

Question # 01 :-

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$$

$$\lambda = 4$$

Solution :-

$$[A - \lambda I]x = 0$$

$$= \begin{bmatrix} 3-4 & 0 & -1 \\ 2 & 3-4 & 1 \\ -3 & 4 & 5-4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & -1 \\ 2 & -1 & 1 \\ -3 & 4 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & -1 & 1 & 0 \\ -3 & 4 & 1 & 0 \end{array} \right]$$

$$R_2 + 2R_1, \quad R_3 - 3R_1$$

$$\begin{bmatrix} -1 & 0 & -1 & | & 0 \\ 0 & -1 & -1 & | & 0 \\ 0 & 4 & 4 & | & 0 \end{bmatrix}$$

$$R_3 + 4R_2$$

$$\begin{bmatrix} -1 & 0 & -1 & | & 0 \\ 0 & -1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

x_3 : free column.

The system has non trivial solution so $\lambda = 4$ is the eigen value of $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$

$$\therefore x_3 = a$$

$$R_2 \Rightarrow -x_2 - a = 0$$

$$\boxed{x_2 = -a}$$

$$R_1 \Rightarrow -x_1 - x_3 = 0$$

$$\Rightarrow \boxed{x_1 = -a}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -a \\ -a \\ a \end{bmatrix} \text{ or } \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \rightarrow \text{eigen vector}$$

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Question # 02

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix}$$

Solution:-

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 1 & 1 \\ 0 & 5-\lambda & 0 \\ -2 & 0 & 7-\lambda \end{vmatrix} = 0$$

$$= 1[0 + 2(5-\lambda)] - 0 + (7-\lambda)(3-\lambda)(5-\lambda)$$

$$= 10 - 2\lambda + (7-\lambda)(15 - 5\lambda - 3\lambda + \lambda^2)$$

$$= 10 - 2\lambda + (7-\lambda)(15 - 8\lambda + \lambda^2)$$

$$= 10 + 2\lambda + 105 - 56\lambda + 7\lambda^2 - 15\lambda + 8\lambda^2 - \lambda^3$$

$$= -\lambda^3 + 15\lambda^2 - 73\lambda + 115 = 0$$

$$= -(\lambda^3 - 15\lambda^2 + 73\lambda - 115) = 0$$

$$\lambda^3 - 15\lambda^2 + 73\lambda - 115 = 0$$

$$\lambda = 3.58$$

$$\lambda = 6.41$$

$$\lambda = 5$$

$$\lambda = 5, 3.58, 6.41 \text{ etc.}$$

Question # 3

$$b_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \quad b_3 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \quad x = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$$

Solution:-

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 3 & 8 & 3 & -2 \end{array} \right] \quad \begin{array}{l} R_2 - R_1, \\ R_3 - 3R_1, \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 2 & 0 & -2 \end{array} \right] \quad R_3 + R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & -2 & -2 \end{array} \right]$$

$$R_3 \Rightarrow -2x_3 = -2$$

$$\boxed{x_3 = 1}$$

$$R_2 \Rightarrow -2x_3 - 2x_3 = 0$$

$$\boxed{x_2 = -1}$$

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$$R_1 \Rightarrow x_1 + 2x_2 + x_3 = 0$$

$$x_1 - 2 + 1 = 0$$

$$\boxed{x_1 = 1}$$

$$[x]_B = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \text{Ans}$$

Question # 04

$$A = \begin{bmatrix} 2 & 1 & 4 \\ -1 & 2 & -2 \\ 2 & 0 & -5 \end{bmatrix} \quad y = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

Solution:-

Breaking matrix into vectors.

$$u_1 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, u_3 = \begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$$

checking the vectors are orthogonal or not.

$$U_1 \cdot U_2 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$U_1 \cdot U_2 = 2 - 2 + 0 = 0$$

$$U_2 \cdot U_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$$

$$U_2 \cdot U_3 = 4 - 4 + 0 = 0$$

$$U_1 \cdot U_3 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$$

$$U_1 \cdot U_3 = 8 + 2 - 10 = 0$$

Vectors are orthogonal to each other.

$$y = C_1 U_1 + C_2 U_2 + C_3 U_3$$

For C_1

$$C_1 \frac{y \cdot U_1}{U_1 \cdot U_1} = \frac{3}{9} = \frac{1}{3}$$

For C_2

$$C_2 = \frac{y U_2}{U_2 \cdot U_2} = \frac{5}{5} = 1$$

For C_3

$$C_3 = \frac{y U_3}{U_3 U_3} = \frac{-30}{45} = -\frac{2}{3}$$

$$y = C_1 U_1 + C_2 U_2 + C_3 U_3$$

$$y = \frac{1}{3} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - \frac{2}{3} \begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$$

Ans