

### Question # 01:-

$$2x_1 - 3x_2 + 2x_3 - x_4 = 0$$

$$-x_1 + 4x_2 + 2x_3 - 3x_4 = 0 \quad 1$$

$$2x_2 - 3x_3 + 5x_4 = -3$$

$$= \left[ \begin{array}{cccc|c} 2 & -3 & 2 & -1 & 0 \\ -1 & 4 & 2 & -3 & 1 \\ 0 & 2 & -3 & 5 & -3 \end{array} \right]$$

$$R_1 \div 2$$

$$= \left[ \begin{array}{cccc|c} 1 & -3/2 & 1 & -1/2 & 0 \\ -1 & 4 & 2 & -3 & 1 \\ 0 & 2 & -3 & 5 & -3 \end{array} \right]$$

$$R_2 + R_1$$

$$= \left[ \begin{array}{cccc|c} 1 & -3/2 & 1 & -1/2 & 0 \\ 0 & 5/2 & 3 & -7/2 & 1 \\ 0 & 2 & -3 & 5 & -3 \end{array} \right]$$

$$R_2 \times 2/5$$

$$= \left[ \begin{array}{cccc|c} 1 & -3/2 & 1 & -1/2 & 0 \\ 0 & 1 & 6/5 & -7/5 & 2/5 \\ 0 & 2 & -3 & 5 & -3 \end{array} \right]$$

$$R_3 - 2R_2, \quad R_1 + 3/2 R_2$$

$$= \left[ \begin{array}{cccc|c} 1 & 0 & 14/5 & -13/5 & 3/5 \\ 0 & 1 & 6/5 & -7/5 & 2/5 \\ 0 & 0 & -27/5 & 39/5 & -19/5 \end{array} \right]$$

$$R_3 \times -5/27$$

$$= \left[ \begin{array}{cccc|c} 1 & 0 & 14/5 & -13/5 & 3/5 \\ 0 & 1 & 6/5 & -7/5 & 2/5 \\ 0 & 0 & 1 & -13/9 & 19/27 \end{array} \right]$$

$$= \left[ \begin{array}{cccc|c} R_2 - \frac{6}{5} R_3, & R_1 - \frac{14}{5} R_3 & & & \\ 1 & 0 & 0 & 13/9 & -37/27 \\ 0 & 1 & 0 & 1/3 & -4/9 \\ 0 & 0 & 1 & -13/9 & 19/27 \end{array} \right]$$

$x_4$  is free

$$\text{let } x_4 = 9$$

$$\Rightarrow x_3 - \frac{13}{9} x_4 = \frac{19}{27}$$

$$x_3 - \frac{13}{9} (9) = \frac{19}{27}$$

$$x_3 = \frac{19}{27} + 13$$



$$x_3 = \frac{370}{27}$$

$$\Rightarrow x_2 + 1/3x_4 = -4/9$$

$$x_2 + 1/3(9) = -\frac{4}{9}$$

$$x_2 = -4/9 - 3$$

$$x_2 = \frac{-31}{9}$$

$$x_1 + \frac{13x_4}{9} = \frac{-37}{27}$$

$$x_1 + \frac{13}{9}(9) = \frac{-37}{27}$$

$$x_1 = \frac{-37}{27} - 13$$

$$x_1 = \frac{-388}{27}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -388/27 \\ -31/9 \\ 370/27 \\ 9 \end{bmatrix} \quad \text{Ans.}$$

**Question # 02:**  $76I_1 - 25I_2 - 50I_3 = 10$   
 $-25I_1 + 56I_2 + I_3 = 0$   
 $-50I_1 + I_2 + 106I_3 = 0$

$$= \left[ \begin{array}{ccc|c} 76 & -25 & -50 & 10 \\ -25 & 56 & -1 & 0 \\ -50 & -1 & 106 & 0 \end{array} \right]$$

$76R_2 + 25R_1, \quad 76R_3 + 50R_1$

$$= \left[ \begin{array}{ccc|c} 76 & -25 & -50 & 10 \\ 0 & 3631 & -1326 & 250 \\ 0 & -1326 & 5556 & 500 \end{array} \right]$$

$3631R_3 + 1326R_2$

$$= \left[ \begin{array}{ccc|c} 76 & -25 & -50 & 10 \\ 0 & 3631 & -1326 & 250 \\ 0 & 0 & 18415560 & 2147000 \end{array} \right]$$

