University of Southampton

COMP2208: Intelligent Systems

Comparison of Search Methods

Huw Jones
27618153
hcbj1g15@soton.ac.uk

November 30, 2016

1 Approach

In order to analyse the differences in scalability, I decided to build a framework that would allow me to minimise the time spent on writing code. At the moment, I am most familiar with Java, therefore that is the language I chose to build my solution in. My code is nowhere near "good" or optimised (in terms of real time running, not nodes expanded), but it works.

I decided to implement a monitoring thread to print out the current search status. This means that the number of nodes evaluated, how long the search has been running (in real time), and the amount of memory currently in use.

1.1 The Setup

I tried to keep data structures simple and minimalistic. The state of the puzzle is stored in a **Node**. A **Node** has a **Grid** (which represents the state of the puzzle), a parent node reference, a depth (used for IDS), and a priority (used for A*). A **Grid** has a width/height, a 2D array of characters (representing blocks), and a HashMap that maps characters to their position.

1.2 The Framework

I created a framework whereby the program parameters can be manipulated via the command line.

```
-e, --exit [STATE]     Specifies exit state.
-h, --height [HEIGHT]  Sets the grid height.
    --help             Prints this help message.
-i, --interval [TIME]  Sets the refresh interval (in ms) - for monitoring search status.
-r, --ran [STATE]      Specifies the seed used for the pseudo-random number.
-s, --start [STATE]    Specifies the start state
-t, --type [TYPE]      Specifies the search type:
                       BFS - Breadth First Search
                       DFS - Depth First Search
                       IDS - Iterative Deepening Search
                       A* - A* Heuristic Search
-w, --width [WIDTH]    Sets the grid width.
```

This framework allowed me to create scripts to automate my searching. It also allowed me to inject different start/finish grids, as well as injecting different grid sizes - all without having to rewrite any of my code.

In addition, I allowed the input of the random number seed. This helped during debugging my program. If a DFS search didn't work with one random number, I could provide the random number and debug that case.

1.3 The Organisation

My code was organised as following. All my code is available in Appendix A.

```
|-- BlocksWorld.java
|-- blocksworld
|   |-- Block.java
|   |-- Grid.java
|   |-- GridController.java
|   |-- Node.java
|   |-- Pair.java
|   |-- Position.java
|   |-- exceptions
|   |   |-- InvalidBlockIDException.java
|   |   |-- InvalidDirectionException.java
|   |   \-- InvalidPositionException.java
|   \-- search
|       |-- AStar.java
|       |-- BFS.java
|       |-- DFS.java
|       |-- IDS.java
|       \-- Search.java
|-- run.sh
\-- test.sh
```

I chose to organise my code this way to make it easier for me to add features and group common features together.

1.4 The Scripts

I created a couple of shell scripts to aid my data collection and semi-automate running puzzles. The most useful script, `test.sh` (see Appendix A.16), took a CSV file of puzzle states and executed a given search type on the entire set, then dumped the output to a file. It saved a long time of running lots of puzzles against all the searches. `run.sh` (see Appendix A.15) is effectively a script that aliases the running of my java program.

2 Evidence

A **Grid** state is represented in a grid of dimensions $H \times W$. The agent is represented by a '`*`', blanks are represented by '`-`', and blocks are represented by a lowercase letter. In the evidence provided, the number above a **Grid** state is the node number.

When a search is running, the current status of the search is displayed. This include the number of nodes evaluated (time complexity), the length of real time the search has been running, and the amount of used memory (space complexity).

2.1 Breadth First Search

Appendix B.1 shows the order that the program evaluated nodes. It is evident that BFS is working correctly as the tiles appear to jump around if the nodes are being read in number order. Here, the first layer of the tree are nodes 1 & 2. Nodes 3 to 5 are the second layer children of node 1. Nodes 6 to 8 are the second layer children of node 2. Nodes 9 through 11 are the third layer children of node 3. And so forth for the remainder of the nodes shown.

Example output from a BFS search running is shown in Appendix C.1. Here, you can see the memory usage, my implementation of BFS fits the expected space complexity. Since the

2.2 Depth First Search

The order of nodes evaluated is shown in Appendix B.2. With these set of nodes, the movement of the agent is fluid from state to state. Therefore, it can be concluded that the implementation of DFS is working correctly.

Appendix C.2 shows the trace of a DFS running. It shows how few nodes were evaluated (in comparison to BFS), but then the solution is inherently long (compared to the optimal solution). I think this conclusively proves that my implementation of DFS is working correctly.

2.3 Iterative Deepening Search

My implementation of IDS logs when the maximum search depth increases. In the output log - as shown in Appendix B.3 - it can be seen that the nodes processed follow the expected order for IDS. When the depth is increased, it is evident that the search restarts again from the root node and proceeds to search down to the maximum depth.

In addition, the memory usage (space complexity), as shown in the output in Appendix C.3, seems to follow no trend. I believe this is due to the nodes on the fringe (at maximum depth) being removed as they aren't a solution. This would explain why the footprint of IDS stays so small when it is running.

2.4 A* Heuristic Search

With A* Heuristic Search, it is more difficult to prove that the algorithm is working correctly. Appendix B.4 shows the output log for A* Search. It is more difficult to see how my implementation of A* prioritises its node selection, however, I believe it to be working correctly. Appendix C.4 shows that very few nodes were evaluated (compared to other optimal searches such as BFS), and yet the optimal solution was found.

3 Scalability

I use the term **complexity** synonymously with difficulty in regards to searching throughout this report. In this report, the difficulty/complexity number is the length of the optimal solution for a problem.

To investigate the scalability of all 4 searches, I decided to start with the given problem and work backwards. The base (given) problem required a minimum of 14 moves to solve and since my searches printed out the list of states to complete the puzzle, I used this printout to form puzzles of different complexities. This is how I controlled the "Problem Difficulty".

As mentioned in Section 1.4, I wrote a script to automate running puzzles of different complexities. I used the output from the base puzzle to create a CSV of states to test. These states ranged from a complexity of 1, to 14 (the base problem). I ran this test script of all 4 of the search types.

The graph in Appendix D.3 displays my findings. It is important to note that I've used a logarithmic scale on the y axis to enhance the readability of the graph.

The most striking element of this graph is how "random" the line for DFS is. This shows that the complexity of DFS is not directly linked to the difficulty of the puzzle. This is to be expected for DFS as there is no "real" logic to how it searches – it tries random moves until it finds a solution.

The lines for BFS/IDS seem to intertwine and have the same gradient. Having the same gradient shows that both search methods increase at roughly the same exponential rate (in regards to time complexity against problem difficulty). There are times where IDS time complexity is less than BFS complexity. This is most probably due to the location of the solution node. Since BFS searches left to right, top to bottom and IDS searches from top to bottom, left to right, if the solution is on the bottom left of the tree, IDS will find it before BFS.

Finally, it's important to note how dramatic the difference between A* and BFS/IDS are in terms of time complexity. With A* the heuristics make all the difference between an efficient search and an inefficient search. In this investigation, my heuristic function is inadmissible as it is the sum of the Manhattan Distance, Depth of the Node, and the number of blocks in the incorrect position. With this heuristic function, the time complexity for A* increases exponentially with the problem difficulty, but not to the same extent as that of BFS/IDS. At a problem with difficulty of "14", A* had a time complexity 3 orders of magnitude smaller than BFS/IDS.

Overall, this graph shows the striking difference in time complexities. I can safely conclude that the best search method, of the 4 compared, is A* Heuristic Search. It is evident that using the heuristic function mentioned previously, A* search has a time complexity magnitudes smaller, than that of IDS/BFS. This effectively rules IDS/BFS out. DFS, although sometimes providing good time complexity for some random seeds, is ultimately a poor choice if you are wanting to find the optimal solution.

4 Extras & Limitations

4.1 Extras

4.1.1 Complexity vs Problem Size

I decided to go a little further whilst investigating the complexity for different problem sizes. I collected data for puzzles on different grid sizes. I used grids sized 2x2, 3x3 and 5x5.

The graphs for these grids are available in Appendix D.1, Appendix D.2, Appendix D.3, and Appendix D.4.

These graphs only reinforce my conclusions in the previous section.

With DFS, the conclusions of randomness and being erratic are shown conclusively. For example, a problem with difficulty of 1 (1 move to solve the puzzle) took DFS 2,604,225 evaluations to find a solution – the solution was 2,604,224 moves too long (compared to the optimal solution). But on the other hand, a problem that took 3.9 million evaluations using A* (as yet to be solved using DFS/IDS), solved by DFS by evaluating only 826,179 nodes. However, the major caveat being the solution from A* was 21 moves long, and the solution from DFS was 826,179 moves long.

4.1.2 Space Complexity vs Time Complexity

In this section, I decided to look at the trends between Time Complexity (number of nodes evaluated) and Space Complexity (indicated by RAM usage). The graphs are included in the Appendix. Where a trendline has been plotted, it should in theory have a y-intercept of 0, however, due to the overhead of loaded classes and the JVM, this base usage is in fact approximately 30MB.

BFS Complexity The graph in Appendix D.5 shows a fairly linear trend. As the time complexity increases, the space complexity increases at the same rate. This is to be completely expected because a Breadth First Search does not allow for nodes to be removed from memory.

DFS Complexity The graph in Appendix D.6 shows a linear trend. As the time complexity increases, the space complexity increases at roughly the same rate. This graph is not as conclusive due to the massive steps every so often. However, I attribute this to garbage collection.

To minimise interruptions and reduce CPU usage, garbage collection does not run all the time, but periodically. There are several measures that are watched and trigger garbage collection; these include time intervals, heap size. In the case of this graph, I believe garbage collection was being trigger when the heap grew too big. Then garbage collection kicked in and removed a lot of junk (unused/orphaned objects) from the heap.

It is obvious that my implementation of DFS is not optimal, otherwise this memory profile would be less obvious. Anyway, I believe that this graph shows that the time complexity and space complexity are linearly linked.

IDS Complexity The graph in Appendix D.7 shows absolutely not trend whatsoever. Unlike the dodgy garbage collection in the DFS graph (Appendix D.6) I think garbage collection has worked really well for IDS.

It is noted that the maximum memory usage is 151MB. This was when IDS had evaluated just over 11 million nodes. IDS works by searching top to bottom until it reaches a limit, then left to right. If it does not find a solution, it increases its depth limit and tries again. Because of this, when IDS hits the limit on the left side of the tree, those nodes are removed from memory as it travels to the right hand side. This means that IDS has a very small space complexity footprint – and this is reflected in my findings.

A* Heuristic Complexity The graph in Appendix D.8 shows a linear trend. I attribute thickness of the "line" to be due to garbage collection – yet again.

A* works by expanding nodes, evaluating their "goodness" (also know as priority), and then expanding nodes from the "good" nodes. This strategy minimises the amount of nodes that are looked at, but means that almost all nodes end up being stored in a Priority Queue. Therefore, as the number of nodes expanded increases, then the space complexity footprint will also increase. I strongly believe that my graph shows this to be true.

4.2 Limitations

To produce my data for scalability, I only tested 1 grid for each problem difficulty. If I had more forethought and time, I would've tested the searches against a number of different permutations for each problem difficulty. This would exclude rotated and reflected puzzles, but would include unique different puzzles. For example, I only tested the searches against the puzzle labelled A, I would have liked to test against other puzzles with a difficulty of 2 as well – some examples are shown below.

```
A:      B:      C:
----    --*-    ----
-*--    --a-    a---
ba--    -b--    *b--
-c--    -c--    -c--
```

Although I do not think that testing against different states with the same difficulty would have affected my conclusions, I would still have preferred to use more data.

This report only looked at square grids. However, my code did not actually limit the input to have to be square in proportion. I definitely could have investigated the effect that different sized grids had on the effectiveness of the searches, but considering this report's purpose was to compare and contrast the effectiveness of different tree search algorithms, I figured this would have been a little too far out of the scope of the report. In addition, I don't think that non-square grids would have had any affect on my outcomes – but that is a claim to be investigated more thoroughly in the future.

