BAHRIA UNIVERSITY (KARACHI CAMPUS)

Department of Software Engineering.

Assignment 03 (Spring 2023)

Course Title: Linear Algebra	**Course Code: GSC-121**
Class: BSE 2B	**Shift: Morning**
Course Instructor: Muhammad Imran Afridi	**Time: 3 Days**
Submission: 03 June 2023	**Max Marks: 05 Points**

Assignment No 03

Submitted By:

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Section: _____2 B_____

Question #01: Evaluate the system of Equations by Cramer's Rule:

$$2x_1 - 2x_2 + x_3 = 1$$
$$x_1 - 3x_2 + 2x_3 = -1$$
$$3x_2 - x_3 = 0$$

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ 0 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}$$

$$|A| = \begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ 0 & 3 & -1 \end{bmatrix}$$

$$= 2(3-6) + 2(-1-0) + 1(3-0)$$
$$= 2(-3) + 2(-1) + 1(3)$$
$$= -6 - 2 + 3$$
$$= -5$$

$$A_1 = \begin{bmatrix} 1 & -2 & 1 \\ -1 & -3 & 2 \\ 0 & 3 & -1 \end{bmatrix}$$

$$= 1(3-6) + 2(1-0) + 1(-3-0)$$
$$= 1(-3) + 2(1) + 1(-3)$$
$$= -3 + 2 - 3$$
$$= -4$$

$$|A_2| = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= 2(1-0) - 1(-1-0) + 1(0-0)$$
$$= 2(1) - 1(-1) + 1(0)$$
$$= 2 + 1 + 0$$
$$= 3$$

$$|A_3| = \begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & -1 \\ 0 & 3 & 0 \end{bmatrix}$$

$$= 2(0+3) + 2(0-0) + 1(3+0)$$
$$= 2(3) + 2(0) + 1(3)$$
$$= 6 + 0 + 3$$
$$= 9$$

$$\frac{|A_1|}{|A|} = \frac{-4}{-5} = \frac{4}{5} \quad \text{Ans}$$

$$\frac{|A_2|}{|A|} = \frac{-3}{5} \quad \text{Ans}$$

$$\frac{|A_3|}{|A|} = \frac{9}{5} \quad \text{Ans}$$

Question #02: Determine the values of 'a'

such that $\left\{ \begin{bmatrix} 1 \\ a \end{bmatrix}, \begin{bmatrix} a \\ a+2 \end{bmatrix} \right\}$ are linearly independent

$$\begin{bmatrix} 1 & a & | & 0 \\ a & a+2 & | & 0 \end{bmatrix}$$

$$R_2 - a R_1$$

$$\begin{bmatrix} 1 & a & | & 0 \\ 0 & a+2-a^2 & | & 0 \end{bmatrix}$$

$$-a^2 + a + 2 = 0$$

$$a^2 - a - 2 = 0$$

$$a^2 - 2a + a - 2 = 0$$
$$a(a-2) + 1(a-2) = 0$$
$$(a+1)(a-2) = 0$$

$$a + 1 = 0 \qquad\qquad\qquad a - 2 = 0$$

$$[a = -1] \qquad\qquad_{sr} \quad [a = 2] \qquad Ans$$

The given system of Equation is linearly in dependant except for there two values.

Question # 03 : Evaluate the bases for Null A and Col A.

$$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$$

$$f = \left[\begin{array}{cccc|c} -2 & 4 & -2 & -4 & 0 \\ 2 & -6 & -3 & 1 & 0 \\ -3 & 8 & 2 & -3 & 0 \end{array}\right]$$

$$R_2 + R_1$$

$$\begin{bmatrix} -2 & 4 & -2 & -4 & 0 \\ 0 & -2 & -5 & -3 & 0 \\ -3 & 8 & 2 & -3 & 0 \end{bmatrix}$$

$$2R_3 - 3R_1$$

$$\begin{bmatrix} -2 & 4 & -2 & -4 & 0 \\ 0 & -2 & -5 & -3 & 0 \\ 0 & 4 & 10 & 6 & 0 \end{bmatrix}$$

$$R_3 + 2R_2$$

$$\begin{bmatrix} -2 & 4 & -2 & -4 & 0 \\ 0 & -2 & -5 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

free variable.

let $x_3 = a$, $x_4 = b$

$R_2 \Rightarrow$ $-2x_2 - 5x_3 - 3x_4 = 0$

$\Rightarrow -2x_2 - 5a - 3b = 0$

$\Rightarrow \boxed{x_2 = \dfrac{5a + 3b}{-2}}$

$R_3 \Rightarrow$ $-2x_1 + 4 \left[\dfrac{5a + 3b}{-2} \right] - 2a - 4b = 0$

$\Rightarrow \quad -2x_1 + \dfrac{20a}{-2} + 12b + 4a + 8b = 0$

$\Rightarrow \quad 2x_2 = \dfrac{24a + 20b}{-2}$

$\Rightarrow \quad x_2 = \dfrac{\overset{6}{24}a + \overset{5}{20}b}{-\cancel{2}}$

$\boxed{x_2 = -6a - 5b}$

Null $A = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -6a - 5b \\ -5/2\,a - 3/2\,b \\ a \\ b \end{bmatrix}$

$= a \begin{bmatrix} -6 \\ -5/2 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} -5 \\ -3/2 \\ 0 \\ 1 \end{bmatrix}$

$\underbrace{\qquad\qquad\qquad\qquad}_{[\text{Basis for Null } A]}$

$$Col \; A = \left\{ \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix} , \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix} \right\}$$

Ans

[Bases for col A].

Question # 04, The set $B = \{ 2+3t, \; 1-2t+t^2$
$4t+2t^2+5t^3 \}$ is a bases for P_3, find
the Co-ordinate vecfor of $p(t)=3+4t-2t^2$
$+10t^3$.

$$\begin{bmatrix} 2 & 1 & 0 & | & 3 \\ 3 & -2 & 4 & | & 4 \\ 0 & 1 & 2 & | & -2 \\ 0 & 0 & 5 & | & 10 \end{bmatrix}$$

$2R_2 - 3R_1$

$$\begin{bmatrix} 2 & 1 & 0 & | & 3 \\ 0 & -7 & 8 & | & -1 \\ 0 & 1 & 2 & | & 2 \\ 0 & 0 & 5 & | & 10 \end{bmatrix}$$

$7R_3 + R_2$

$$\begin{bmatrix} 2 & 1 & 0 & 3 \\ 0 & -7 & 8 & -1 \\ 0 & 0 & 22 & -15 \\ 0 & 0 & 5 & 10 \end{bmatrix}$$

$$22 R_4 - 5 R_3$$

$$\begin{bmatrix} 2 & 1 & 0 & 3 \\ 0 & -7 & 8 & -1 \\ 0 & 0 & 22 & -15 \\ 0 & 0 & 0 & 295 \end{bmatrix} \quad (\text{No solution}).$$

No solution because of R_4, co-ordinates
vector is not possible to be fain d.