

DEMO ARXIV TEMPLATE

A PREPRINT

H. Sherry Zhang 

Department of Econometrics and Business Statistics
Monash University
Melbourne, VIC
huize.zhang@monash.edu

Collaborators

Department of Econometrics and Business Statistics
Monash University
Melbourne, VIC

November 23, 2022

ABSTRACT

- indexes, useful, quantify severity, early monitoring,
- A huge number of indexes have been proposed by domain experts, however, a large majority of them are not being adopted, reused, and compared in research or in practice.
- One of the reasons for this is the plenty of indexes are quite complex and there is no obvious easy-to-use implementation to apply them to user's data.
- The paper describes a general pipeline framework to construct indexes from spatio-temporal data,
- This allows all the indexes to be constructed through a uniform data pipeline and different indexes to vary on the details of each step in the data pipeline and their orders.
- The pipeline proposed aim to smooth the workflow of index construction through breaking down the complicated steps proposed by various indexes into small building blocks shared by most of the indexes.
- The framework will be demonstrated with drought indexes as examples, but applicable in general to environmental indexes constructed from multivariate spatio-temporal data

Keywords indexes • data pipeline • software design

1 Introduction

Why index is useful, why people care about indexes

incorporate the following in why using index: multiple pieces of information (variables) that need to be taken into account

Many concepts relevant to decision making cannot be directly measured, however, they are crucial for resource allocation, early prevention, and other operational purpose. For example, fire authorities would be interested to quantify fire risk since bushfires can have a huge impact on monetary loss, health, and the local ecosystem. Climatologists would be interested in monitoring the change in global climate since variability in atmospheric and oceanic conditions has a direct impact on global weather and climate. Usually this concept of interest is associated with more than one variables and these variables need to be integrated to make decisions on the subject matter. A common approach to quantify concepts like these is to construct an index using these relevant variables. This allows researchers to compare the quantity of interest across entities (i.e. countries, regions) and also cross time.

Define what is an index, what is not

In this article, an index is defined as a tool to quantify a concept of interest that does not have a direct measure. The concept of interest doesn't have a direct measure can because it is impractical to measure at the population level. For example, it would be nearly impossible to include all the available stocks in the market to characterise stock market behavior, so indexes like Dow Jones Industrial Average, S&P 500, and Nasdaq Composite select a representative set of stocks to measure the overall market behavior. Also belonging to this category are the economic indexes like the Consumer Price Index, where price changes of a basket of items are weighted to measure inflation. The lack of direct measure could also because the concept itself is an unobservable human construction, rather than a physical quantity that can be measured. Many natural hazard and social concepts falls into this category. This includes drought indexes constructed from meteorological, agricultural, hydrological, and social-economic variables, e.g. Standardised Precipitation Index (SPI) (McKee et al. 1993) and Aggregated Drought Index (ADI) (Keyantash and Dracup 2004) among others. Social development indexes like Human Development Index (United Nations Development Programme 2022) and Global Liveability Index (Economist Intelligence Unit 2019) measure various aspects of the quality of human capital and urban life.

still need to tweak the tone a bit: "they are called index, they are not the index we will talk about"

Despite many quantity having the term *index* in their name, they cannot be technically classified as indexes according to the definition given above. The reason for these quantities to lose their index memberships is that they are variables can be accurately measured given the instrument precision. This includes quantities like precipitation of the driest month or percentage of days when maximum temperature is below 10th percentile. They are measures of precipitation and percentage of days under specific conditions (dries month, maximum temperature below 10th percentile). They are variables, or indicators, that can be used to construct indexes but are not indexes themselves. Similarly, a set of remote sensing indexes are not indexes, since they are measures of electromagnetic wave reflectance. This includes Normalized Difference Vegetation Index (NDVI) (Tucker 1979), derived from the ratio of difference over sum on two segments in the spectrum, also called band: near-infrared (NIR) and red. So are the "indexes" derived from NDVI, e.g. Vegetation Condition Index (Kogan 1995). Notice that this does not exclude all the construction derived from remotes sensor variables to be valid indexes. For example, Vegetation Drought Response Index (Brown et al. 2008) is a valid index since it integrates climate, satellite, and biophysical variables to quantify vegetation stress.

