Problem 1: Hamilton Path in DAGs (10 points)
Show that the Directed Hamilton Path problem can be solved in polynomial time in directed acyclic graphs. Give an efficient algorithm, justify correctnedd and running time.

Problem 2: Decision implies Construction (10 points)
Suppose that someone gives you a polynomial-time algorithm to decide 3SAT. Describe how to use this algorithm to find a satisfying assignment in polynomial time (if such an assignment exists.)

Problem 3: Vertex Cover, Greedy Heuristic (10 points)

Consider the following heuristic to solve the vertex-cover problem. Repeatedly select a vertex of highest degree, and remove all of its incident edges. Give an example to show that this heuristic does not have an approximation ratio of 2.

Problem 4: Hamilton Cycles in Undirected and Directed Graphs (10 points)
Recall that a Hamilton Cycle in a directed graph is a cycle that visits all vertices of the graph exactly once. Similarly, a Hamilton Cycle in an undirected graph is a cycle that visits all vertices of the graph exactly once. In class and in DPV book we showed that 3SAT$\leq_P$Directed Hamilton Cycle. Show that Directed Hamilton Cycle $\leq_P$ Undirected Hamilton Cycle.

Problem 5: Hamilton Paths (10 points)
Recall that a Hamilton Cycle in a directed graph is a cycle that visits all vertices of the graph exactly once. Similarly, a Hamilton Path in a directed graph is a path that starts from some vertex v, ends in some vertex $u \neq v$, and visits all vertices of the graph exactly once. In class we showed that 3SAT$\leq_P$Directed Hamilton Cycle. Show that 3SAT$\leq_P$ Directed Hamilton Path.

Problem 6: Hardess of TSP Approximation (10 points)
In class we showed that Undirected Hamilton Cycle $\leq_P$ TSP.
Define the 2-Approx-TSP problem as follows:
Input: G, the complete graph on n vertices, cost $c(e) > 0$ for every edge of G.
Let OPT be the cost of a minimum cost cycle that visits every vertex exactly once.
Output: A cycle c of G that visits all the vertices of G exactly once and has cost at most 2OPT.
Show that Undirected Hamilton Cycle $\leq_P$ 2-Approx-TSP.

Problem 7: Cliques (10 points)
Let $G(V, E)$ be an undirected graph.
Definition: Let $X \subseteq V$ be a subset of the vertices of G. We say that X is a *clique* if and only if, for all vertices $u \in X$ and $v \in X$, the edge $\{u, v\} \in E$ ie there is an edge from u to v in the graph G. We say that the *size* of the clique is the number of vertices in X: $|X|$.
Clique is the following problem:
Input: Unirected graph $G(V, E)$ and a number k with $1 \leq k \leq n$.
Output: YES if the graph G has a clique of size *at least* k.
NO if all cliques of G have size strictly smaller than k.
Show that Independent Set$\leq_P$Clique.

Problem 8: Stingy SAT (10 points)
STINGY SAT is the following problem: given a set of clauses (each a disjunction of literals) and an integer k with $1 \leq k \leq n$, find a satisfying assignment in which at most k variables are true, if such an assignment exists. Prove that STINGY SAT is NP-complete. (Hint: Reduce SAT to Stingy SAT.)

Problem 9: Subgraph Isomorphism (10 points)
Subgraph Isomrphism is the following problem:
Input: Undirected graph $G(V, E)$, and undirected graph $H(V', E')$, with $|V'| \leq |V|$.
Output: YES if G has a subgraph isomorphic to H, ie for some $V' \subseteq V$, the subgraph of G induced by the vertices in V' is isomorphic to G'.
NO if G has no subgraph isomorphic to V'.
Prove that Subgraph Isomorphism is NP-complete.

Problem 10: Hardness of Set Cover (10 points)
Recall the unweighted Set Cover problem. $X = \{e_1, e_2, \ldots, e_n\}$ is a ground set of n elements. $F = \{S_1, S_2, \ldots, S_m\}$ is a collection of subsets of X, ie $S_j \subseteq X$, for $1 \leq j \leq m$. A *set cover* C is a subset of F such that every element of X belongs to a set in C. Given k, such that $1 \leq k \leq m$, we want to know if there exists a set cover that has at most k sets.
Recall the unweighted Vertex Cover (VC) problem. $G(V, E)$ is an undirected graph. A *vertex cover* C_V is a subset of V such that every edge in E has at least one of its endpoints in C_V. Given k, such that $1 \leq k \leq |V|$, we want to know if there exists a vertex cover that has at most k vertices.
Show that VC$\leq_P$Set-Cover.