

# Team notebook

October 5, 2021

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# 1 Algorithms

## 1.1 Mo's algorithm on trees

---

```
/**
problems:
- https://codeforces.com/gym/101161 problem E
```

```
*/
void flat(vector<vector<edge>> &g, vector<int> &a,
          vector<int> &le, vector<int> &ri, vector<int> &cost,
          int node, int pi, int &ts, int w) {

    cost[node] = w;
    le[node] = ts;
    a[ts] = node;
    ts++;
    for (auto e : g[node]) {
        if (e.to == pi) continue;
        flat(g, a, le, ri, cost, e.to, node, ts, e.w);
    }
    ri[node] = ts;
    a[ts] = node;
    ts++;
}

/**
 * Case when the cost is in the edges.
 */
void compute_queries(vector<vector<edge>> &g) {
    // g is undirected
    int n = g.size();

    lca_tree.init(g, 0);

    vector<int> a(2 * n), le(n), ri(n), cost(n);
    // a: nodes in the flatten array
    // le: left id of the given node
    // ri: right id of the given node
    // cost: cost of the edge from the node to the parent

    int ts = 0; // timestamp
    flat(g, a, le, ri, cost, 0, -1, ts, 0);

    int q; cin >> q;
    vector<query> queries(q);
    for (int i = 0; i < q; i++) {
        int u, v;
        cin >> u >> v;
        u--; v--;
        int lca = lca_tree.query(u, v);
        if (le[u] > le[v])
            swap(u, v);
    }
}
```

```

queries[i].id = i;
queries[i].lca = lca;
queries[i].u = u;
queries[i].v = v;
if (lca == u) {
    queries[i].a = le[u] + 1;
    queries[i].b = le[v];
} else {
    queries[i].a = ri[u];
    queries[i].b = le[v];
}
}
solve_mo(queries, a, le, cost); // this is the usal algorithm
}

```

---

## 1.2 Mo's algorithm

---

```

const int MN = 5 * 100000 + 1;
const int SN = 708;

struct Query {
    int a, b, id;
    Query() {}
    Query(int x, int y, int i) : a(x), b(y), id(i) {}

    bool operator<(const Query &o) const {
        if (a / SN != o.a / SN) return a < o.a;
        return a / SN & 1 ? b < o.b : b > o.b;
    }
};

struct DS {
    DS() : {}

    void Insert(int x) {}

    void Erase(int x) {}

    long long Query() {}
};

Query s[MN];
int ans[MN];

```

```

DS active;

int main() {
    int n;
    cin >> n;
    vector<int> a(n);
    for (auto &i : a) cin >> i;

    int q;
    cin >> q;
    for (int i = 0; i < q; ++i) {
        int b, e;
        cin >> b >> e;
        b--;
        e--;
        s[i] = Query(b, e, i);
    }

    sort(s, s + q);

    int i = 0;
    int j = -1;
    for (int k = 0; k < (int)q; ++k) {
        int L = s[k].a;
        int R = s[k].b;

        while (j < R) active.Insert(a[++j]);
        while (j > R) active.Erase(a[j--]);
        while (i < L) active.Erase(a[i++]);
        while (i > L) active.Insert(a[--i]);

        ans[s[k].id] = active.Query();
    }

    for (int i = 0; i < q; ++i) {
        cout << ans[i] << endl;
    }

    return 0;
};

```

---

## 1.3 sliding window

---

```

/*
 * Given an array ARR and an integer K, the problem boils down to
 * computing for each index i: min(ARR[i], ARR[i-1], ..., ARR[i-K+1]).
 * if mx == true, returns the maximum.
 * http://people.cs.uct.ac.za/~ksmith/articles/sliding_window_minimum.html
 */

vector<int> sliding_window_minmax(vector<int> & ARR, int K, bool mx) {
    deque< pair<int, int> > window;
    vector<int> ans;
    for (int i = 0; i < ARR.size(); i++) {
        if (mx) {
            while (!window.empty() && window.back().first <= ARR[i])
                window.pop_back();
        } else {
            while (!window.empty() && window.back().first >= ARR[i])
                window.pop_back();
        }
        window.push_back(make_pair(ARR[i], i));

        while(window.front().second <= i - K)
            window.pop_front();

        ans.push_back(window.front().first);
    }
    return ans;
}

```

---

## 2 DP Optimizations

### 2.1 convex hull trick

```

/**
 * Problems:
 * http://codeforces.com/problemset/problem/319/C
 * http://codeforces.com/contest/311/problem/B
 * https://csacademy.com/contest/archive/task/squared-ends
 * http://codeforces.com/contest/932/problem/F
 */

struct line {

```

```

    long long m, b;
    line (long long a, long long c) : m(a), b(c) {}
    long long eval(long long x) {
        return m * x + b;
    }
};

long double inter(line a, line b) {
    long double den = a.m - b.m;
    long double num = b.b - a.b;
    return num / den;
}

/**
 * min m_i * x_j + b_i, for all i.
 * x_j <= x_{j + 1}
 * m_i >= m_{j + 1}
 */
struct ordered_cht {
    vector<line> ch;
    int idx; // id of last "best" in query
    ordered_cht() {
        idx = 0;
    }

    void insert_line(long long m, long long b) {
        line cur(m, b);
        // new line's slope is less than all the previous
        while (ch.size() > 1 &&
            (inter(cur, ch[ch.size() - 2]) >= inter(cur, ch[ch.size() - 1]))) {
            // f(x) is better in interval [inter(ch.back(), cur), inf)
            ch.pop_back();
        }

        ch.push_back(cur);
    }

    long long eval(long long x) { // minimum
        // current x is greater than all the previous x,
        // if that is not the case we can make binary search.
        idx = min<int>(idx, ch.size() - 1);
        while (idx + 1 < (int)ch.size() && ch[idx + 1].eval(x) <=
            ch[idx].eval(x))
            idx++;
    }
};

```

```

    return ch[idx].eval(x);
}
};

/**
 * Dynammic convex hull trick
 * */

typedef long long int64;
typedef long double float128;

const int64 is_query = -(1LL<<62), inf = 1e18;

struct Line {
    int64 m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        if (!s) return 0;
        int64 x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};

struct HullDynamic : public multiset<Line> { // will maintain upper hull
    for maximum
    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if (z == end()) return y->m == x->m && y->b <= x->b;
        return (float128)(x->b - y->b)*(z->m - y->m) >= (float128)(y->b -
            z->b)*(y->m - x->m);
    }
    void insert_line(int64 m, int64 b) {
        auto y = insert({ m, b });
        y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
        if (bad(y)) { erase(y); return; }
        while (next(y) != end() && bad(next(y))) erase(next(y));
        while (y != begin() && bad(prev(y))) erase(prev(y));
    }
};

```

```

}

int64 eval(int64 x) {
    auto l = *lower_bound((Line) { x, is_query });
    return l.m * x + l.b;
}
};

```

## 2.2 divide and conquer

```

/**
 * recurrence:
 *   dp[k][i] = min dp[k-1][j] + c[i][j - 1], for all j > i;
 *
 * "comp" computes dp[k][i] for all i in O(n log n) (k is fixed)
 *
 * Problems:
 *   https://icpc.kattis.com/problems/branch
 *   http://codeforces.com/contest/321/problem/E
 * */

void comp(int l, int r, int le, int re) {
    if (l > r) return;

    int mid = (l + r) >> 1;

    int best = max(mid + 1, le);
    dp[cur][mid] = dp[cur ^ 1][best] + cost(mid, best - 1);
    for (int i = best; i <= re; i++) {
        if (dp[cur][mid] > dp[cur ^ 1][i] + cost(mid, i - 1)) {
            best = i;
            dp[cur][mid] = dp[cur ^ 1][i] + cost(mid, i - 1);
        }
    }

    comp(l, mid - 1, le, best);
    comp(mid + 1, r, best, re);
}

```

## 2.3 dp on trees

```

/**
 * This trick is very useful when doing DP on trees, basically, you can
 * save
 * the answer for each node as if it was the root of the tree. Partial
 * results
 * are also stored in order to query subtrees (taking the root and
 * exclude some
 * child).
 *
 * problems:
 * - http://codeforces.com/gym/101161, problem I : Sky tax
 * - http://codeforces.com/contest/791/problem/D
 */

struct edge {
    int to, p_id;
    edge(int a, int b) : to(a), p_id(b) {}
};

struct state {
    bool seen;
    long long missing;
    long long total;
    vector<long long> partial;

    state() { clear(); }

    void clear() {
        seen = false;
        missing = 0;
        total = 0;
        partial.clear();
    }
};

void add_edge(int u, int v) {
    int id_u_v = g[u].size();
    int id_v_u = g[v].size();
    g[u].emplace_back(v, id_v_u); // id of the parent in the child's list
    (g[v][id] -> u)
    g[v].emplace_back(u, id_u_v); // id of the parent in the child's list
    (g[u][id] -> v)
}

```

```

int go(int node, int id_parent) {

    state &s = dp[node];

    if (!s.seen) {
        int ans = 1;
        s.partial.assign(g[node].size(), 0); // create the list of partial
        results.
        for (int i = 0; i < int(g[node].size()); i++) {
            int to = g[node][i].to;
            int pid = g[node][i].p_id;
            if (i != id_parent) {
                int tmp = go(to, pid);
                ans += tmp;
                s.partial[i] = tmp;
            }
        }

        s.missing = id_parent;
        s.total = ans;
        s.seen = true;

        return ans;
    } else {

        if (s.missing == id_parent) { // the same id_parent than before, so
            we can not complete the results yet
            return s.total;
        }

        if (s.missing != -1) { // only one missing and is different of
            'id_parent'
            int tmp = go(g[node][s.missing].to, g[node][s.missing].p_id);
            s.partial[s.missing] = tmp;
            s.total += tmp;
            s.missing = -1;
        }

        int extra = (id_parent == -1) ? 0 : s.partial[id_parent];
        return s.total - extra;
    }
}

```

## 3 Data structures

### 3.1 STL Treap

---

```
#include <ext/rope> //header with rope
using namespace std;
using namespace __gnu_cxx; //namespace with rope and some additional stuff
int main()
{
    ios_base::sync_with_stdio(false);
    rope <int> v; //use as usual STL container
    int n, m;
    cin >> n >> m;
    for(int i = 1; i <= n; ++i)
        v.push_back(i); //initialization
    int l, r;
    for(int i = 0; i < m; ++i)
    {
        cin >> l >> r;
        --l, --r;
        rope <int> cur = v.substr(l, r - l + 1);
        v.erase(l, r - l + 1);
        v.insert(v.mutable_begin(), cur);
    }
    for(rope <int>::iterator it = v.mutable_begin(); it !=
        v.mutable_end(); ++it)
        cout << *it << " ";
    return 0;
}
```

---

### 3.2 STL order statistics tree II

---

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace std;
using namespace __gnu_pbds;

typedef tree<int,null_type,less<int>,rb_tree_tag,
tree_order_statistics_node_update> order_set;
```

```
order_set X;

int get(int y) {
    int l=0,r=1e9+1;
    while(l<r) {
        int m=l+((r-l)>>1);
        if(m-X.order_of_key(m+1)<y)
            l=m+1;
        else
            r=m;
    }
    return l;
}
```

```
main(){
    ios::sync_with_stdio(0);
    cin.tie(0);
    int n,m;
    cin>>n>>m;

    for(int i=0;i<m;i++) {
        char a;
        int b;
        cin>>a>>b;
        if(a=='L')
            cout<<get(b)<<endl;
        else
            X.insert(get(b));
    }
}
```

