Smile Interpolation

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1 Motivations

Using a smile is useful to define the distribution of a financial underlying at a given date

Theorem 1.1 (Breeden-Litzenberger formula)**.** *Let S be a financial asset. Let ϕ be the probability density function of S_T. Then*

$$\phi(K) = \frac{d^2 C^T}{dK^2} \tag{1}$$

where C^T is the T-forward price of a call of strike K

Let us assume that we have a smooth parametrisation of the smile $K \mapsto \sigma(K)$ for a given maturity T.

Proposition 1.2. *With the above notations*

$$\phi(K) = \frac{\partial^2 C^T}{\partial K^2} + \sigma''(K)\frac{\partial C^T}{\partial \sigma} + \sigma'(K)^2 \frac{\partial^2 C^T}{\partial \sigma^2} \tag{2}$$

If we want the density function to be a $\mathcal{C}^1$ function of the strike K, $K \mapsto \sigma(K)$ has to be at least a $\mathcal{C}^3$ function

2 Interpolation via a polynomial function of degree 5