What is the challenges with current index construction

see if there is any paper describing this type of pains

useful to reference tidy data and tidy model that makes the workflow on modelling tidy somewhere in introduction

Currently, index construction lacks a standardised workflow. It is often up to researchers or research institutions to decide whether to provide open source code on the new indexes, what would be the best user interface for other researchers to use the new indexes, and how easily the new indexes can be compared with other existing indexes. This makes the computation lack transparency and indexes cumbersome to experiment with:

- Researchers who wish to validate the indexes calculated from large institutes need to reinvent the wheels themselves since the source code used for computing is often not available for public consumption;
- Open-source code provided by research groups has a narrow margin for exploring other options outside the provided;
- Similar steps used by different indexes are difficult to spot since the design of the user interface for indexes often includes all the steps under a single function call; and
- It is generally hard to inspect intermediate results during the index construction if users wish to check the output of a certain step.

what can be done if people adopt this pipeline/ why it is beneficial?

This paper proposes a data pipeline for index construction. By recognising the common steps shared by many indexes, we develop a pipeline that breaks down index construction into multiple modules and allow operations in various modules to be combined like building blocks to construct indexes. The pipeline approach is general while adaptable to most index construction. It allows indexes to be created, studied, and compared in a structured tidy form and enables statistical analysis of indexes to be performed easily: More specifically, it enables researchers to 1) validate the indexes calculated from external organisations, 2) unify various indexes under the same framework for computing, 3) swap or adjust individual steps in the index construction to study their contribution, 4) calculate uncertainty on indexes through bootstrap or others, 5) enhance existing indexes through comparing and studying their statistical properties, and finally, 6) propose new indexes from combining different steps in existing indexes.

who would benefit from this paper

This work is of interest to researchers actively developing new indexes since it encourages new indexes to be delivered in an easy-to-reproduce design. It would also provide analysts who wish to compute a range of indexes in their analysis a uniform interface to build relevant indexes from raw data. For statisticians and software developing engineers, this work frames the process of index construction in a more user-oriented workflow and could motivate similar research for other process in scientific computing.

The rest of the paper is structured as follows: Section 2 reviews the concept of data pipeline in R. The pipeline framework for index construction is presented in Section 3. Section 4 explains how to include a new building block in each pipeline module. Examples are given in Section 5 to demonstrate the index construction with the pipeline built.

2 Data pipeline

Think about if there is another word for data pipeline

Why you should care about pipeline

Data pipeline is not a new concept to computing. It refers to a set of data processing elements connected in series, where the output of one element is the input of the next one. Wickham et al. (2009) argues that whether made explicit or not, the pipeline has to be presented in every graphics program. The paper also argues that breaking down graphic rendering into steps is beneficial for understanding the implementation and comparing between different graphic systems. The discussion on pipeline construction is well documented in early interactive graphics software: Buja et al. (1988), Sutherland et al. (2000), and Xie, Hofmann, and Cheng (2014) and their pipeline steps include non-linear transformation, variable standardization, randomization and dimension reduction.

What is pipeline, its underlying software design philosophy, and how these are reflected in R

One of the most commonly known pipeline examples is perhaps the Unix pipeline where programs can be concatenated with | to flow the output from the last program into the next program, i.e.

```
command 1 | command 2 | command 3 | ...
```

To solve a complex problem, the Unix system builds simple programs that do one thing well and work well together. This design is also reflected in the tidyverse ecosystem in R. To solve a complicated data problem using tidyverse, analysts typically build the solution using a collection of tools from the tidyverse toolbox. The data object can flow smoothly from one command to the next, safeguarded by the tidy data format (Wickham 2014), which prescribes three rules on how to lay out tabular data. The tidyverse tools also embrace a strong human-centered design where function names are intuitive and easy to reference through autocomplete. With the tidyverse design principle in mind, the tidymodel suite enables analysts to build machine learning models through the data pipeline. It includes typical tasks required in machine learning like data resampling, feature engineering, model fitting, model tuning, and model evaluation. An advantage of tidymodel pipeline over separate software for individual models is that analysts no longer need to write model-specific syntax to work with each model, but pipeline-specific syntax that is applicable to all the models implemented in tidymodel. This allows users to easily experiment with a collection of machine learning models.

Constructing indexes would also benefit from pipeline and embracing the aforementioned design philosophy.

