Assignment # 01

Q1: Find solution set of given set of equation by Gauss Jordan method.

$$2x_1 - 3x_2 + 2x_3 - x_4 = 0$$
$$-x_1 + 4x_2 + 2x_3 - 3x_4 = 1$$
$$2x_2 - 3x_3 + 5x_4 = -3$$

Solution :-

$$\Rightarrow \left[\begin{array}{cccc|c} 2 & -3 & 2 & -1 & 0 \\ -1 & 4 & 2 & -3 & 1 \\ 0 & 2 & -3 & 5 & -3 \end{array}\right]$$

$R_2 \leftrightarrow R_1$

$$\Rightarrow \left[\begin{array}{cccc|c} -1 & 4 & 2 & -3 & 1 \\ 2 & -3 & 2 & -1 & 0 \\ 0 & 2 & -3 & 5 & -3 \end{array}\right]$$

$R_1 / -1$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -4 & -2 & 3 & -1 \\ 2 & -3 & 2 & -1 & 0 \\ 0 & 2 & -3 & 5 & -3 \end{array}\right]$$

$R_2 - 2R_1$

$$\begin{array}{lcccc|c} R_2 \longrightarrow & 2 & -3 & 2 & -1 & 0 \\ 2R_1 \longrightarrow & -2 & \mp 8 & \mp 4 & -6 & \mp 2 \\ \hline R_2 \longrightarrow & 0 & 5 & 6 & -7 & 2 \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -4 & -2 & 3 & -1 \\ 0 & 5 & 6 & -7 & 2 \\ 0 & 2 & -3 & 5 & -3 \end{array}\right]$$

$5R_3 - 2R_2$

$$\Rightarrow \begin{array}{lcccc|c} 5R_3 \longrightarrow & 0 & 10 & -15 & 25 & -15 \\ 2R_2 \longrightarrow & 0 & \pm 10 & \pm 12 & \mp 14 & \pm 4 \\ \hline R_3 \longrightarrow & 0 & 0 & -27 & 39 & -19 \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -4 & -2 & 3 & -1 \\ 0 & 5 & 6 & -7 & 2 \\ 0 & 0 & -27 & 39 & -19 \end{array}\right]$$

$R_2/5$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -4 & -2 & 3 & -1 \\ 0 & 1 & 6/5 & -7/5 & 2/5 \\ 0 & 0 & -27 & 39 & -19 \end{array}\right]$$

$R_3/-27$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -4 & -2 & 3 & -1 \\ 0 & 1 & 6/5 & -7/5 & 2/5 \\ 0 & 0 & 1 & -13/9 & 19/27 \end{array}\right]$$

$R_2 - \frac{6}{5} R_3$

$$\begin{array}{lcccc|c} R_2 \longrightarrow & 0 & 1 & 6/5 & -7/5 & 2/5 \\ \frac{6}{5}R_3 \longrightarrow & 0 & 0 & \pm 6/5 & \mp 26/15 & \pm 38/45 \\ \hline R_2 \longrightarrow & 0 & 1 & 0 & 1/3 & -4/9 \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -4 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1/3 & -4/9 \\ 0 & 0 & 1 & -13/9 & 19/27 \end{array}\right]$$

$R_1 + 2R_3$

$$\Rightarrow \begin{array}{rcccc|c} R_1 \rightarrow & 1 & -4 & -2 & 3 & -1 \\ 2R_3 \rightarrow & 0 & 0 & 2 & -26/9 & 38/27 \\ \hline R_1 \rightarrow & 1 & -4 & 0 & 1/9 & 11/27 \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & -4 & 0 & 1/9 & 11/27 \\ 0 & 1 & 0 & 1/3 & -4/9 \\ 0 & 0 & 1 & -13/9 & 19/27 \end{array}\right]$$

$R_1 + 4R_2$

$$\begin{array}{rcccc|c} R_1 \rightarrow & 1 & -4 & 0 & 1/9 & 11/27 \\ 4R_2 \rightarrow & 0 & 4 & 0 & 4/3 & -16/9 \\ \hline R_1 \rightarrow & 1 & 0 & 0 & 13/9 & -37/27 \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 13/9 & -37/27 \\ 0 & 1 & 0 & 1/3 & -4/9 \\ 0 & 0 & 1 & -13/9 & 19/27 \end{array}\right]$$