Appendices

A Code

A.1 BlocksWorld.java

```
import blocksworld.Grid;
import blocksworld.GridController;
import blocksworld.Pair;
import blocksworld.Position;
import blocksworld.exceptions.InvalidBlockIDException;
import blocksworld.exceptions.InvalidPositionException;
import blocksworld.search.*;
import org.omg.CORBA.DynAnyPackage.Invalid;

import java.text.ParseException;
import java.util.Arrays;
import java.util.List;

/**
 * BlocksWorld
 *
 * @author Huw Jones
 * @since 08/10/2016
 */
public class BlocksWorld {

    public static void main(String[] args) {
        List<String> argList = Arrays.asList(args);

        BlocksWorld.header();

        try {
            if (argList.contains("--help")) {
                help();
                return;
            }

            int width = -1;
            int height = -1;
            int refresh = -1;

            Long seed = null;
            if (argList.contains("--height") || argList.contains("-h")) {
                int index = (argList.contains("-h")) ? argList.indexOf("-h") : argList.indexOf
                    ("--height");
                height = getInt(argList, index);
            }
            if (argList.contains("--width") || argList.contains("-w")) {
                int index = (argList.contains("-w")) ? argList.indexOf("-w") : argList.indexOf
                    ("--width");
                width = getInt(argList, index);
            }

            if (argList.contains("--interval") || argList.contains("-i")) {
                int index = (argList.contains("-i")) ? argList.indexOf("-i") : argList.indexOf
                    ("--interval");
                refresh = getInt(argList, index);
            }

            if (width == -1 || height == -1) {
                System.out.println("Please specify width/height.");
                return;
            }

            if (argList.contains("--ran") || argList.contains("-r")) {
                int index = (argList.contains("-r")) ? argList.indexOf("-r") : argList.indexOf
                    ("--ran");
                seed = getSeed(argList, index);
            }

            if (!argList.contains("--type") && !argList.contains("-t")) {
                System.out.println("Please specify a search type. (See help for more details)"
                    );
```

```
            return;
        }

        int index = (argList.contains("-t")) ? argList.indexOf("-t") : argList.indexOf("--
            type");
        try {

            String type = argList.get(index + 1);
            Grid startGrid = null;
            Grid exitGrid = null;
            if (argList.contains("--start") || argList.contains("-s")){
                int startIndex = (argList.contains("-s")) ? argList.indexOf("-s") :
                    argList.indexOf("--start");
                startGrid = parseState(argList.get(startIndex + 1), width, height);
            }
            if(argList.contains("--exit") || argList.contains("-e")) {
                int exitIndex = (argList.contains("-e")) ? argList.indexOf("-e") : argList
                    .indexOf("--exit");
                exitGrid = parseState(argList.get(exitIndex + 1), width, height);
            }

            search(type, startGrid, exitGrid, seed, refresh);
        } catch (ArrayIndexOutOfBoundsException e) {
            System.out.println("No option was specified for " + argList.get(index));
        }
    } catch (ParseException ex) {
        System.out.println("Failed to read input: " + ex.getMessage());
    } catch (ArrayIndexOutOfBoundsException ex) {
        System.out.println("No argument specified: " + ex.getMessage());
    } catch (IllegalArgumentException ex) {
        System.out.println("No input provided for option: " + ex.getMessage());
    }
}

/**
 * Prints out programme header info
 */
private static void header() {
    System.out.println("Usage: BlocksWorld [OPTION]...");
    System.out.println("COMP2208 BlocksWorld Search Tool.\n");
}

/**
 * Prints out help
 */
private static void help() {
    BlocksWorld.header();
    System.out.println("Arguments:");
    System.out.println("  -e, --exit [STATE]\tSpecifies exit state.");
    System.out.println("  -h, --height [HEIGHT]\tSets the grid height.");
    System.out.println("      --help\t\tPrints this help message.");
    System.out.println("  -i, --interval [TIME]\tSets the refresh interval (in ms) - for
        monitoring search status.");
    System.out.println("  -r, --ran [STATE]\tSpecifies the seed used for the pseudo-random
        number.");
    System.out.println("  -s, --start [STATE]\tSpecifies the start state");
    System.out.println("  -t, --type [TYPE]\tSpecifies the search type:\r\n\t\tBFS - 
        Breadth First Search\r\n\t\tDFS - Depth First Search\r\n\t\tIDS - Iterative 
        Deepening Search\r\n\t\tA* - A* Heuristic Search");
    System.out.println("  -w, --width [WIDTH]\tSets the grid width.");
}

private static int getInt(List<String> args, int argIndex) throws ParseException,
    ArrayIndexOutOfBoundsException, IllegalArgumentException {
    String widthStr = args.get(argIndex + 1);
    if (widthStr.substring(0, 1).equals("'")) {
        throw new IllegalArgumentException("No option was specified for " + args.get(
            argIndex));
    }
    return Integer.parseInt(widthStr);
}

private static long getSeed(List<String> args, int argIndex) throws ParseException,
    ArrayIndexOutOfBoundsException, IllegalArgumentException {
    String seedStr = args.get(argIndex + 1);
    return Long.parseLong(seedStr);
```

```
    }

    private static Grid parseState(String state, int src_width, int src_height) throws
        ParseException {
        List<String> substrs = Arrays.asList(state.split(":"));
        try {
            int width = Integer.parseInt(substrs.get(0));
            int height = Integer.parseInt(substrs.get(1));
            if(width != src_width || height != src_height){
                throw new ParseException("Height/Width in state does not match provided height
                    /width.", 0);
            }
            Grid g = GridController.createGrid(width, height);

            String row;
            char symbol;
            for (int i = 2; i < substrs.size(); i++) {
                row = substrs.get(i);
                for (int x = 0; x < g.getWidth(); x++) {
                    symbol = row.charAt(x);
                    try {
                        if (symbol >= 'a' && symbol <= 'z') {
                            g.placeBlock(symbol, new Position(x, i - 2));
                        } else if (symbol == '*') {
                            try {
                                g.placeAgent(x, i - 2);
                            } catch (InvalidBlockIDException ex){
                                System.out.println("Multiple agents detected... skipping");
                            }
                        }
                    } catch (InvalidPositionException e) {
                        e.printStackTrace();
                    }
                }
            }
            return g;
        } catch (NumberFormatException ex) {
            throw new ParseException(ex.getMessage(), 0);
        }
    }

    private static void search(String type, Grid startState, Grid exitState, Long seed, int
        refreshTime) {
        Search search = null;
        switch (type) {
            case "BFS":
                search = new BFS();
                break;
            case "DFS":
                search = new DFS();
                break;
            case "IDS":
                search = new IDS();
                break;
            case "A*":
                search = new AStar();
                break;
            default:
                header();
                System.out.println(String.format("Type '%s' was not recognised.", type));
                return;
        }
        if (search == null) return;
        if (startState != null) {
            search.setStartState(startState);
        }
        if (exitState != null) {
            search.setExitState(exitState);
        }
        if(seed != null){
            search.setSeed(seed);
        }
        if(refreshTime != -1L){
            search.setRefreshTime(refreshTime);
        }
        search.run();
```

```
    }
}
```

A.2 Grid.java

```
package blocksworld;

import blocksworld.exceptions.InvalidBlockIDException;
import blocksworld.exceptions.InvalidPositionException;

import java.util.ArrayList;
import java.util.HashMap;
import java.util.NoSuchElementException;
import java.util.stream.Collectors;

/**
 * Grid
 * Holds the state of the grid
 *
 * @author Huw Jones
 * @since 08/10/2016
 */
public class Grid {

    private int width;
    private int height;
    private char[][] grid;
    private HashMap<Character, Position> blocks;

    /**
     * Creates a new grid with a specified width and height
     *
     * @param width  Width of new grid
     * @param height Height of new grid
     */
    public Grid(int width, int height) {
        this.width = width;
        this.height = height;
        this.grid = new char[width][height];
        this.blocks = new HashMap<>();
    }

    /**
     * Returns the width of the grid
     *
     * @return Grid Width
     */
    public int getWidth() {
        return width;
    }

    /**
     * Returns the height of the grid
     *
     * @return Grid Height
     */
    public int getHeight() {
        return height;
    }

    /**
     * Places the agent to the grid at position (x, y)
     *
     * @param x x-coord
     * @param y y-coord
     * @throws InvalidPositionException Thrown if the given position is invalid
     */
    public void placeAgent(int x, int y) throws InvalidPositionException,
        InvalidBlockIDException {
        placeAgent(new Position(x, y));
    }

    /**
     * Places the agent to the grid at position P(x, y)
     *
     * @param position Position P(x, y)
     * @throws InvalidPositionException Thrown if the given position is invalid
     */
    public void placeAgent(Position position) throws InvalidPositionException,
```

```
        InvalidBlockIDException {
        if(this.blocks.containsKey('*')){
            throw new InvalidBlockIDException('*');
        }
        this.placeBlock('*', position);
    }

    /**
     * Adds a block to the grid at position (x, y) with the ID blockID.
     * Throws an exception if the position is invalid
     *
     * @param blockID  ID of the block, must be unique
     * @param position Position of the block
     * @throws InvalidPositionException Thrown if the position is invalid
     */
    public void placeBlock(char blockID, Position position) throws InvalidPositionException {
        if (!isPositionValid(position)) {
            throw new InvalidPositionException(position);
        }
        try {
            this.grid[position.getX()][position.getY()] = blockID;
        } catch (ArrayIndexOutOfBoundsException ex) {
            ex.printStackTrace();
        }

        // Prevent adding null chars to the block HashMap
        if (blockID == Character.MIN_VALUE) return;
        this.blocks.put(blockID, position);
    }

    /**
     * Returns whether a position is valid on the grid
     *
     * @param position Position to check
     * @return False if position is not valid
     */
    public boolean isPositionValid(Position position) {
        return isPositionValid(position.getX(), position.getY());
    }

    /**
     * Returns whether a position is valid on the grid
     *
     * @param x x-coord
     * @param y y-coord
     * @return False if position is not valid
     */
    public boolean isPositionValid(int x, int y) {
        return (x < this.width && y < this.height && x >= 0 && y >= 0);
    }

    /**
     * Places a block on the grid
     *
     * @param block Block to place
     * @throws InvalidPositionException Thrown if the position is invalid
     */
    public void placeBlock(Block block) throws InvalidPositionException {
        placeBlock(block.getID(), block.getPosition());
    }

    /**
     * Gets the agent block
     *
     * @return Agent block
     * @throws NoSuchElementException If the block was not found
     */
    public Block getAgent() throws NoSuchElementException {
        return getBlock('*');
    }

    /**
     * Gets the Block with blockID
     *
     * @param blockID Block to fetch
     * @return Block
```

```
     * @throws NoSuchElementException If block was not found
     */
    public Block getBlock(char blockID) throws NoSuchElementException {
        if (!this.blocks.containsKey(blockID)) {
            throw new NoSuchElementException("No such block with ID: " + blockID);
        }
        return new Block(blockID, this.blocks.get(blockID));
    }

    /**
     * Gets the list of blocks in the grid
     *
     * @return List of Blocks
     */
    public ArrayList<Block> getBlocks() {
        // Map the HashMap to an ArrayList<Block>
        return this.blocks.entrySet().stream().map(map -> new Block(map.getKey(), map.getValue
            ())).collect(Collectors.toCollection(ArrayList::new));
    }

    @Override
    public String toString() {
        String grid = "";
        try {
            Character block;
            for (int y = 0; y < this.height; y++) {
                for (int x = 0; x < this.width; x++) {
                    block = this.grid[x][y];
                    grid += (block != Character.MIN_VALUE) ? block : "-";

                }
                grid += "\n";
            }
        } catch (Exception ex){
            ex.printStackTrace();
        }
        return grid;
    }
}
```

