

# Classification algorithms for fetal QRS extraction in abdominal ECG signals

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## Abstract.

Fetal heart rate monitoring through non-invasive electrocardiography is of great relevance in clinical practice to supervise the fetal health during pregnancy. However, the analysis of fetal ECG is considered a challenging problem for biomedical and signal processing communities. This is mainly due to the low signal-to-noise ratio of fetal ECG and the difficulties in cancellation of maternal QRS complexes, motion, etc. This paper presents a survey of different unsupervised classification algorithms for the detection of fetal QRS complexes from abdominal ECG signals. Concretely, clustering algorithms are applied to classify signal features into noise, maternal QRS complexes and fetal QRS complexes. Hierarchical, k-means, k-medoids, fuzzy c-means, and dominant sets were the selected algorithms for this work. A MATLAB GUI has been developed to automatically apply the clustering algorithms and display FHR monitoring. Real abdominal ECG signals have been used for this study, which validate the proposed method and show high efficiency.

**Keywords:** Abdominal ECG, fetal heart rate, clustering algorithms, MATLAB, GUI.

## 1 Introduction

One the most important ways to detect cardiac anomalies in early stages of the fetus heart forming and supervise its well-being is the monitoring of the fetal heart activity [1]. The fetal electrocardiogram (FECG) [2] signal may not be directly measurable, and it has to be determined from the measurement of a composite signal as for example the abdominal ECG. Noninvasive fetal electrocardiography consists in the signal recording by using surface electrodes placed on the abdomen of a pregnant woman. It has huge potential applications, but presents some drawbacks mainly due to abdominal recordings of fetal ECG have lower signal-to-noise ratio (SNR) as compared with the invasive procedure. The significant amount of noise comes from fetal brain activity, muscle contractions, mother electromyogram (EMG) and respiration, movement artifacts and etc. Moreover, the considerably higher amplitude of the maternal ECG (MECG) components as compared to the FECG components makes difficult the extraction of fetal information. Discrete Wavelet Transform (DWT) [3] can

be used for the suppression of different types of noise including DC levels and wandering [4], [5]. Different approaches have been also proposed for the extraction of the FECG and the detection of parameters from this signal [6], [7]. However, the AECG signal processing can be oriented to extraction of information such as FHR, but avoiding the processing required by MECG removing. In the present paper, different clustering algorithms are studied to be applied on the denoised AECG signals [5] for fetal QRS detection. From this study, a new approach for FHR extraction using AECG signals is proposed which uses single-channel, does not require the use of a maternal ECG reference signal and does not need removing the maternal components.

## 2 Clustering overview

Clustering algorithms are used for data partitioning into a certain number of clusters [8]. Most researchers describe a cluster by considering the internal homogeneity and the external separation [9], *i.e.*, patterns in the same cluster should be similar to each other, while patterns in different clusters should not. Some simple mathematical descriptions of several clustering methods [10] are presented at this review.

### 2.1 Clustering procedure

The general scheme of a clustering procedure is shown in Fig. 1.

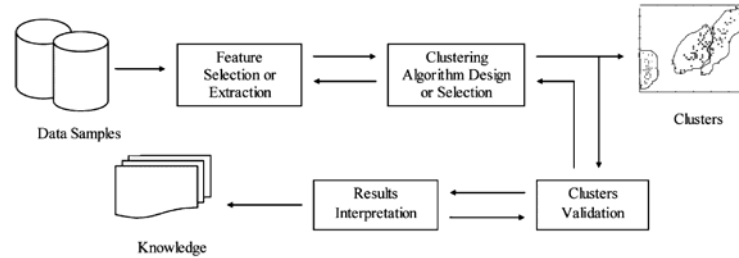


Fig. 1. General scheme of a clustering procedure [8].

- **Feature selection or extraction.** Feature selection chooses distinguishing features from a set of candidates, while feature extraction utilizes some transformations to generate useful and novel features from the original ones [11], [12]. In general, ideal features should be used to distinguish patterns belonging to different clusters, immune to noise, easy to extract and easy to interpret.
- **Clustering algorithm design or selection.** This step is usually combined with the selection of a corresponding proximity measure and the construction of a criterion function. Once a proximity measure is chosen, the construction of a clustering criterion function makes the partition of clusters to be an optimization problem. There is no clustering algorithm which can be universally used to solve all problems. Therefore, it is important to carefully investigate the characteristics of the problem at hand, in order to select or design an appropriate clustering strategy.

- **Cluster validation.** Given a data set, each clustering algorithm can always generate a division, no matter whether the structure exists or not. Moreover, different approaches usually lead to different clusters; and even for the same algorithm, parameter identification or the presentation order of input patterns may affect the final results. Therefore, effective evaluation standards and criteria are important to provide the users with a degree of confidence in the clustering results.
- **Results interpretation.** The final goal of clustering is to provide users with meaningful insights from the original data, so that they can effectively solve the problems. Experts in the relevant fields will interpret the data partition. Further analysis may be required to guarantee the reliability of extracted knowledge.

## 2.2 Distance measures

Given a set of patterns,  $X = \{x_1, \dots, x_j, \dots, x_N\}$ , where  $x_j = (x_{j1}, x_{j2}, \dots, x_{jd})^T \in R^d$  each measure  $x_{ji}$  is said to be a feature. The distance or dissimilarity function is the main tool to measure similarity between features and it is defined to satisfy the following conditions [8]:

1. Symmetry:  $D(x_i, x_j) = D(x_j, x_i)$
2. Positivity:  $D(x_i, x_j) \geq 0$  for all  $x_i$  and  $x_j$ .
3. Minkowski inequality:  $D(x_i, x_j) \leq D(x_i, x_k) + D(x_k, x_j) \quad \forall x_i, x_j, x_k$
4. Reflexivity:  $D(x_i, x_j) = 0$  if  $x_i = x_j$ .