$\downarrow$

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let $x_4 = 9$

$R_3 \Rightarrow \quad x_3 - \frac{13}{9} x_4 = \frac{19}{27}$

$$x_3 - \frac{13}{9}(9) = \frac{19}{27}$$

$$x_3 = \frac{19}{27} + 13$$

$$x_3 = \frac{370}{27}$$

$R_2 \Rightarrow \quad x_2 + \frac{1}{3} x_4 = \cancel{\frac{37}{27}} \; \frac{-4}{9}$

$$x_2 = \frac{-4}{9} - 3$$

$$x_2 = -31/9$$

$R_1 \Rightarrow \quad x_1 + \frac{13}{9} x_4 = \frac{-37}{27}$

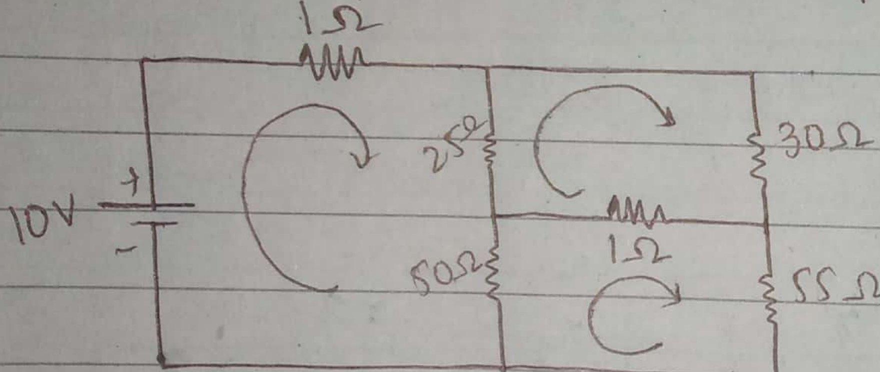
$$x_1 = \frac{-37}{27} - 13$$

$$x_1 = \frac{-388}{27}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -388/27 \\ -31/9 \\ 370/27 \\ 9 \end{bmatrix}$$

Answer! *

Q2 Solve the given circuit by row echelon (Gauss elimination form)



$$\begin{aligned} 76 I_1 - 25 I_2 - 50 I_3 &= 10 \\ -25 I_1 + 56 I_2 - I_3 &= 0 \\ -50 I_1 - I_2 + 106 I_3 &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 76 & -25 & -50 & 10 \\ -25 & 56 & -1 & 0 \\ -50 & -1 & 106 & 0 \end{array}\right]$$

$76R_2 + 25R_1$ $\quad$ $76R_3 + 50R_1$

$$\Rightarrow \left[\begin{array}{ccc|c} 76 & -25 & -50 & 10 \\ 0 & 3631 & -1326 & 250 \\ 0 & -1326 & 5556 & 500 \end{array}\right]$$

$R_3/2$

$$\Rightarrow \left[\begin{array}{ccc|c} 76 & -25 & -50 & 10 \\ 0 & 3631 & -1326 & 250 \\ 0 & -663 & 2778 & 250 \end{array}\right]$$

$3631 R_3 + 663 R_2$

$$\Rightarrow \left[\begin{array}{ccc|c} 76 & -25 & -50 & 10 \\ 0 & 3631 & -1326 & 250 \\ 0 & 0 & 9207780 & 1073500 \end{array}\right]$$

$R_3 \Rightarrow \quad 9207780 I_3 = 1073500$

$$I_3 = \frac{2825}{24231}$$

$R_2 \Rightarrow \quad 3631 I_2 - 1326 I_3 = 250$

$$3631 I_2 - 1326\left(\frac{2825}{24231}\right) = 250$$

$$I_2 = \frac{900}{8077} \quad 0.042$$

$R_1 \Rightarrow 76I_1 - 25I_2 - 50I_3 = 10$

$$76I_1 - 25(0.042) - 50\left(\frac{2825}{24231}\right) = 10$$

$$I_1 = 0.22$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0.22 \\ 0.042 \\ 2825/24231 \end{bmatrix}$$ Answer!

