Question # 1:- solution set by LU factorization

$$2x_1 - 2x_2 + x_3 = 1$$
$$x_1 - 3x_2 + 2x_3 = -1$$
$$3x_2 - x_3 = 0$$

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ 0 & 3 & -1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ 0 & 3 & -1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}}_{L} \cdot \underbrace{\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}}_{U}$$

$$A = L \cdot U$$

$$\begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ 0 & 3 & -1 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21}U_{11} & l_{21}U_{12} + U_{22} & l_{21}U_{13} + U_{33} \\ l_{31}U_{11} & l_{31}U_{12} + l_{32}U_{22} & l_{31}U_{13} + l_{32}U_{23} + U_{33} \end{bmatrix}$$

Comparing

$U_{11} = 2$ | $U_{12} = -2$ | $U_{13} = 1$

$l_{21} U_{11} = 1$

$l_{21}(2) = 1$

$\boxed{l_{21} = \frac{1}{2}}$

$l_{21} U_{12} + U_{22} = -3$

$(1/2)(-2) + U_{22} = -3$

$-1 + U_{22} = -3$

$\boxed{U_{22} = -2}$

$l_{21} U_{13} + U_{23} = 2$

$(1/2)(1) + U_{23} = 2$

$1/2 + U_{23} = 2$

$\boxed{U_{23} = 3/2}$

$l_{31} U_{11} = 0$

$l_{31}(2) = 0$

$\boxed{l_{31} = 0}$

$l_{31} U_{12} + l_{32} U_{22} = 3$

$(0) U_{12} + l_{32}(-2) = 3$

$\boxed{l_{32} = -3/2}$

$l_{31} U_{13} + l_{32} U_{23} + U_{33} = -1$

$(0) U_{13} + \left(\frac{-3}{2}\right)\left(\frac{3}{2}\right) + U_{33} = -1$

$\frac{-9}{4} + U_{33} = -1$

$U_{33} = \frac{-4+9}{4}$

$\boxed{U_{33} = \frac{5}{4}}$

Hence,

$$\begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ 0 & 3 & -1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & -3/2 & 1 \end{bmatrix}}_{L} \cdot \underbrace{\begin{bmatrix} 2 & -2 & 1 \\ 0 & -2 & 3/2 \\ 0 & 0 & 5/4 \end{bmatrix}}_{U}$$

we have

$U \cdot x = y$ —— 1

$L \cdot y = b$ —— 2

Taking 2

$L \cdot y = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & -3/2 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$R_1 \Rightarrow \boxed{y_1 = 1}$

$R_2 \Rightarrow 1/2 y_1 + y_2 = -1$

$$\frac{1}{2} + y_2 = -1$$

$$y_2 = \frac{-2-1}{2}$$

$$\boxed{y_2 = \frac{-3}{2}}$$

$R_3 \Rightarrow \frac{-3}{2} y_2 + y_3 = 0$

$$\frac{-3}{2}\left(\frac{-3}{2}\right) + y_3 = 0$$

$$\frac{+9}{4} + y_3 = 0$$

$$\boxed{y_3 = -\frac{9}{4}}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3/2 \\ -9/4 \end{bmatrix}$$

Taking (2)

$$U \cdot x = y$$

$$\begin{bmatrix} 2 & -2 & 1 \\ 0 & -2 & 3/2 \\ 0 & 0 & 5/4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3/2 \\ -9/4 \end{bmatrix}$$

$R_3 \Rightarrow \quad 5/\cancel{4}x_3 = -9/\cancel{4}$

$$\boxed{x_3 = -9/5}$$

$R_2 \Rightarrow \quad -2x_2 + 3/2x_3 = -3/2$

$$-2x_2 + 3/2(-9/5) = -3/2$$

$$-2x_2 - \frac{27}{10} = \frac{-3}{2}$$

$$\frac{-20x_2 - 27}{10} = \frac{-3}{2}$$

$$-20x_2 - 27 = -15$$

$$\cancel{20}^{5}x_2 = -\cancel{12}^{3}$$

$$\boxed{x_2 = -\frac{3}{5}}$$

$R_1 \Rightarrow \quad 2x_1 - 2x_2 + x_3 = 1$

$$2x_1 - 2(-3/5) - 9/5 = 1$$

$$2x_1 = 1 + 9/5 - 6/5$$

$$2x_1 = \frac{8}{5} \Rightarrow \boxed{x_1 = 4/5}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4/5 \\ -3/5 \\ -9/5 \end{bmatrix} \quad \text{Ans}$$

Question #2:- Inverse of matrices (if possible).