In index construction, data pipeline is often presented in a workflow diagram in the research paper to illustrate how the raw data is transformed into the final indexes. This agrees with Wickham's argument on the presence of the data pipeline, however, more often than not, the pipeline is not made explicit in the software. Often the time, all the steps are lumped into a single wrapper function, rather than being split into smaller, modulated functions. This increases the cost of maintaining and understanding the code base, gives analysts little freedom to customise the indexes for specific needs, and hinders reusing existing code for building new indexes. A pipeline approach unites a range of indexes under a single data pipeline and analysts can compose indexes from pipeline steps like building Legos from individual bricks. In this workflow, analysts are not limited by indexes that have been already proposed and can easily combine pipeline steps to compose novel indexes. Analysis of the indexes (i.e. calculation of uncertainty) is also feasible by adding external code into the pipeline.

3 A pipeline for building statistical indexes

3.1 How does the pipeline constructin of an index look like?

Consider a commonly used drought index: Standardized Precipitation-Evapotranspiration Index (SPEI) (Vicente-Serrano, Beguería, and López-Moreno 2010). Its construction involves: 1) calculating potential evapotranspiration (PET) from average temperature, t_{avg} , and its difference, d , with precipitation, $prcp$, 2) aggregating difference series with a time scale, 3) fitting the aggregated series with a chosen distribution to get density values, and 4) converting the density values to normal quantiles. Under the pipeline approach, SPEI will be constructed as:

```
DATA %>%
  trans_pet(method = thornthwaite, var = tavg) %>%
  aggregate(scale = 12, var = d) %>%
  normalise(dist = gamma, fit_method = lmom) %>%
  augment(var = .agg_prcp)
```

Change the examples to two scales values, compare how indexes look like

Here each command corresponds to one step described above with additional arguments specific to the step. The pipeline construction produces the same index value as command like `spei(...)`, as shown in Figure 1, but with additional benefit:

- Multiple scales and multiple distributions can be fitted using the `c(...)` syntax to compare index values constructed from different parameterisations;
- Intermediate results can be checked after each step; and
- Additional steps and analysis can be wired into the workflow for index diagnostics and customised user need.

everyone is providing single command function, this is also the same for spi

may not worthwhile if only one index, as long as start changing

just want to calculate index, single function is fine

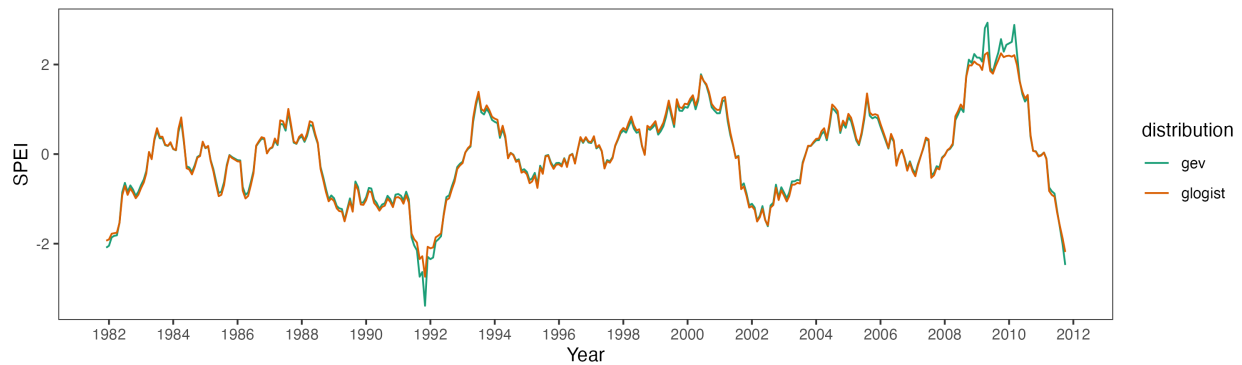


Figure 1: Standardised Precipitation Evapotranspiration Index (SPEI) calculated with two distribution fits in Step 3 described above: generalised logistic (`glogist`) and generalised extreme value (`gev`) distribution, using the `wichita` data from the package `SPEI`.

