Q1

Solution :-

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix} \qquad \lambda = 4.$$

$$A - \lambda I = 0$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & -1 \\ 2 & -1 & 1 \\ -3 & 4 & 1 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & -1 & 1 & 0 \\ -3 & 4 & 1 & 0 \end{array}\right]$$

$$R_2 + 2R_1, \; R_3 - 3R_1$$

$$\begin{array}{lcccc} R_2 \rightarrow & 2 & -1 & 1 & 0 \\ 2R_1 \rightarrow & -2 & 0 & -2 & 0 \\ \hline & 0 & -1 & -1 & 0 \text{ new } R_2. \\ R_3 \rightarrow & -3 & 4 & 1 & 0 \\ -3R_1 \rightarrow & 3 & 0 & 3 & 0 \\ \hline & 0 & 4 & 4 & \text{— new } R_3. \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 4 & 4 & 0 \end{array}\right]$$

$R_3 + 4R_2$

$R_3 \rightarrow \quad 0 \quad 4 \quad 4 \quad 0$

$4R_2 \rightarrow \quad 0 \quad -4 \quad -4 \quad 0$

$\quad\quad 0 \quad 0 \quad 0 \quad 0 \quad$ new R_3

$$\Rightarrow \left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The system has a non-trivial solution so, $\lambda = 4$ is an eigen value of A.

x_3 is free

let $x_3 = a$

$R_2 \Rightarrow -x_2 - x_3 = 0 \qquad R_1 \Rightarrow -x_1 - x_3 = 0$

$-x_2 - a = 0 \qquad -x_1 - a = 0$

$\boxed{x_2 = -a} \qquad \boxed{x_1 = -a}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -a \\ -a \\ a \end{bmatrix} = a \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

So, $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ is an eigen vector.

Q2

Solution:-

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 3 & 1 & 1 \\ 0 & 5 & 0 \\ -2 & 0 & 7 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 1 & 1 \\ 0 & 5-\lambda & 0 \\ -2 & 0 & 7-\lambda \end{vmatrix} = 0$$

Expanding by R_2.

$$\Rightarrow 0 \begin{vmatrix} 1 & 1 \\ 0 & 7-\lambda \end{vmatrix} - (5-\lambda) \begin{vmatrix} 3-\lambda & 1 \\ -2 & 7-\lambda \end{vmatrix} + 0 \begin{vmatrix} 3-\lambda & 1 \\ -2 & 0 \end{vmatrix} = 0$$

$\Rightarrow 0 + (-5+\lambda)\left[(3-\lambda)(7-\lambda) - (-2)\right] + 0 = 0$

$\Rightarrow -5+\lambda\left[21 - 3\lambda - 7\lambda + \lambda^2 + 2\right] = 0$

$\Rightarrow -5+\lambda\left[\lambda^2 - 10\lambda + 23\right] = 0$

$\Rightarrow -5\lambda^2 + 50\lambda - 115 + \lambda^3 - 10\lambda^2 + 23\lambda = 0$

$\Rightarrow \lambda^3 - 15\lambda^2 + 73\lambda - 115 = 0 \longrightarrow$ Characteristic equation

$\Rightarrow \lambda^3 - 15\lambda^2 + 73\lambda - 115 \longrightarrow$ Characteristic polynomial

$\lambda = 5 \quad , \quad \lambda = 5 \pm \sqrt{2}$

Ans

Q3

Solution :-

$b_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \; b_2 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \; b_3 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} \quad x = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 3 & 8 & 3 & -2 \end{array}\right]$

$R_3 - 3R_1 \, , \; R_2 - R_1$

$R_3 \rightarrow 3 \quad 8 \quad 3 \quad -2$ $\qquad$ $R_2 \rightarrow 1 \quad 0 \quad -1 \quad 0$

$-3R_1 \rightarrow -3 \quad -6 \quad -3 \quad 0$ $\qquad$ $-R_1 \rightarrow -1 \quad -2 \quad -1 \quad 0$

$0 \quad 2 \quad 0 \quad -2$ new R_3 $\qquad$ $0 \quad -2 \quad -2 \quad 0$ new R_2

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 2 & 0 & -2 \end{array}\right]$

$R_3 + R_2$

$R_3 \rightarrow 0 \quad 2 \quad 0 \quad -2$

$+R_2 \rightarrow 0 \quad -2 \quad -2 \quad 0$

$0 \quad 0 \quad -2 \quad -2$ new R_3

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & -2 & -2 \end{array}\right]$

$R_3 \Rightarrow -2x_3 = -2$

$\boxed{x_3 = 1}$

$R_2 \Rightarrow -2x_2 - 2x_3 = 0$

$-2x_2 - 2 = 0$

$\boxed{x_2 = -1}$

$R_1 \Rightarrow x_1 + 2x_2 + x_3 = 0$

$x_1 + 2(-1) + 1 = 0$ so,

$\boxed{x_1 = 1}$

$[x]_B = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ Ans

Q4

Solution :-

$A = \begin{bmatrix} 2 & 1 & 4 \\ -1 & 2 & -2 \\ 2 & 0 & -5 \end{bmatrix} \qquad y = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$

let, $u_1 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \; u_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \; u_3 \begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$

$u_1 \cdot u_2 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \Rightarrow u_1 \cdot u_2 = 0$

$u_1 \cdot u_3 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix} \Rightarrow u_1 \cdot u_3 = 0$

$u_2 \cdot u_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix} \Rightarrow u_2 \cdot u_3 = 0$

The columns of the matrix are orthogonal to each other.

$y = Ax = c_1u_1 + c_2u_2 + c_3u_3$ —— (A)

For c_1 :-

$c_1 = \dfrac{y \cdot u_1}{u_1 \cdot u_1}$

$c_1 = \dfrac{3}{9} = \boxed{\dfrac{1}{3}}$

$$C_2 = \frac{y \cdot u_2}{u_2 \cdot u_2}$$

$$C_2 = \frac{5}{5} \Rightarrow \boxed{C_2 = 1}$$

$$C_3 = \frac{y \cdot u_3}{u_3 \cdot u_3}$$

$$C_3 = \frac{-30}{45} \Rightarrow \boxed{\frac{-2}{3}}$$

<u>Putting</u> values of C_1, C_2 & C_3 in eq (A)

$$y = \frac{1}{3}\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - \frac{2}{3}\begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$$

Ans

x ——— x