A.3 GridController.java

```
package blocksworld;

import blocksworld.exceptions.InvalidBlockIDException;
import blocksworld.exceptions.InvalidDirectionException;
import blocksworld.exceptions.InvalidPositionException;

import java.util.ArrayList;

/**
 * Grid Controller
 *
 * @author Huw Jones
 * @since 08/10/2016
 */
public class GridController {

    public static Grid placeBlock(Grid grid, char blockID, Position position) throws
        InvalidPositionException {
        grid.placeBlock(blockID, position);
        return grid;
    }

    public static Grid placeBlock(Grid grid, char blockID, int x, int y) throws
        InvalidPositionException {
        grid.placeBlock(blockID, new Position(x, y));
        return grid;
    }

    public static Grid placeBlock(Grid grid, Block block) throws InvalidPositionException {
        grid.placeBlock(block.getID(), block.getPosition());
        return grid;
    }

    public static Grid placeAgent(Grid grid, int x, int y) throws InvalidPositionException,
        InvalidBlockIDException {
        grid.placeAgent(x, y);
        return grid;
    }

    public static Grid placeAgent(Grid grid, Position position) throws
        InvalidPositionException, InvalidBlockIDException {
        grid.placeAgent(position);
        return grid;
    }

    public static Grid move(Grid grid, DIRECTION direction) throws InvalidDirectionException {
        if (!canMove(grid, direction))
            throw new InvalidDirectionException(direction, grid.getAgent().getPosition(),
                getNewAgentPosition(grid, direction));

        // Get position where agent is *going* to move to
        Position newAgentPosition = getNewAgentPosition(grid, direction);

        Grid newGrid = GridController.createGrid(grid.getWidth(), grid.getHeight());

        ArrayList<Block> oldBlocks = grid.getBlocks();
        ArrayList<Block> newBlocks = new ArrayList<>();

        Block oldBlock = null;
        Block oldAgent = null;

        for (Block block : oldBlocks) {
            if (block.getID() == '*') {
                oldAgent = block;
            } else if (block.getPosition().equals(newAgentPosition)) {
                oldBlock = block;
            } else {
                newBlocks.add(block);
            }
        }
        if (oldAgent != null) {
            newBlocks.add(new Block('*', newAgentPosition));
            if (oldBlock != null) {
                newBlocks.add(new Block(oldBlock.getID(), oldAgent.getPosition()));
```

```
            }
        }

        for (Block block : newBlocks) {
            try {
                newGrid.placeBlock(block);
            } catch (InvalidPositionException e) {
                e.printStackTrace();
            }
        }

        return newGrid;
    }

    public static boolean canMove(Grid grid, DIRECTION direction) {
        try {
            getNewAgentPosition(grid, direction);
        } catch (InvalidDirectionException ex) {
            return false;
        }
        return true;
    }

    private static Position getNewAgentPosition(Grid grid, DIRECTION direction) throws
        InvalidDirectionException {
        Position oldPosition = grid.getAgent().getPosition();
        int old_x = oldPosition.getX();
        int old_y = oldPosition.getY();

        int new_x = -1;
        int new_y = -1;

        switch (direction) {
            case NORTH:
                new_x = old_x;
                new_y = old_y - 1;
                break;
            case EAST:
                new_x = old_x + 1;
                new_y = old_y;
                break;
            case SOUTH:
                new_x = old_x;
                new_y = old_y + 1;
                break;
            case WEST:
                new_x = old_x - 1;
                new_y = old_y;
                break;
        }

        Position newPosition = new Position(new_x, new_y);

        if (!grid.isPositionValid(new_x, new_y))
            throw new InvalidDirectionException(direction, oldPosition, newPosition);

        return newPosition;
    }

    public static Grid createGrid(int width, int height) {
        return new Grid(width, height);
    }

    public enum DIRECTION {
        NORTH,
        EAST,
        SOUTH,
        WEST
    }
}
```

A.4 Node.java

```
package blocksworld;

/**
 * Node
 * Stores a grid state
 *
 * @author Huw Jones
 * @since 08/10/2016
 */
public class Node {

    private Grid grid = null;
    private Node parent;
    private int depth;
    private Integer priority = null;

    private Node() {
        this.depth = 0;
    }

    public Node(Node parent) {
        this.parent = parent;
        // Cheap way to calculate depth of this node
        this.depth = parent.getDepth() + 1;
    }

    public int getDepth() {
        return depth;
    }

    public static Node createRootNode() {
        return new Node();
    }

    public Grid getGrid() {
        return grid;
    }

    public void setGrid(Grid grid) {
        if (grid != null) this.grid = grid;
    }

    public int getPriority() {
        return priority;
    }

    public void setPriority(int priority) {
        if (this.priority == null) {
            this.priority = priority;
        }
    }

    public Node getParent() {
        return parent;
    }
}
```

A.5 Pair.java

```
package blocksworld;

/**
 * Holds a Pair of Values
 *
 * @author Huw Jones
 * @since 08/10/2016
 */
public class Pair<K, V> {

    private K key;
    private V value;

    public Pair(K key, V value) {
        this.key = key;
        this.value = value;
    }

    public K getKey() {
        return key;
    }

    public V getValue() {
        return value;
    }

    @Override
    public boolean equals(Object obj) {
        if (!(obj instanceof Pair)) return false;
        Pair p = (Pair) obj;
        return this.key == p.key && this.value == p.value;
    }

    @Override
    public String toString() {
        return String.format("(%s, %s)", key, value);
    }
}
```

A.6 Position.java

```
package blocksworld;

/**
 * Effectively a Co-Ordinate in 2D space
 *
 * @author Huw Jones
 * @since 04/11/2016
 */
public class Position extends Pair<Integer, Integer> {
    public Position(int x, int y) {
        super(x, y);
    }

    public int getX(){
        return this.getKey();
    }

    public int getY(){
        return this.getValue();
    }

    public Position subtract(Position position){
        return new Position(
                this.getX() - position.getX(),
                this.getY() - position.getY()
        );
    }
}
```

A.7 InvalidBlockIDException.java

```
package blocksworld.exceptions;

/**
 * Thrown if an invalid Block ID was specified
 *
 * @author Huw Jones
 * @since 08/10/2016
 */
public class InvalidBlockIDException extends Exception {

    /**
     * Constructs a new exception with {@code null} as its detail message.
     * The cause is not initialized, and may subsequently be initialized by a
     * call to {@link #initCause}.
     */
    public InvalidBlockIDException(char c) {
        super("Invalid BlockID: " + c);
    }
}
```

A.8 InvalidDirectionException.java

```
package blocksworld.exceptions;

import blocksworld.GridController;
import blocksworld.Pair;

/**
 * Thrown if an invalid direction was given
 *
 * @author Huw Jones
 * @since 08/10/2016
 */
public class InvalidDirectionException extends Exception {

    public InvalidDirectionException(GridController.DIRECTION d, Pair oldPos, Pair newPos) {
        super("Cannot move in direction: " + d + ", from position " + oldPos + ", to " +
            newPos);
    }
}
```

A.9 InvalidPositionException.java

```
package blocksworld.exceptions;

import blocksworld.Position;

/**
 * Thrown if an invalid position was specified
 *
 * @author Huw Jones
 * @since 08/10/2016
 */
public class InvalidPositionException extends Exception {

    public InvalidPositionException(int x, int y) {
        super(String.format("Invalid position at (%d, %d)", x, y));
    }

    public InvalidPositionException(Position position) {
        this(position.getX(), position.getY());
    }
}
```

A.10 AStar.java

```
package blocksworld.search;

import blocksworld.*;
import blocksworld.exceptions.InvalidDirectionException;

import java.util.*;

/**
 * A* Search
 *
 * @author Huw Jones
 * @since 27/11/2016
 */
public class AStar extends Search {

    PriorityQueue<Node> nodeQueue;

    /**
     * Set up the initial environment before running the search
     */
    @Override
    protected void preRun() {
        this.nodeQueue = new PriorityQueue<>(new PriorityComparator());
        this.rootNode = Node.createRootNode();
        this.rootNode.setGrid(this.startGrid);
    }

    /**
     * Where the actual search runs
     */
    @Override
    protected void runSearch() throws Exception {
        ArrayList<GridController.DIRECTION> directions = new ArrayList<>(4);
        Arrays.stream(GridController.DIRECTION.values()).forEach(directions::add);

        this.currentNode = rootNode;
        while_loop:
        while (true) {
            numberOfNodes++;

            // Check the if the node satisfies the exit condition
            if (this.checkExitCondition(currentNode.getGrid())) {
                completed(currentNode);
                break;
            }

            for (GridController.DIRECTION direction : directions) {
                try {
                    // Process the move and store the new state in the node
                    Node newNode = new Node(currentNode);
                    newNode.setGrid(
                            GridController.move(
                                    newNode.getParent().getGrid(),
                                    direction
                            )
                    );

                    // Calculate the node heuristic score and add it to the queue
                    newNode.setPriority(
                            calculatePriority(newNode)
                    );
                    nodeQueue.add(newNode);
                } catch (InvalidDirectionException e) {
                }
            }
            // Process the next node
            nextNode();
        }
    }

    @Override
    protected void nextNode() {
        currentNode = nodeQueue.poll();
    }
```

```
    /**
     * Calculates the priority (heuristic score) of the node
     *
     * @param node Node to calculate score for
     * @return Score for that node
     */
    private int calculatePriority(Node node) {
        int score = 0;
        score += getManhattanDistance(node.getGrid());
        score += getTilesInCorrectPlace(node.getGrid());
        score += node.getDepth();
        return score;
    }

    /**
     * Calculates the Manhattan Distance Heuristic
     *
     * @param grid Grid to calculate
     * @return score
     */
    private int getManhattanDistance(Grid grid) {
        int score = 0;
        ArrayList<Block> blocks = grid.getBlocks();
        for (Block block : blocks) {
            try {
                // Get the difference between the exit position and the current block position
                Position difference = this.exitGrid
                        .getBlock(block.getID())
                        .getPosition()
                        .subtract(block.getPosition());
                // Add the X/Y distance from target block position (distance not displacement,
                     hence Math.abs)
                score += Math.abs(difference.getX());
                score += Math.abs(difference.getY());
            } catch (NoSuchElementException ex) {
            }
        }
        return score;
    }

    /**
     * Calculates the number of tiles in the correct place
     *
     * @param grid Grid to calculate
     * @return score
     */
    private int getTilesInCorrectPlace(Grid grid) {
        ArrayList<Block> blocks = grid.getBlocks();
        int score = 0;
        for (Block block : blocks) {
            try {
                // Increment score for every incorrectly positioned block
                if (!this.exitGrid.getBlock(block.getID()).getPosition().equals(block.
                    getPosition())) score++;
            } catch (NoSuchElementException ex) {

            }
        }
        return score;
    }

    /**
     * Finds the highest priority node (node with lowest score)
     * Used to sort the PriorityQueue
     */
    private class PriorityComparator implements Comparator<Node> {
        @Override
        public int compare(Node o1, Node o2) {
            return o1.getPriority() - o2.getPriority();
        }
    }
}
```

A.11 BFS.java

```
package blocksworld.search;

import blocksworld.GridController;
import blocksworld.GridController.DIRECTION;
import blocksworld.Node;
import blocksworld.Pair;
import blocksworld.exceptions.InvalidDirectionException;

import java.util.Queue;
import java.util.concurrent.ConcurrentLinkedQueue;

/**
 * Breadth First Search
 *
 * @author Huw Jones
 * @since 21/10/2016
 */
public class BFS extends Search {

    private Queue<Pair<Node, DIRECTION>> nodeQueue;

    /**
     * Set up the initial environment before running the search
     */
    @Override
    protected void preRun() {
        this.nodeQueue = new ConcurrentLinkedQueue<>();
        this.rootNode = Node.createRootNode();
        this.rootNode.setGrid(this.startGrid);
    }

    /**
     * Where the actual search runs
     */
    @Override
    protected void runSearch() {
        this.currentNode = rootNode;

        while (true) {
            if (currentDirection != null) {
                try {
                    currentNode.setGrid(
                            GridController.move(
                                    currentNode.getParent().getGrid(),
                                    currentDirection
                            )
                    );
                    numberOfNodes++;
                    if (this.checkExitCondition(currentNode.getGrid())) {
                        completed(currentNode);
                        break;
                    }
                } catch (InvalidDirectionException e) {
                    nextNode();
                    continue;
                }
            }

            for (DIRECTION direction : DIRECTION.values()) {
                nodeQueue.add(new Pair<>(new Node(currentNode), direction));
            }
            nextNode();
        }
    }

    @Override
    protected void nextNode() {
        currentPair = nodeQueue.poll();
        currentNode = currentPair.getKey();
        currentDirection = currentPair.getValue();
    }
}
```