```
/**
Input
20 7
L 5
D 5
L 4
L 5
D 5
L 4
L 5
```

```
Output
5
4
```

```
6
4
7
***/
```

---

### 3.3 STL order statistics tree

---

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <bits/stdc++.h>
```

```
using namespace __gnu_pbds;
using namespace std;
```

```
typedef
tree<
    pair<int,int>,
    null_type,
    less<pair<int,int>>,
    rb_tree_tag,
    tree_order_statistics_node_update>
ordered_set;
```

```
main()
{
    ios::sync_with_stdio(0);
    cin.tie(0);
    int n;
    int sz=0;
    cin>>n;
    vector<int> ans(n,0);

    ordered_set t;
    int x,y;
    for(int i=0;i<n;i++)
    {
        cin>>x>>y;
        ans[t.order_of_key({x,++sz})]++;
        t.insert({x,sz});
    }

    for(int i=0;i<n;i++)
        cout<<ans[i]<<'\\n';
```

```
}
```

```
/**
Input
5
1 1
5 1
7 1
3 3
5 5
```

```
Output
1
2
1
1
0
***/
```

---

### 3.4 binary index tree

---

```
struct binary_index_tree {
    int n;
    int t[2 * N];

    void add(int where, long long what){
        for (where++; where <= n; where += where & -where){
            t[where] += what;
        }
    }

    void add(int from, int to, long long what) {
        add(from, what);
        add(to + 1, -what);
    }

    long long query(int where){
        long long sum = t[0];
        for (where++; where > 0; where -= where & -where){
            sum += t[where];
        }
        return sum;
    }
}
```



```
};
```

---

### 3.5 dsu

---

```
struct Dsu {
    vector<int> p;

    Dsu(int n) {
        p.resize(n);
        for (int i = 0; i < n; i++) {
            p[i] = i;
        }
    }

    int Find(int x) { return x == p[x] ? x : p[x] = Find(p[x]); }

    int Join(int x, int y) {
        int px = Find(x), py = Find(y);
        if (px == py) return 0;
        p[px] = py;
        return 1;
    }
};
```

---

### 3.6 hash table

---

```
/**
 * Micro hash table, can be used as a set.
 * Very efficient vs std::set
 */

const int MN = 1001;
struct ht {
    int _s[(MN + 10) >> 5];
    int len;
    void set(int id) {
        len++;
        _s[id >> 5] |= (1LL << (id & 31));
    }
    bool is_set(int id) {
        return _s[id >> 5] & (1LL << (id & 31));
    }
};
```

---

```
}
};
```

---

### 3.7 heavy light decomposition

---

```
// Heavy-Light Decomposition
struct TreeDecomposition {
    vector<int> g[MAXN], c[MAXN];
    int s[MAXN]; // subtree size
    int p[MAXN]; // parent id
    int r[MAXN]; // chain root id
    int t[MAXN]; // index used in segtree/bit/...
    int d[MAXN]; // depth
    int ts;

    void dfs(int v, int f) {
        p[v] = f;
        s[v] = 1;
        if (f != -1) d[v] = d[f] + 1;
        else d[v] = 0;

        for (int i = 0; i < g[v].size(); ++i) {
            int w = g[v][i];
            if (w != f) {
                dfs(w, v);
                s[v] += s[w];
            }
        }
    }

    void hld(int v, int f, int k) {
        t[v] = ts++;
        c[k].push_back(v);
        r[v] = k;

        int x = 0, y = -1;
        for (int i = 0; i < g[v].size(); ++i) {
            int w = g[v][i];
            if (w != f) {
                if (s[w] > x) {
                    x = s[w];
                    y = w;
                }
            }
        }
    }
};
```

---

```

    }
}
if (y != -1) {
    hld(y, v, k);
}

for (int i = 0; i < g[v].size(); ++i) {
    int w = g[v][i];
    if (w != f && w != y) {
        hld(w, v, w);
    }
}
}

void init(int n) {
    for (int i = 0; i < n; ++i) {
        g[i].clear();
    }
}

void add(int a, int b) {
    g[a].push_back(b);
    g[b].push_back(a);
}

void build() {
    ts = 0;
    dfs(0, -1);
    hld(0, 0, 0);
}
};

```

### 3.8 persistent array

```

struct node {
    node *l, *r;
    int val;

    node (int x) : l(NULL), r(NULL), val(x) {}
    node () : l(NULL), r(NULL), val(-1) {}
};

typedef node* pnode;

```

```

pnode update(pnode cur, int l, int r, int at, int what) {
    pnode ans = new node();

    if (cur != NULL) {
        *ans = *cur;
    }
    if (l == r) {
        ans->val = what;
        return ans;
    }
    int m = (l + r) >> 1;
    if (at <= m) ans->l = update(ans->l, l, m, at, what);
    else ans->r = update(ans->r, m + 1, r, at, what);
    return ans;
}

int get(pnode cur, int l, int r, int at) {
    if (cur == NULL) return 0;
    if (l == r) return cur->val;
    int m = (l + r) >> 1;
    if (at <= m) return get(cur->l, l, m, at);
    else return get(cur->r, m + 1, r, at);
}

```

### 3.9 persistent seg tree

```

/**
 * Problems:
 * http://codeforces.com/contest/813/problem/E
 *
 * Important:
 * When using lazy propagation remember to create new
 * versions for each push_down operation!!!
 */

struct node {
    node *l, *r;
    long long acc;
    int flip;

    node (int x) : l(NULL), r(NULL), acc(x), flip(0) {}
    node () : l(NULL), r(NULL), acc(0), flip(0) {}
}

```

```

};

typedef node* pnode;

pnode create(int l, int r) {
    if (l == r) return new node();
    pnode cur = new node();
    int m = (l + r) >> 1;
    cur->l = create(l, m);
    cur->r = create(m + 1, r);
    return cur;
}

pnode copy_node(pnode cur) {
    pnode ans = new node();
    *ans = *cur;
    return ans;
}

void push_down(pnode cur, int l, int r) {
    assert(cur);
    if (cur->flip) {
        int len = r - l + 1;
        cur->acc = len - cur->acc;
        if (cur->l) {
            cur->l = copy_node(cur->l);
            cur->l->flip ^= 1;
        }
        if (cur->r) {
            cur->r = copy_node(cur->r);
            cur->r->flip ^= 1;
        }
        cur->flip = 0;
    }
}

int get_val(pnode cur) {
    assert(cur);
    assert((cur->flip) == 0);
    if (cur) return cur->acc;
    return 0;
}

pnode update(pnode cur, int l, int r, int at, int what) {
    pnode ans = copy_node(cur);

```

```

    if (l == r) {
        assert(l == at);
        ans->acc = what;
        ans->flip = 0;
        return ans;
    }
    int m = (l + r) >> 1;
    push_down(ans, l, r);
    if (at <= m) ans->l = update(ans->l, l, m, at, what);
    else ans->r = update(ans->r, m + 1, r, at, what);

    push_down(ans->l, l, m);
    push_down(ans->r, m + 1, r);
    ans->acc = get_val(ans->l) + get_val(ans->r);
    return ans;
}

pnode flip(pnode cur, int l, int r, int a, int b) {
    pnode ans = new node();

    if (cur != NULL) {
        *ans = *cur;
    }
    if (l > b || r < a)
        return ans;

    if (l >= a && r <= b) {
        ans->flip ^= 1;
        push_down(ans, l, r);
        return ans;
    }

    int m = (l + r) >> 1;
    ans->l = flip(ans->l, l, m, a, b);
    ans->r = flip(ans->r, m + 1, r, a, b);
    push_down(ans->l, l, m);
    push_down(ans->r, m + 1, r);
    ans->acc = get_val(ans->l) + get_val(ans->r);
    return ans;
}

long long get_all(pnode cur, int l, int r) {
    assert(cur);
    push_down(cur, l, r);
    return cur->acc;
}

```

```

}

void traverse(pnode cur, int l, int r) {
    if (!cur) return;
    cout << l << " - " << r << " : " << (cur->acc) << " " << (cur->flip)
        << endl;
    traverse(cur->l, l, (l + r) >> 1);
    traverse(cur->r, 1 + ((l + r) >> 1), r);
}

```

### 3.10 persistent trie

// both tries can be tested with the problem:  
<http://codeforces.com/problemset/problem/916/D>

```

// Persistent binary trie (BST for integers)
const int MD = 31;

struct node_bin {
    node_bin *child[2];
    int val;

    node_bin() : val(0) {
        child[0] = child[1] = NULL;
    }
};

typedef node_bin* pnode_bin;

pnode_bin copy_node(pnode_bin cur) {
    pnode_bin ans = new node_bin();
    if (cur) *ans = *cur;
    return ans;
}

pnode_bin modify(pnode_bin cur, int key, int inc, int id = MD) {
    pnode_bin ans = copy_node(cur);
    ans->val += inc;
    if (id >= 0) {
        int to = (key >> id) & 1;
        ans->child[to] = modify(ans->child[to], key, inc, id - 1);
    }
}

```

```

}

return ans;
}

int sum_smaller(pnode_bin cur, int key, int id = MD) {
    if (cur == NULL) return 0;
    if (id < 0) return 0; // strictly smaller
    // if (id == - 1) return cur->val; // smaller or equal

    int ans = 0;
    int to = (key >> id) & 1;
    if (to) {
        if (cur->child[0]) ans += cur->child[0]->val;
        ans += sum_smaller(cur->child[1], key, id - 1);
    } else {
        ans = sum_smaller(cur->child[0], key, id - 1);
    }
    return ans;
}

// Persistent trie for strings.
const int MAX_CHILD = 26;
struct node {
    node *child[MAX_CHILD];
    int val;

    node() : val(-1) {
        for (int i = 0; i < MAX_CHILD; i++) {
            child[i] = NULL;
        }
    }
};

typedef node* pnode;

pnode copy_node(pnode cur) {
    pnode ans = new node();
    if (cur) *ans = *cur;
    return ans;
}

pnode set_val(pnode cur, string &key, int val, int id = 0) {
    pnode ans = copy_node(cur);
    if (id >= int(key.size())) {
        ans->val = val;
    }
}

```

```

} else {
    int t = key[id] - 'a';
    ans->child[t] = set_val(ans->child[t], key, val, id + 1);
}
return ans;
}

pnode get(pnode cur, string &key, int id = 0) {
    if (id >= int(key.size()) || !cur)
        return cur;
    int t = key[id] - 'a';
    return get(cur->child[t], key, id + 1);
}

```

### 3.11 segment tree

```

/**
 * Taken from: http://codeforces.com/blog/entry/18051
 * */

const int MN = 1e5; // limit for array size

struct seg_tree {
    int n; // array size
    int t[2 * MN];

    seg_tree(int _n) : n(_n) {}

    void clear() {
        memset(t, 0, sizeof t);
    }

    void build() { // build the tree
        for (int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
    }

    // Single modification, range query.
    void modify(int p, int value) { // set value at position p
        for (t[p += n] = value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];
    }

    int query(int l, int r) { // sum on interval [l, r)
        int res = 0;

```

```

        for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
            if (l&1) res += t[l++];
            if (r&1) res += t[--r];
        }
        return res;
    }
};

```

// Range modification, single query.

```

void modify(int l, int r, int value) {
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) t[l++] += value;
        if (r&1) t[--r] += value;
    }
}

```

```

int query(int p) {
    int res = 0;
    for (p += n; p > 0; p >>= 1) res += t[p];
    return res;
}

```

```

/**
 * If at some point after modifications we need to inspect all the
 * elements in the array, we can push all the modifications to the
 * leaves using the following code. After that we can just traverse
 * elements starting with index n. This way we reduce the complexity
 * from O(n log(n)) to O(n) similarly to using build instead of n
 * modifications.
 * */

```

```

void push() {
    for (int i = 1; i < n; ++i) {
        t[i<<1] += t[i];
        t[i<<1|1] += t[i];
        t[i] = 0;
    }
}

