LINEAR ALGEBRA ASSIGNMENT 01

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Section:- BSE-2B

Q NO:01:-

$$2x_1 - 3x_2 + 2x_3 - x_4 = 0$$
$$-x_1 + 4x_2 + 2x_3 - 3x_4 = 1$$
$$2x_2 - 3x_3 + 5x_4 = -3$$

$$= \left[\begin{array}{cccc|c} 2 & -3 & 2 & -1 & 0 \\ -1 & 4 & 2 & -3 & 1 \\ 0 & 2 & -3 & 5 & -3 \end{array}\right]$$

$R_1 \div 2 :-$

$$= \left[\begin{array}{cccc|c} 1 & -\frac{3}{2} & 1 & -\frac{1}{2} & 0 \\ -1 & 4 & 2 & -3 & 1 \\ 0 & 2 & -3 & 5 & -3 \end{array}\right]$$

$R_2 + R_1 :-$

$$= \left[\begin{array}{cccc|c} 1 & -\frac{3}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & \frac{5}{2} & 3 & -\frac{7}{2} & 1 \\ 0 & 2 & -3 & 5 & -3 \end{array}\right]$$

$R_2 \times \frac{2}{5} :-$

$$= \left[\begin{array}{cccc|c} 1 & -\frac{3}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{6}{5} & -\frac{7}{5} & \frac{2}{5} \\ 0 & 2 & -3 & 5 & -3 \end{array}\right]$$

$R_3 - 2R_2 \;,\; R_1 + \frac{3}{2}R_2 :-$

$$= \begin{bmatrix} 1 & 0 & 14/5 & -13/5 & \bigm| & 3/5 \\ 0 & 1 & 6/5 & -7/5 & \bigm| & 2/5 \\ 0 & 0 & -27/5 & 39/5 & \bigm| & -19/5 \end{bmatrix}$$

$R_3 \times -5/27 :-$

$$= \begin{bmatrix} 1 & 0 & 14/5 & -13/5 & \bigm| & 3/5 \\ 0 & 1 & 6/5 & -7/5 & \bigm| & 2/5 \\ 0 & 0 & 1 & -13/9 & \bigm| & 19/27 \end{bmatrix}$$

$R_2 - 6/5 R_3, \ R_1 - 14/5 R_3 :-$

$$= \begin{bmatrix} 1 & 0 & 0 & 13/9 & \bigm| & -37/27 \\ 0 & 1 & 0 & 1/3 & \bigm| & -4/9 \\ 0 & 0 & 1 & -13/9 & \bigm| & 19/27 \end{bmatrix}$$

x_4 is free

let $x_4 = 9$

$\Rightarrow \quad x_3 - \dfrac{13}{9} x_4 = \dfrac{19}{27}$

$x_3 - \dfrac{13}{9}(9) = \dfrac{19}{27}$

$x_3 = \dfrac{19}{27} + 13$

$x_3 = \dfrac{370}{27}$

$\Rightarrow$

$x_2 + \dfrac{1}{3} x_4 = -\dfrac{4}{9}$

$x_2 + \dfrac{1}{3}(9) = -\dfrac{4}{9}$

$x_2 = -\dfrac{4}{9} - 3$

$x_2 = \dfrac{-31}{9}$

$$\frac{x_1 + 13\, x_4}{9} = \frac{-37}{27}$$

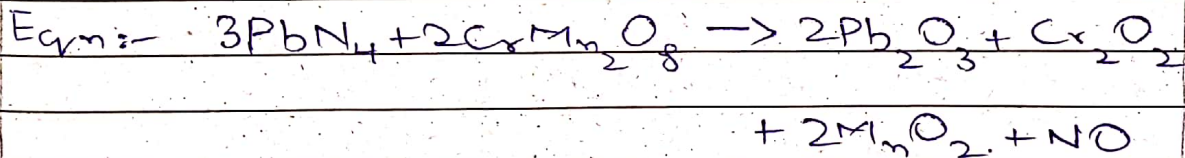$$x_1 + \frac{13\,(\alpha)}{\alpha} = \frac{-37}{27}$$