A.12 DFS.java

```
package blocksworld.search;

import blocksworld.GridController;
import blocksworld.GridController.DIRECTION;
import blocksworld.Node;
import blocksworld.Pair;
import blocksworld.exceptions.InvalidDirectionException;

import java.util.ArrayList;
import java.util.Arrays;
import java.util.Collections;
import java.util.Stack;

/**
 * Depth First Search
 *
 * @author Huw Jones
 * @since 27/10/2016
 */
public class DFS extends Search {

    private Stack<Pair<Node, DIRECTION>> nodeStack;

    @Override
    protected void preRun() {
        this.nodeStack = new Stack<>();
        this.rootNode = Node.createRootNode();
        this.rootNode.setGrid(this.startGrid);
    }

    @Override
    protected void runSearch() {
        // Init
        ArrayList<DIRECTION> directions = new ArrayList<>(4);
        Arrays.stream(DIRECTION.values()).forEach(directions::add);

        currentNode = rootNode;
        while (true) {



            if (currentDirection != null) {
                try {
                    currentNode.setGrid(
                            GridController.move(
                                    currentNode.getParent().getGrid(),
                                    currentDirection
                            )
                    );
                    numberOfNodes++;
                    if (this.checkExitCondition(currentNode.getGrid())) {
                        completed(currentNode);
                        break;
                    }
                } catch (InvalidDirectionException e) {
                    nextNode();
                    continue;
                }
            }

            Collections.shuffle(directions, this.random);

            for (DIRECTION direction : directions) {
                nodeStack.push(new Pair<>(new Node(currentNode), direction));
            }

            nextNode();
        }
    }

    @Override
    protected void nextNode() {
        currentPair = nodeStack.pop();
        currentNode = currentPair.getKey();
```

```
            currentDirection = currentPair.getValue();
        }
    }
```

A.13 IDS.java

```
package blocksworld.search;

import blocksworld.GridController;
import blocksworld.GridController.DIRECTION;
import blocksworld.Node;
import blocksworld.Pair;
import blocksworld.exceptions.InvalidDirectionException;

import java.util.ArrayList;
import java.util.Arrays;
import java.util.Collections;
import java.util.Stack;

/**
 * Iterative Deepening Search
 *
 * @author Huw Jones
 * @since 12/11/2016
 */
public class IDS extends Search {

    private Stack<Pair<Node, DIRECTION>> nodeStack;
    private ArrayList<DIRECTION> directions;
    private int depth;

    /**
     * Set up the initial environment before running the search
     */
    @Override
    protected void preRun() {
        this.nodeStack = new Stack<>();
        this.rootNode = Node.createRootNode();
        this.rootNode.setGrid(this.startGrid);
        directions = new ArrayList<>(4);
        Arrays.stream(DIRECTION.values()).forEach(directions::add);

        currentNode = rootNode;
        currentPair = null;
        currentDirection = null;
    }

    /**
     * Where the actual search runs
     */
    @Override
    protected void runSearch() {
        while (true) {
            // Check if we've hit the depth limit
            if (currentNode.getDepth() > depth) {
                // If we have no more nodes in the stack, increase the depth
                if(this.nodeStack.size() == 0) {
                    increaseDepth();
                } else {
                    // Otherwise continue processing the stack
                    nextNode();
                }
                continue;
            }



            if (currentDirection != null) {
                try {
                    // Process the move and store the new state in the node
                    currentNode.setGrid(
                            GridController.move(
                                    currentNode.getParent().getGrid(),
                                    currentDirection
                            )
                    );
                    numberOfNodes++;
                    // Check if the grid meets the exit condition, if so, exit the search
                    if (this.checkExitCondition(currentNode.getGrid())) {
                        completed(currentNode);
```

```
                    break;
                }
            } catch (InvalidDirectionException e) {
                if(nodeStack.size() == 0){
                    increaseDepth();
                } else {
                    nextNode();
                }
                continue;
            }
        }

        Collections.shuffle(directions, this.random);

        // Push new directions on the stack to be processed
        for (DIRECTION direction : directions) {
            nodeStack.push(new Pair<>(new Node(currentNode), direction));
        }

        nextNode();
    }
    System.out.println("Max Iterative Depth:");
    System.out.println(depth);
}

/**
 * Gets the next node off of the stack
 */
@Override
protected void nextNode() {
    currentPair = nodeStack.pop();
    currentNode = currentPair.getKey();
    currentDirection = currentPair.getValue();
}

/**
 * Increases the depth of the search
 */
private void increaseDepth(){
    // Reset the search environment
    preRun();
    // Increment depth
    depth++;
    // Log new depth
    System.out.println("\r\nDepth increased: "+ depth);
}
}
```

A.14 Search.java

```
package blocksworld.search;

import blocksworld.*;
import blocksworld.exceptions.InvalidBlockIDException;
import blocksworld.exceptions.InvalidPositionException;

import java.text.NumberFormat;
import java.util.List;
import java.util.Locale;
import java.util.Random;
import java.util.Stack;
import java.util.stream.Collectors;

/**
 * Abstract Search Class
 *
 * @author Huw Jones
 * @since 11/10/2016
 */
public abstract class Search {

    protected Grid startGrid;
    protected long randomSeed;
    protected Random random;
    protected long numberOfNodes = 0;
    protected Grid exitGrid;
    protected Node rootNode;
    protected Node currentNode;
    protected Pair<Node, GridController.DIRECTION> currentPair;
    protected GridController.DIRECTION currentDirection = null;
    private boolean completed = false;
    private long startTime;
    private int refreshTime = 100;

    public Search() {
        this.randomSeed = new Random().nextLong();
        this.buildGrid();
        this.createExitGrid();
        Runtime.getRuntime().addShutdownHook(new Thread() {
            @Override
            public void run() {
                System.out.print("\r\n\r\n");
            }
        });
    }

    /**
     * Builds the default 4x4 grid
     */
    private void buildGrid() {
        startGrid = GridController.createGrid(4, 4);
        try {
            GridController.placeBlock(startGrid, 'a', 0, 3);
            GridController.placeBlock(startGrid, 'b', 1, 3);
            GridController.placeBlock(startGrid, 'c', 2, 3);
            GridController.placeAgent(startGrid, 3, 3);
        } catch (InvalidPositionException | InvalidBlockIDException e) {
            e.printStackTrace();
        }
    }

    /**
     * Builds the defualt exit grid state
     */
    void createExitGrid() {
        this.exitGrid = GridController.createGrid(this.startGrid.getWidth(), this.startGrid.
            getHeight());

        try {
            GridController.placeBlock(exitGrid, 'a', 1, 1);
            GridController.placeBlock(exitGrid, 'b', 1, 2);
            GridController.placeBlock(exitGrid, 'c', 1, 3);
        } catch (InvalidPositionException e) {
            e.printStackTrace();
```

```
        }
    }

    /**
     * Sets the random number seed
     * @param seed Seed
     */
    public void setSeed(long seed) {
        this.randomSeed = seed;
    }

    /**
     * Runs the search
     */
    public void run() {
        System.out.println("Creating random seed...");
        this.random = new Random(this.randomSeed);
        System.out.println(String.format("Random seed: %d.", this.randomSeed));
        System.out.println("Running Search::preRun");
        this.preRun();
        System.out.println("Start State:");
        System.out.println(this.startGrid.toString());
        System.out.println("Exit State:");
        System.out.println(this.exitGrid.toString());
        System.out.println("Running Search::runSearch");
        try {
            Thread t = new Thread(new Monitor(), "MonitorThread");
            t.setDaemon(true);
            this.startTime = System.nanoTime();
            t.start();
            this.runSearch();
        } catch (Exception ex) {
            System.out.println("Error running search.");
            ex.printStackTrace();
        }
    }

    /**
     * Called when a search completes.
     * It dumps the solution and stats to console
     * @param exitNode Node that solves the puzzle
     */
    protected void completed(Node exitNode) {
        this.completed = true;
        System.out.println("\r\n\r\n==========");
        System.out.println("Solution found.");

        System.out.println("Solution as follows:");
        System.out.println(this.getSolution(exitNode));

        System.out.println("\r\n==========");
        System.out.println("Start State:\r\n");
        System.out.println(this.startGrid.toString());
        System.out.println("==========");
        System.out.println("Exit State:\r\n");
        System.out.println(this.exitGrid.toString());
        System.out.println("==========");
        System.out.println("Random Seed:");
        System.out.println(this.randomSeed);
        System.out.println("==========");
        System.out.println("Nodes Expanded:");
        System.out.println(this.numberOfNodes);
        System.out.println("==========");
    }

    /**
     * Set up the initial environment before running the search
     */
    abstract protected void preRun();

    /**
     * Where the actual search runs
     */
    abstract protected void runSearch() throws Exception;

    /**
```

```
     * Builds the solution from the exit node
     * @param endNode End Node that is in the exit state
     * @return String that is the solution
     */
    public String getSolution(Node endNode) {
        // Using a stack so we can reverse the order of the nodes easier
        Stack<String> states = new Stack<>();
        Node currentNode = endNode;

        // Dump all nodes on the stack whilst the parent isn't null
        // Only the root node has a null parent
        do {
            if (currentNode != null) {
                if (currentNode.getGrid() != null) {
                    states.add(currentNode.getGrid().toString());
                }
            }
        } while ((currentNode = currentNode.getParent()) != null);

        // Count moves whilst looping through the stack and append grid state to the sting
        StringBuilder builder = new StringBuilder();
        String currentString;
        int moves = 0;
        while (states.size() != 0) {
            currentString = states.pop();
            builder.append("\n");
            builder.append(moves);
            builder.append(":");
            builder.append("\n");
            builder.append(currentString);
            moves++;
        }
        return builder.toString();
    }

    /**
     * Checks whether or not a grid meets the exit criteria
     * @param grid Grid to check
     * @return true if the exit condition has been reached
     */
    protected boolean checkExitCondition(Grid grid) {
        // Lambda to get blocks (excluding the agent "*")
        List<Block> blocks = grid.getBlocks().stream().filter(b -> b.getID() != '*').collect(
            Collectors.toList());

        // Assume we're complete
        boolean exitReached = true;
        Block comparisonBlock;

        // Loop through the block and AND the matching block result
        for (Block block : blocks) {
            comparisonBlock = this.exitGrid.getBlock(block.getID());
            exitReached &= comparisonBlock.getPosition().equals(block.getPosition());
        }

        return exitReached;
    }

    /**
     * Gets the monitor thread interval refresh time
     * @return Interval refresh time (in ms)
     */
    public int getRefreshTime() {
        return refreshTime;
    }

    /**
     * Sets the refresh interval
     * @param time time in ms
     */
    public void setRefreshTime(int time) {
        this.refreshTime = time;
    }

    /**
     * Sets the start grid state
```

```
     * @param startGrid Grid to start from
     */
    public void setStartState(Grid startGrid) {
        this.startGrid = startGrid;
    }

    /**
     * Sets the exit grid state
     * @param exitGrid Grid that forms the exit conditions
     */
    public void setExitState(Grid exitGrid) {
        this.exitGrid = exitGrid;
    }

    /**
     * Gets the next node
     */
    protected abstract void nextNode();

    /**
     * Monitor Thread (provides ongoing stats of the search in console)
     */
    private class Monitor implements Runnable {
        @Override
        public void run() {
            long time;
            long minutes;
            long seconds;
            long memory;
            while (!completed) {
                time = System.nanoTime() - startTime;
                memory = (Runtime.getRuntime().totalMemory() - Runtime.getRuntime().freeMemory
                    ()) / 1048576;
                seconds = time / 1000000000;
                minutes = seconds / 60;
                seconds -= minutes * 60;
                System.out.print(String.format("\rExpanded Nodes: %12s\t\tElapsed Time [%s:%s
                    ]\t\tUsed Memory: %6sMB",
                        NumberFormat.getNumberInstance(Locale.getDefault()).format(
                            numberOfNodes),
                        String.format("%2d", minutes).replace(' ', '0'),
                        String.format("%2d", seconds).replace(' ', '0'),
                        NumberFormat.getNumberInstance(Locale.getDefault()).format(memory)));
                try {
                    Thread.sleep(Search.this.getRefreshTime());
                } catch (InterruptedException e) {
                }
            }
        }
    }
}
```

A.15 run.sh

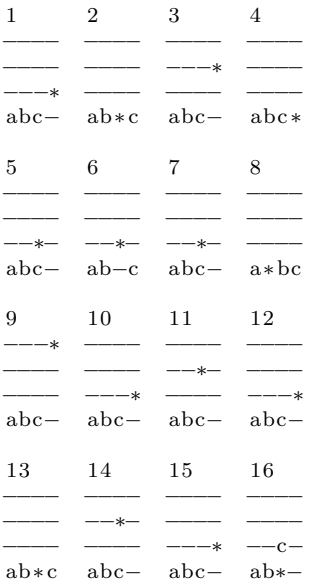
```
#!/bin/bash
java -cp "out/production/COMP2208" -XX:+UseG1GC BlocksWorld $*
```