```

// Non commutative combiner functions.

```

void modify(int p, const S& value) {
    for (t[p += n] = value; p >>= 1; ) t[p] = combine(t[p<<1], t[p<<1|1]);
}

```

```

S query(int l, int r) {
    S resl, resr;
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) resl = combine(resl, t[l++]);
        if (r&1) resr = combine(t[--r], resr);
    }
    return combine(resl, resr);
}

/**
 * segment tree for intervals
 *
 */

const int MN = 100000 + 100;
struct seg_tree {
    int val[MN * 4 + 4];
    int pending[MN * 4 + 4];

    seg_tree() {
        memset(val, -1, sizeof val);
        memset(pending, -1, sizeof pending);
    }

    void propagate(int node, int b, int e) {
        if (pending[node] != -1) {
            val[node] = pending[node];
            if (b < e) {
                pending[node << 1] = pending[node];
                pending[node << 1 | 1] = pending[node];
            }
            pending[node] = -1;
        }
    }

    void set(int node, int b, int e, int from, int to, int v) {
        if (b > to || e < from) return;

        if (b >= from && e <= to) {
            pending[node] = v;
            propagate(node, b, e);
            return;
        }
    }
}

```

```

    int mid = (b + e) >> 1;

    set(node << 1, b, mid, from, to, v);
    set(node << 1 | 1, mid + 1, e, from, to, v);
}

int query(int node, int b, int e, int pos) {
    propagate(node, b, e);

    if (b == e && b == pos) {
        return val[node];
    }

    int mid = (b + e) >> 1;
    if (pos <= mid)
        return query(node << 1, b, mid, pos);
    return query(node << 1 | 1, mid + 1, e, pos);
}

void set(int from, int to, int v) {
    return set(1, 0, MN - 1, from, to, v);
}

int query(int pos) {
    return query(1, 0, MN - 1, pos);
}
};

```

---

### 3.12 sparse table

---

```

// RMQ.
const int MN = 100000 + 10; // Max number of elements
const int ML = 18; // ceil(log2(MN));

struct st {
    int data[MN];
    int M[MN][ML];
    int n;

    void init(const vector<int> &d) {

```

```

n = d.size();
for (int i = 0; i < n; ++i)
    data[i] = d[i];

build();
}

void build() {
    for (int i = 0; i < n; ++i)
        M[i][0] = data[i];
    for (int j = 1, p = 2, q = 1; p <= n; ++j, p <= 1, q <= 1)
        for (int i = 0; i + p - 1 < n; ++i)
            M[i][j] = max(M[i][j - 1], M[i + q][j - 1]);
}

int query(int b, int e) {
    int k = log2(e - b + 1);
    return max(M[b][k], M[e + 1 - (1<<k)][k]);
}
};

```

### 3.13 splay tree

```

using namespace std;
#include<bits/stdc++.h>
#define D(x) cout<<x<<endl;

typedef int T;

struct node{
    node *left, *right, *parent;
    T key;
    node (T k) : key(k), left(0), right(0), parent(0) {}
};

struct splay_tree{

    node *root;

    void right_rot(node *x) {
        node *p = x->parent;
        if (x->parent = p->parent) {
            if (x->parent->left == p) x->parent->left = x;

```

```

            if (x->parent->right == p) x->parent->right = x;
        }
        if (p->left = x->right) p->left->parent = p;
        x->right = p;
        p->parent = x;
    }
}

void left_rot(node *x) {
    node *p = x->parent;
    if (x->parent = p->parent) {
        if (x->parent->left == p) x->parent->left = x;
        if (x->parent->right == p) x->parent->right = x;
    }
    if (p->right = x->left) p->right->parent = p;
    x->left = p;
    p->parent = x;
}

void splay(node *x, node *fa = 0) {

    while( x->parent != fa and x->parent != 0) {
        node *p = x->parent;
        if (p->parent == fa)
            if (p->right == x)
                left_rot(x);
            else
                right_rot(x);
        else {
            node *gp = p->parent; //grand parent
            if (gp->left == p)
                if (p->left == x)
                    right_rot(x), right_rot(x);
                else
                    left_rot(x), right_rot(x);
            else
                if (p->left == x)
                    right_rot(x), left_rot(x);
                else
                    left_rot(x), left_rot(x);
        }
    }
    if (fa == 0) root = x;
}

```

```

void insert(T key) {

```

```

node *cur = root;
node *pcur = 0;
while (cur) {
    pcur = cur;
    if (key > cur->key) cur = cur->right;
    else cur = cur->left;
}
cur = new node(key);
cur->parent = pcur;
if (!pcur) root = cur;
else if (key > pcur->key) pcur->right = cur;
else pcur->left = cur;
splay(cur);
}

node *find(T key) {
    node *cur = root;
    while (cur) {
        if (key > cur->key) cur = cur->right;
        else if (key < cur->key) cur = cur->left;
        else return cur;
    }
    return 0;
}

splay_tree(){ root = 0;};
};

```

---

### 3.14 trie

```

const int MN = 26; // size of alphabet
const int MS = 100010; // Number of states.

struct trie{
    struct node{
        int c;
        int a[MN];
    };

    node tree[MS];
    int nodes;

    void clear(){

```

```

        tree[nodes].c = 0;
        memset(tree[nodes].a, -1, sizeof tree[nodes].a);
        nodes++;
    }

    void init(){
        nodes = 0;
        clear();
    }

    int add(const string &s, bool query = 0){
        int cur_node = 0;
        for(int i = 0; i < s.size(); ++i){
            int id = gid(s[i]);
            if(tree[cur_node].a[id] == -1){
                if(query) return 0;
                tree[cur_node].a[id] = nodes;
                clear();
            }
            cur_node = tree[cur_node].a[id];
        }
        if(!query) tree[cur_node].c++;
        return tree[cur_node].c;
    }
};

```

---

### 3.15 wavelet tree

```

// this can be tested in the problem:
// http://www.spoj.com/problems/ILKQUERY/

struct wavelet {
    vector<int> values, ori;
    vector<int> map_left, map_right;
    int l, r, m;
    wavelet *left, *right;
    wavelet() : left(NULL), right(NULL) {}
    wavelet(int a, int b, int c) : l(a), r(b), m(c), left(NULL),
        right(NULL) {}
};

wavelet *init(vector<int> &data, vector<int> &ind, int lo, int hi) {

```



```

if (lo > hi || (data.size() == 0)) return NULL;
int mid = ((long long)(lo) + hi) / 2;
if (lo + 1 == hi) mid = lo; // handle negative values

wavelet *node = new wavelet(lo, hi, mid);

vector<int> data_l, data_r, ind_l, ind_r;
int ls = 0, rs = 0;
for (int i = 0; i < int(data.size()); i++) {
    int value = data[i];
    if (value <= mid) {
        data_l.emplace_back(value);
        ind_l.emplace_back(ind[i]);
        ls++;
    } else {
        data_r.emplace_back(value);
        ind_r.emplace_back(ind[i]);
        rs++;
    }
    node->map_left.emplace_back(ls);
    node->map_right.emplace_back(rs);
    node->values.emplace_back(value);
    node->ori.emplace_back(ind[i]);
}

if (lo < hi) {
    node->left = init(data_l, ind_l, lo, mid);
    node->right = init(data_r, ind_r, mid + 1, hi);
}
return node;
}

int kth(wavelet *node, int to, int k) {
    // returns the kth element in the sorted version of (a[0], ..., a[to])
    if (node->l == node->r) return node->m;
    int c = node->map_left[to];
    if (k < c)
        return kth(node->left, c - 1, k);
    return kth(node->right, node->map_right[to] - 1, k - c);
}

int pos_kth_occurrence(wavelet *node, int val, int k) {
    // returns the position on the original array of the kth occurrence of
    // the value "val"
    if (!node) return -1;

```

```

if (node->l == node->r) {
    if (int(node->ori.size()) <= k)
        return -1;
    return node->ori[k];
}

if (val <= node->m)
    return pos_kth_occurrence(node->left, val, k);
return pos_kth_occurrence(node->right, val, k);
}

```

---

## 4 Geometry

### 4.1 center 2 points + raiious

```

vector<point> find_center(point a, point b, long double r) {
    point d = (a - b) * 0.5;
    if (d.dot(d) > r * r) {
        return vector<point> ();
    }
    point e = b + d;
    long double fac = sqrt(r * r - d.dot(d));
    vector<point> ans;
    point x = point(-d.y, d.x);
    long double l = sqrt(x.dot(x));
    x = x * (fac / l);
    ans.push_back(e + x);
    x = point(d.y, -d.x);
    x = x * (fac / l);
    ans.push_back(e + x);
    return ans;
}

```

---

### 4.2 closest pair problem

```

struct point {
    double x, y;
    int id;
    point() {}

```

```

    point (double a, double b) : x(a), y(b) {}
};

double dist(const point &o, const point &p) {
    double a = p.x - o.x, b = p.y - o.y;
    return sqrt(a * a + b * b);
}

double cp(vector<point> &p, vector<point> &x, vector<point> &y) {
    if (p.size() < 4) {
        double best = 1e100;
        for (int i = 0; i < p.size(); ++i)
            for (int j = i + 1; j < p.size(); ++j)
                best = min(best, dist(p[i], p[j]));
        return best;
    }

    int ls = (p.size() + 1) >> 1;
    double l = (p[ls - 1].x + p[ls].x) * 0.5;
    vector<point> xl(ls), xr(p.size() - ls);
    unordered_set<int> left;
    for (int i = 0; i < ls; ++i) {
        xl[i] = x[i];
        left.insert(x[i].id);
    }
    for (int i = ls; i < p.size(); ++i) {
        xr[i - ls] = x[i];
    }

    vector<point> yl, yr;
    vector<point> pl, pr;
    yl.reserve(ls); yr.reserve(p.size() - ls);
    pl.reserve(ls); pr.reserve(p.size() - ls);
    for (int i = 0; i < p.size(); ++i) {
        if (left.count(y[i].id))
            yl.push_back(y[i]);
        else
            yr.push_back(y[i]);

        if (left.count(p[i].id))
            pl.push_back(p[i]);
        else
            pr.push_back(p[i]);
    }

```

```

    double dl = cp(pl, xl, yl);
    double dr = cp(pr, xr, yr);
    double d = min(dl, dr);
    vector<point> yp; yp.reserve(p.size());
    for (int i = 0; i < p.size(); ++i) {
        if (fabs(y[i].x - l) < d)
            yp.push_back(y[i]);
    }
    for (int i = 0; i < yp.size(); ++i) {
        for (int j = i + 1; j < yp.size() && j < i + 7; ++j) {
            d = min(d, dist(yp[i], yp[j]));
        }
    }
    return d;
}

double closest_pair(vector<point> &p) {
    vector<point> x(p.begin(), p.end());
    sort(x.begin(), x.end(), [](const point &a, const point &b) {
        return a.x < b.x;
    });
    vector<point> y(p.begin(), p.end());
    sort(y.begin(), y.end(), [](const point &a, const point &b) {
        return a.y < b.y;
    });
    return cp(p, x, y);
}

```

---

### 4.3 squares

---

```

typedef long double ld;

const ld eps = 1e-12;
int cmp(ld x, ld y = 0, ld tol = eps) {
    return (x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1;
}

struct point{
    ld x, y;
    point(ld a, ld b) : x(a), y(b) {}
    point() {}
};