$$x_1 = \frac{-37}{27} - 13$$

$$x_1 = \frac{-388}{27}$$

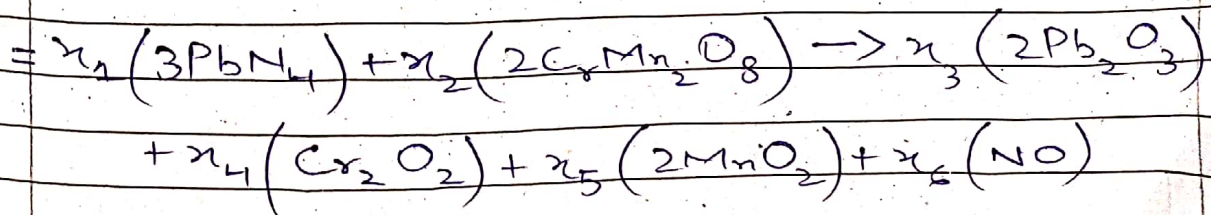$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -388/27 \\ -31/9 \\ 370/27 \\ 9 \end{bmatrix} \quad \underline{\underline{Answer.}}$$

Q:03 Balance the chemical eqn by Row reduction to Echelon form.

Eqn:- $3PbN_4 + 2Cr Mn_2 O_8 \rightarrow 2Pb_2 O_3 + Cr_2 O_2$
$$+ 2Mn O_2 + NO$$

Soln:- Vector formation :-

$$\begin{bmatrix} Pb \\ N \\ Cr \\ Mn \\ O \end{bmatrix}$$

$$= x_1\left(3PbN_4\right) + x_2\left(2Cr Mn_2 O_8\right) \rightarrow x_3\left(2Pb_2 O_3\right)$$
$$+ x_4\left(Cr_2 O_2\right) + x_5\left(2Mn O_2\right) + x_6\left(NO\right)$$

$$= x_1 \begin{bmatrix} 3 \\ 12 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 4 \\ 16 \end{bmatrix} \rightarrow x_3 \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 6 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 2 \end{bmatrix}$$

$$+ x_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 4 \end{bmatrix} + x_6 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \left[\begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & -2 & 0 & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & 0 \end{array} \right]$$

$R_2 - 4R_1 :-$

$$= \left[\begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & -2 & 0 & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & 0 \end{array} \right]$$

$R_2 \longleftrightarrow R_5 :-$

$$= \left[\begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & 0 \end{array} \right]$$

$$R_3 \div 2 \ , \ R_4 \div 2 :-$$

$$= \begin{bmatrix} 3 & 0 & -4 & 0 & 0 & 0 & | & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & | & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & | & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & | & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & | & 0 \end{bmatrix}$$

$$-16R_3 + R_2 \ , \ -8R_4 + R_2$$

$$= \begin{bmatrix} 3 & 0 & -4 & 0 & 0 & 0 & | & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & | & 0 \\ 0 & 0 & -6 & 14 & -4 & -1 & | & 0 \\ 0 & 0 & -6 & -2 & 4 & -1 & | & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & | & 0 \end{bmatrix}$$

$$-R_4 + R_3 \ , \ 6R_5 + 16R_3$$

$$= \begin{bmatrix} 3 & 0 & -4 & 0 & 0 & 0 & | & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & | & 0 \\ 0 & 0 & -6 & 14 & -4 & -1 & | & 0 \\ 0 & 0 & 0 & 16 & -8 & 0 & | & 0 \\ 0 & 0 & 0 & 224 & -64 & -22 & | & 0 \end{bmatrix}$$

$$R_4 \div 8 \ , \ R_5 \div 2$$

$$= \begin{bmatrix} 3 & 0 & -4 & 0 & 0 & 0 & | & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & | & 0 \\ 0 & 0 & -6 & 14 & -4 & -1 & | & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 & | & 0 \\ 0 & 0 & 0 & 112 & -32 & -11 & | & 0 \end{bmatrix}$$

$-6R_1 + 4R_3 , -R_2 + R_3$

$$= \begin{bmatrix} -18 & 0 & 0 & 56 & -16 & -4 & | & 0 \\ 0 & -16 & 0 & 16 & 0 & 0 & | & 0 \\ 0 & 0 & -6 & 14 & -4 & -1 & | & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 & | & 0 \\ 0 & 0 & 0 & 112 & -32 & -11 & | & 0 \end{bmatrix}$$

$R_1 \div -2 , R_2 \div -8$

$$= \begin{bmatrix} 9 & 0 & 0 & -28 & 8 & 2 & | & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & | & 0 \\ 0 & 0 & -6 & 14 & -4 & -1 & | & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 & | & 0 \\ 0 & 0 & 0 & 112 & -32 & -11 & | & 0 \end{bmatrix}$$