A.16 test.sh

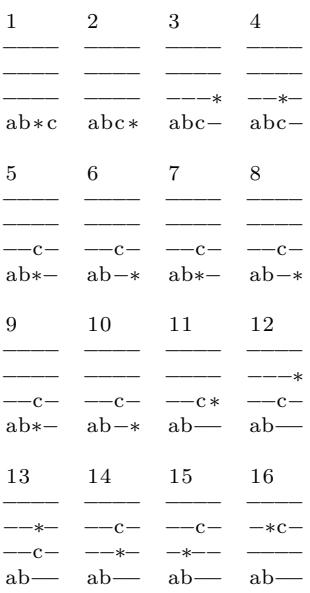
```
#!/bin/bash
fFlag=false;
tFlag=false;
nFlag=false;
output=".";
testNumber="";
interval=100;

while getopts 'i:n:t:f:o:' flag; do
  case "${flag}" in
    i) interval="${OPTARG}" ;;
    o) output="${OPTARG}" ;;
    t) type="${OPTARG}" ; tFlag=true ;;
    f) file="${OPTARG}" ; fFlag=true ;;
    n) testNumber="${OPTARG}" ; nFlag=true;;
    *) echo "Unexpected option ${flag}" ; exit ;
  esac
done

if ! $fFlag
then
  echo "File of states must be provided with -f!";
  exit 1
fi

if ! $tFlag
then
  echo "Search type must be provided with -t!";
  exit 1
fi

if ! $nFlag
then
  echo "Test number must be provided with -n!";
  exit 1
fi

cat "$file" | awk '
BEGIN{
  RS = "\n";
  FS = ",";
}
{
  output="'$output'/'$type'-"$1".txt";
  split($2, parts, ":");
  height=parts[2];
  width=parts[1];
  if($1 == "" || $1 == "'$testNumber'") {
    if(NF == 2){
      system("./run.sh -i '$interval' -w \""width"\" -h \""height"\" -t \"'$type'\" -s \""$2"\
          " > "output);
    } else {
      system("./run.sh -i '$interval' -w "width" -h "height" -t \"'$type'\" -s \""$2"\" -e \""
          $3"\" > "output);
    }
  }
}
'
```

B Output Evidence

B.1 Breadth First Search



B.2 Depth First Search



B.3 Iterative Deepening Search

```
Depth increased: 1
1       2
————    ————
————    ————
————    ———*
ab*c    abc—

Depth increased: 2
3       4       5       6
————    ————    ————    ————
————    ————    ————    ————
————    ————    ————    ——*—
ab*c    abc*    a*bc    ab—c

7       8       9       10
————    ————    ————    ————
————    ————    ————    ———*
———*    ————    ——*—    ————
abc—    abc*    abc—    abc—

Depth increased: 3
11      12      13      14
————    ————    ————    ————
————    ————    ————    ————
———*    ————    ———*    ————
abc—    abc*    abc—    ab*c

15      16      17
————    ————    ————
————    ————    ————
——*—    ——c—    —*——
abc—    ab*—    abc—
```

B.4 A* Heuristic Search

```
1       2       3       4
————    ————    ————    ————
————    ————    ————    ———*
———*    ————    ——*—    ————
abc—    ab*c    abc—    abc—

5       6       7       8
————    ————    ————    ————
————    ————    ————    ————
————    ————    —*——    —b——
abc*    abc*    abc—    a*c—

9       10      11      12
————    ————    ————    ————
————    ————    ————    ————
—b——    —b——    —b*—    —b——
ac*—    *ac—    ac——    ac—*

13      14      15      16
———*    ————    ————    ————
————    ————    ————    ————
————    —b——    ———*    ———*
abc—    ac*—    abc—    abc—
```

C Example Output

C.1 Breadth First Search

```
Running Search::preRun
Running Search::runSearch

Expanded Nodes:        29,378    Elapsed Time [00:01]    Used Memory:      58MB
Expanded Nodes:        71,154    Elapsed Time [00:02]    Used Memory:      77MB
Expanded Nodes:       124,017    Elapsed Time [00:03]    Used Memory:     130MB
Expanded Nodes:       166,634    Elapsed Time [00:04]    Used Memory:     120MB
Expanded Nodes:       236,683    Elapsed Time [00:05]    Used Memory:     167MB
Expanded Nodes:       288,347    Elapsed Time [00:06]    Used Memory:     331MB
Expanded Nodes:       336,500    Elapsed Time [00:07]    Used Memory:     286MB
Expanded Nodes:       398,896    Elapsed Time [00:08]    Used Memory:     282MB
Expanded Nodes:       458,833    Elapsed Time [00:09]    Used Memory:     462MB
Expanded Nodes:       496,093    Elapsed Time [00:10]    Used Memory:     355MB
Expanded Nodes:       574,817    Elapsed Time [00:11]    Used Memory:     603MB
Expanded Nodes:       625,648    Elapsed Time [00:12]    Used Memory:     528MB
Expanded Nodes:       687,264    Elapsed Time [00:13]    Used Memory:     479MB
Expanded Nodes:       765,974    Elapsed Time [00:14]    Used Memory:     724MB
Expanded Nodes:       819,442    Elapsed Time [00:15]    Used Memory:     644MB
Expanded Nodes:       909,066    Elapsed Time [00:16]    Used Memory:     670MB
Expanded Nodes:       998,278    Elapsed Time [00:17]    Used Memory:     692MB
Expanded Nodes:     1,105,581    Elapsed Time [00:18]    Used Memory:     765MB
Expanded Nodes:     1,179,148    Elapsed Time [00:19]    Used Memory:     981MB
Expanded Nodes:     1,295,061    Elapsed Time [00:20]    Used Memory:   1,079MB
Expanded Nodes:     1,370,995    Elapsed Time [00:21]    Used Memory:   1,054MB
Expanded Nodes:     1,440,684    Elapsed Time [00:22]    Used Memory:   1,011MB
Expanded Nodes:     1,547,352    Elapsed Time [00:23]    Used Memory:   1,074MB
Expanded Nodes:     1,654,448    Elapsed Time [00:24]    Used Memory:   1,138MB
Expanded Nodes:     1,765,672    Elapsed Time [00:25]    Used Memory:   1,213MB
Expanded Nodes:     1,856,803    Elapsed Time [00:26]    Used Memory:   1,492MB
Expanded Nodes:     1,930,003    Elapsed Time [00:27]    Used Memory:   1,444MB
Expanded Nodes:     2,012,078    Elapsed Time [00:28]    Used Memory:   1,435MB
Expanded Nodes:     2,100,007    Elapsed Time [00:29]    Used Memory:   1,444MB
Expanded Nodes:     2,212,357    Elapsed Time [00:30]    Used Memory:   1,520MB
Expanded Nodes:     2,325,635    Elapsed Time [00:31]    Used Memory:   1,596MB
Expanded Nodes:     2,438,910    Elapsed Time [00:32]    Used Memory:   1,673MB
Expanded Nodes:     2,549,779    Elapsed Time [00:33]    Used Memory:   1,749MB
Expanded Nodes:     2,664,120    Elapsed Time [00:34]    Used Memory:   1,826MB
Expanded Nodes:     2,766,546    Elapsed Time [00:35]    Used Memory:   2,153MB
Expanded Nodes:     2,849,977    Elapsed Time [00:36]    Used Memory:   2,130MB
Expanded Nodes:     2,970,872    Elapsed Time [00:37]    Used Memory:   2,230MB
Expanded Nodes:     3,075,584    Elapsed Time [00:38]    Used Memory:   2,283MB
Expanded Nodes:     3,178,468    Elapsed Time [00:39]    Used Memory:   2,328MB
Expanded Nodes:     3,262,110    Elapsed Time [00:40]    Used Memory:   2,314MB
Expanded Nodes:     3,349,678    Elapsed Time [00:41]    Used Memory:   2,313MB
Expanded Nodes:     3,450,024    Elapsed Time [00:42]    Used Memory:   2,362MB
Expanded Nodes:     3,564,528    Elapsed Time [00:43]    Used Memory:   2,439MB
Expanded Nodes:     3,669,360    Elapsed Time [00:44]    Used Memory:   2,772MB
Expanded Nodes:     3,775,457    Elapsed Time [00:45]    Used Memory:   2,822MB
Expanded Nodes:     3,843,688    Elapsed Time [00:46]    Used Memory:   2,761MB
Expanded Nodes:     3,949,632    Elapsed Time [00:47]    Used Memory:   2,812MB
Expanded Nodes:     4,040,206    Elapsed Time [00:48]    Used Memory:   2,821MB
Expanded Nodes:     4,127,753    Elapsed Time [00:49]    Used Memory:   2,821MB
Expanded Nodes:     4,239,930    Elapsed Time [00:50]    Used Memory:   2,897MB
Expanded Nodes:     4,352,605    Elapsed Time [00:51]    Used Memory:   2,973MB
Expanded Nodes:     4,464,891    Elapsed Time [00:52]    Used Memory:   3,049MB
Expanded Nodes:     4,564,968    Elapsed Time [00:53]    Used Memory:   3,363MB
Expanded Nodes:     4,651,719    Elapsed Time [00:54]    Used Memory:   3,356MB
Expanded Nodes:     4,747,240    Elapsed Time [00:55]    Used Memory:   3,380MB
Expanded Nodes:     4,842,474    Elapsed Time [00:56]    Used Memory:   3,407MB
Expanded Nodes:     4,941,967    Elapsed Time [00:57]    Used Memory:   3,438MB
Expanded Nodes:     5,027,298    Elapsed Time [00:58]    Used Memory:   3,433MB
Expanded Nodes:     5,140,339    Elapsed Time [00:59]    Used Memory:   3,509MB
Expanded Nodes:     5,252,009    Elapsed Time [01:00]    Used Memory:   3,585MB
Expanded Nodes:     5,364,664    Elapsed Time [01:01]    Used Memory:   3,661MB
Expanded Nodes:     5,477,345    Elapsed Time [01:02]    Used Memory:   3,738MB
Expanded Nodes:     5,568,689    Elapsed Time [01:03]    Used Memory:   4,025MB
Expanded Nodes:     5,674,481    Elapsed Time [01:04]    Used Memory:   4,082MB
Expanded Nodes:     5,744,161    Elapsed Time [01:08]    Used Memory:   4,027MB
Expanded Nodes:     5,812,563    Elapsed Time [01:09]    Used Memory:   3,969MB
Expanded Nodes:     5,905,378    Elapsed Time [01:10]    Used Memory:   4,260MB
Expanded Nodes:     5,992,149    Elapsed Time [01:11]    Used Memory:   4,256MB
Expanded Nodes:     6,067,506    Elapsed Time [01:12]    Used Memory:   4,214MB
```

```
Expanded Nodes:     6,150,992     Elapsed Time [01:13]     Used Memory:   4,198MB
Expanded Nodes:     6,263,272     Elapsed Time [01:14]     Used Memory:   4,275MB
Expanded Nodes:     6,360,382     Elapsed Time [01:15]     Used Memory:   4,581MB
Expanded Nodes:     6,405,473     Elapsed Time [01:16]     Used Memory:   4,444MB
Expanded Nodes:     6,488,545     Elapsed Time [01:17]     Used Memory:   4,428MB
Expanded Nodes:     6,600,002     Elapsed Time [01:18]     Used Memory:   4,505MB
Expanded Nodes:     6,696,811     Elapsed Time [01:19]     Used Memory:   4,808MB
Expanded Nodes:     6,758,005     Elapsed Time [01:20]     Used Memory:   4,724MB
Expanded Nodes:     6,825,442     Elapsed Time [01:21]     Used Memory:   4,658MB
Expanded Nodes:     6,939,339     Elapsed Time [01:22]     Used Memory:   4,734MB
Expanded Nodes:     7,051,978     Elapsed Time [01:23]     Used Memory:   4,768MB
Expanded Nodes:     7,163,761     Elapsed Time [01:24]     Used Memory:   4,844MB
Expanded Nodes:     7,274,912     Elapsed Time [01:25]     Used Memory:   4,921MB
Expanded Nodes:     7,313,015     Elapsed Time [01:31]     Used Memory:   5,039MB
Expanded Nodes:     7,337,856     Elapsed Time [01:32]     Used Memory:   5,118MB
Expanded Nodes:     7,454,059     Elapsed Time [01:33]     Used Memory:   5,201MB
Expanded Nodes:     7,539,038     Elapsed Time [01:34]     Used Memory:   5,090MB
Expanded Nodes:     7,637,113     Elapsed Time [01:35]     Used Memory:   5,106MB
Expanded Nodes:     7,722,598     Elapsed Time [01:36]     Used Memory:   5,111MB
Expanded Nodes:     7,833,920     Elapsed Time [01:37]     Used Memory:   5,187MB
Expanded Nodes:     7,873,575     Elapsed Time [01:43]     Used Memory:   5,300MB
Expanded Nodes:     7,951,934     Elapsed Time [01:44]     Used Memory:   5,264MB
Expanded Nodes:     8,048,079     Elapsed Time [01:45]     Used Memory:   5,567MB
Expanded Nodes:     8,137,070     Elapsed Time [01:46]     Used Memory:   5,526MB
Expanded Nodes:     8,229,954     Elapsed Time [01:47]     Used Memory:   5,542MB

================================
Solution found.
Solution as follows:

0:
____
____
____
abc*

1:
____
____
___*
abc_

2:
____
____
__*_
abc_

3:
____
____
_*__
abc_

4:
____
____
_b__
a*c_

5:
____
____
_b__
*ac_

6:
____
____
*b__
_ac_

7:
____
____
b*__
```

```
−ac−

8:
————
————
ba——
−*c−

9:
————
————
ba——
−c*−

10:
————
————
ba*−
−c——

11:
————
−−*−
ba——
−c——

12:
————
−*−−
ba——
−c——

13:
————
−a——
b*−−
−c——

14:
————
−a——
*b——
−c——

==========
Start State:

————
————
————
abc*

==========
Exit State:

————
−a——
−b——
−c——

==========
Random Seed:
−2679893794501661041
==========
Nodes Expanded:
8,318,621
==========
```