```

```

struct square{
    ld x1, x2, y1, y2,
        a, b, c;
    point edges[4];
    square(ld _a, ld _b, ld _c) {
        a = _a, b = _b, c = _c;
        x1 = a - c * 0.5;
        x2 = a + c * 0.5;
        y1 = b - c * 0.5;
        y2 = b + c * 0.5;
        edges[0] = point(x1, y1);
        edges[1] = point(x2, y1);
        edges[2] = point(x2, y2);
        edges[3] = point(x1, y2);
    }
};

ld min_dist(point &a, point &b) {
    ld x = a.x - b.x,
        y = a.y - b.y;
    return sqrt(x * x + y * y);
}

bool point_in_box(square s1, point p) {
    if (cmp(s1.x1, p.x) != 1 && cmp(s1.x2, p.x) != -1 &&
        cmp(s1.y1, p.y) != 1 && cmp(s1.y2, p.y) != -1)
        return true;
    return false;
}

bool inside(square &s1, square &s2) {
    for (int i = 0; i < 4; ++i)
        if (point_in_box(s2, s1.edges[i]))
            return true;

    return false;
}

bool inside_vert(square &s1, square &s2) {
    if ((cmp(s1.y1, s2.y1) != -1 && cmp(s1.y1, s2.y2) != 1) ||
        (cmp(s1.y2, s2.y1) != -1 && cmp(s1.y2, s2.y2) != 1))
        return true;
    return false;
}

```

```

bool inside_hori(square &s1, square &s2) {
    if ((cmp(s1.x1, s2.x1) != -1 && cmp(s1.x1, s2.x2) != 1) ||
        (cmp(s1.x2, s2.x1) != -1 && cmp(s1.x2, s2.x2) != 1))
        return true;
    return false;
}

ld min_dist(square &s1, square &s2) {
    if (inside(s1, s2) || inside(s2, s1))
        return 0;

    ld ans = 1e100;
    for (int i = 0; i < 4; ++i)
        for (int j = 0; j < 4; ++j)
            ans = min(ans, min_dist(s1.edges[i], s2.edges[j]));

    if (inside_hori(s1, s2) || inside_hori(s2, s1)) {
        if (cmp(s1.y1, s2.y2) != -1)
            ans = min(ans, s1.y1 - s2.y2);
        else
            if (cmp(s2.y1, s1.y2) != -1)
                ans = min(ans, s2.y1 - s1.y2);
    }

    if (inside_vert(s1, s2) || inside_vert(s2, s1)) {
        if (cmp(s1.x1, s2.x2) != -1)
            ans = min(ans, s1.x1 - s2.x2);
        else
            if (cmp(s2.x1, s1.x2) != -1)
                ans = min(ans, s2.x1 - s1.x2);
    }

    return ans;
}

```

#### 4.4 triangles

Let  $a$ ,  $b$ ,  $c$  be length of the three sides of a triangle.

$$p = (a + b + c) * 0.5$$

The inradius is defined by:

$$iR = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

The radius of its circumcircle is given by the formula:

$$cR = \frac{abc}{\sqrt{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}}$$

## 5 Graphs

### 5.1 SCC kosaraju

---

```

struct SCC {
    vector<vector<int> > g, gr;
    vector<bool> used;
    vector<int> order, component;
    int total_components;

    SCC(vector<vector<int> > &adj) {
        g = adj;
        int n = g.size();
        gr.resize(n);
        for (int i = 0; i < n; i++)
            for (auto to : g[i])
                gr[to].push_back(i);

        used.assign(n, false);
        for (int i = 0; i < n; i++)
            if (!used[i])
                GenTime(i);

        used.assign(n, false);
        component.assign(n, -1);
        total_components = 0;
        for (int i = n - 1; i >= 0; i--) {
            int v = order[i];
            if (!used[v]) {
                vector<int> cur_component;
                Dfs(cur_component, v);
                for (auto node : cur_component)
                    component[node] = total_components;
                total_components++;
            }
        }
    }
}

```

---

```

    }
}

void GenTime(int node) {
    used[node] = true;
    for (auto to : g[node])
        if (!used[to])
            GenTime(to);
    order.push_back(node);
}

void Dfs(vector<int> &cur, int node) {
    used[node] = true;
    cur.push_back(node);
    for (auto to : gr[node])
        if (!used[to])
            Dfs(cur, to);
}

vector<vector<int>> CondensedGraph() {
    vector<vector<int>> ans(total_components);
    for (int i = 0; i < int(g.size()); i++) {
        for (int to : g[i]) {
            int u = component[i], v = component[to];
            if (u != v)
                ans[u].push_back(v);
        }
    }
    return ans;
}
};

```

---

### 5.2 board

---

```

struct board {
    int n, m, r;
    board(int a, int b, int c = 1) : n(a), m(b), r(c) {}

    long long frec(int x, int y) {
        // returns how many squares of r x r contain the cell (x, y)
        long long a = min(x, n - r) - max(x - r + 1, 0) + 1;
        long long b = min(y, m - r) - max(y - r + 1, 0) + 1;
        return a * b;
    }
}

```

```

}

bool valid(int x, int y) {
    return x >= 0 && x < n && y >= 0 && y < m;
}
};

```

---

## 5.3 bridges

---

```

struct Graph {
    vector<vector<Edge>> g;
    vector<int> vi, low, d, pi, is_b;
    int bridges_computed;

    int ticks, edges;

    Graph(int n, int m) {
        g.assign(n, vector<Edge>());
        is_b.assign(m, 0);
        vi.resize(n);
        low.resize(n);
        d.resize(n);
        pi.resize(n);
        edges = 0;
        bridges_computed = 0;
    }

    void AddEdge(int u, int v) {
        g[u].push_back(Edge(v, edges));
        g[v].push_back(Edge(u, edges));
        edges++;
    }

    void Dfs(int u) {
        vi[u] = true;
        d[u] = low[u] = ticks++;
        for (int i = 0; i < (int)g[u].size(); ++i) {
            int v = g[u][i].to;
            if (v == pi[u]) continue;
            if (!vi[v]) {
                pi[v] = u;
                Dfs(v);
                if (d[u] < low[v]) is_b[g[u][i].id] = true;
            }
        }
    }
};

```

```

        low[u] = min(low[u], low[v]);
    } else {
        low[u] = min(low[u], d[v]);
    }
}
}

// Multiple edges from a to b are not allowed.
// (they could be detected as a bridge).
// If you need to handle this, just count
// how many edges there are from a to b.
void CompBridges() {
    fill(pi.begin(), pi.end(), -1);
    fill(vi.begin(), vi.end(), 0);
    fill(low.begin(), low.end(), 0);
    fill(d.begin(), d.end(), 0);
    ticks = 0;
    for (int i = 0; i < (int)g.size(); ++i)
        if (!vi[i]) Dfs(i);
    bridges_computed = true;
}

map<int, vector<Edge>> BridgesTree() {
    if (!bridges_computed) CompBridges();
    int n = g.size();
    Dsu dsu(g.size());
    for (int i = 0; i < n; i++)
        for (auto e : g[i])
            if (!is_b[e.id]) dsu.Join(i, e.to);

    map<int, vector<Edge>> tree;
    for (int i = 0; i < n; i++)
        for (auto e : g[i])
            if (is_b[e.id])
                tree[dsu.Find(i)].emplace_back(dsu.Find(e.to), e.id);

    return tree;
}
};

```

---

## 5.4 dijkstra

---

```

struct edge {
    int to;
    long long w;
    edge () {}
    edge (int a, long long b) : to(a), w(b) {}
    bool operator < (const edge &o) const {
        return w > o.w;
    }
};

typedef vector<vector<edge>> graph;

const long long inf = 1000000LL * 100000000LL;

pair<vector<int>, vector<long long>> dijkstra(graph &g, int start) {
    int n = g.size();
    vector<long long> d(n, inf);
    vector<int> p(n, -1);
    d[start] = 0;
    priority_queue<edge> q;
    q.push(edge(start, 0));

    while (!q.empty()) {
        int node = q.top().to;
        long long dist = q.top().w;
        q.pop();

        if (dist > d[node]) continue;

        for (int i = 0; i < (int)g[node].size(); i++) {
            int to = g[node][i].to;
            long long w_extra = g[node][i].w;

            if (dist + w_extra < d[to]) {
                p[to] = node;
                d[to] = dist + w_extra;
                q.push(edge(to, d[to]));
            }
        }
    }

    return {p, d};
}

```

## 5.5 directed mst

```

const int inf = 1000000 + 10;

struct edge {
    int u, v, w;
    edge() {}
    edge(int a, int b, int c) : u(a), v(b), w(c) {}
};

/**
 * Computes the minimum spanning tree for a directed graph
 * - edges : Graph description in the form of list of edges.
 *   each edge is: From node u to node v with cost w
 * - root : Id of the node to start the DMST.
 * - n     : Number of nodes in the graph.
 * */

int dmst(vector<edge> &edges, int root, int n) {
    int ans = 0;
    int cur_nodes = n;
    while (true) {
        vector<int> lo(cur_nodes, inf), pi(cur_nodes, inf);
        for (int i = 0; i < edges.size(); ++i) {
            int u = edges[i].u, v = edges[i].v, w = edges[i].w;
            if (w < lo[v] and u != v) {
                lo[v] = w;
                pi[v] = u;
            }
        }

        lo[root] = 0;
        for (int i = 0; i < lo.size(); ++i) {
            if (i == root) continue;
            if (lo[i] == inf) return -1;
        }

        int cur_id = 0;
        vector<int> id(cur_nodes, -1), mark(cur_nodes, -1);
        for (int i = 0; i < cur_nodes; ++i) {
            ans += lo[i];
            int u = i;
            while (u != root and id[u] < 0 and mark[u] != i) {
                mark[u] = i;
                u = pi[u];
            }
        }
    }
}

```

```

    if (u != root and id[u] < 0) { // Cycle
        for (int v = pi[u]; v != u; v = pi[v])
            id[v] = cur_id;
        id[u] = cur_id++;
    }
}

if (cur_id == 0)
    break;

for (int i = 0; i < cur_nodes; ++i)
    if (id[i] < 0) id[i] = cur_id++;

for (int i = 0; i < edges.size(); ++i) {
    int u = edges[i].u, v = edges[i].v, w = edges[i].w;
    edges[i].u = id[u];
    edges[i].v = id[v];
    if (id[u] != id[v])
        edges[i].w -= lo[v];
}

cur_nodes = cur_id;
root = id[root];
}

return ans;
}

```

## 5.6 eulerian path

// Taken from <https://github.com/lbv/pc-code/blob/master/code/graph.cpp>  
 // Eulerian Trail

```

struct Euler {
    ELV adj; IV t;
    Euler(ELV Adj) : adj(Adj) {}
    void build(int u) {
        while(! adj[u].empty()) {
            int v = adj[u].front().v;
            adj[u].erase(adj[u].begin());
            build(v);
        }
        t.push_back(u);
    }
}

```

```

};

bool eulerian_trail(IV &trail) {
    Euler e(adj);
    int odd = 0, s = 0;
    /*
        for (int v = 0; v < n; v++) {
            int diff = abs(in[v] - out[v]);
            if (diff > 1) return false;
            if (diff == 1) {
                if (++odd > 2) return false;
                if (out[v] > in[v]) start = v;
            }
        }
    */
    e.build(s);
    reverse(e.t.begin(), e.t.end());
    trail = e.t;
    return true;
}

```

## 5.7 karp min mean cycle

```

/**
 * Finds the min mean cycle, if you need the max mean cycle
 * just add all the edges with negative cost and print
 * ans * -1
 *
 * test: uva, 11090 - Going in Cycle!!
 * */

```

```

const int MN = 1000;
struct edge{
    int v;
    long long w;
    edge(){} edge(int v, int w) : v(v), w(w) {}
};

```

```

long long d[MN][MN];
// This is a copy of g because increments the size
// pass as reference if this does not matter.
int karp(vector<vector<edge>> > g) {
    int n = g.size();

```

```

g.resize(n + 1); // this is important

for (int i = 0; i < n; ++i)
    if (!g[i].empty())
        g[n].push_back(edge(i,0));
++n;

for(int i = 0; i < n; ++i)
    fill(d[i], d[i] + (n+1), INT_MAX);

d[n - 1][0] = 0;

for (int k = 1; k <= n; ++k) for (int u = 0; u < n; ++u) {
    if (d[u][k - 1] == INT_MAX) continue;
    for (int i = g[u].size() - 1; i >= 0; --i)
        d[g[u][i].v][k] = min(d[g[u][i].v][k], d[u][k - 1] + g[u][i].w);
}

bool flag = true;

for (int i = 0; i < n && flag; ++i)
    if (d[i][n] != INT_MAX)
        flag = false;

if (flag) {
    return true; // return true if there is no a cycle.
}

double ans = 1e15;

for (int u = 0; u + 1 < n; ++u) {
    if (d[u][n] == INT_MAX) continue;
    double W = -1e15;

    for (int k = 0; k < n; ++k)
        if (d[u][k] != INT_MAX)
            W = max(W, (double)(d[u][n] - d[u][k]) / (n - k));

    ans = min(ans, W);
}

// printf("%.2lf\n", ans);
cout << fixed << setprecision(2) << ans << endl;

return false;

```

```

}

```

## 5.8 konig's theorem

In any bipartite graph, the number of edges in a maximum matching equals the number of vertices in a minimum vertex cover

## 5.9 minimum path cover in DAG

Given a directed acyclic graph  $G = (V, E)$ , we are to find the minimum number of vertex-disjoint paths to cover each vertex in  $V$ .