$\Rightarrow 18415560x_3 = 2147000$

$x_3 = \frac{107350}{920778} = \frac{2825}{24231}$

$920778 \quad 24231$



$$\Rightarrow 3631x_2 - 1326x_3 = 250$$

$$3631x_2 - 1326 \left[ \frac{2825}{24231} \right] = 250$$

$$3631x_2 - \frac{3745950}{24231} = 250$$

$$x_2 = \frac{900}{8077}$$

$$\Rightarrow 76x_1 - 25x_2 - 50x_3 = 10$$

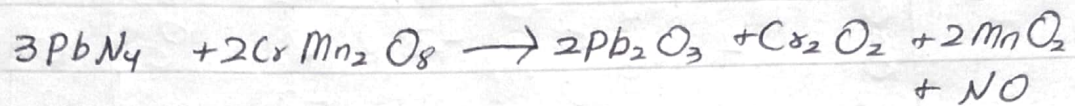
$$76x_1 - 25 \left( \frac{900}{8077} \right) - 50 \left( \frac{2825}{24231} \right) = 1$$

$$76x_1 - \frac{22500}{8077} - \frac{141250}{24231} = 10$$

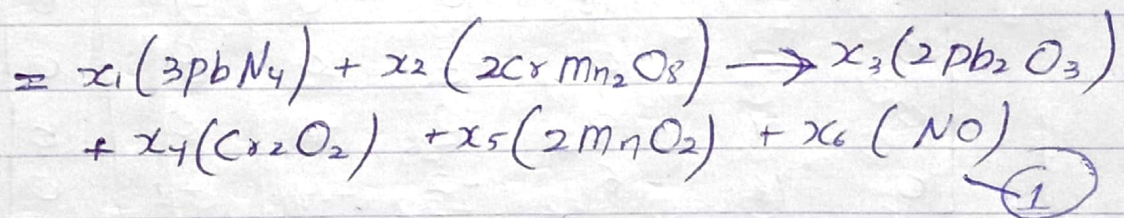
$$x_1 = \frac{5935}{24231}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{5935}{24231} \\ \frac{900}{8077} \\ \frac{2825}{24231} \end{bmatrix}$$

### Question # 3:-



$$\begin{bmatrix} \text{Pb} \\ \text{N} \\ \text{Cr} \\ \text{Mn} \\ \text{O} \end{bmatrix}$$



$$= x_1 \begin{bmatrix} 3 \\ 12 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 4 \\ 16 \end{bmatrix} \rightarrow x_3 \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 6 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 2 \end{bmatrix}$$

$$+ x_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 4 \end{bmatrix} + x_6 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$



$$= \left[ \begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & -2 & 0 & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & 0 \end{array} \right]$$

$$R_2 - 4R_1$$

$$\left[ \begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & -2 & 0 & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & 0 \end{array} \right]$$

$$R_2 \leftrightarrow R_5$$

$$\left[ \begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & 0 \end{array} \right]$$

$$R_3 \div 2, R_4 \div 2$$

$$\left[ \begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & 0 \end{array} \right]$$

$$-16R_3 + R_2, -8R_4 + R_2$$

$$= \left[ \begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & 0 \\ 0 & 0 & -6 & 14 & -4 & -1 & 0 \\ 0 & 0 & -6 & -2 & 4 & -1 & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & 0 \end{array} \right]$$

$$-R_4 + R_3, 6R_5 + 16R_3$$

$$= \left[ \begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & 0 \\ 0 & 0 & -6 & 14 & -4 & -1 & 0 \\ 0 & 0 & 0 & 16 & -8 & 0 & 0 \\ 0 & 0 & 0 & 224 & -64 & -22 & 0 \end{array} \right]$$

$$R_4 \div 8, R_5 \div 2$$



$$= \left[ \begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & 0 \\ 0 & 0 & -6 & 14 & -4 & -1 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 112 & -32 & -11 & 0 \end{array} \right]$$

$$-6R_1 + 4R_2, -R_2 + R_3$$

$$= \left[ \begin{array}{cccccc|c} -18 & 0 & 0 & 56 & -16 & -4 & 0 \\ 0 & -16 & 0 & 16 & 0 & 0 & 0 \\ 0 & 0 & -6 & 14 & -4 & -1 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 112 & -32 & -11 & 0 \end{array} \right]$$

$$R_1 \div -2, R_2 \div -8$$

$$= \left[ \begin{array}{cccccc|c} 9 & 0 & 0 & -28 & 8 & 2 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -6 & 14 & -4 & -1 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 112 & -32 & -11 & 0 \end{array} \right]$$

$$R_1 + 14R_4, R_2 + R_4, R_3 - 7R_4, R_5 - 56R_4$$

$$= \left[ \begin{array}{cccccc|c} 9 & 0 & 0 & 0 & -6 & 2 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -6 & 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 24 & -11 & 0 \end{array} \right]$$

$$4R_1 + R_5, \quad 24R_2 + R_5, \quad -8R_3 + R_5, \quad 24R_4 + R_5$$

$$= \left[ \begin{array}{cccccc|c} 36 & 0 & 0 & 0 & 0 & -3 & 0 \\ 0 & 48 & 0 & 0 & 0 & -11 & 0 \\ 0 & 0 & 48 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 48 & 0 & -11 & 0 \\ 0 & 0 & 0 & 0 & 24 & -11 & 0 \end{array} \right]$$

$$R_1 \div 3, \quad R_3 \div 3$$

$$= \left[ \begin{array}{cccccc|c} 12 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 48 & 0 & 0 & 0 & -11 & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 48 & 0 & -11 & 0 \\ 0 & 0 & 0 & 0 & 24 & -11 & 0 \end{array} \right]$$

