Question # 01:-

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix} \quad \lambda = 4$$

solution:-

$$|A - \lambda I| = 0$$

$$= \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-4 & 0 & -1 \\ 2 & 3-4 & 1 \\ -3 & 4 & 5-4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & -1 \\ 2 & -1 & 1 \\ -3 & 4 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & -1 & 1 & 0 \\ -3 & 4 & 1 & 0 \end{array}\right]$$

$R_2 + 2R_1$, $R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 4 & 4 & 0 \end{array}\right]$$

$R_3 + 4R_2$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

x_3 = free column.

The system has non trival solution so $\lambda = 4$ is the eigen value of the given set.

$\therefore x_3 = a$

$R_2 = -x_2 - a = 0$

$\boxed{x_2 = -a}$

$R_1 \Rightarrow -x_1 + x_3 = 0$

$\boxed{x_1 = -a}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -a \\ -a \\ a \end{bmatrix} \text{ or } \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \longrightarrow \text{eigenvato.}$$

Question # 02:

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 3-\lambda & 1 & 1 \\ 0 & 5-\lambda & 0 \\ -2 & 0 & 7-\lambda \end{bmatrix} = 0$$

$$= 1[0+2(5-\lambda)] - 0 + (7-\lambda)[(3-\lambda)(5-\lambda)]$$

$$= 10 - 2\lambda + (7-\lambda)[15 - 5\lambda - 3\lambda + \lambda^2]$$

$$= 10 - 2\lambda + (7-\lambda)[15 - 8\lambda + \lambda^2]$$

$$= 10 + 2\lambda + 105 - 56\lambda + 7\lambda^2 - 15\lambda + 8\lambda^2 - \lambda^3$$

$$= -\lambda^3 + 15\lambda^2 - 73\lambda + 115 = 0$$

$$= -(\lambda^3 - 15\lambda^2 + 73\lambda - 115) = 0$$

$$\lambda^3 - 15\lambda^2 + 73\lambda - 115 = 0$$

$\lambda = 3.58$

$\lambda = 6.41$

$\lambda = 5$

$\boxed{\lambda = 5,\ 3.58,\ 6.41}$ Ans

Question # 3.

$b_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ $b_2 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$, $b_3 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$, $x = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$

solution

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 3 & 8 & 3 & -2 \end{array}\right] \quad R_2 - R_1,\ R_3 - 3R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 2 & 0 & -2 \end{array}\right] \quad R_3 + R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & -2 & -2 \end{array}\right]$$

$R_3 \Rightarrow \quad -2x_3 = -2$

$x_3 = 1$

$R_2 \Rightarrow \quad -2x_2 - 2x_3 = 0$

$\boxed{x_2 = -1}$

$R_1 \Rightarrow \quad x_1 + 2x_2 + x_3 = 0$

$x_1 - 2 + 1 = 0$

$\boxed{x_1 = 1}$

$$[x]_{13} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \text{ Ans.}$$

Question # 04:-

$$A = \begin{bmatrix} 2 & 1 & 4 \\ -1 & 2 & -2 \\ 2 & 0 & 5 \end{bmatrix} \quad y = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$$

Breaking matrix into vector

$$v_1 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$$

Checking the vectors are orthogonal or not

$$u_1 \cdot u_2 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$U_1 \cdot U_2 = 2-2+0 = 0$$

$$U_2 \cdot U_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$$

$$U_2 - U_3 = 4-4+0 = 0$$

$$U_1 \cdot U_3 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$$

$$U_1 \cdot U_3 = 8+2-10 = 0$$

Vectors are orthogonal to each other

$$\therefore y = c_1 U_1 + c_2 U_2 + C_3 U_3.$$

for C_1

$$C_1 = \frac{yU_1}{U_1U_1} = \frac{3}{9} = \frac{1}{3}$$

for C_2

$$C_2 = \frac{yU_2}{U_2 \cdot U_2} = \frac{5}{5} = 1$$

for C_3

$$C_3 = \frac{yU_3}{U_3U_3} = \frac{-30}{45} = \frac{-2}{3}$$

$$y = C_1U_1 + C_2U_2 + C_3U_3$$

$$y = \frac{1}{3}\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - \frac{2}{3}\begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$$

Ans.