3.2 Pipeline steps for constructing indices

constructing time series index should also be encapsulated in the framework

Figure 2 presents the index construction pipeline from the raw data to the final indexes. The available intermediate steps include temporal processing, spatial aggregation, variable transformation, scaling, dimension reduction, normalisation, benchmarking, and simplification. These steps do not have to be arranged in sequence and not all the steps are required to present in every index construction. Spatial aggregation and temporal processing are operations of the data in space and time. Variable transformation, scaling, and dimension reduction concerns operations on the multivariate aspect of the data. Normalisation, benchmarking, and simplification can be viewed as the “post-processing” of indexes, which addresses the cross-comparison across different indexes and time, and index communication.

- divide into pre and post-processing

More steps are involved before the mathematical construction of indexes. This includes 1) defining a concept of interest, being useful for decision making and 2) finding the relevant and measurable variables to characterise the concept. These choices define the scope of the indexes and the data availability for constructing an index but not the construction itself, hence not included in the pipeline framework. The framework proposed in this paper assumes the objective of an index has been defined and relevant variables have been collected and quality controlled.

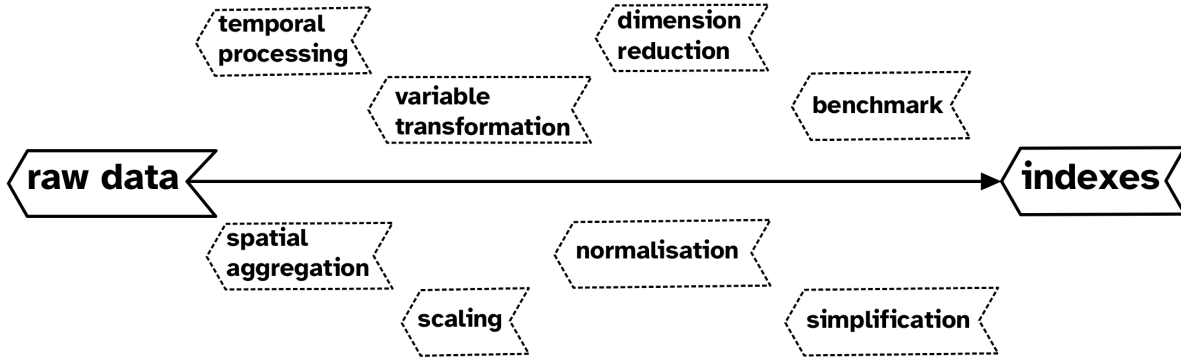


Figure 2: Diagram of pipeline steps for index construction.

Raw data

Discussion: whether we should allow raw data to have different spatial/ temporal resolution: This would require the notation \mathbf{s} and \mathbf{t} to depend on p

Initially my thought was to raw data only after aligning the resolution: Another section on original data directly downloaded, can have different spatial resolution, temporal granularity, data quality problem. After processing them and align them together they become the “raw data”

Once variables are collected, they can have different spatial resolution and temporal granularity. Let $\mathbf{x}(\mathbf{s}; \mathbf{t})$ denote the raw data that contain the spatial, temporal, and multivariate aspect: the spatial component $\mathbf{s} = (s_1, s_2, \dots, s_n)'$ is defined in the 2D space: $\mathbf{s} \in \mathcal{D}_s \subseteq \mathbb{R}^2$, the temporal component $\mathbf{t} = (t_1, t_2, \dots, t_J)'$ is defined in the 1D space: $\mathbf{t} \in \mathcal{D}_t \subseteq \mathbb{R}$. Expanding on the multivariate aspect, the data can be written as $\mathbf{x}(\mathbf{s}; \mathbf{t}) = (x_1(\mathbf{s}; \mathbf{t}), x_2(\mathbf{s}; \mathbf{t}), \dots, x_P(\mathbf{s}; \mathbf{t}))'$

Temporal processing

Temporal processing concerns the operations on the temporal aspect of the data. It can be represented by $f_\psi(x(\mathbf{s}; \mathbf{t}))$, where $\psi \in \Psi \subseteq \mathbb{R}^{d_\psi}$ is the parameters associated with the temporal operation and d_ψ is the number of parameter of ψ .

- purpose is to aggregate across time, smooth?

An example of temporal processing is to aggregate the data, $x(s_i; t_j)$, into k -month sum. This can be used to reflect how the accumulated values change across time. In the drought index SPI, precipitation can be aggregated at different scales to reflect short-term, mid-term, and long-term drought severity. With $\psi = k$, the aggregated series can be written as $x(s_i; t_{j'}) = \sum_{j=j'-k+1}^{j'} x(s_i; t_j)$ where j' is the time index after the aggregation.