C.2 Depth First Search

```
Running Search::preRun
Running Search::runSearch

Expanded Nodes:          2,766     Elapsed Time [00:00]     Used Memory:        1MB
Expanded Nodes:         15,459     Elapsed Time [00:00]     Used Memory:        3MB
Expanded Nodes:         33,489     Elapsed Time [00:00]     Used Memory:       13MB
Expanded Nodes:         49,812     Elapsed Time [00:00]     Used Memory:       55MB
Expanded Nodes:         68,246     Elapsed Time [00:01]     Used Memory:       57MB
Expanded Nodes:         85,650     Elapsed Time [00:01]     Used Memory:       75MB

================================
Solution found.
Solution as follows:
Expanded Nodes:         92,972     Elapsed Time [00:01]     Used Memory:       69MB
0:
____
____
____
abc*

1:
____
____
____
ab*c

2:
____
____
__*_
ab_c

3:
____
____
_*__
ab_c

4:
____
_*__
____
ab_c

5:
____
__*_
____
ab_c

6:
__*_
____
____
ab_c

7:
___*
____
____
ab_c

8:
____
___*
____
ab_c

....
....
....
....

92965:
```

```
————
−a∗−
bc——
————

92966:
————
−a−∗
bc——
————

92967:
————
−a∗−
bc——
————

92968:
————
−a——
bc∗−
————

92969:
————
−a——
bc——
−−∗−

92970:
————
−a——
bc——
−∗−−

92971:
————
−a——
b∗−−
−c——

92972:
————
−a——
∗b——
−c——


==========
Start State:

————
————
————
abc∗

==========
Exit State:

————
−a——
−b——
−c——

==========
Random Seed:
−6659880257389911118
==========
Nodes Expanded:
92972
==========
```

C.3 Iterative Deepening Search

```
Running Search::preRun
Running Search::runSearch

Depth increased: 1

Depth increased: 2

Depth increased: 3

Depth increased: 4

Depth increased: 5
Expanded Nodes:           189    Elapsed Time [00:00]    Used Memory:

Depth increased: 6
Expanded Nodes:           870    Elapsed Time [00:00]    Used Memory:

Depth increased: 7
Expanded Nodes:         4,122    Elapsed Time [00:00]    Used Memory:

Depth increased: 8
Expanded Nodes:        15,569    Elapsed Time [00:00]    Used Memory:

Depth increased: 9
Expanded Nodes:        44,117    Elapsed Time [00:01]    Used Memory:     87MB

Depth increased: 10
Expanded Nodes:       162,212    Elapsed Time [00:02]    Used Memory:    140MB

Depth increased: 11
Expanded Nodes:       302,767    Elapsed Time [00:03]    Used Memory:    143MB
Expanded Nodes:       465,493    Elapsed Time [00:04]    Used Memory:     32MB

Depth increased: 12
Expanded Nodes:       649,869    Elapsed Time [00:05]    Used Memory:      4MB
Expanded Nodes:       835,458    Elapsed Time [00:06]    Used Memory:    138MB
Expanded Nodes:     1,018,926    Elapsed Time [00:07]    Used Memory:    104MB
Expanded Nodes:     1,167,648    Elapsed Time [00:08]    Used Memory:    116MB
Expanded Nodes:     1,342,004    Elapsed Time [00:09]    Used Memory:     57MB
Expanded Nodes:     1,524,357    Elapsed Time [00:10]    Used Memory:     25MB
Expanded Nodes:     1,699,719    Elapsed Time [00:11]    Used Memory:    121MB

Depth increased: 13
Expanded Nodes:     1,824,296    Elapsed Time [00:12]    Used Memory:     71MB
Expanded Nodes:     1,966,793    Elapsed Time [00:13]    Used Memory:     50MB
Expanded Nodes:     2,162,291    Elapsed Time [00:14]    Used Memory:     57MB
Expanded Nodes:     2,348,533    Elapsed Time [00:15]    Used Memory:     37MB
Expanded Nodes:     2,540,465    Elapsed Time [00:16]    Used Memory:     34MB
Expanded Nodes:     2,732,738    Elapsed Time [00:17]    Used Memory:     31MB
Expanded Nodes:     2,920,399    Elapsed Time [00:18]    Used Memory:     16MB
Expanded Nodes:     3,103,913    Elapsed Time [00:19]    Used Memory:    137MB
Expanded Nodes:     3,295,237    Elapsed Time [00:20]    Used Memory:    131MB
Expanded Nodes:     3,489,259    Elapsed Time [00:21]    Used Memory:    135MB
Expanded Nodes:     3,667,882    Elapsed Time [00:22]    Used Memory:     89MB
Expanded Nodes:     3,835,332    Elapsed Time [00:23]    Used Memory:     10MB
Expanded Nodes:     4,022,254    Elapsed Time [00:24]    Used Memory:    141MB
Expanded Nodes:     4,214,688    Elapsed Time [00:25]    Used Memory:    141MB
Expanded Nodes:     4,403,704    Elapsed Time [00:26]    Used Memory:    127MB
Expanded Nodes:     4,595,737    Elapsed Time [00:27]    Used Memory:    124MB
Expanded Nodes:     4,787,754    Elapsed Time [00:28]    Used Memory:    122MB
Expanded Nodes:     4,969,529    Elapsed Time [00:29]    Used Memory:     85MB
Expanded Nodes:     5,147,124    Elapsed Time [00:30]    Used Memory:     44MB
Expanded Nodes:     5,341,106    Elapsed Time [00:31]    Used Memory:     40MB
Expanded Nodes:     5,515,405    Elapsed Time [00:32]    Used Memory:    133MB
Expanded Nodes:     5,676,501    Elapsed Time [00:33]    Used Memory:     40MB

Depth increased: 14
Expanded Nodes:     5,853,237    Elapsed Time [00:34]    Used Memory:    133MB
Expanded Nodes:     6,045,152    Elapsed Time [00:35]    Used Memory:    129MB
Expanded Nodes:     6,228,239    Elapsed Time [00:36]    Used Memory:     99MB
Expanded Nodes:     6,396,529    Elapsed Time [00:37]    Used Memory:     24MB

================================
Solution found.
```

```
Solution as follows:

0:
————
————
————
abc*

1:
————
————
———*
abc—

2:
————
————
——*—
abc—

3:
————
————
—*——
abc—

4:
————
————
—b——
a*c—

5:
————
————
—b——
*ac—

6:
————
————
*b——
—ac—

7:
————
————
b*——
—ac—

8:
————
————
ba——
—*c—

9:
————
————
ba——
—c*—

10:
————
————
ba*—
—c——

11:
————
——*—
ba——
—c——

12:
————
```

```
-*--
ba--
-c--

13:
----
-a--
b*--
-c--

14:
----
-a--
*b--
-c--
```

```
Start State:

----
----
----
abc*
```

```
Exit State:

----
-a--
-b--
-c--
```

```
Random Seed:
-2736763515611179222
```

```
Nodes Expanded:
6430966
```

```
Max Iterative Depth:
14
```

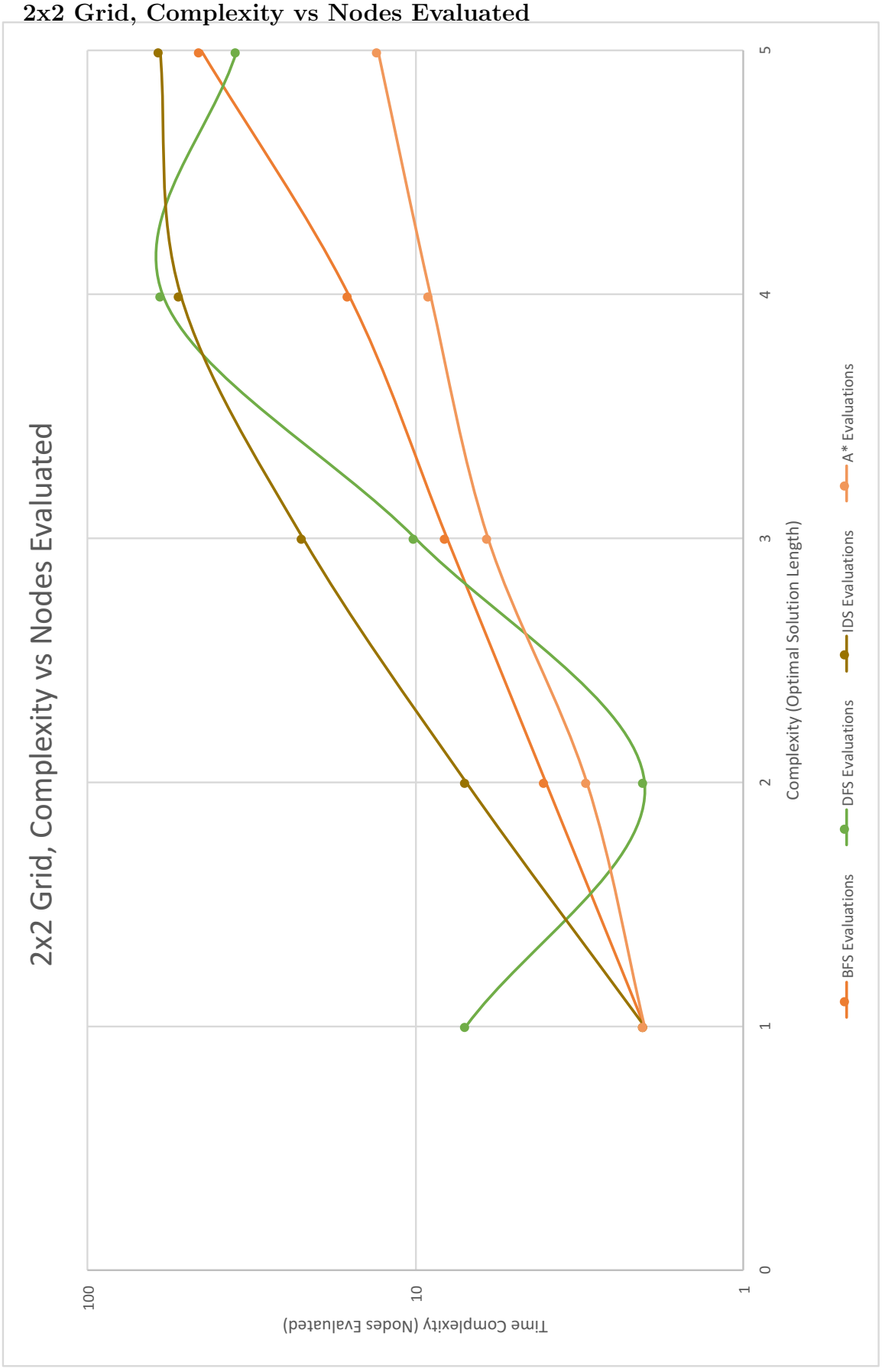
C.4 A* Heuristic Search

```
Running Search::preRun
Running Search::runSearch

Expanded Nodes:              0     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:             38     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:            170     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:            427     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:            741     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:            972     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          1,449     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          2,042     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          2,403     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          2,627     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          3,200     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          3,759     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          4,309     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          4,706     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          5,089     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          5,480     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          5,833     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          6,215     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          6,609     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          7,013     Elapsed Time [00:00]     Used Memory:
Expanded Nodes:          7,438     Elapsed Time [00:01]     Used Memory:     93MB
Expanded Nodes:          7,548     Elapsed Time [00:01]     Used Memory:
Expanded Nodes:          8,169     Elapsed Time [00:01]     Used Memory:
Expanded Nodes:          8,785     Elapsed Time [00:01]     Used Memory:
Expanded Nodes:          8,997     Elapsed Time [00:01]     Used Memory:

==========
Solution found.
Solution as follows:

0:
____
____
____
abc*

1:
____
____
___*
abc_

2:
____
____
__*_
abc_

3:
____
____
_*__
abc_

4:
____
____
_b__
a*c_

5:
____
____
_b__
*ac_

6:
____
____
*b__
_ac_
```