$R_1 + 14R_4 , R_2 + R_4 , R_3 - 7R_4 , R_5 - 56R_4$

$$= \begin{bmatrix} 9 & 0 & 0 & 0 & -6 & 2 & | & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & | & 0 \\ 0 & 0 & -6 & 0 & 3 & -1 & | & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & 24 & -11 & | & 0 \end{bmatrix}$$

$4R_1 + R_5 , 24R_2 + R_5 , -8R_3 + R_5 , 24R_4 + R_5$

$$= \begin{bmatrix} 36 & 0 & 0 & 0 & 0 & -3 & | & 0 \\ 0 & 48 & 0 & 0 & 0 & -11 & | & 0 \\ 0 & 0 & 48 & 0 & 0 & -3 & | & 0 \\ 0 & 0 & 0 & 48 & 0 & -11 & | & 0 \\ 0 & 0 & 0 & 0 & 24 & -11 & | & 0 \end{bmatrix}$$

$R_1 \div 3, \; R_3 \div 3$

$$= \begin{bmatrix} 12 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 48 & 0 & 0 & 0 & -11 & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 48 & 0 & -11 & 0 \\ 0 & 0 & 0 & 0 & 24 & -11 & 0 \end{bmatrix}$$

$R_1 \div 12, \; R_2 \div 48, \; R_3 \div 16, \; R_4 \div 48, \; R_5 \div 24$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -\frac{1}{12} & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{11}{48} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{16} & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{11}{48} & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{11}{24} & 0 \end{bmatrix}$$

let $x_6 = 48$

$\Rightarrow x_5 - \frac{11}{24}(48) = 0$

$x_5 - 22 = 0$

$\boxed{x_5 = 22}$

$\Rightarrow x_4 - \frac{11}{48} x_6 = 0$

$x_4 - \frac{11}{48}(48) = 0$

$\boxed{x_4 = 11}$

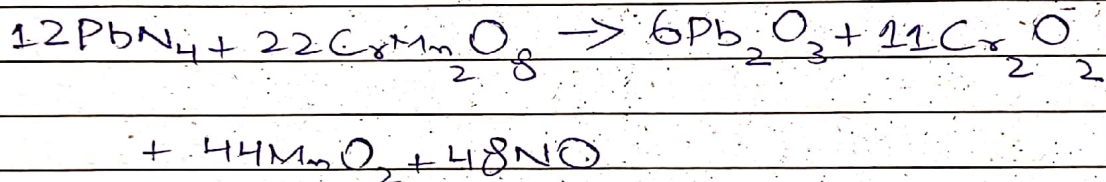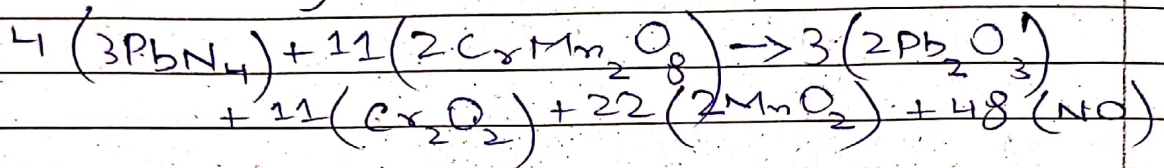$\Rightarrow x_3 - \frac{1}{16}(48) = 0$

$x_3 = 0 \quad \boxed{x_3 = 3}$

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$$\Rightarrow x_2 - \frac{11}{48}(48) = 0$$

$$x_2 = 11$$

$$\Rightarrow x_1 - \frac{1}{10}(48) = 0$$

$$\boxed{x_1 = 4}$$

now putting values in eq :-

$$4\,(3PbN_4) + 11\,(2Cr\,Mn_2\,O_8) \rightarrow 3\,(2Pb_2\,O_3)$$
$$+ 11\,(Cr_2O_2) + 22\,(2MnO_2) + 48\,(NO)$$

$$12PbN_4 + 22Cr\,Mn_2\,O_8 \rightarrow 6Pb_2O_3 + 11Cr_2\,O_2$$

$$+ 44MnO_2 + 48NO$$

$$L \cdot H \cdot S = R \cdot H \cdot S$$

L.H.S	R.H.S
$Pb = 12$	$Pb = 12$
$N = 48$	$N = 48$
$Cr = 22$	$Cr = 22$
$Mn = 44$	$Mn = 44$
$O = 176$	$O = 176$