We can construct a bipartite graph  $G' = (V_{out} \cup V_{in}, E')$  from  $G$ , where :

$$V_{out} = \{v \in V : v \text{ has positive out-degree}\}$$

$$V_{in} = \{v \in V : v \text{ has positive in-degree}\}$$

$$E' = \{(u, v) \in V_{out} \times V_{in} : (u, v) \in E\}$$

Then it can be shown, via König's theorem, that  $G'$  has a matching of size  $m$  if and only if there exists  $n - m$  vertex-disjoint paths that cover each vertex in  $G$ , where  $n$  is the number of vertices in  $G$  and  $m$  is the maximum cardinality bipartite matching in  $G'$ .

Therefore, the problem can be solved by finding the maximum cardinality matching in  $G'$  instead.

**NOTE:** If the paths are not necessarily disjoint, find the transitive closure and solve the problem for disjoint paths.

## 5.10 planar graph (euler)

Euler's formula states that if a finite, connected, planar graph is drawn in the plane without any edge intersections, and  $v$  is the number of vertices,  $e$  is the number of edges and  $f$  is the number of faces (regions bounded by edges, including the outer, infinitely large region), then:

$$f + v = e + 2$$

It can be extended to non connected planar graphs with  $c$  connected components:

$$f + v = e + c + 1$$



## 5.11 query with lca

```

struct lowest_ca {
    int T[MN], L[MN], W[MN];
    int P[MN][ML], MI[MN][ML], MA[MN][ML];

    void dfs(vector<vector<edge>> &g, int root, int pi = -1) {
        if (pi == -1) {
            L[root] = W[root] = 0;
            T[root] = -1;
        }
        for (int i = 0; i < (int)g[root].size(); ++i) {
            int to = g[root][i].v;
            if (to != pi) {
                T[to] = root;
                W[to] = g[root][i].w;
                L[to] = L[root] + 1;
                dfs(g, to, root);
            }
        }
    }

    void init(vector<vector<edge>> &g, int root) {
        // g is undirected
        dfs(g, root);
        int N = g.size(), i, j;

        for (i = 0; i < N; i++) {
            for (j = 0; 1 << j < N; j++) {
                P[i][j] = -1;
                MI[i][j] = inf;
            }
        }

        for (i = 0; i < N; i++) {
            P[i][0] = T[i];
            MI[i][0] = W[i];
        }

        for (j = 1; 1 << j < N; j++)
            for (i = 0; i < N; i++)
                if (P[i][j - 1] != -1) {
                    P[i][j] = P[P[i][j - 1]][j - 1];
                    MI[i][j] = min(MI[i][j - 1], MI[P[i][j - 1]][j - 1]);
                }
    }
}

```

```

}

int query(int p, int q) {
    int tmp, log, i;

    int mmin = inf;
    if (L[p] < L[q])
        tmp = p, p = q, q = tmp;

    for (log = 1; 1 << log <= L[p]; log++);
    log--;

    for (i = log; i >= 0; i--)
        if (L[p] - (1 << i) >= L[q]) {
            mmin = min(mmin, MI[p][i]);
            p = P[p][i];
        }

    if (p == q)
        // return p;
        return mmin;

    for (i = log; i >= 0; i--)
        if (P[p][i] != -1 && P[p][i] != P[q][i]) {
            mmin = min(mmin, min(MI[p][i], MI[q][i]));
            p = P[p][i], q = P[q][i];
        }

    // return T[p];
    return min(mmin, min(MI[p][0], MI[q][0]));
}

int get_child(int p, int q) { // p is ancestor of q
    if (p == q) return -1;

    int i, log;
    for (log = 1; 1 << log <= L[q]; log++) {}
    log--;

    for (i = log; i >= 0; i--)
        if (L[q] - (1 << i) > L[p]) {
            q = P[q][i];
        }

    assert(P[q][0] == p);
}

```

```

    return q;
}

int is_ancestor(int p, int q) {
    if (L[p] >= L[q])
        return false;

    int dist = L[q] - L[p];

    int cur = q;
    int step = 0;
    while (dist) {
        if (dist & 1)
            cur = P[cur][step];
        step++;
        dist >>= 1;
    }

    return cur == p;
}
};

```

## 5.12 tarjan scc

```

const int MN = 20002;

struct tarjan_scc {
    int scc[MN], low[MN], d[MN], stacked[MN];
    int ticks, current_scc;
    deque<int> s; // used as stack.

    tarjan_scc() {}

    void init () {
        memset(scc, -1, sizeof scc);
        memset(d, -1, sizeof d);
        memset(stacked, 0, sizeof stacked);
        s.clear();
        ticks = current_scc = 0;
    }

    void compute(vector<vector<int> > &g, int u) {
        d[u] = low[u] = ticks++;

```

```

        s.push_back(u);
        stacked[u] = true;
        for (int i = 0; i < g[u].size(); ++i) {
            int v = g[u][i];
            if (d[v] == -1)
                compute(g, v);
            if (stacked[v]) {
                low[u] = min(low[u], low[v]);
            }
        }

        if (d[u] == low[u]) { // root
            int v;
            do {
                v = s.back(); s.pop_back();
                stacked[v] = false;
                scc[v] = current_scc;
            } while (u != v);
            current_scc++;
        }
    }
};

```

## 5.13 two sat (with kosaraju)

```

/**
 * Given a set of clauses (a1 v a2)^(a2 v a3)....
 * this algorithm find a solution to it set of clauses.
 * test: http://lightoj.com/volume\_showproblem.php?problem=1251
 */

#include<bits/stdc++.h>
using namespace std;
#define MAX 100000
#define endl '\n'

vector<int> G[MAX];
vector<int> GT[MAX];
vector<int> Ftime;
vector<vector<int> > SCC;
bool visited[MAX];
int n;

```

```

void dfs1(int n){
    visited[n] = 1;

    for (int i = 0; i < G[n].size(); ++i) {
        int curr = G[n][i];
        if (visited[curr]) continue;
        dfs1(curr);
    }

    Ftime.push_back(n);
}

void dfs2(int n, vector<int> &scc) {
    visited[n] = 1;
    scc.push_back(n);

    for (int i = 0; i < GT[n].size(); ++i) {
        int curr = GT[n][i];
        if (visited[curr]) continue;
        dfs2(curr, scc);
    }
}

void kosaraju() {
    memset(visited, 0, sizeof visited);

    for (int i = 0; i < 2 * n ; ++i) {
        if (!visited[i]) dfs1(i);
    }

    memset(visited, 0, sizeof visited);
    for (int i = Ftime.size() - 1; i >= 0; i--) {
        if (visited[Ftime[i]]) continue;
        vector<int> _scc;
        dfs2(Ftime[i], _scc);
        SCC.push_back(_scc);
    }
}

/**
 * After having the SCC, we must traverse each scc, if in one SCC are -b
 * y b, there is not a solution.

```

```

* Otherwise we build a solution, making the first "node" that we find
* truth and its complement false.
**/

```

```

bool two_sat(vector<int> &val) {
    kosaraju();
    for (int i = 0; i < SCC.size(); ++i) {
        vector<bool> tmpvisited(2 * n, false);
        for (int j = 0; j < SCC[i].size(); ++j) {
            if (tmpvisited[SCC[i][j] ^ 1]) return 0;
            if (val[SCC[i][j]] != -1) continue;
            else {
                val[SCC[i][j]] = 0;
                val[SCC[i][j] ^ 1] = 1;
            }
            tmpvisited[SCC[i][j]] = 1;
        }
    }
    return 1;
}

```

// Example of use

```

int main() {

    int m, u, v, nc = 0, t; cin >> t;
    // n = "nodes" number, m = clauses number

    while (t--) {
        cin >> m >> n;
        Ftime.clear();
        SCC.clear();
        for (int i = 0; i < 2 * n; ++i) {
            G[i].clear();
            GT[i].clear();
        }

        // (a1 v a2) = (a1 -> a2) = (a2 -> a1)
        for (int i = 0; i < m ; ++i) {
            cin >> u >> v;
            int t1 = abs(u) - 1;
            int t2 = abs(v) - 1;
            int p = t1 * 2 + ((u < 0)? 1 : 0);
            int q = t2 * 2 + ((v < 0)? 1 : 0);

```

```

    G[p ^ 1].push_back(q);
    G[q ^ 1].push_back(p);
    GT[p].push_back(q ^ 1);
    GT[q].push_back(p ^ 1);
}

vector<int> val(2 * n, -1);
cout << "Case " << ++nc << ": ";
if (two_sat(val)) {
    cout << "Yes" << endl;
    vector<int> sol;
    for (int i = 0; i < 2 * n; ++i)
        if (i % 2 == 0 and val[i] == 1)
            sol.push_back(i / 2 + 1);
    cout << sol.size() ;

    for (int i = 0; i < sol.size(); ++i) {
        cout << " " << sol[i];
    }
    cout << endl;
} else {
    cout << "No" << endl;
}
}
return 0;
}

```

## 6 Math

### 6.1 Lucas theorem

For non-negative integers  $m$  and  $n$  and a prime  $p$ , the following congruence relation holds :

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p},$$

where :

$$m = m_k p^k + m_{k-1} p^{k-1} + \dots + m_1 p + m_0,$$

and :

$$n = n_k p^k + n_{k-1} p^{k-1} + \dots + n_1 p + n_0$$

are the base  $p$  expansions of  $m$  and  $n$  respectively. This uses the convention that  $\binom{m}{n} = 0$  if  $m < n$ .