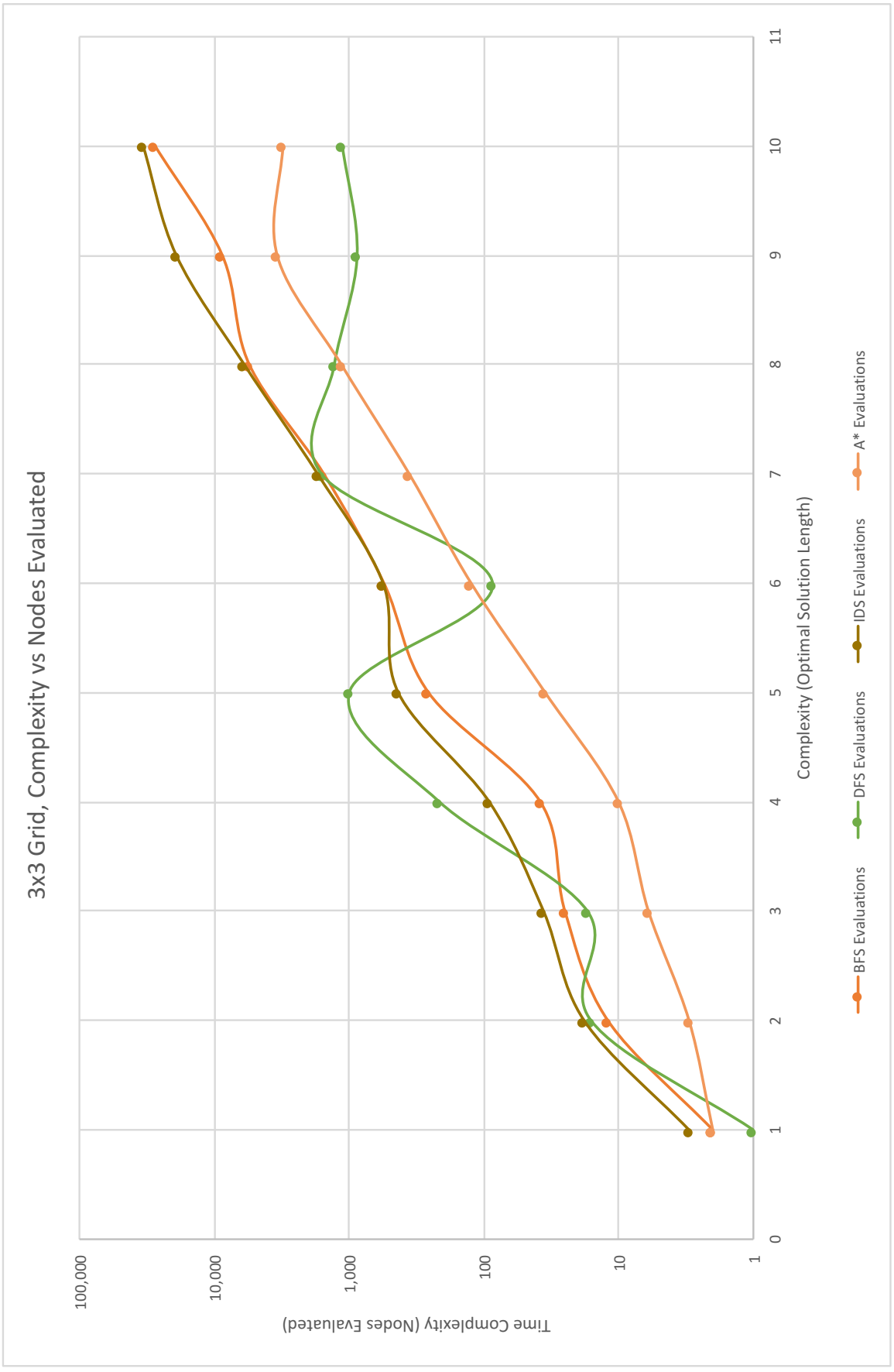
```
7:
————
————
b*——
—ac—

8:
————
————
ba——
—*c—

9:
————
————
ba——
—c*—

10:
————
————
ba*—
—c——

11:
————
——*—
ba——
—c——

12:
————
—*——
ba——
—c——

13:
————
—a——
b*——
—c——

14:
————
—a——
*b——
—c——

==========
Start State:

————
————
————
abc*

==========
Exit State:

————
—a——
—b——
—c——

==========
Random Seed:
7965083047300516107
==========
Nodes Expanded:
9,491
==========
```

D Graphs

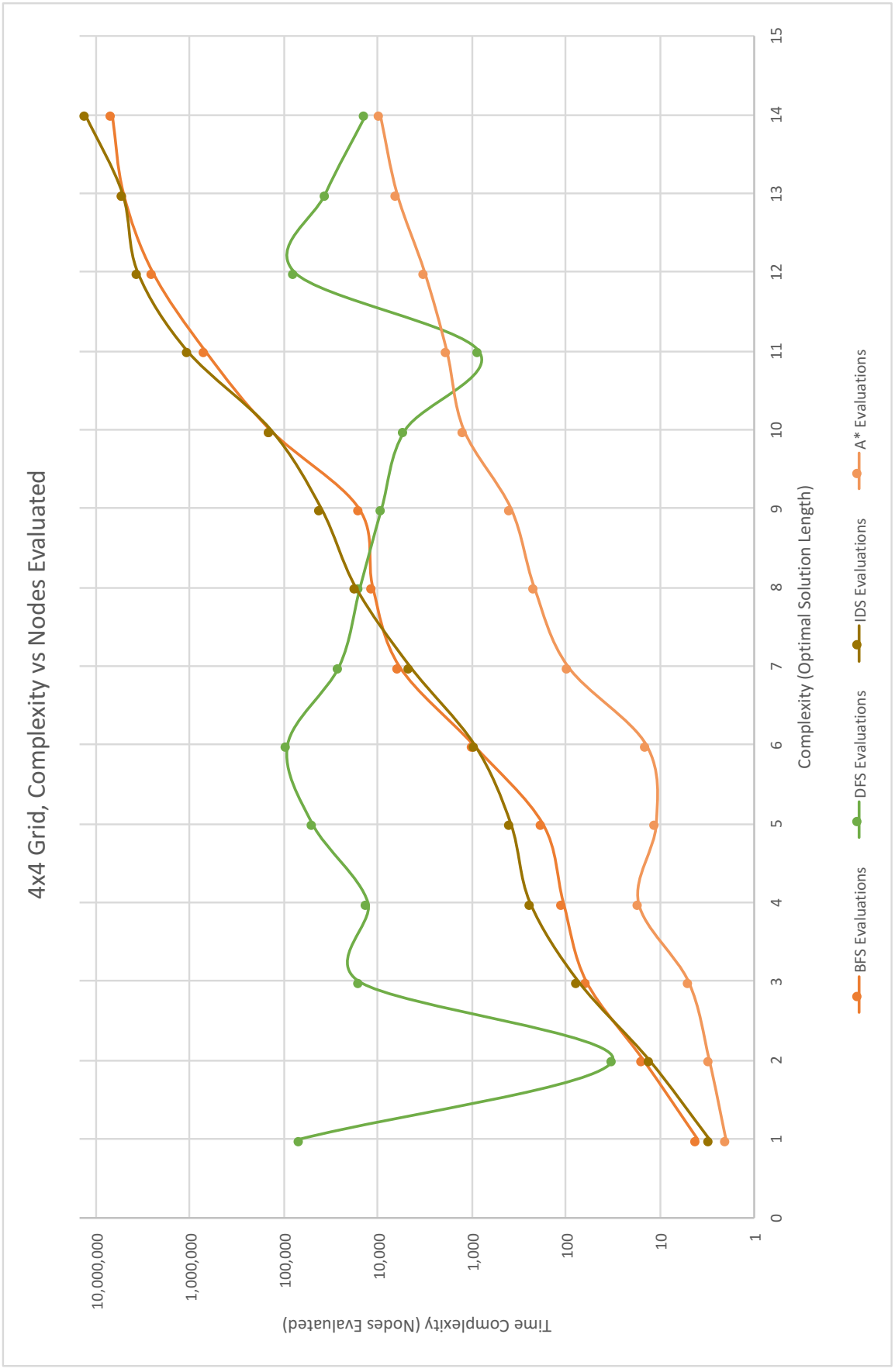
D.1 2x2 Grid, Complexity vs Nodes Evaluated



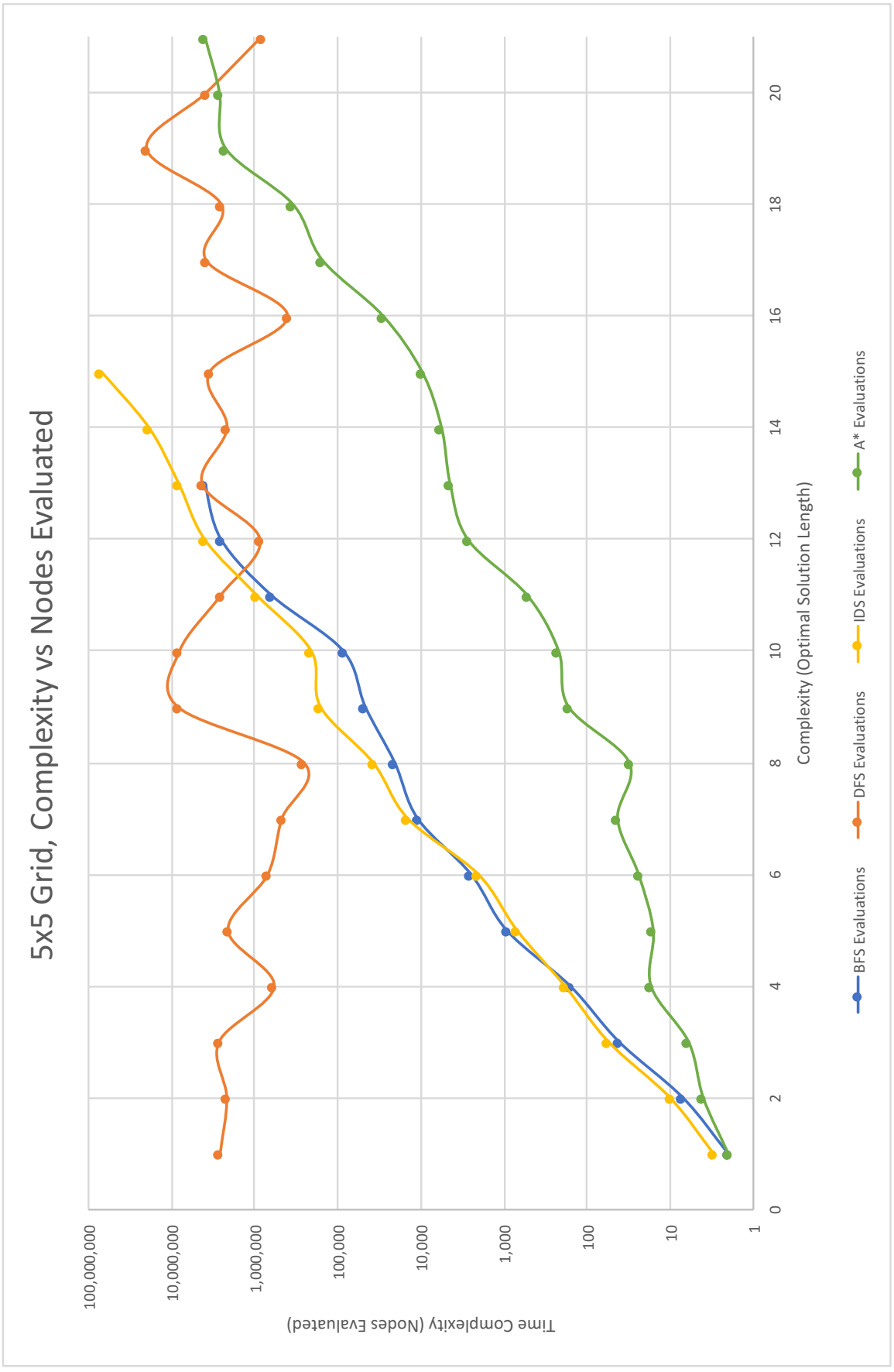
D.2 3x3 Grid, Complexity vs Nodes Evaluated



