i.

The data here unfortunately provide only years of work experience during marriage. Since many women undoubtedly have experience prior to marriage, we have here an econometric "initial conditions" problem (Heckman (1981*b*)). In the form it takes here, the "initial wage", W_1^*, is a function of prior work experience and hence is stochastic. As is evident from equation (1′), it will have a distribution that is non-normal. It is also correlated with the error term. However, to a great extent the inclusion of a separate equation for W_1^*, with its own error term, is a way of using an instrumental variable for W_1^*. This procedure has some similarity to an instrumental-variable technique suggested by Heckman (1981*b*, p. 188) as a solution to the problem.

The data set available for the estimation of the three equations is the National Longitudinal Survey of Young Women. The young women in this survey were aged 14–24 in 1968, the initial year of the survey, and have been interviewed periodically since then. For the purpose of this study, only data for married women are selected and only for those periods during which they were married. About 1500 women fulfill this criterion of having at least one period of data during which they were married. The survey years available are those from 1968 to 1975. However, no labour supply data are available for 1974, hence because no wage equation can be estimated for that year or the one following (because lagged employment status is required), both 1974 and 1975 are deleted from the wage and employment-status equation. However, these periods are retained for the fertility equation.[9] In addition, retrospective fertility questions allow us to obtain fertility data on marriages dating back to 1965; consequently up to 11 periods (1965–1975) of fertility data are available. Up to 6 periods (1968–1973) of wage and employment data are available, periods that may fall into anywhere from the first to ninth year of marriage.[10]

The independent variables in the equation are all directly available from the data save the wealth variable, Y. The hourly wage rate is available (for workers only) and various exogenous taste variables are available for V and human-capital variables for Z. In V are included years of education, race (=1 if nonwhite), and birth year (i.e. cohort). In the set of human-capital variables is included years of education, race, education of the father and mother, and year of marriage (to proxy the secular growth of productivity). The value of exogenous wealth is calculated as the sum of (a) family asset value at the beginning of marriage, and (b) the present value of the husband's lifetime earnings. The first of these is directly available from the survey data. For the second, an instrumental variable is created by first estimating a life-cycle profile with the age-earnings data of the husbands in the sample, and by then using the predicted time profiles from this regression to calculate the present value of each husband's earning stream 30 years into the future, discounted at a rate of 5%. Details of the results are available from the author.[11]

A final problem with all fertility and labour supply models estimated on annual data is that the relevant time period for sequential fertility decisions is longer than 12 months. Time elapses between the decision to conceive and the point of conception, between the point of conception and an actual birth, and between the time of a birth and the time fecundity is reattained. The stochastic processes underlying these time periods are quite complex (Sheps and Mencken (1974)) and cannot be modelled here. However, to account for the fact that a birth in one year is much less likely if a birth has occurred in the previous year, the variable B_{t-1} will be entered into equations (5) and (6). Hence fertility and labour supply decisions are made conditional upon whether a birth has occurred in the previous year, creating another form of state dependence.

3. RESULTS

The results of the estimation are shown in Table I. The fertility and employment equations will be discussed first. The coefficients on the time variables indicate that fertility rates rise in the first three years of marriage and fall thereafter at a decreasing rate. At the end of 11 years, the fertility index is falling at a rate of 0·05 per year which, at the means, corresponds to a drop of 0·014 per year in the probability of a birth. Employment rates follow a somewhat different pattern, dropping at an increasing rate over the first six years of marriage, then rebounding. As might be expected, this implies that employment rates turn up before completed family size is reached (fertility rates are still positive, implying that the stock of children is still rising). No doubt the declining rate of childbirth reduces the amount of home-time required for child care, as older children require less time.

The results also indicate that the increases in the level of the wage profile decrease and increase, respectively, the profiles of fertility and labour supply. A one-percent increase in the wage profile decreases the probability of a birth by about 0·0006 and increases the probability of working by about 0·005, both measured at the means of the dependent variable.[12] Increases in lifetime wealth, however, decrease both profiles. While a decrease in the labour-supply profile is expected, that in the fertility profile is not necessary. Negative income effects on fertility are often found in cross section, and apparently do not disappear in a panel context. A large literature is devoted to an examination of these effects (e.g. Becker and Lewis (1974)).