### 6.2 counting

```

const int MN = 1e5 + 100;
long long fact[MN];

void fill_fact() {
    fact[0] = 1;
    for (int i = 1; i < MN; i++) {
        fact[i] = mult(fact[i - 1], i);
    }
}

long long perm_rep(vector<int> &frec) {
    int total = 0;
    long long den = 1;
    for (int i = 0; i < (int)frec.size(); i++) {
        den = mult(den, mod_inv(fact[frec[i]]));
        total += frec[i];
    }
    return mult(fact[total], den);
}

```

### 6.3 cumulative sum of divisors

/\*\*  
The function SOD(n) (sum of divisors) is defined  
as the summation of all the actual divisors of  
an integer number n. For example,

$$\text{SOD}(24) = 2+3+4+6+8+12 = 35.$$

The function CSOD(n) (cumulative SOD) of an integer n, is defined as  
below:

$$\text{csod}(n) = \sum_{i=1}^n \text{sod}(i)$$

It can be computed in  $O(\sqrt{n})$ :

\*/

```

long long csod(long long n) {
    long long ans = 0;
    for (long long i = 2; i * i <= n; ++i) {
        long long j = n / i;

```

```

    ans += (i + j) * (j - i + 1) / 2;
    ans += i * (j - i);
}
return ans;
}

```

---

## 6.4 fft

---

```

/**
 * Fast Fourier Transform.
 * Useful to compute convolutions.
 * computes:
 *   C(f star g)[n] = sum_m(f[m] * g[n - m])
 * for all n.
 * test: icpc live archive, 6886 - Golf Bot
 * */

using namespace std;
#include<bits/stdc++.h>
#define D(x) cout << #x " = " << (x) << endl
#define endl '\n'

const int MN = 262144 << 1;
int d[MN + 10], d2[MN + 10];

const double PI = acos(-1.0);

struct cpx {
    double real, image;
    cpx(double _real, double _image) {
        real = _real;
        image = _image;
    }
    cpx(){}
};

cpx operator + (const cpx &c1, const cpx &c2) {
    return cpx(c1.real + c2.real, c1.image + c2.image);
}

cpx operator - (const cpx &c1, const cpx &c2) {

```

```

    return cpx(c1.real - c2.real, c1.image - c2.image);
}

cpx operator * (const cpx &c1, const cpx &c2) {
    return cpx(c1.real*c2.real - c1.image*c2.image, c1.real*c2.image +
        c1.image*c2.real);
}

int rev(int id, int len) {
    int ret = 0;
    for (int i = 0; (1 << i) < len; i++) {
        ret <= 1;
        if (id & (1 << i)) ret |= 1;
    }
    return ret;
}

cpx A[1 << 20];

void FFT(cpx *a, int len, int DFT) {
    for (int i = 0; i < len; i++)
        A[rev(i, len)] = a[i];
    for (int s = 1; (1 << s) <= len; s++) {
        int m = (1 << s);
        cpx wm = cpx(cos( DFT * 2 * PI / m), sin(DFT * 2 * PI / m));
        for(int k = 0; k < len; k += m) {
            cpx w = cpx(1, 0);
            for(int j = 0; j < (m >> 1); j++) {
                cpx t = w * A[k + j + (m >> 1)];
                cpx u = A[k + j];
                A[k + j] = u + t;
                A[k + j + (m >> 1)] = u - t;
                w = w * wm;
            }
        }
    }
    if (DFT == -1) for (int i = 0; i < len; i++) A[i].real /= len,
        A[i].image /= len;
    for (int i = 0; i < len; i++) a[i] = A[i];
    return;
}

cpx in[1 << 20];

void solve(int n) {

```

```

memset(d, 0, sizeof d);
int t;
for (int i = 0; i < n; ++i) {
    cin >> t;
    d[t] = true;
}
int m;
cin >> m;
vector<int> q(m);
for (int i = 0; i < m; ++i)
    cin >> q[i];

for (int i = 0; i < MN; ++i) {
    if (d[i])
        in[i] = cpx(1, 0);
    else
        in[i] = cpx(0, 0);
}

FFT(in, MN, 1);
for (int i = 0; i < MN; ++i) {
    in[i] = in[i] * in[i];
}
FFT(in, MN, -1);

int ans = 0;
for (int i = 0; i < q.size(); ++i) {
    if (in[q[i]].real > 0.5 || d[q[i]]) {
        ans++;
    }
}
cout << ans << endl;
}

int main() {
    ios_base::sync_with_stdio(false); cin.tie(NULL);
    int n;
    while (cin >> n)
        solve(n);
    return 0;
}

```

## 6.5 fibonacci properties

Let A, B and n be integer numbers.

$$k = A - B \quad (1)$$

$$F_A F_B = F_{k+1} F_A^2 + F_k F_A F_{A-1} \quad (2)$$

$$\sum_{i=0}^n F_i^2 = F_{n+1} F_n \quad (3)$$

$ev(n)$  = returns 1 if  $n$  is even.

$$\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - ev(n) \quad (4)$$

$$\sum_{i=0}^n F_i F_{i-1} = \sum_{i=0}^{n-1} F_i F_{i+1} \quad (5)$$

## 6.6 polynomials

```

const double pi = acos(-1);
struct poly {
    deque<double> coef;
    double x_lo, x_hi;

    double evaluate(double x) {
        double ans = 0;
        for (auto it : coef)
            ans = (ans * x + it);
        return ans;
    }

    double volume(double x, double dx=1e-6) {
        dx = (x_hi - x_lo) / 1000000.0;
        double ans = 0;
        for (double ix = x_lo; ix <= x; ix += dx) {
            double rad = evaluate(ix);
            ans += pi * rad * rad * dx;
        }
        return ans;
    }
}

```

```
};
```

---

## 6.7 sigma function

the sigma function is defined as:

$$\sigma_x(n) = \sum_{d|n} d^x$$

when  $x = 0$  is called the divisor function, that counts the number of positive divisors of  $n$ .

Now, we are interested in find

$$\sum_{d|n} \sigma_0(d)$$

if  $n$  is written as prime factorization:

$$n = \prod_{i=1}^k P_i^{e_k}$$

we can demonstrate that:

$$\sum_{d|n} \sigma_0(d) = \prod_{i=1}^k g(e_k + 1)$$

where  $g(x)$  is the sum of the first  $x$  positive numbers:

$$g(x) = (x * (x + 1)) / 2$$

## 7 Matrix

### 7.1 matrix

```
const int MN = 111;
const int mod = 10000;
```

```
struct matrix {
    int r, c;
    int m[MN][MN];
```

```
    matrix(int _r, int _c) : r(_r), c(_c) {
```

```
        memset(m, 0, sizeof m);
    }
```

```
void print() {
    for (int i = 0; i < r; ++i) {
        for (int j = 0; j < c; ++j)
            cout << m[i][j] << " ";
        cout << endl;
    }
}
```

```
int x[MN][MN];
matrix & operator *= (const matrix &o) {
    memset(x, 0, sizeof x);
    for (int i = 0; i < r; ++i)
        for (int k = 0; k < c; ++k)
            if (m[i][k] != 0)
                for (int j = 0; j < c; ++j) {
                    x[i][j] = (x[i][j] + (m[i][k] * o.m[k][j]) % mod) % mod;
                }
    memcpy(m, x, sizeof(m));
    return *this;
}
};
```

```
void matrix_pow(matrix b, long long e, matrix &res) {
    memset(res.m, 0, sizeof res.m);
    for (int i = 0; i < b.r; ++i)
        res.m[i][i] = 1;

    if (e == 0) return;
    while (true) {
        if (e & 1) res *= b;
        if ((e >>= 1) == 0) break;
        b *= b;
    }
}
```

---

## 8 Misc

### 8.1 Template Java

---

```

import java.io.*;
import java.util.StringTokenizer;

public class Template {

    public static void main(String []args) throws IOException {
        Scanner in = new Scanner(System.in);
        OutputStreamWriter out = new OutputStreamWriter(System.out);
        Task solver = new Task();
        solver.solve(in, out);
        out.close();
    }
}

class Task{
    public void solve(Scanner in, OutputStreamWriter out){

    }
}

class Scanner{
    public BufferedReader reader;
    public StringTokenizer st;

    public Scanner(InputStream stream){
        reader = new BufferedReader(new InputStreamReader(stream));
        st = null;
    }

    public String next(){
        while(st == null || !st.hasMoreTokens()){
            try{
                String line = reader.readLine();
                if(line == null) return null;
                st = new StringTokenizer(line);
            }catch (Exception e){
                throw (new RuntimeException());
            }
        }
        return st.nextToken();
    }

    public int nextInt(){

```

```

        return Integer.parseInt(next());
    }
    public long nextLong(){
        return Long.parseLong(next());
    }
    public double nextDouble(){
        return Double.parseDouble(next());
    }
}

class OutputStreamWriter{
    BufferedWriter writer;

    public OutputStreamWriter(OutputStream stream){
        writer = new BufferedWriter(new OutputStreamWriter(stream));
    }

    public void print(int i) throws IOException {
        writer.write(i);
    }

    public void print(String s) throws IOException {
        writer.write(s);
    }

    public void print(char []c) throws IOException {
        writer.write(c);
    }
    public void close() throws IOException {
        writer.close();
    }
}

```

---

## 8.2 dates

---

```

//
// Time - Leap years
//

// A[i] has the accumulated number of days from months previous to i
const int A[13] = { 0, 0, 31, 59, 90, 120, 151, 181, 212, 243, 273, 304,
334 };

```



```
// same as A, but for a leap year
const int B[13] = { 0, 0, 31, 60, 91, 121, 152, 182, 213, 244, 274, 305,
335 };
// returns number of leap years up to, and including, y
int leap_years(int y) { return y / 4 - y / 100 + y / 400; }
bool is_leap(int y) { return y % 400 == 0 || (y % 4 == 0 && y % 100 !=
0); }
// number of days in blocks of years
const int p400 = 400*365 + leap_years(400);
const int p100 = 100*365 + leap_years(100);
const int p4 = 4*365 + 1;
const int p1 = 365;
int date_to_days(int d, int m, int y)
{
    return (y - 1) * 365 + leap_years(y - 1) + (is_leap(y) ? B[m] : A[m]) +
        d;
}
void days_to_date(int days, int &d, int &m, int &y)
{
    bool top100; // are we in the top 100 years of a 400 block?
    bool top4;   // are we in the top 4 years of a 100 block?
    bool top1;   // are we in the top year of a 4 block?

    y = 1;
    top100 = top4 = top1 = false;

    y += ((days-1) / p400) * 400;
    d = (days-1) % p400 + 1;

    if (d > p100*3) top100 = true, d -= 3*p100, y += 300;
    else y += ((d-1) / p100) * 100, d = (d-1) % p100 + 1;

    if (d > p4*24) top4 = true, d -= 24*p4, y += 24*4;
    else y += ((d-1) / p4) * 4, d = (d-1) % p4 + 1;

    if (d > p1*3) top1 = true, d -= p1*3, y += 3;
    else y += (d-1) / p1, d = (d-1) % p1 + 1;

    const int *ac = top1 && (!top4 || top100) ? B : A;
    for (m = 1; m < 12; ++m) if (d <= ac[m + 1]) break;
    d -= ac[m];
}
```

## 8.3 fraction

---

```
struct frac{
    long long x, y;
    frac(long long a, long long b) {
        long long g = __gcd(a, b);
        x = a / g;
        y = b / g;
    }
    bool operator < (const frac &o) const {
        return (x * o.y < y * o.x);
    }
};
```

---

## 8.4 io

---

```
// taken from :
https://github.com/lbv/pc-code/blob/master/solved/c-e/diablo/diablo.cpp
// this is very fast as well :
https://github.com/lbv/pc-code/blob/master/code/input.cpp

typedef unsigned int u32;
#define BUF 524288
struct Reader {
    char buf[BUF]; char b; int bi, bz;
    Reader() { bi=bz=0; read(); }
    void read() {
        if (bi==bz) { bi=0; bz = fread(buf, 1, BUF, stdin); }
        b = bz ? buf[bi++] : 0; }
    void skip() { while (b > 0 && b <= 32) read(); }
    u32 next_u32() {
        u32 v = 0; for (skip(); b > 32; read()) v = v*10 + b-48; return v; }
    int next_int() {
        int v = 0; bool s = false;
        skip(); if (b == '-') { s = true; read(); }
        for (; 48<=b&&b<=57; read()) v = v*10 + b-48; return s ? -v : v; }
    char next_char() { skip(); char c = b; read(); return c; }
};
```