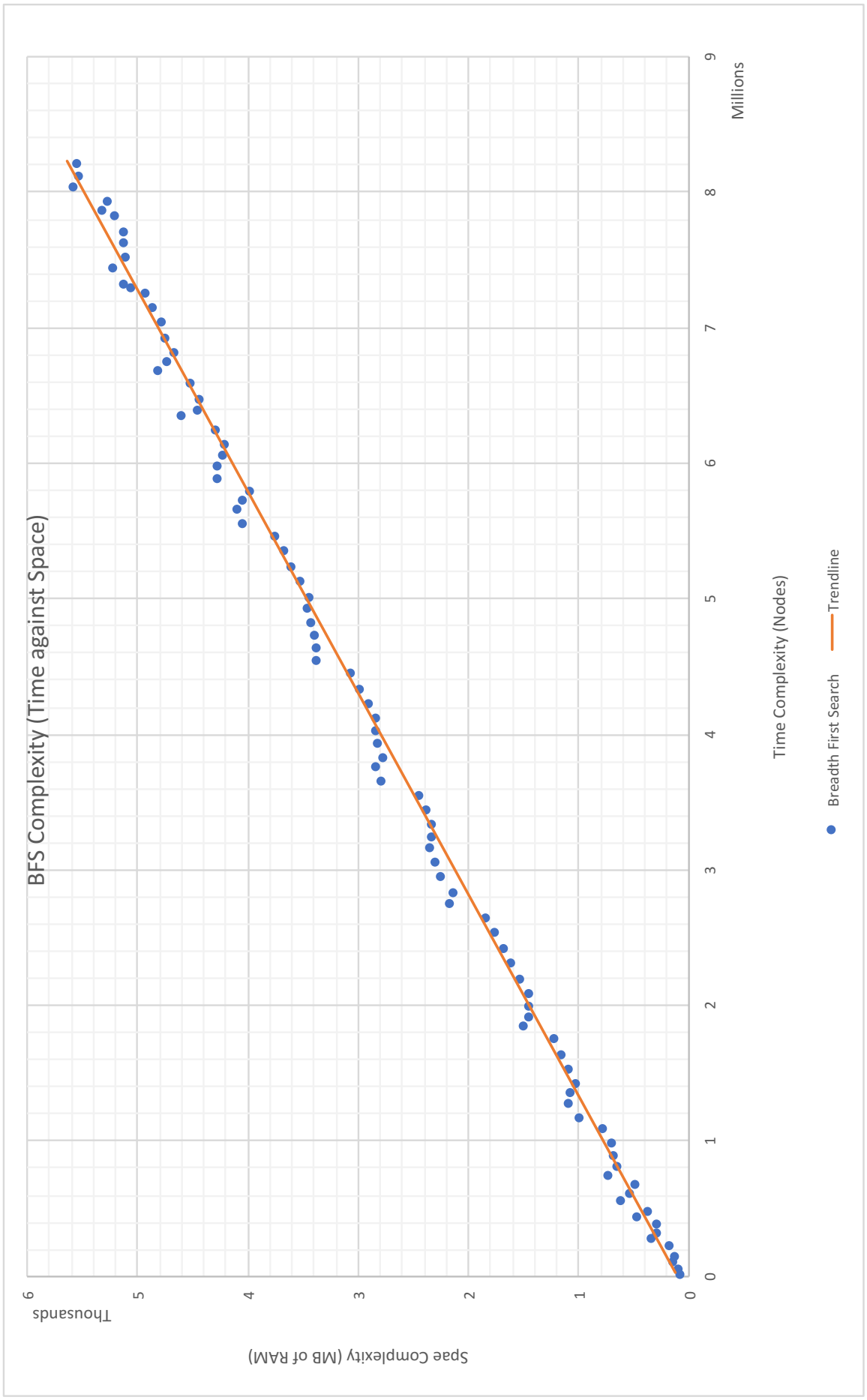
D.3 4x4 Grid, Complexity vs Nodes Evaluated



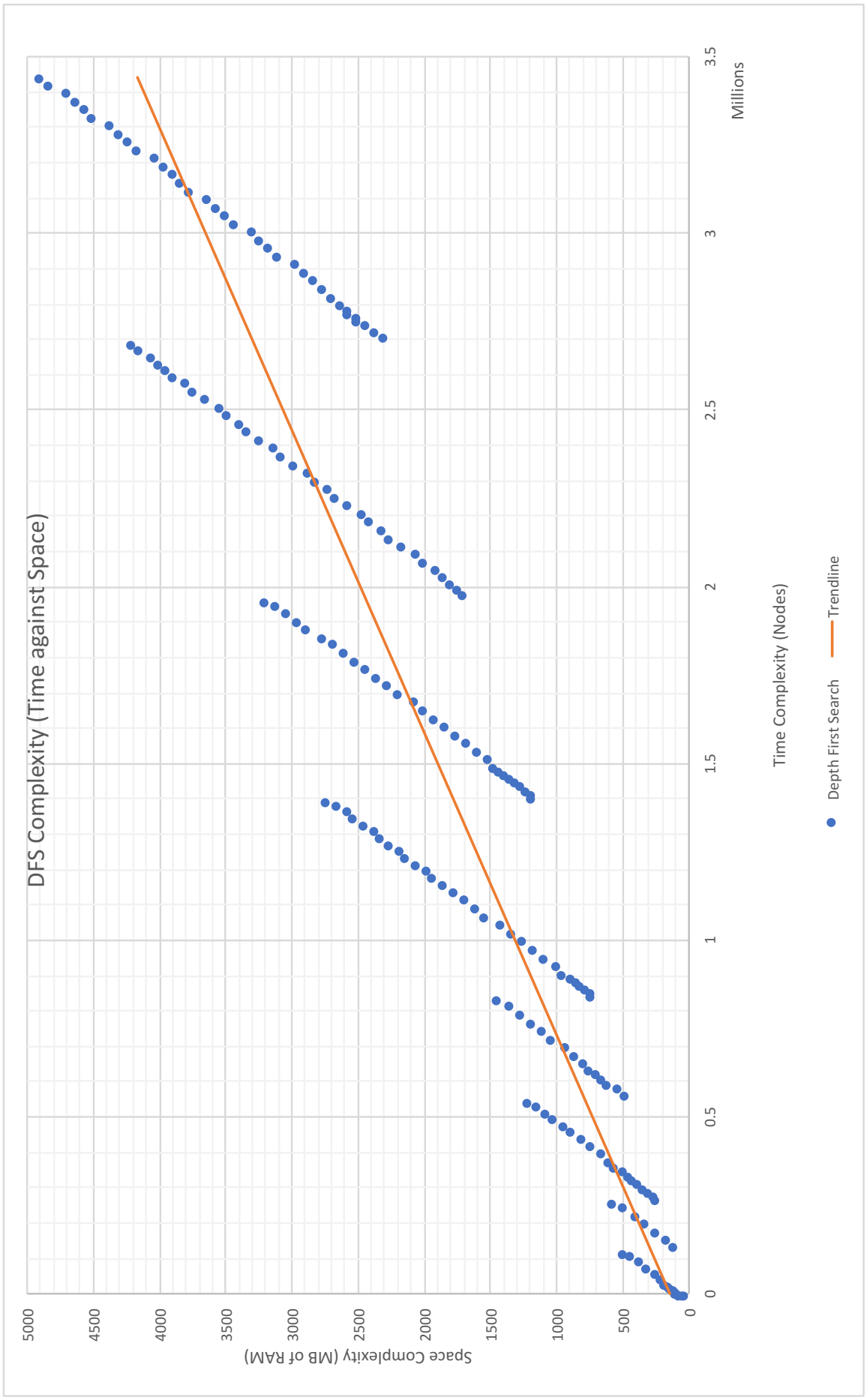
D.4 5x5 Grid, Complexity vs Nodes Evaluated



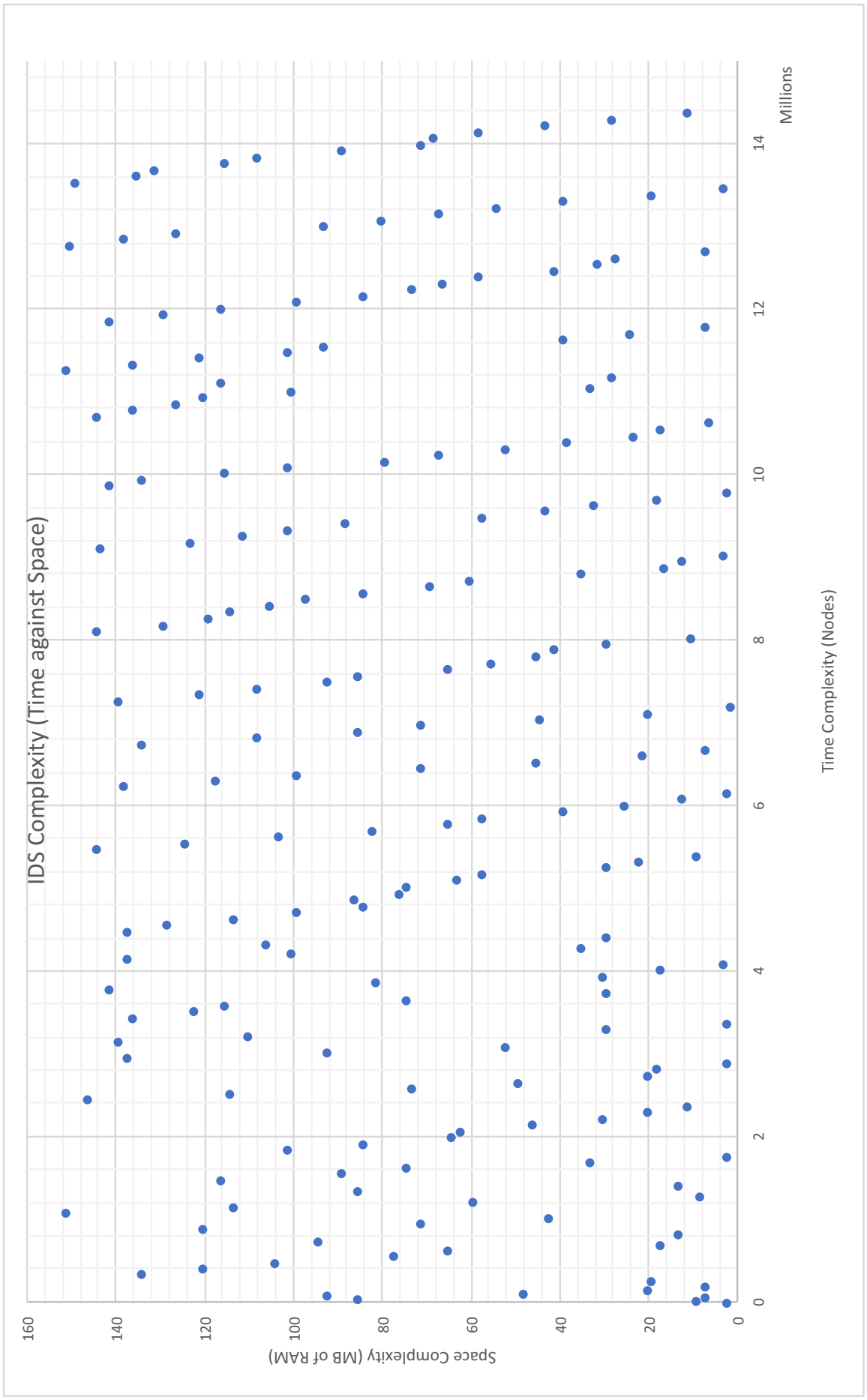
D.5 BFS Complexity (Time vs Space)



D.6 DFS Complexity (Time vs Space)



D.7 IDS Complexity (Time vs Space)



D.8 A* Complexity (Time vs Space)