(i): $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 0 & 1 & 4 \end{bmatrix}$ —— (1)

$$A^{-1} = \frac{1}{|A|} \text{ Adjoint of } A$$

<u>for $|A|$:</u>

Expanding (1) w.r.t C_1

$$|A| = 1 \begin{vmatrix} 0 & 3 \\ 1 & 4 \end{vmatrix} - 2 \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix}$$

$$= (-3) - 2(-4-2)$$

$$= -3 + 12$$

$$\boxed{|A| = 9}$$

Now for Adjoint of A.

$$\begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

R.W

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 0 & 1 & 4 \end{bmatrix}$$

$C_{11} = +(-3) = -3$

$C_{12} = -(-6) = 6$

$C_{13} = +(-3) = -3$

$C_{21} = -(8) = -8$

$C_{22} = +(4) = 4$

$C_{23} = -(-1) = 1$

$C_{31} = +(2) = 2$

$C_{32} = -(1) = -1$

$C_{33} = +(2) = 2$

Hence $\text{Adj } A = \begin{bmatrix} -3 & -8 & 2 \\ 6 & 4 & -1 \\ -3 & 1 & 2 \end{bmatrix}$

so $A^{-1} = \dfrac{1}{|A|}$ Adjoint of A

$$A^{-1} = \frac{1}{9} \begin{bmatrix} -3 & -8 & 2 \\ 6 & 4 & -1 \\ -3 & 1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1/3 & -8/9 & 2/9 \\ 2/3 & 4/9 & -1/9 \\ -1/3 & 1/9 & 2/9 \end{bmatrix}$$

Ans

ii): $B = \begin{bmatrix} 3 & -4 & 3 \\ 6 & 9 & 6 \\ -7 & 8 & -7 \end{bmatrix}$ —— (i)

$B^{-1} = \frac{1}{|B|}$ Adj of B

for $|B|$:

Expanding (i) w.r.t R_1:

$$|B| = 3\begin{vmatrix} 9 & 6 \\ 8 & -7 \end{vmatrix} + 4\begin{vmatrix} 6 & 6 \\ -7 & -7 \end{vmatrix} + 3\begin{vmatrix} 6 & 9 \\ -7 & 8 \end{vmatrix}$$

$\Rightarrow 3[-63-48] + 4[-42+42] + 3[48+63]$

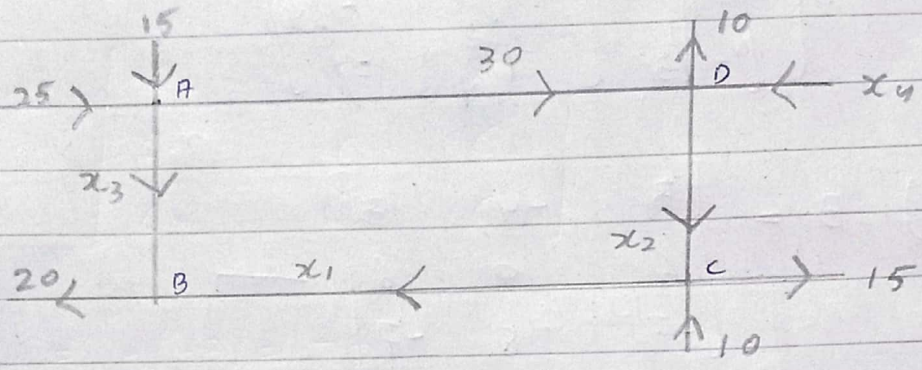
$\Rightarrow 3(-111) + 3(111)$

$\Rightarrow -333 + 333$

$|B| = 0$

since $|B| = 0$ so the inverse of matrics is not possible.

Q4) Question # 3:-



flow in = flow out

A ⇒ $15 + 25 = x_3 + 30$ ——— (1)

B ⇒ $x_3 + x_1 = 20$ ——— (2)

C ⇒ $10 + x_2 = 15 + x_1$ ——— (3)

D ⇒ $30 + x_4 = 10 + x_2$ ——— (4)

from Eq # (1)

$$15 + 25 = x_3 + 30$$

$$x_3 = 15 + 25 - 30$$

$$\boxed{x_3 = 10}$$

from Eq # (2)

$$x_1 + x_3 = 20$$

$x_1 + 10 = 20$ $\quad \because x_3 = 10$

$\boxed{x_1 = 10}$

from Eq # 3.