Q3 Balance the given unbalance chemical equation by Row reduction to Echelon form.

$$3PbN_4 + 2CrMn_2O_8 \longrightarrow 2Pb_2O_3 + Cr_2O_2 + 2MnO_2 + NO$$

$$\begin{bmatrix} Pb \\ N \\ Cr \\ Mn \\ O \end{bmatrix}$$

$$x_1(3PbN_4) + x_2(2CrMn_2O_8) \longrightarrow x_3(2Pb_2O_3) + x_4(Cr_2O_2) + x_5(2MnO_2) + x_6(NO)$$

$$\Rightarrow \left[\begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & -2 & 0 & 0 \\ 0 & 16 & -6 & -2 & -4 & -1 & 0 \end{array}\right]$$

$R_2 - 4R_1$

$$\begin{array}{l|cccccc|c}
R_2 \rightarrow & 12 & 0 & 0 & 0 & 0 & -1 & 0 \\
4R_1 \rightarrow & -12 & 0 & \mp 16 & 0 & 0 & 0 & 0 \\
\hline
 & 0 & 0 & 16 & 0 & 0 & -1 & 0
\end{array}$$

$$\left[\begin{array}{cccccc|c}
3 & 0 & -4 & 0 & 0 & 0 & 0 \\
0 & 0 & 16 & 0 & 0 & -1 & 0 \\
0 & 2 & 0 & -2 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & -2 & 0 & 0 \\
0 & 16 & -6 & -2 & -4 & -1 & 0
\end{array}\right]$$

$R_3 \longleftrightarrow R_2$

$$\left[\begin{array}{cccccc|c}
3 & 0 & -4 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & -2 & 0 & 0 & 0 \\
0 & 0 & 16 & 0 & 0 & -1 & 0 \\
0 & 4 & 0 & 0 & -2 & 0 & 0 \\
0 & 16 & -6 & -2 & -4 & -1 & 0
\end{array}\right]$$

$R_5 - 8R_2$ $\qquad$ $R_4 - 2R_2$

$$\begin{array}{l|cccccc|c}
R_5 \rightarrow & 0 & 16 & -6 & -2 & -4 & -1 & 0 \\
8R_2 \rightarrow & 0 & \pm 16 & 0 & \mp 16 & 0 & 0 & 0 \\
\hline
R_5 \rightarrow & 0 & 0 & -6 & 14 & -4 & -1 & 0
\end{array}$$

$$\begin{array}{l|cccccc|c}
R_4 \rightarrow & 0 & 4 & 0 & 0 & -2 & 0 & 0 \\
2R_2 \rightarrow & 0 & \pm 4 & 0 & \mp 4 & 0 & 0 & 0 \\
\hline
R_4 \rightarrow & 0 & 0 & 0 & 4 & -2 & 0 & 0
\end{array}$$

$$\Rightarrow \left[\begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 4 & -2 & 0 & 0 \\ 0 & 0 & -6 & 14 & -4 & -1 & 0 \end{array}\right]$$

$16R_5 + 6R_3$

$$\Rightarrow \left[\begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2\cancel{0} & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 4\cancel{8} & -2 & 0 & 0 \\ 0 & 0 & 0 & 224 & -64 & -22 & 0 \end{array}\right]$$

$R_5 - 56R_4$

$$\Rightarrow \left[\begin{array}{cccccc|c} 3 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 4 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 48 & -22 & 0 \end{array}\right]$$

$R_5/48$, $R_1/3$, $R_2/2$, $R_3/16$, $R_4/4$

$$\Rightarrow \left[\begin{array}{cccccc|c} 1 & 0 & -4/3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/16 & 0 \\ 0 & 0 & 0 & 1 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -11/24 & 0 \end{array}\right]$$