The rest of the results in the fertility and labour-supply equations are also of interest. Neither birth year (cohort) nor education has a significant effect on either profile, holding constant the level of the wage profile and the value of wealth. The lack of significance in the cohort variable is of interest because it implies that the upward shifts in employment rates and downward shifts in fertility rates for recent cohorts are more a result of shifts in the wage profile than of changes in tastes. Likewise, whereas education is often entered into these types of equations without a wage variable, leaving the interpretation of its coefficient ambiguous, the results here suggest that its effect is a result more of its impact on the wage profile than of its direct impact on tastes. The rest of the results indicate further that nonwhites both have more children and work more; that having had a birth the previous period does indeed lower the probability of having a birth this period and the probability of working this period; that the permanent component explains a large portion of the variance in labour supply though not in fertility;[13] and that the unobserved components of the two equations have a strong negative correlation, as should be expected.

The results of the wage equation indicate that nonwhites, surprisingly, have higher initial wages than whites, holding constant education and other variables. This may be a result of the greater labour-force commitment of nonwhite women, though it is still somewhat puzzling. Education of the wife has a positive effect on her wages, as does the education of her father; but the education of the mother has a negative effect. The impact of secular growth, proxied by the year of marriage, indicates that the wage profile shifts

TABLE I

Full-information maximum-likelihood (FIML) *coefficient estimates*[1]

	Fertility	Labour supply		Log-wage
Time:—[2]				
1–3	0·402*	−0·010	Race	0·031*
	(0·029)	(0·025)		(0·012)
4–6	−0·108*	−0·039*		
	(0·021)	(0·019)	Education	0·087*
7–9	−0·106*	0·118*		(0·003)
	(0·031)	(0·033)		
10–11	−0·054	[3]	Father's education	0·004*
	(0·107)			(0·002)
ln W_1^*	−0·234*	1·353*	Mother's education	−0·003*
	(0·103)	(0·124)		(0·002)
ln Wealth	−0·075*	−0·021	Marriage year	0·049*
	(0·038)	(0·045)		(0·003)
Cohort	0·001	−0·001	γ	0·043*
	(0·010)	(0·013)		(0·006)
Race	0·093*	0·204*	δ	−0·004
	(0·047)	(0·061)		(0·004)
Education	−0·002	−0·001	Constant	−3·867*
	(0·015)	(0·18)		(0·206)
B_{t-1}	−0·515*	−0·273*	Standard Error	0·291*
	(0·050)	(0·047)		(0·003)
Constant	−1·546*	−0·688	ρ	0·534*
	(0·588)	(0·762)		(0·012)
ρ[4]	[5]	0·493*		
		(0·026)		
Cross-equation correlation=	−0·559*			
	(0·026)			

Notes:

1. Variables (means) are: B_t (0·20); S_t (0·40); ln W_t (0·65), in 1967 dollars; ln W_1^* (0·45), in 1967 dollars; Wealth (1·01), in hundreds of thousands of 1967 dollars; Cohort (48·9); Race (0·18), where 1 = nonwhite; Years of education (11·3); Father's education (9·8); Mother's education (10·4); Year of marriage (69·2).
2. Coefficients denote the slope of the profile in the interval bounded by the years indicated.
3. No observation.
4. Fraction of error variance accounted for by the permanent component.
5. Estimated at zero.

Standard errors in parentheses.
*Significant at the 10% level.

upward at a rate of about 5% per year in real terms. Further, an additional year of work experience raises the wage by a little more than 4%, indicating substantial gains in future earning power. However, wages appear to depreciate in no significant amount through non-work; this is probably because the depreciation in skills is outweighed by general economy-wide increases in wages. Thus the gross return and the net return to work experience are quite close.[14]

Several other econometric tests were tried out. First, as suggested by Heckman (1981*a*), the factor loadings were allowed to vary over time rather than stay constant as the traditional permanent-transitory process estimated thus far requires. Allowing the loadings to vary in effect allows the percent of variance explained by the "permanent" component to change over time. When this was tested here, it was found that (a) the fertility and wage components changed insignificantly over time (staying at zero and 0·534, respectively), and (b) the labour-supply component grew slightly over the period at a rate of about 0·06 on average (its mean is still 0·493). In any case, the rest of the estimated parameters were little affected. Another set of tests included the unemployment rate, first in the wage equation alone, and then in all three equations. Its coefficients were uniformly insignificant and did not affect the rest of the coefficients in the equations. Third, a series of interactions with time were tested. It was found that both higher wealth values and higher wage rates increase the slope of the fertility profile and decrease the slope of the labour-supply profile. Also, nonwhites were found to have shallower fertility profiles and steeper labour-supply profiles. These interactions were significant statistically; others tried were insignificant, primarily because the other variables (education, cohort, etc.) are insignificant in any case.