Spatial aggregation

Sometimes, it can be beneficial to borrow information from nearby spatial locations or there is the need to combine raster data from satellite images with vector ground measures. Spatial aggregation can be written as $g_\theta(x(\mathbf{s}; \mathbf{t}))$, where $\theta \in \Theta \subseteq \mathbb{R}^{d_\theta}$ is the spatial operation parameters and d_θ is the number of parameter of θ .

- θ can be a constant or a derived from the data
- 3 by 3 aggregation, another purpose: the spatial grid is too fine, aggregate to higher level.

Variable transformation

Variable transformation can be necessary for different reasons: 1) to change the shape of the variable, 2) to stabilise the variance.

mention modelling? -> satisfy model assumption

A variable transformation changes the shape of the variable. It can be written as $h_\tau(x(\mathbf{s}; \mathbf{t}))$, where $\tau \in T \subseteq \mathbb{R}^{d_\tau}$ is the potential parameter in the transformation, if any, and d_τ is the number of parameter of τ . Variable transformation involve square root transformation, log transformation, and also include those non-linear transformation.

Scaling

Sometimes, scaling, as a specific type of variable transformation, is merged into dimension reduction as a pre-step, i.e. transform the covariance matrix into correlation matrix before applying the Principal Component Analysis (PCA), but it worth to be separated into its own step to make it explicit. Its main different from variable transformation is that scaling does not change the shape of the data and it can usually be written in the form of

$$[x(s_i; t_j) - \alpha]/\gamma.$$

Figure 3 shows a range of operations on a simulated variable. For example, In a z-score standardisation, $\alpha = \bar{x}(s; t)$ and $\gamma = \sigma(s; t)$ are used to standardise the data by the its mean and standard deviation across all time and all space units. Another example is to scale the variable into a unit interval, which can be written as $\alpha = \min[x(s_i, t_j)]$ and $\gamma = \max[x(s_i, t_j)] - \min[x(s_i, t_j)]$. While being a common technique in data pre-processing, scaling has an impact on the outliers of the data and analysts need to be cautious on checking its effect.

Dimension reduction

When multiple variables are collected in space and time to construct indices, they are often summarised with dimension reduction. Dimension reduction $x_{p^*}(\mathbf{s}; \mathbf{t}) \rightarrow x_p(\mathbf{s}; \mathbf{t})$, where $p^* = 1, 2, \dots, P^*$, $p = 1, 2, \dots, P$, and $P^* < P$. The most commonly used dimension reduction technique Principal Component Analysis (PCA), also called Empirical Orthogonal Function (EOF), can be written as $x_{p^*}(\mathbf{s}; \mathbf{t}) = \sum_{p=1}^P \lambda_p x_p(\mathbf{s}; \mathbf{t})$ where λ_p is the loading of the PC1, derived from maximising the variance of the data given the constraint $\sum_{p=1}^P \lambda_p^2 = 1$.

Normalising

The purpose of normalising is for cross-comparison using the normal score. This step can get criticism from analysts for being unnecessarily forcing the scale to the normal scale. This step is usually in the end of the pipeline when the data have been processed into a univariate series. When the data are not distributed normally, a distributed is usually fitted before converting to the normal score via the normal reverse CDF function: $\Phi^{-1}(\cdot): \Phi^{-1}[F_\eta(x(\mathbf{s}; \mathbf{t}))]$, where $\eta \in H \subseteq \mathbb{R}^{d_\eta}$ is the distribution parameter and d_η is the number of parameter of η . Sometimes, to account for the monthly variation in the data, the fitting is done separately for each month and the normalising step can be written as $\Phi^{-1}[F_\eta^m(x(\mathbf{s}; t_{j^*}))]$, where j^* is all the indexes that satisfy $j^* \bmod 12 = m$ for each $m = 0, 1, \dots, 11$.

- In some cases, the distribution and inverse normal can be multivariate, i.e. use copula.

Benchmarking

Benchmarking sets a constant value to allow the constructed index to be compared across time. Here we denote it with $u[x(s_i, t_j)]$ where u is a scalar of interest in the index constructed, could be a constant or a function of the data, i.e. mean.