---

## 9 Number theory

### 9.1 convolution

---

```
typedef long long int LL;
typedef pair<LL, LL> PLL;

inline bool is_pow2(LL x) {
    return (x & (x-1)) == 0;
}

inline int ceil_log2(LL x) {
    int ans = 0;
    --x;
    while (x != 0) {
        x >>= 1;
        ans++;
    }
    return ans;
}

/* Returns the convolution of the two given vectors in time proportional
   to n*log(n).
   * The number of roots of unity to use nroots_unity must be set so that
   the product of the first
   * nroots_unity primes of the vector nth_roots_unity is greater than the
   maximum value of the
   * convolution. Never use sizes of vectors bigger than 2^24, if you need
   to change the values of
   * the nth roots of unity to appropriate primes for those sizes.
   */
vector<LL> convolve(const vector<LL> &a, const vector<LL> &b, int
    nroots_unity = 2) {
    int N = 1 << ceil_log2(a.size() + b.size());
    vector<LL> ans(N,0), fA(N), fB(N), fC(N);
    LL modulo = 1;
    for (int times = 0; times < nroots_unity; times++) {
        fill(fA.begin(), fA.end(), 0);
        fill(fB.begin(), fB.end(), 0);
        for (int i = 0; i < a.size(); i++) fA[i] = a[i];
        for (int i = 0; i < b.size(); i++) fB[i] = b[i];
        LL prime = nth_roots_unity[times].first;
        LL inv_modulo = mod_inv(modulo % prime, prime);
        LL normalize = mod_inv(N, prime);
```

```
        ntfft(fA, 1, nth_roots_unity[times]);
        ntfft(fB, 1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++) fC[i] = (fA[i] * fB[i]) % prime;
        ntfft(fC, -1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++) {
            LL curr = (fC[i] * normalize) % prime;
            LL k = (curr - (ans[i] % prime) + prime) % prime;
            k = (k * inv_modulo) % prime;
            ans[i] += modulo * k;
        }
        modulo *= prime;
    }
    return ans;
}
```

---

### 9.2 crt

---

```
/**
 * Chinese remainder theorem.
 * Find z such that z % x[i] = a[i] for all i.
 */
long long crt(vector<long long> &a, vector<long long> &x) {
    long long z = 0;
    long long n = 1;
    for (int i = 0; i < x.size(); ++i)
        n *= x[i];

    for (int i = 0; i < a.size(); ++i) {
        long long tmp = (a[i] * (n / x[i])) % n;
        tmp = (tmp * mod_inv(n / x[i], x[i])) % n;
        z = (z + tmp) % n;
    }

    return (z + n) % n;
}
```

---

### 9.3 diophantine equations

---

```
long long gcd(long long a, long long b, long long &x, long long &y) {
    if (a == 0) {
        x = 0;
```

```

    y = 1;
    return b;
}
long long x1, y1;
long long d = gcd(b % a, a, x1, y1);
x = y1 - (b / a) * x1;
y = x1;
return d;
}

bool find_any_solution(long long a, long long b, long long c, long long
    &x0,
    long long &y0, long long &g) {
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % g) {
        return false;
    }

    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}

void shift_solution(long long &x, long long &y, long long a, long long b,
    long long cnt) {
    x += cnt * b;
    y -= cnt * a;
}

long long find_all_solutions(long long a, long long b, long long c,
    long long minx, long long maxx, long long miny,
    long long maxy) {
    long long x, y, g;
    if (!find_any_solution(a, b, c, x, y, g)) return 0;
    a /= g;
    b /= g;

    long long sign_a = a > 0 ? +1 : -1;
    long long sign_b = b > 0 ? +1 : -1;

    shift_solution(x, y, a, b, (minx - x) / b);
    if (x < minx) shift_solution(x, y, a, b, sign_b);
    if (x > maxx) return 0;

```

```

    long long lx1 = x;

    shift_solution(x, y, a, b, (maxx - x) / b);
    if (x > maxx) shift_solution(x, y, a, b, -sign_b);
    long long rx1 = x;

    shift_solution(x, y, a, b, -(miny - y) / a);
    if (y < miny) shift_solution(x, y, a, b, -sign_a);
    if (y > maxy) return 0;
    long long lx2 = x;

    shift_solution(x, y, a, b, -(maxy - y) / a);
    if (y > maxy) shift_solution(x, y, a, b, sign_a);
    long long rx2 = x;

    if (lx2 > rx2) swap(lx2, rx2);
    long long lx = max(lx1, lx2);
    long long rx = min(rx1, rx2);

    if (lx > rx) return 0;
    return (rx - lx) / abs(b) + 1;
}

```

## 9.4 discrete logarithm

---

```

// Computes x which  $a^x = b \pmod n$ .

```

```

long long d_log(long long a, long long b, long long n) {
    long long m = ceil(sqrt(n));
    long long aj = 1;
    map<long long, long long> M;
    for (int i = 0; i < m; ++i) {
        if (!M.count(aj))
            M[aj] = i;
        aj = (aj * a) % n;
    }

    long long coef = mod_pow(a, n - 2, n);
    coef = mod_pow(coef, m, n);
    // coef =  $a^{-m}$ 
    long long gamma = b;
    for (int i = 0; i < m; ++i) {

```

```

    if (M.count(gamma)) {
        return i * m + M[gamma];
    } else {
        gamma = (gamma * coef) % n;
    }
}
return -1;
}

```

---

## 9.5 ext euclidean

```

void ext_euclid(long long a, long long b, long long &x, long long &y,
    long long &g) {
    x = 0, y = 1, g = b;
    long long m, n, q, r;
    for (long long u = 1, v = 0; a != 0; g = a, a = r) {
        q = g / a, r = g % a;
        m = x - u * q, n = y - v * q;
        x = u, y = v, u = m, v = n;
    }
}

```

---

## 9.6 highest exponent factorial

```

int highest_exponent(int p, const int &n){
    int ans = 0;
    int t = p;
    while(t <= n){
        ans += n/t;
        t*=p;
    }
    return ans;
}

```

---

## 9.7 miller rabin

```

const int rounds = 20;

// checks whether a is a witness that n is not prime, 1 < a < n

```

---

```

bool witness(long long a, long long n) {
    // check as in Miller Rabin Primality Test described
    long long u = n - 1;
    int t = 0;
    while (u % 2 == 0) {
        t++;
        u >>= 1;
    }
    long long next = mod_pow(a, u, n);
    if (next == 1) return false;
    long long last;
    for (int i = 0; i < t; ++i) {
        last = next;
        next = mod_mul(last, last, n);
        if (next == 1) {
            return last != n - 1;
        }
    }
    return next != 1;
}

```

```

// Checks if a number is prime with prob 1 - 1 / (2 ^ it)
// D(miller_rabin(9999999999999997LL) == 1);
// D(miller_rabin(9999999999971LL) == 1);
// D(miller_rabin(7907) == 1);
bool miller_rabin(long long n, int it = rounds) {
    if (n <= 1) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    for (int i = 0; i < it; ++i) {
        long long a = rand() % (n - 1) + 1;
        if (witness(a, n)) {
            return false;
        }
    }
    return true;
}

```

---

## 9.8 mod integer

```

template<class T, T mod>
struct mint_t {

```

---

```

T val;
mint_t() : val(0) {}
mint_t(T v) : val(v % mod) {}

mint_t operator + (const mint_t& o) const {
    return (val + o.val) % mod;
}
mint_t operator - (const mint_t& o) const {
    return (val - o.val) % mod;
}
mint_t operator * (const mint_t& o) const {
    return (val * o.val) % mod;
}
};

```

```
typedef mint_t<long long, 998244353> mint;
```

## 9.9 mod inv

```

long long mod_inv(long long n, long long m) {
    long long x, y, gcd;
    ext_euclid(n, m, x, y, gcd);
    if (gcd != 1)
        return 0;
    return (x + m) % m;
}

```

## 9.10 mod mul

```

// Computes (a * b) % mod
long long mod_mul(long long a, long long b, long long mod) {
    long long x = 0, y = a % mod;
    while (b > 0) {
        if (b & 1)
            x = (x + y) % mod;
        y = (y * 2) % mod;
        b /= 2;
    }
    return x % mod;
}

```

## 9.11 mod pow

```

// Computes ( a ^ exp ) % mod.
long long mod_pow(long long a, long long exp, long long mod) {
    long long ans = 1;
    while (exp > 0) {
        if (exp & 1)
            ans = mod_mul(ans, a, mod);
        a = mod_mul(a, a, mod);
        exp >>= 1;
    }
    return ans;
}

```

## 9.12 number theoretic transform

```

typedef long long int LL;
typedef pair<LL, LL> PLL;

```

/\* The following vector of pairs contains pairs (prime, generator)  
 \* where the prime has an Nth root of unity for N being a power of two.  
 \* The generator is a number g s.t  $g^{p-1} \equiv 1 \pmod{p}$   
 \* but is different from 1 for all smaller powers \*/

```

vector<PLL> nth_roots_unity {
    {1224736769, 330732430}, {1711276033, 927759239}, {167772161, 167489322},
    {469762049, 343261969}, {754974721, 643797295}, {1107296257, 883865065}};

```

```

PLL ext_euclid(LL a, LL b) {
    if (b == 0)
        return make_pair(1, 0);
    pair<LL, LL> rc = ext_euclid(b, a % b);
    return make_pair(rc.second, rc.first - (a / b) * rc.second);
}

```

```

//returns -1 if there is no unique modular inverse
LL mod_inv(LL x, LL modulo) {
    PLL p = ext_euclid(x, modulo);
    if ( (p.first * x + p.second * modulo) != 1 )
        return -1;
    return (p.first + modulo) % modulo;
}

```

```
//Number theory fft. The size of a must be a power of 2
void ntfft(vector<LL> &a, int dir, const PLL &root_unity) {
    int n = a.size();
    LL prime = root_unity.first;
    LL basew = mod_pow(root_unity.second, (prime-1) / n, prime);
    if (dir < 0) basew = mod_inv(basew, prime);
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        LL w = 1;
        for (int i = 0; i < mh; i++) {
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                LL x = (a[j] - a[k] + prime) % prime;
                a[j] = (a[j] + a[k]) % prime;
                a[k] = (w * x) % prime;
            }
            w = (w * basew) % prime;
        }
        basew = (basew * basew) % prime;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^= k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
}
```

### 9.13 pollard rho factorize

```
long long pollard_rho(long long n) {
    long long x, y, i = 1, k = 2, d;
    x = y = rand() % n;
    while (1) {
        ++i;
        x = mod_mul(x, x, n);
        x += 2;
        if (x >= n) x -= n;
        if (x == y) return 1;
        d = __gcd(abs(x - y), n);
        if (d != 1) return d;
        if (i == k) {
            y = x;
            k *= 2;
        }
    }
}
```

```
    }
}
return 1;
}

// Returns a list with the prime divisors of n
vector<long long> factorize(long long n) {
    vector<long long> ans;
    if (n == 1)
        return ans;
    if (miller_rabin(n)) {
        ans.push_back(n);
    } else {
        long long d = 1;
        while (d == 1)
            d = pollard_rho(n);
        vector<long long> dd = factorize(d);
        ans = factorize(n / d);
        for (int i = 0; i < dd.size(); ++i)
            ans.push_back(dd[i]);
    }
    return ans;
}
```