Implications

A. *Profile shapes*

The nature of the model makes it difficult to see the implications of the results by direct inspection of the coefficients. Not only do the coefficients on the fertility and employment-status variables measure only the effect on the relevant latent indexes rather than on the probabilities, but the lagged fertility rate and the lagged cumulative work experience variables mean that changes in many of the variables, such as education, have cumulative impacts that are quite different from their direct, contemporaneous impacts. Therefore it is necessary to evaluate by simulation the birth and employment probabilities and how they change with changes in the independent variables.

Figure 1 shows several profiles. The three profiles labelled A are obtained by predicting the mean probabilities of births and employment at each point in time and the log of the hourly wage rate, using the mean of the independent variables.[15] As the figure indicates, fertility rates rise from around 0·10 at the beginning of marriage to a peak of 0·28 in the third year, after which they taper off and reach 0·10 by the end of 11 years.

Employment rates dip down from 0·44 at the beginning of marriage to a trough of 0·37 after 6 years, after which they rise up to around 0·50 by the end of 9 years. Wage rates increase over the entire life cycle at the mean, but they grow at a decreasing rate during the first three or four years of marriage. This is simply a result of the falling employment rates over that period. Afterwards wages grow fairly steadily at about 2% per year.

The effect of an increase in education of one year is also shown in the figure. Its effects on fertility are minimal and not drawn; the birth probability falls by about one-half of one percent in each year. However, employment profiles are shifted upward in a roughly parallel fashion and the log-wage profile shifts up as well. The declining rate of wage growth in the early years is considerably dampened at the higher education level. Similar shifts occur when other of the variables in the wage equation, such as the year of marriage, change. The only difference between education effects and other effects of this type is the direct effect of education in the fertility and employment equations, which is trivial in magnitude. Also, a 10% increase in the level of the initial wage (i.e. a 0·10 increase in the constant term) lowers the annual birth probability by about one-half a percentage point, and raises the annual employment probability by about 0·05. At the