$R_2 + R_4$

$$\Rightarrow \left[\begin{array}{cccccc|c} 1 & 0 & -4/3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/16 & 0 \\ 0 & 0 & 0 & 1 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -11/24 & 0 \end{array}\right]$$

$R_1 + \frac{4}{3} R_3$

$$\begin{array}{lcccccc|c} R_1 \rightarrow & 1 & 0 & -4/3 & 0 & 0 & 0 & 0 \\ 4/3\,R_3 \rightarrow & 0 & 0 & 4/3 & 0 & 0 & -1/12 & 0 \\ \hline R_1 \rightarrow & 1 & 0 & 0 & 0 & 0 & -1/12 & 0 \end{array}$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -1/12 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/16 & 0 \\ 0 & 0 & 0 & 1 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -11/24 & 0 \end{array}\right]$$

$R_4 + \frac{1}{2} R_5$ $\qquad$ $R_2 + \frac{1}{2} R_2$

$$\begin{array}{lcccccc|c} R_4 \rightarrow & 0 & 0 & 0 & 1 & -1/2 & 0 & 0 \\ 1/2\,R_5 \rightarrow & 0 & 0 & 0 & 0 & 1/2 & -11/48 & 0 \\ \hline R_4 \rightarrow & 0 & 0 & 0 & 1 & 0 & -11/48 & 0 \end{array}$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -1/12 & 0 \\ 0 & 1 & 0 & 0 & 0 & -11/48 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/16 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/48 & 0 \\ 0 & 0 & 0 & 0 & 1 & -11/24 & 0 \end{array}\right]$$

$\Rightarrow$ let $x_6 = 12$

$R_5 \Rightarrow \quad x_5 - \frac{11}{24} x_6 = 0$

$$x_5 = \frac{11}{24}(12)$$

$$x_5 = \frac{11}{2}$$

$R_4 \Rightarrow \quad x_4 - \frac{11}{48} x_6 = 0$

$$x_4 = \frac{11}{48}(12)$$

$$x_4 = \frac{11}{4}$$

$R_3 \Rightarrow \quad x_3 - \frac{1}{16} x_6 = 0$

$$x_3 = \frac{3}{4}$$

$R_2 \Rightarrow \quad x_2 = \frac{11}{48}(12) = \frac{11}{4}$

$R_1 \Rightarrow \quad x_1 - \frac{1}{12}x_6 = 0$

$x_1 = 1$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 1 \\ 11/4 \\ 3/4 \\ 11/4 \\ 11/2 \\ 12 \end{bmatrix}$$

By Placing these values the equation will be balanced

Q4 For what values of h the given v is a linear combination of b_1, b_2, b_3.

$v = [h, 1, 3]$ $b_1 = [3 \ 2 \ 0]$

$b_2 = [-1, 4, 3]$ $b_3 = [1 \ -2 \ 2]$

$$\Rightarrow \left[\begin{array}{ccc|c} 3 & -1 & 1 & h \\ 2 & 4 & -2 & 1 \\ 0 & 3 & 2 & 3 \end{array}\right]$$

$3R_2 - 2R_1$

$$\begin{array}{lcccc|c} 3R_2 & \rightarrow & 6 & 12 & -6 & 3 \\ 2R_1 & \rightarrow & \underset{-}{6} & \underset{+}{-}2 & \pm 2 & \pm 2h \\ \hline R_2 & \rightarrow & 0 & 14 & -8 & 3-2h \end{array}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 3 & -1 & 1 & h \\ 0 & 14 & -8 & 3-2h \\ 0 & 3 & 2 & 3 \end{array}\right]$$

$14R_3 - 3R_2$

$$\Rightarrow \left[\begin{array}{ccc|c} 3 & -1 & 1 & h \\ 0 & 14 & -8 & 3-2h \\ 0 & 0 & 52 & 33+6h \end{array}\right]$$

$$33 \cancel{+6h} \cancel{\neq} 0$$

$$h \cancel{\neq -\frac{11}{2}}$$

For all ~~val~~ real values of h the system is consistent, so v is a linear combination of b_1, b_2, b_3 at all real values of h.