Simplification

In public communication, indexes are often delivered in categorical grades, along with its underlying numerical values. The simplification step, sometimes can also be called discretisation, prescribes how the continuous index values are converted into the discrete grades. This process can be written with the piece-wise function:

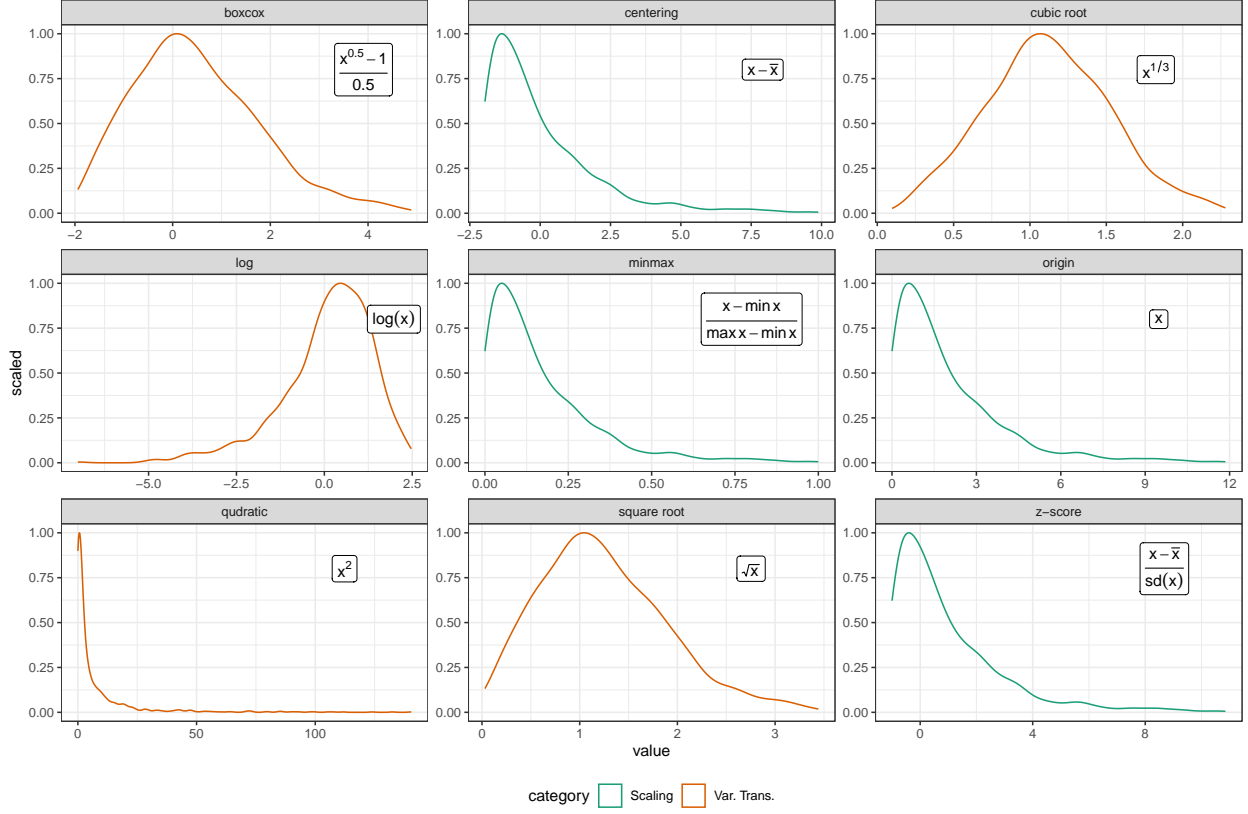


Figure 3: Comparison of scaling and variable transformation.

$$\begin{cases} C_0 & c_1 \leq (s_i; t_j) < c_0 \\ C_1 & c_2 \leq x(s_i; t_j) < c_1 \\ C_2 & c_3 \leq x(s_i; t_j) < c_2 \\ \dots & \\ C_z & c_z \leq x(s_i; t_j) \end{cases}$$

where C_0, C_1, \dots, C_z are the categories and c_0, c_1, \dots, c_z are the threshold value in each category. In SPI, drought are sorted into four categories: mild drought: $[-0.99, 0]$; moderate drought: $[-1.49, -1]$; severe drought: $[-1.99, -1.5]$, and extreme drought: $[-\infty, -2]$. In this case, C_0, C_1, C_2, C_3 are the drought categories: mild, moderate, severe, and extreme drought ($z = 3$) and $c_0 = 0, c_1 = -1, c_2 = -1.5, c_3 = -2$ are the cutoff value for each class.