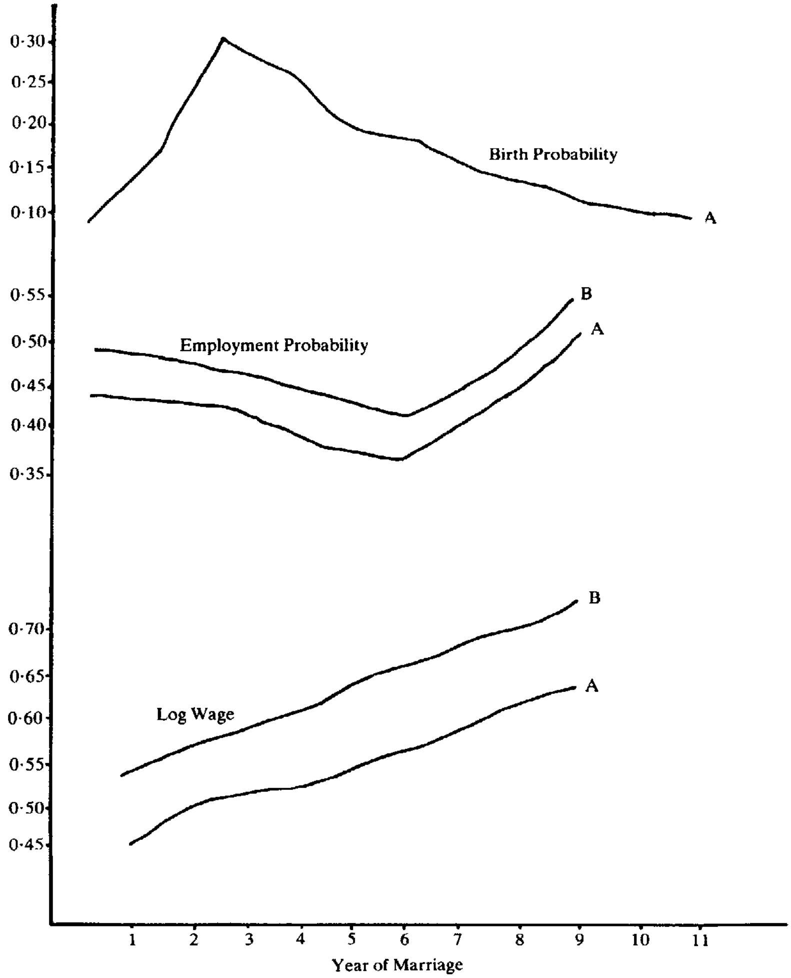

FIGURE I
Simulated profiles

mean fertility and employment rates, these imply wage elasticities of 25 and 125%, respectively. The 10% upward shift in the initial wage also increases the wage profile, by ten percent at the beginning of course, but by 12% by the end of 9 years.

B. *Indirect cost of children*

The framework of the model could permit a straightforward calculation of the present and future foregone earnings of child-bearing. A change in the stream of fertility rates shifts the profile of employment rates and hence the profile of wage rates.[16] A higher rate of births shifts the employment profile downward and hence the wage profile downward. The effect tapers off over time, but wages are permanently lowered a certain amount by each additional birth. For example, if no children are had within the 11 years, wages grow at the mean by about $2\frac{1}{2}$–3% per year. A woman who has a child in the first year has a much lower employment rate temporarily and, consequently, an initially much slower growth rate of wages, less than one percent a year. If no more children are had, employment rates after three years or so return to higher levels and wage rates attain the same growth rate as that of the woman who has had no child, but the wage profile is at a lower absolute level. Additional children thereafter have further effects in lowering the wage profile. With a longer panel data set than is available here, a full lifetime wage profile could be calculated for each type of fertility profile, and a discounted present value of foregone earnings could be obtained.

C. *Time-series trends in fertility and female employment*

The model does a better job in backcasting employment rates over the postwar period than fertility rates. Major postwar changes have occurred in female education levels, male income, and wages, which may explain fertility and employment trends. Mean female education increased by 1·3 years from 1950 to 1960 and by 1·2 years more from 1960 to 1970. Mean real male income (a proxy for wealth) grew by 21 and 25% in these two periods, respectively, and the hourly wage rate of females aged 20–24, a proxy for W_1, grew by 5% and 24% in the two respective periods.[17] Using these growth rates to simulate birth probabilities for 22-year-olds in the fifties and sixties, thereby approximating the changes in the level of the fertility profile over those periods, yields a slight decrease in the birth probability of about half-a-percent during the fifties and a larger decrease of 0·02 over the sixties. Thus the model has no explanation for the postwar baby boom and seems to underestimate the declines in fertility since the early sixties. The lack of explanation of the baby boom is not surprising given the estimated unimportance of either the wealth coefficient, upon which Butz and Ward (1980) rely for an explanation of the boom, or the cohort coefficient, upon which Richard Easterlin (1973) relies for an explanation of the boom.

The model does much better with employment rates. A simulation backcasts employment rates for young women to have risen by a scant two percent in the fifties, compared to an actual change of essentially zero, and an increase in the employment rate of about 0·12 in the sixties, compared to an actual change of 0·10. The relative success in explaining employment comes directly from the explanatory power of the wage rate, which rose very slowly in the fifties and quite rapidly in the sixties, even for young women with presumably little work experience.

D. *Sources of growth in the female wage*

A related question of interest is the source of growth in the female wage over the postwar period, especially the reason for the rapid growth in the sixties.[18] For example, the growth in female employment in the sixties is only partly a result of wage growth; it is also a cause inasmuch as it increases cumulative work experience. This creates a type of chicken-and-egg problem of causality. However, if the rate of return to work experience is close to the 5% estimated in the model here, it is unlikely that the induced effects of employment growth on the wage could approach in magnitude the growth rate of the initial, base wage of 24%. Even if the 0·12 increase in the employment rate in the sixties had steadily incremented the work-experience variable for all age groups over the entire decade, cumulative work experience would have risen by only 1·2 years for the average woman. This would add only about 6% wage growth to the 24% base rate.