### 9.14 primes

```
namespace primes {
    const int MP = 100001;
    bool sieve[MP];
    long long primes[MP];
    int num_p;
    void fill_sieve() {
        num_p = 0;
        sieve[0] = sieve[1] = true;
        for (long long i = 2; i < MP; ++i) {
            if (!sieve[i]) {
                primes[num_p++] = i;
                for (long long j = i * i; j < MP; j += i)
                    sieve[j] = true;
            }
        }
    }
}
```

```

// Finds prime numbers between a and b, using basic primes up to sqrt(b)
// a must be greater than 1.
vector<long long> seg_sieve(long long a, long long b) {
    long long ant = a;
    a = max(a, 3LL);
    vector<bool> pmap(b - a + 1);
    long long sqrt_b = sqrt(b);
    for (int i = 0; i < num_p; ++i) {
        long long p = primes[i];
        if (p > sqrt_b) break;
        long long j = (a + p - 1) / p;
        for (long long v = (j == 1) ? p + p : j * p; v <= b; v += p) {
            pmap[v - a] = true;
        }
    }
    vector<long long> ans;
    if (ant == 2) ans.push_back(2);
    int start = a % 2 ? 0 : 1;
    for (int i = start, I = b - a + 1; i < I; i += 2)
        if (pmap[i] == false)
            ans.push_back(a + i);
    return ans;
}

vector<pair<int, int>> factor(int n) {
    vector<pair<int, int>> ans;
    if (n == 0) return ans;
    for (int i = 0; primes[i] * primes[i] <= n; ++i) {
        if ((n % primes[i]) == 0) {
            int expo = 0;
            while ((n % primes[i]) == 0) {
                expo++;
                n /= primes[i];
            }
            ans.emplace_back(primes[i], expo);
        }
    }

    if (n > 1) {
        ans.emplace_back(n, 1);
    }
    return ans;
}
}

```

## 9.15 totient sieve

---

```

for (int i = 1; i < MN; i++)
    phi[i] = i;

for (int i = 1; i < MN; i++)
    if (!sieve[i]) // is prime
        for (int j = i; j < MN; j += i)
            phi[j] -= phi[j] / i;

```

---

## 9.16 totient

---

```

long long totient(long long n) {
    if (n == 1) return 0;
    long long ans = n;
    for (int i = 0; primes[i] * primes[i] <= n; ++i) {
        if ((n % primes[i]) == 0) {
            while ((n % primes[i]) == 0) n /= primes[i];
            ans -= ans / primes[i];
        }
    }
    if (n > 1) {
        ans -= ans / n;
    }
    return ans;
}

```

---

# 10 Strings

## 10.1 Incremental Aho Corasick

---

```

class IncrementalAhoCorasick {
    static const int Alphabets = 26;
    static const int AlphabetBase = 'a';
    struct Node {
        Node *fail;
        Node *next[Alphabets];
        int sum;
        Node() : fail(NULL), next={}, sum(0) {}
    };
};

```

```

struct String {
    string str;
    int sign;
};

public:
    //totalLen = sum of (len + 1)
    void init(int totalLen) {
        nodes.resize(totalLen);
        nNodes = 0;
        strings.clear();
        roots.clear();
        sizes.clear();
        que.resize(totalLen);
    }

    void insert(const string &str, int sign) {
        strings.push_back(String{ str, sign });
        roots.push_back(nodes.data() + nNodes);
        sizes.push_back(1);
        nNodes += (int)str.size() + 1;
        auto check = [&]() { return sizes.size() > 1 && sizes.end()[-1] ==
            sizes.end()[-2]; };
        if(!check())
            makePMA(strings.end() - 1, strings.end(), roots.back(), que);
        while(check()) {
            int m = sizes.back();
            roots.pop_back();
            sizes.pop_back();
            sizes.back() += m;
            if(!check())
                makePMA(strings.end() - m * 2, strings.end(), roots.back(), que);
        }
    }

    int match(const string &str) const {
        int res = 0;
        for(const Node *t : roots)
            res += matchPMA(t, str);
        return res;
    }

private:

```

```

static void makePMA(vector<String>::const_iterator begin,
    vector<String>::const_iterator end, Node *nodes, vector<Node*>
    &que) {
    int nNodes = 0;
    Node *root = new(&nodes[nNodes++]) Node();
    for(auto it = begin; it != end; ++it) {
        Node *t = root;
        for(char c : it->str) {
            Node *&n = t->next[c - AlphabetBase];
            if(n == nullptr)
                n = new(&nodes[nNodes++]) Node();
            t = n;
        }
        t->sum += it->sign;
    }
    int qt = 0;
    for(Node *&n : root->next) {
        if(n != nullptr) {
            n->fail = root;
            que[qt++] = n;
        } else {
            n = root;
        }
    }
    for(int qh = 0; qh != qt; ++qh) {
        Node *t = que[qh];
        int a = 0;
        for(Node *n : t->next) {
            if(n != nullptr) {
                que[qt++] = n;
                Node *r = t->fail;
                while(r->next[a] == nullptr)
                    r = r->fail;
                n->fail = r->next[a];
                n->sum += r->next[a]->sum;
            }
            ++a;
        }
    }
}

static int matchPMA(const Node *t, const string &str) {
    int res = 0;
    for(char c : str) {
        int a = c - AlphabetBase;

```



```

    while(t->next[a] == nullptr)
        t = t->fail;
    t = t->next[a];
    res += t->sum;
}
return res;
}

vector<Node> nodes;
int nNodes;
vector<String> strings;
vector<Node*> roots;
vector<int> sizes;
vector<Node*> que;
};

int main() {
    int m;
    while(~scanf("%d", &m)) {
        IncrementalAhoCorasic iac;
        iac.init(600000);
        rep(i, m) {
            int ty;
            char s[300001];
            scanf("%d%s", &ty, s);
            if(ty == 1) {
                iac.insert(s, +1);
            } else if(ty == 2) {
                iac.insert(s, -1);
            } else if(ty == 3) {
                int ans = iac.match(s);
                printf("%d\n", ans);
                fflush(stdout);
            } else {
                abort();
            }
        }
    }
    return 0;
}

```

## 10.2 minimal string rotation

---

```
// Lexicographically minimal string rotation
```

```

int lmsr() {
    string s;
    cin >> s;
    int n = s.size();
    s += s;
    vector<int> f(s.size(), -1);
    int k = 0;
    for (int j = 1; j < 2 * n; ++j) {
        int i = f[j - k - 1];
        while (i != -1 && s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1])
                k = j - i - 1;
            i = f[i];
        }
        if (i == -1 && s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1]) {
                k = j;
            }
            f[j - k] = -1;
        } else {
            f[j - k] = i + 1;
        }
    }
    return k;
}

```

---

## 10.3 suffix array

---

```

/**
 * O(n log^2(n))
 * See http://web.stanford.edu/class/cs97si/suffix-array.pdf for reference
 */

struct entry{
    int a, b, p;
    entry(){}
    entry(int x, int y, int z): a(x), b(y), p(z){}
    bool operator < (const entry &o) const {
        return (a == o.a) ? (b == o.b) ? (p < o.p) : (b < o.b) : (a < o.a);
    }
};

```

```

struct SuffixArray{
    const int N;
    string s;
    vector<vector<int> > P;
    vector<entry> M;

    SuffixArray(const string &s) : N(s.length()) , s(s), P(1, vector<int>
        (N, 0)), M(N) {
        for (int i = 0; i < N; ++i)
            P[0][i] = (int) s[i];

        for (int skip = 1, level = 1; skip < N; skip *= 2, level++) {
            P.push_back(vector<int>(N, 0));
            for (int i = 0 ; i < N; ++i) {
                int next = ((i + skip) < N) ? P[level - 1][i + skip] : -10000;
                M[i] = entry(P[level - 1][i], next, i);
            }
            sort(M.begin(), M.end());
            for (int i = 0; i < N; ++i)
                P[level][M[i].p] = (i > 0 and M[i].a == M[i - 1].a and M[i].b ==
                    M[i - 1].b) ? P[level][M[i - 1].p] : i;
        }
    }

    vector<int> getSuffixArray(){
        vector<int> &rank = P.back();
        vector<pair<int, int> > inv(rank.size());
        for (int i = 0; i < rank.size(); ++i)
            inv[i] = make_pair(rank[i], i);
        sort(inv.begin(), inv.end());
        vector<int> sa(rank.size());
        for (int i = 0; i < rank.size(); ++i)
            sa[i] = inv[i].second;
        return sa;
    }

    // returns the length of the longest common prefix of s[i...L-1] and
    // s[j...L-1]
    int lcp(int i, int j) {
        int len = 0;
        if (i == j) return N - i;
        for (int k = P.size() - 1; k >= 0 && i < N && j < N; --k) {
            if (P[k][i] == P[k][j]) {
                i += 1 << k;

```

```

                j += 1 << k;
                len += 1 << k;
            }
        }
        return len;
    }
};

```

## 10.4 suffix automaton

```

/*
 * Suffix automaton:
 * This implementation was extended to maintain (online) the
 * number of different substrings. This is equivalent to compute
 * the number of paths from the initial state to all the other
 * states.
 *
 * The overall complexity is O(n)
 * can be tested here:
 *   https://www.urionlinejudge.com.br/judge/en/problems/view/1530
 */

struct state {
    int len, link;
    long long num_paths;
    map<int, int> next;
};

const int MN = 200011;
state sa[MN << 1];
int sz, last;
long long tot_paths;

void sa_init() {
    sz = 1;
    last = 0;
    sa[0].len = 0;
    sa[0].link = -1;
    sa[0].next.clear();
    sa[0].num_paths = 1;
    tot_paths = 0;
}

```

```

void sa_extend(int c) {
    int cur = sz++;
    sa[cur].len = sa[last].len + 1;
    sa[cur].next.clear();
    sa[cur].num_paths = 0;
    int p;
    for (p = last; p != -1 && !sa[p].next.count(c); p = sa[p].link) {
        sa[p].next[c] = cur;
        sa[cur].num_paths += sa[p].num_paths;
        tot_paths += sa[p].num_paths;
    }

    if (p == -1) {
        sa[cur].link = 0;
    } else {
        int q = sa[p].next[c];
        if (sa[p].len + 1 == sa[q].len) {
            sa[cur].link = q;
        } else {
            int clone = sz++;
            sa[clone].len = sa[p].len + 1;
            sa[clone].next = sa[q].next;
            sa[clone].num_paths = 0;
            sa[clone].link = sa[q].link;
            for (; p != -1 && sa[p].next[c] == q; p = sa[p].link) {
                sa[p].next[c] = clone;
                sa[q].num_paths -= sa[p].num_paths;
                sa[clone].num_paths += sa[p].num_paths;
            }
            sa[q].link = sa[cur].link = clone;
        }
    }
    last = cur;
}

```

## 10.5 z algorithm

```

using namespace std;
#include<bits/stdc++.h>

```

```

vector<int> compute_z(const string &s){
    int n = s.size();
    vector<int> z(n,0);
    int l,r;
    r = l = 0;
    for(int i = 1; i < n; ++i){
        if(i > r) {
            l = r = i;
            while(r < n and s[r - l] == s[r])r++;
            z[i] = r - l;r--;
        }else{
            int k = i-l;
            if(z[k] < r - i +1) z[i] = z[k];
            else {
                l = i;
                while(r < n and s[r - l] == s[r])r++;
                z[i] = r - l;r--;
            }
        }
    }
    return z;
}

int main(){

    //string line;cin>>line;
    string line = "alfalfa";
    vector<int> z = compute_z(line);

    for(int i = 0; i < z.size(); ++i ){
        if(i)cout<<" ";
        cout<<z[i];
    }
    cout<<endl;

    // must print "0 0 0 4 0 0 1"

    return 0;
}

```