Discretise the continuous index into a few labelled categories. For communicating the severity of natural hazard to general public.

uniform workflow to work with index construction.

- illustration
- math notation
- benefit of the pipeline approach
 - index diagnostic
 - uncertainty

4 Incorporating new buliding blocks into the pipeline

5 Examples

5.1 Constructing Standardised Precipitation Index (SPI)

- a basic workflow and congruence with results in the SPEI pkg
- allow multiple distribution fit
- allow bootstrap uncertainty

5.2 Calculating SPEI with raster data

Reference

- Brown, Jesslyn F., Brian D. Wardlow, Tsegaye Tadesse, Michael J. Hayes, and Bradley C. Reed. 2008. “The Vegetation Drought Response Index (VegDRI): A New Integrated Approach for Monitoring Drought Stress in Vegetation.” *GIScience & Remote Sensing* 45 (1): 16–46. <https://doi.org/10.2747/1548-1603.45.1.16>.
- Buja, A, D Asimov, C Hurley, and JA McDonald. 1988. “Elements of a Viewing Pipeline for Data Analysis.” In *Dynamic Graphics for Statistics*, 277–308. Wadsworth, Belmont.
- Economist Intelligence Unit. 2019. “The Global Liveability Index 2019.” The Economist. <https://www.cbeinternational.ca/pdf/Liveability-Free-report-2019.pdf>.
- Keyantash, John A., and John A. Dracup. 2004. “An Aggregate Drought Index: Assessing Drought Severity Based on Fluctuations in the Hydrologic Cycle and Surface Water Storage.” *Water Resources Research* 40 (9). <https://doi.org/10.1029/2003WR002610>.
- Kogan, F. N. 1995. “Application of Vegetation Index and Brightness Temperature for Drought Detection.” *Advances in Space Research*, Natural Hazards: Monitoring and Assessment Using Remote Sensing Technique, 15 (11): 91–100. [https://doi.org/10.1016/0273-1177\(95\)00079-T](https://doi.org/10.1016/0273-1177(95)00079-T).
- McKee, Thomas B, Nolan J Doesken, John Kleist, et al. 1993. “The Relationship of Drought Frequency and Duration to Time Scales.” In *Proceedings of the 8th Conference on Applied Climatology*, 17:179–83. 22. Boston, MA, USA.
- Sutherland, Peter, Anthony Rossini, Thomas Lumley, Nicholas Lewin-Koh, Julie Dickerson, Zach Cox, and Dianne Cook. 2000. “Orca: A Visualization Toolkit for High-Dimensional Data.” *Journal of Computational and Graphical Statistics* 9 (3): 509–29. <https://www.jstor.org/stable/1390943>.
- Tucker, Compton J. 1979. “Red and Photographic Infrared Linear Combinations for Monitoring Vegetation.” *Remote Sensing of Environment* 8 (2): 127–50. [https://doi.org/10.1016/0034-4257\(79\)90013-0](https://doi.org/10.1016/0034-4257(79)90013-0).
- United Nations Development Programme. 2022. “Human Development Report 2021-22.” New York. <http://report.hdr.undp.org>.
- Vicente-Serrano, Sergio M., Santiago Beguería, and Juan I. López-Moreno. 2010. “A Multiscalar Drought Index Sensitive to Global Warming: The Standardized Precipitation Evapotranspiration Index.” *Journal of Climate* 23 (7): 1696–1718. <https://journals.ametsoc.org/view/journals/clim/23/7/2009jcli2909.1.xml>.
- Wickham, Hadley. 2014. “Tidy Data.” *Journal of Statistical Software* 59 (September): 1–23. <https://doi.org/10.18637/jss.v059.i10>.
- Wickham, Hadley, Michael Lawrence, Dianne Cook, Andreas Buja, Heike Hofmann, and Deborah F. Swayne. 2009. “The Plumbing of Interactive Graphics.” *Computational Statistics* 24 (2): 207–15. <https://doi.org/10.1007/s00180-008-0116-x>.
- Xie, Yihui, Heike Hofmann, and Xiaoyue Cheng. 2014. “Reactive Programming for Interactive Graphics.” *Statistical Science* 29 (2): 201–13. <https://www.jstor.org/stable/43288470?seq=1>.