The sources of growth in the base are also of interest. The growth rate of W_1 is most likely a result of secular productivity growth and shifts in the labour-demand curve (proxied by the year of marriage variable) and of increases in the level of human capital in the female population (proxied by the education variable). In addition, the wage rate, both at young and old ages, can trend because of changes in selectivity bias. As employment rates rise, the truncation of the wage schedule by non-workers lessens; hence wages will fall even if the underlying market-wage schedule itself is stationary. The selection-bias term in the log-wage equation is:

$$\rho\sigma_e f(z)/[1-F(z)]$$

where ρ is the correlation between the wage and labour-supply error terms, $F(z)$ is the probability of working, and $f(z)$ is the corresponding normal probability density. Using the parameters of the model estimated here, plus the changes in observed employment rates over the sixties, yields a decline in this term of 0·03. Thus the selectivity-corrected market-wage schedule rose about 27 instead of the observed 24%.

The 27% base growth rate cannot be reliably allocated between secular productivity growth and human-capital growth with the results of the model here, for productivity growth in the late sixties was much higher than in the early sixties. In fact, the 0·05 coefficient on the year-of-marriage variable implies a 50% increase in the base wage over the decade, if extrapolated backwards. However, the contribution of education is approximately 10% ($0{\cdot}08\times 1{\cdot}2$). If the education contribution were correct, about 17% would be left for secular productivity growth. Hence we could conclude that wage growth of 17% arose from labour-demand shifts, 10% from human-capital growth, and perhaps 6% from increased work experience.

4. SUMMARY

The results of the estimation of the life-cycle model in this paper provide support for the notion that shifts in the level of the lifetime wage profile are associated with shifts in the lifetime profiles of fertility rates and female employment rates. The effect is somewhat stronger on the employment profile than on the fertility profile. However, wealth effects on both profiles are quite small and low in significance. Birth cohort and education appear to have little effect on the profiles, independent of wage and wealth effects.

Extensions of the model are available in many directions. The application of the model to aggregate time-series data rather than contemporary micro-data would permit a more accurate determination of the causes of postwar changes in fertility, employment and wages. Shifts in the wage profile, both in level and in the rate of return to work experience and to education, may have caused many of the shifts in the levels and slopes of the fertility and employment profiles. Also, a wide variety of statistical extensions of the model are possible, as so to account for uncertainty and the impact of unexpected life-cycle events, and so as to provide a richer stochastic structure.

APPENDIX

To evaluate the likelihood function we must evaluate the joint probability of observing up to 33 events for each individual:

$$P_i = \text{Prob}\,(B_1, \ldots, B_{T_i^b}, S_1, \ldots, S_{T_i^s}, W_1, \ldots, W_{T_i^w})$$

where W_t is now the log of the wage rate, which enters the probability only for working periods. The probability of observing these events must be related to the probabilities of observing the error terms in equations (4)–(6) in the text. Specifically, rewrite equations (4)–(6) as follows:

$$B_t = 1 \quad \text{if } u'_t > A_t; \qquad B_t = 0 \text{ otherwise}$$

$$S_t = 1 \quad \text{if } v'_t > \hat{A}_t; \qquad S_t = 0 \text{ otherwise}$$

$$W_t = C_t + e'_t,$$

where $e'_t = \mu_w + e_t$ and where:

$$A_t = -a_0 - a_1 f(t) - a_2 Z\eta - a_3 Y - Va_4 - a_5 B_{t-1}$$

$$\hat{A}_t = -b_0 - b_1 f(t) - b_2 Z\eta - b_3 Y - Vb_4 - b_5 B_{t-1}$$

$$C_t = Z\eta + \gamma \textstyle\sum_{\tau=1}^{t-1} S_\tau - \delta(t-1).$$

Note the inclusion of the lagged fertility rate. Hence the joint probability of the events can be written:

$$P_i = \int_{LB_1}^{UB_1} \cdots \int_{LB_{T_i^b}}^{UB_{T_i^b}} \int_{LS_1}^{US_1} \cdots \int_{LS_{T_i^s}}^{US_{T_i^s}}$$