$$R_1 \div 12, \quad R_2 \div 48, \quad R_3 \div 16, \quad R_4 \div 48, \quad R_5 \div 24$$



$$= \left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -1/12 & 0 \\ 0 & 1 & 0 & 0 & 0 & -11/48 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/16 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/48 & 0 \\ 0 & 0 & 0 & 0 & 1 & -11/24 & 0 \end{array} \right]$$

$$\text{let } x_6 = 48$$

$$\Rightarrow x_5 - \frac{11}{24} (48) = 0$$

$$x_5 - 22 = 0$$

$$\boxed{x_5 = 22}$$

$$\Rightarrow x_4 - \frac{11}{48} x_6 = 0$$

$$x_4 - \frac{11}{48} (48) = 0$$

$$\boxed{x_4 = 11}$$

$$\Rightarrow x_3 - 1/16 (48) = 0$$

$$\boxed{x_3 = 3}$$

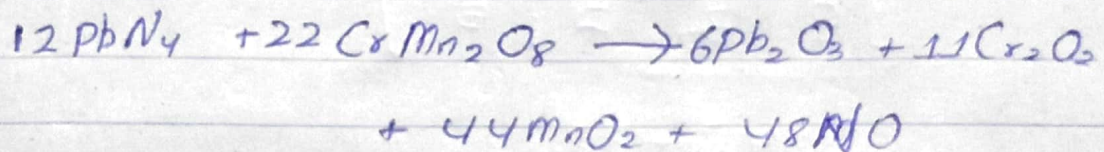
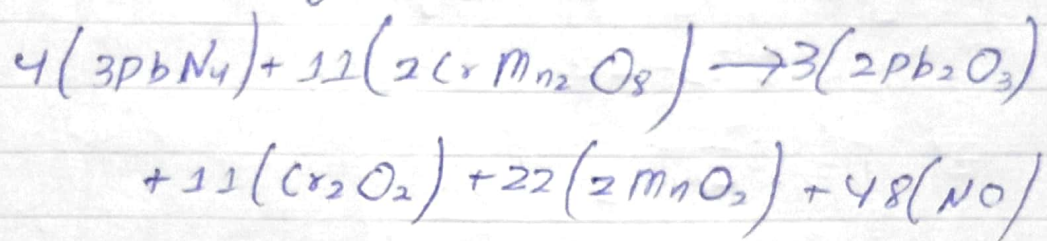
$$\Rightarrow x_2 - \frac{11}{48} (48) = 0$$

$$\boxed{x_2 = 11}$$

$$\Rightarrow x_1 - 1/10 (48) = 0$$

$$\boxed{x_1 = 4}$$

Now putting values in eq (1)



$$\text{C.H.S} = \text{R.H.S}$$

$$\text{Pb} = 12$$

$$\text{Pb} = 12$$

$$\text{N} = 48$$

$$\text{N} = 48$$

$$\text{Cr} = 22$$

$$\text{Cr} = 22$$

$$\text{Mn} = 44$$

$$\text{Mn} = 44$$

$$\text{O} = 176$$

$$\text{O} = 176$$



Question # 04:-  $b_1 \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + b_2 \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} + b_3 \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} h \\ 1 \\ 3 \end{bmatrix}$

$$= \left[ \begin{array}{ccc|c} 3 & -1 & 1 & h \\ 2 & 4 & -2 & 1 \\ 0 & 3 & 2 & 3 \end{array} \right]$$

$$3R_2 - 2R_1$$

$$= \left[ \begin{array}{ccc|c} 3 & -1 & 1 & h \\ 0 & 14 & -8 & -2h+3 \\ 0 & 3 & 2 & 3 \end{array} \right]$$

$$14R_3 - 3R_2$$

$$= \left[ \begin{array}{ccc|c} 3 & -1 & 1 & h \\ 0 & 14 & -8 & -2h+3 \\ 0 & 0 & 52 & 6h+33 \end{array} \right]$$

$$\Rightarrow -6h + 33 = 52$$

$$6h = 52 - 33$$

$$6h = 19$$

$$h = 19/6$$

$$\Rightarrow 52b_3 = 6h + 33$$

$$52b_3 = 6 \left[ \frac{19}{6} \right] + 33$$

$$52b_3 = 52$$

$$b_3 = 1$$

$$14b_2 - 8 = -2 \left( \frac{19}{6} \right) + 3$$

$$14b_2 = -\frac{10}{3} + 8$$

$$14b_2 = 1/3$$

$$3b_1 - 1/3 + 1 = 19/6$$

$$3b_1 = 19/6 + 1/3 - 1$$

$$b_1 = \frac{5}{2} \times \frac{1}{3}$$

$$b_1 = 5/6$$

$$\frac{5}{6} \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 19/6 \\ 1 \\ 3 \end{bmatrix}$$

if  $h = \frac{19}{6}$ , then  $V_1$  is a linear combination of  $b_1, b_2, b_3$