Question #01: Evaluate the system of Equations by Cramer's Rule:

$$2x_1 - 2x_2 + x_3 = 1$$
$$x_1 - 3x_2 + 2x_3 = -1$$
$$3x_2 - x_3 = 0$$

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ 0 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}$$

$$|A| = \begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ 0 & 3 & -1 \end{bmatrix}$$

$$= 2(3-6) + 2(-1-0) + 1(3-0)$$
$$= 2(-3) + 2(-1) + 1(3)$$
$$= -6 - 2 + 3$$
$$= -5$$

$$A_1 = \begin{bmatrix} 1 & -2 & 1 \\ -1 & -3 & 2 \\ 0 & 3 & -1 \end{bmatrix}$$

$= 1(3-6) + 2(1-0) + 1(-3-0)$

$= 1(-3) + 2(1) + 1(-3)$

$= -3 + 2 - 3$

$= -4$

$$|A_2| = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

$= 2(1-0) - 1(-1-0) + 1(0-0)$

$= 2(1) - 1(-1) + 1(0)$

$= 2 + 1 + 0$

$= 3$

$$|A_3| = \begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & -1 \\ 0 & 3 & 0 \end{bmatrix}$$

$= 2(0+3) + 2(0-0) + 1(3+0)$

$= 2(3) + 2(0) + 1(3)$

$= 6 + 0 + 3$

$= 9$

$$\frac{|A_1|}{|A|} = \frac{-4}{-5} = \frac{4}{5} \quad \text{Ans}$$

$$\frac{|A_2|}{|A|} = \frac{-3}{5} \quad \text{Ans}$$

$$\frac{|A_3|}{|A|} = -\frac{9}{5} \quad \text{Ans}$$

Question #02: Determine the values of 'a' such that $\left\{ \begin{bmatrix} 1 \\ a \end{bmatrix}, \begin{bmatrix} a \\ a+2 \end{bmatrix} \right\}$ are linearly independ...

$$\left[\begin{array}{cc|c} 1 & a & 0 \\ a & a+2 & 0 \end{array}\right]$$

$R_2 - aR_1$

$$\left[\begin{array}{cc|c} 1 & a & 0 \\ 0 & a+2-a^2 & 0 \end{array}\right]$$

$$-a^2 + a + 2 = 0$$

$$a^2 - a - 2 = 0$$

$$a^2 - 2a + a - 2 = 0$$
$$a(a-2) + 1(a-2) = 0$$
$$(a+1)(a-2) = 0$$

$a + 1 = 0$, $a - 2 = 0$

$\boxed{a = -1}$, $\boxed{a = 2}$ Ans

The given system of Equation is linearly independent except for these two values.

Question # 03: Evaluate the bases for Null A and Col A.

$$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$$

$$A = \left[\begin{array}{cccc|c} -2 & 4 & -2 & -4 & 0 \\ 2 & -6 & -3 & 1 & 0 \\ -3 & 8 & 2 & -3 & 0 \end{array}\right]$$

$R_2 + R_1$

$$\left[\begin{array}{cccc|c} -2 & 4 & -2 & -4 & 0 \\ 0 & -2 & -5 & -3 & 0 \\ -3 & 8 & 2 & -3 & 0 \end{array}\right]$$

$2R_3 - 3R_1$

$$\left[\begin{array}{cccc|c} -2 & 4 & -2 & -4 & 0 \\ 0 & -2 & -5 & -3 & 0 \\ 0 & 4 & 10 & 6 & 0 \end{array}\right]$$

$R_3 + 2R_2$

$$\left[\begin{array}{cccc|c} -2 & 4 & -2 & -4 & 0 \\ 0 & -2 & -5 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

free variable.

let $x_3 = a,\ x_4 = b$

$R_2 \Rightarrow -2x_2 - 5x_3 - 3x_4 = 0$

$\Rightarrow -2x_2 - 5a - 3b = 0$

$\Rightarrow \boxed{x_2 = \frac{5a + 3b}{-2}}$

$R_3 \Rightarrow -2x_1 + 4\left[\frac{5a + 3b}{-2}\right] - 2a - 4b = 0$

$$\Rightarrow -2x_1 + \frac{20a + 12b + 4a + 8b}{-2} = 0$$

$$\Rightarrow 2x_2 = \frac{24a + 20b}{-2}$$

$$\Rightarrow x_2 = \frac{\cancel{24}^{6}a + \cancel{20}^{5}b}{-\cancel{4}}$$

$$[x_2 = -6a - 5b]$$

$$\text{Null } A = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -6a - 5b \\ -5/2\,a - 3/2\,b \\ a \\ b \end{bmatrix}$$

$$= \underbrace{a \begin{bmatrix} -6 \\ -5/2 \\ 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} -5 \\ -3/2 \\ 0 \\ 1 \end{bmatrix}}_{\text{[Basis for Null } A]}$$

$$\text{Col } A = \left\{ \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix} \right\}$$ Ans

[Bases for Col A].

Question # 04: The set $B = \{2+3t,\ 1-2t+t^2,\ 4t+2t^2+5t^3\}$ is a bases for P_3, find the Co-ordinate vector of $p(t) = 3+4t-2t^2+10t^3$.

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 3 & -2 & 4 & 4 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 5 & 10 \end{array}\right]$$

$2R_2 - 3R_1$

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 0 & -7 & 8 & -1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 5 & 10 \end{array}\right]$$

$7R_3 + R_2$

$$\begin{bmatrix} 2 & 1 & 0 & 3 \\ 0 & -7 & 8 & -1 \\ 0 & 0 & 22 & -15 \\ 0 & 0 & 5 & 10 \end{bmatrix}$$

$22R_4 - 5R_3$

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 0 & -7 & 8 & -1 \\ 0 & 0 & 22 & -15 \\ 0 & 0 & 0 & 295 \end{array}\right] \text{(No solution).}$$

No solution because of R_4, Co-ordinates vector is not possible to be found.