$$\times h(u'_1, \ldots, u'_{T_i^b}, v'_1, \ldots, v'_{T_i^s}, e'_1 = W_1 - C_1, \ldots, e'_{T_i^w} = W_{T_i^w} - C_{T_i^w}$$

$$\times dv'_1 \ldots dv'_{T_i^s} du'_1 \ldots du'_{T_i^b}$$

where h is the multivariate normal density of the u'_t, v'_t, and e'_t, and the upper and lower limits of integration are:

$$LB_t = A_t, \quad UB_t = \infty \quad \text{if } B_t = 1;$$

$$LB_t = -\infty, \quad UB_t = A_t \quad \text{if } B_t = 0;$$

$$LS_t = \hat{A}_t, \quad US_t = \infty \quad \text{if } S_t = 1;$$

$$LS_t = -\infty, \quad US_t = \hat{A}_t \quad \text{if } S_t = 0.$$

The computational feasibility of evaluating this large a multinormal integral depends upon the correlational structure assumed for the density h. With no restrictions on the correlation matrix the integrals cannot be evaluated. However, they can be evaluated if a traditional error components, permanent-transitory process such as that given in the main body of the paper is assumed.[19] The evaluation of the integral under these assumptions is made possible by a factorization of the density:[20]

$$h(u'_1, \ldots, u'_{T_i^b}, v'_1, \ldots, v'_{T_i^s}, e'_1, \ldots, e'_{T_i^w})$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(\eta_1, \ldots, \eta_{T_i^b}, \nu_1, \ldots, \nu_{T_i^s}, e_1, \ldots, e_{T_i^w} | \mu_B, \mu_S, \mu_W)$$

$$\times i(\mu_B, \mu_S, \mu_W) d\mu_B d\mu_S d\mu_W$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\textstyle\prod_{t=1}^{T_i^s} g(\eta_t, \nu_t | \mu_B, \mu_S, \mu_W) \prod_{t=T_i^s+1}^{T_i^b} f(\eta_t | \mu_B, \mu_S, \mu_W)$$

$$\times \textstyle\prod_{t=1}^{T_i^w} f(e_t | \mu_B, \mu_S, \mu_W)] i(\mu_B, \mu_S, \mu_W) d\mu_B d\mu_S d\mu_W$$

where g is the bivariate normal density of η_t and ν_t (correlation ρ_2) and f is the univariate normal density of e_t, both conditional upon the three permanent components; and i is the trivariate normal density of μ_B, μ_S, and μ_W, which factors into a univariate μ_W density and a bivariate (μ_B, μ_S) density. The probability integral therefore reduces to:

$$P_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [\prod_{t=1}^{T_i^s} G(A_t, \hat{A}_t, \mu_B, \mu_S, \mu_W) \prod_{t=T_i^s+1}^{T_i^b} F(A_t, \mu_B, \mu_W)$$

$$\times \prod_{t=1}^{T_i^w} f(W_t - C_t - \mu_w)] i(\mu_B, \mu_S, \mu_W) d\mu_B d\mu_S d\mu_W$$

where

$$\begin{aligned} G(A_t, \hat{A}_t, \mu_B, \mu_S, \mu_W) & \\ &= \text{Prob}\,(\eta_t < A_t - a_2\mu_w - \mu_B,\ \nu_t < \hat{A}_t - b_2\mu_w - \mu_s) \quad \text{if } B_t = L_t = 0 \\ &= \text{Prob}\,(\eta_t > A_t - a_2\mu_w - \mu_B,\ \nu_t < \hat{A}_t - b_2\mu_w - \mu_s) \quad \text{if } B_t = 1, L_t = 0 \\ &= \text{Prob}\,(\eta_t < A_t - a_2\mu_w - \mu_B,\ \nu_t > \hat{A}_t - b_2\mu_w - \mu_s) \quad \text{if } B_t = 0, L_t = 1 \\ &= \text{Prob}\,(\eta_t > A_t - a_2\mu_w - \mu_B,\ \nu_t > \hat{A}_t - b_2\mu_w - \mu_s) \quad \text{if } B_t = L_t = 1 \end{aligned}$$

$$\begin{aligned} F(A_t, \mu_B, \mu_W) &= \text{Prob}\,(\eta_t < A_t - a_2\mu_w - \mu_B) \quad \text{if } B_t = 0 \\ &= \text{Prob}\,(\eta_t > A_t - a_2\mu_w - \mu_B) \quad \text{if } B_t = 1. \end{aligned}$$