Q:04 For what values of "h" the given "v" is a linear combination of b_1, b_2 and b_3 where

$$v = \begin{bmatrix} h \\ 1 \\ 3 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}, \quad b_2 = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}, \quad b_3 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

Soln.

$$b_1 \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + b_2 \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} + b_3 \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} h \\ 1 \\ 3 \end{bmatrix}.$$

$$= \begin{bmatrix} 3 & -1 & 1 & h \\ 2 & 4 & -2 & 1 \\ 0 & 3 & 2 & 3 \end{bmatrix}$$

$3R_2 - 2R_1$

$$= \begin{bmatrix} 3 & -1 & 1 & h \\ 0 & 14 & -8 & -2h+3 \\ 0 & 3 & 2 & 3 \end{bmatrix}$$

$14R_3 - 3R_2 :-$

$$= \begin{bmatrix} 3 & -1 & 1 & h \\ 0 & 14 & -8 & -2h+3 \\ 0 & 0 & 52 & 6h+33 \end{bmatrix}$$

$\Rightarrow -6h+33 = 52$

$\qquad 6h = 52 - 33$

$\qquad 6h = 19$

$\qquad h = \dfrac{19}{6}$

$\Rightarrow 52b_3 = 6h + 33$

$\qquad 52b_3 = \cancel{6}\left[\dfrac{19}{\cancel{6}}\right] + 33$

$\qquad 52b_3 = 52$

$\qquad b_3 = 1$

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$$14b_2 - 8 = -2\left(\dfrac{19}{6}\right) + 3$$

$$14b_2 = \dfrac{-10}{3} + 8$$

$$14b_2 = \dfrac{1}{3}$$

$$3b_1 - \dfrac{1}{3} + 1 = \dfrac{19}{6}$$

$$3b_1 = \dfrac{19}{6} + \dfrac{1}{3} - 1$$

$$b_1 = \dfrac{5}{2} \times \dfrac{1}{3}$$

$$b_1 = \dfrac{5}{6}$$

$$\dfrac{5}{6}\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + \dfrac{1}{3}\begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 19/6 \\ 1 \\ 3 \end{bmatrix}$$

If $h = \dfrac{19}{6}$, then V_1 is a linear combination of b_1, b_2 and b_3.

Q:02 Solve circuit by Row Echelon form.

$$76I_1 - 25I_2 - 50I_3 = 10$$
$$-25I_1 + 56I_2 - I_3 = 0$$
$$-50I_1 - I_2 + 106I_3 = 0$$

Solⁿ Eqns are :—

$$= \begin{bmatrix} 76 & -25 & -50 & | & 10 \\ -25 & 56 & -1 & | & 0 \\ -50 & -1 & 106 & | & 0 \end{bmatrix}$$

$$76R_2 + 25R_1 \; , \; 76R_3 + 50R_1$$

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$$= \begin{bmatrix} 76 & -25 & -50 & 10 \\ 0 & 3631 & -1326 & 250 \\ 0 & -1326 & 5556 & 500 \end{bmatrix}$$

$3631R_3 + 1326R_2 :-$

$$= \begin{bmatrix} 76 & -25 & -50 & 10 \\ 0 & 3631 & -1326 & 250 \\ 0 & 0 & 18415560 & 2147000 \end{bmatrix}$$

$\Rightarrow 18415560 x_3 = 2147000$

$$x_3 = \frac{107350}{920778} = \frac{2825}{24231}$$

$\Rightarrow 3631 x_2 - 1326 x_3 = 250$

$3631 x_2 - 1326 \left[\dfrac{2825}{24231} \right]^3 = 250$

$3631 x_2 - \dfrac{3745950}{24231} = 250$

$x_2 = \dfrac{900}{8077}$

$\Rightarrow 76 x_1 - 25 x_2 - 50 x_3 = 10$

$76 x_1 - 25 \left(\dfrac{900}{8077} \right) - 50 \left(\dfrac{2825}{24231} \right) = 10$

$76 x_1 - \dfrac{22500}{8077} - \dfrac{141250}{24231} = 10$

$x_1 = \dfrac{5935}{24231}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \dfrac{5935}{24231} \\ \dfrac{900}{8077} \\ \dfrac{2825}{24231} \end{bmatrix} \quad \text{Answer.}$$