$10 + x_2 = 15 + x_1 \quad \because x_1 = 10$

$x_2 = 15 + 10 - 10$

$\boxed{x_2 = 15}$

from Eq # 4

$30 + x_4 = 10 + x_2$

$x_4 = 10 - 30 + 15 \quad \because x_2 = 15.$

$\boxed{x_4 = -5}$

Now putting values in equations

eq 1 $\Rightarrow$ $15 + 25 = x_3 + 30$

$40 = 10 + 30$

$40 = 40$ proved

eq 2 $\Rightarrow$ $x_3 + x_1 = 20$

$10 + 10 = 20$

$20 = 20$ proved

Eq ③ $\Rightarrow$ $10 + x_2 = 15 + x_1$

$10 + 15 = 15 + 10$

$25 = 25$ proved

Eq # ④ $\Rightarrow$ $30 + x_4 = 10 + x_2$

$30 + (-5) = 10 + 15$

$25 = 25$ proved.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 10 \\ -5 \end{bmatrix}$$ Ans

Question # 4:- find all "X" in R^4 that are mapped into the zero Vector by the transformation $X \longrightarrow AX$ for the given matrix "A"

$$A = \begin{bmatrix} 3 & -4 & 10 & 7 & -4 \\ -5 & -3 & -7 & -11 & 15 \\ 4 & 3 & 5 & 2 & 1 \\ 8 & -7 & 23 & 4 & 15 \end{bmatrix}$$

$Ax = 0$

$$\left[\begin{array}{ccccc|c} 3 & -4 & 10 & 7 & -4 & 0 \\ -5 & -3 & -7 & -11 & 15 & 0 \\ 4 & 3 & 5 & 2 & 1 & 0 \\ 8 & -7 & 23 & 4 & 15 & 0 \end{array}\right]$$

$3R_2 + 5R_1$, $3R_3 - 4R_1$, $3R_4 - 8R_1$

$$\left[\begin{array}{cccccc} 3 & -4 & 10 & 7 & -4 & 0 \\ 0 & -29 & 29 & 2 & 25 & 0 \\ 0 & 25 & -25 & -22 & 19 & 0 \\ 0 & 11 & -11 & 44 & 77 & 0 \end{array}\right]$$

$29R_3 + 25R_2$, $29R_4 - 11R_2$

$$\left[\begin{array}{ccccc|c} 3 & -4 & 10 & 7 & -4 & 0 \\ 0 & -29 & 29 & 2 & 25 & 0 \\ 0 & 0 & 0 & -588 & 886 & 0 \\ 0 & 0 & 0 & -1254 & -2505 & 0 \end{array}\right]$$

$588R_4 - 1254R_3$

$$\begin{array}{c} \left[\begin{array}{ccccc|c} 3 & -4 & 10 & 7 & -4 & 0 \\ 0 & -29 & 29 & 2 & 25 & 0 \\ 0 & 0 & 0 & -588 & 886 & 0 \\ 0 & 0 & 0 & 0 & 361896 & 0 \end{array}\right] \\ \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \end{array}$$

x_3 is free column so let $x_3 = 1$

$R_4 \Rightarrow \quad 361846\, x_5 = 0$

$\boxed{x_5 = 0}$

$R_3 \Rightarrow \quad -588x_4 + 886x_5 = 0$

$-588x_4 = 0$

$\boxed{x_4 = 0}$

$R_2 \Rightarrow \quad -29x_2 + 29x_3 + 2x_4 + 25x_5 = 0$

$-29x_2 + 29x_3 + 0 + 0 = 0$

$29x_3 = 29x_2$

$29(1) = 29(x_2)$

$\boxed{x_2 = 1}$

$R_1 \Rightarrow \quad 3x_1 - 4x_2 + 10x_3 + 7x_4 - 4x_5 = 0$

$3x_1 - 4 + 10 + 0 - 0 = 0$

$3x_1 + 6 = 0$

$\boxed{x_1 = -2}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Answer