This integral can be evaluated relatively efficiently with a Gaussian quadrature algorithm developed by Butler and Moffit (1982) for use in estimating probit equations on cross-section time-series data (see also Heckman (1981*a*) for a discussion of this model). The probability is of the general form

$$P_i = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q(\mu_W, \mu_B, \mu_S) d\mu_W d\mu_B d\mu_S$$

and can be evaluated numerically, as Butler and Moffitt show, by the expansion:

$$P_i \cong \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} w_j w_k w_l Q(x_j, x_k, x_l)$$

where w_j, w_k, w_l are weight factors and x_j, x_k, x_l are points derived by Gauss.

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NOTES

1. See also Devaney (1980) for another paper on time-series aggregate fertility trends and wage-rate influences.
2. Hotz (1979) and Lehrer and Nerlove (1979) have studied life-cycle fertility-labour-supply relationships, although empirically estimating only partially complete models.
3. See Moffitt (1980*a*) for a formal model.
4. S represents only work *after* marriage. There may be some work experience before marriage, although the present data set does not have information on it (see also n. 10).
5. Here K_t is the number of children at time t. Estimating the K_t equation directly is rather difficult since its error term must be truncated from below by definition, since the stock of children is monotonically non-decreasing. See Moffitt (1980*b*) for an estimation of such a model.
6. I here follow Heckman and MaCurdy (1980) and MaCurdy (1980) by ignoring uncertainty. See MaCurdy (1978) for an uncertainty discussion.
7. Of course no allowance for child quality is made here. However, it is of interest to ascertain whether the wealth coefficient in a life-cycle model is negative or positive, ignoring child quality.

8. The necessity to numerically evaluate the integrals made the processing time quite long. Convergence of the likelihood function required several hours of CPU time on the Rutgers University IBM 370 and cost more than $2000 at low priority.

9. There is no necessity in the FIML technique to have the same number of periods in each equation. Unbalanced samples create no problems.

10. There is another initial conditions problem here (Heckman, 1981*b*), for labour supply data are not available for the first one-to-three periods of marriage for women married between 1965 and 1967. Hence part of the data needed to construct the cumulative work experience variable is missing for this relatively small subsample. The procedure followed here is to estimate an instrumental variable for S_t, utilizing the data on the rest of the sample, and to use the predicted values for the missing periods for these individuals.

11. Real 1967 earnings are used throughout. In the regression step all husbands' earnings for all periods are pooled into one cross-section time-series panel and regressed upon age, age squared, education, education squared, and year of marriage (results available from the author). In the prediction step the mean residual over each husband's time periods is readded to the equation, a procedure whose efficiency has been pointed out by Taub (1979).

12. See below for a discussion of the implied elasticities.

13. The zero permanent fertility component is no doubt a result of the lagged fertility variable on the right-hand side.

14. *OLS* estimates of γ and δ are 0·09 and 0·03, respectively. The net return is thus about 0·06.

15. The simulation is cumulatively performed, i.e. the simulated probabilities of birth and employment are fed into the next period's simulation of birth and employment probabilities and wages.

16. These effects must be calculated within the present model as conditional probabilities, i.e. $E(S_t|B_t)$.

17. The age-specific time-series wage data used in this section were kindly provided by William Butz. The male income data and age-specific female employment data were taken from W. Butz and M. Ward, "The Emergence of Countercyclical U.S. Fertility," R-1605-NIH, Rand Corporation, 1977, p. 44.

18. For example, see Devaney (1980, pp. 20–23) for a discussion of these same issues.

19. Although the permanent and transitory errors are assumed independent, numerical integration is still required because the successive error terms for the same individual are correlated (they contain common permanent components). See Heckman (1981*a*).

20. Since $T_i^b > T_i^s$ for all individuals in the sample, the notational setup below shows no observations on labour supply without a fertility observation.

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