Measure	Mathematical expression	Comments
Minkowski distance	$D_{xy} = (\sum_{l=1}^d  x_l - y_l ^n)^{1/n}$	Features with large values and variances tend to dominate over other features
Cityblock distance	$D_{xy} = \sum_{l=1}^d  x_l - y_l $	Special case of Minkowski metric at $n = 1$ . Tend to form hyperrectangular clusters.
Euclidean distance	$D_{xy} = (\sum_{l=1}^d  x_l - y_l ^2)^{1/2}$	The most commonly used metric. Tend to form hyperspherical clusters.
Squared euclidean distance	$D_{xy} = \sum_{l=1}^d  x_l - y_l ^2$	
Chebychev distance	$D_{xy} = \max  x_l - y_l $	Special case of Minkowski metric at $n \rightarrow \infty$
Mahalanobis distance	$D_{xy} = (x - y)^T S^{-1} (x - y)$ S covariance matrix	Invariant to any nonsingular linear transformation. Tend to form hyperellipsoidal clusters.

**Table 1.** Measure table

The distances used in this paper are shown in Table 1. The Minkowski distance [13] is a metric in a normed vector space which can be considered as a generalization of both Euclidean and Cityblock distance. The Euclidean Squared distance metric uses the same equation as the Euclidean distance metric, but does not calculate the square root. As a result, clustering with the Euclidean squared distance metric is faster.

### 2.3 Clustering algorithms

Clustering algorithms are divided into two groups: hierarchical and partitional. Hierarchical clustering attempts to construct a tree-like nested structure partition of  $X = \{H_1, \dots, H_Q\}$  ( $Q \leq N$ ), such that  $C_i \in H_m, C_j \in H_l$  y  $m > l$  imply  $C_i \in C_j$  or  $C_i \cap C_j = \emptyset$  for all  $i, j \neq i, m, l = 1, \dots, Q$ . Linkage-based clustering starts by assigning each data point to a single point cluster. Then, repeatedly, merge the “closest” clusters of the previous clustering, decreasing the number of clusters with each round. If kept going, this algorithm would eventually results in the trivial clustering in which all of the domain points share one large cluster. Then, to clearly define these algorithms we have first to decide how to measure the distance between clusters, and, second, to determine when to stop merging. The most common ways of extending the distance to a measure of distance between domain clusters are:

- Single Linkage Clustering: the distance between clusters is defined by the minimum distance:  $D(C_i, C_j) = \min\{d(x, y) : x \in C_i, y \in C_j\}$ .
- Average linkage Clustering: the distance is defined to be the average distance between a point in one of the clusters and a point in the other:  $D(C_i, C_j) = \frac{1}{|C_i||C_j|} \sum_{x \in C_i, y \in C_j} d(x, y)$ .
- Max linkage Clustering: the distance is the maximum distance between clusters:  $D(C_i, C_j) = \max\{d(x, y) : x \in C_i, y \in C_j\}$ .

Without employing a stopping rule, the outcome of such an algorithm can be described by a clustering dendrogram, *i.e.*, a tree of domain subsets, having the singleton sets in its leaves, and the full domain as its root [14].

On the other hand, hard partitional clustering decomposes a data set  $X$  into a  $K$  disjoint clusters  $C = \{C_1, \dots, C_K\}$  ( $K \leq N$ ) such that:

- (1)  $C_{i_K} \neq \emptyset, i = 1, \dots, K$
- (2)  $\bigcup_{i=1}^K C_i = X$
- (3)  $C_i \cap C_j = \emptyset, i, j = 1, \dots, K, \text{ y } i \neq j$

Contrary to hard partitional clustering (in which each pattern only belongs to one cluster), for fuzzy clustering [15] a pattern may belong to all clusters with a degree of membership  $u_{i,j} \in [0,1]$ , which represents the membership coefficient of the  $j$ -th object in the  $i$ -th cluster and satisfies the following two constraints:  $\sum_{i=1}^C u_{i,j} = 1$  and  $d \sum_{i=1}^C u_{i,j} < N, \forall j$ .

## 3 Clustering technique for fetal QRS extraction

The proposed clustering technique for the extraction of fetal QRS complexes has been modeled using MATLAB. First, a previously proposed wavelet-based preprocessing is applied to the AECG signal in order to remove wandering and noise [5] [7]. Then, the new clustering technique is used for fetal QRS extraction. Finally, the false positive and false negative procedure proposed in [7] is used to correct false-detected

and/or non-detected fetal QRS complexes. This section is devoted to the study of different clustering algorithms for the classification of special features in AECG signals in order to determine the cluster corresponding to fetal QRS complexes.

The first step consists in selecting the signal features allowing us to distinguish patterns belonging to different clusters. After an analysis of the AECG signals, the amplitude difference between a local maximum followed by a local minimum has been selected as the main feature. Extracting this amplitude feature from AECG signals, three different clusters can be obtained. The cluster with a greater value of amplitude indicates that our data corresponds to RS-peaks of maternal ECG complexes. The cluster with an intermediate value corresponds to RS-peaks of a fetal ECG complex. Lastly, the cluster with lower value corresponds to noise or other waves.

### 3.1 Avoiding local minima

Clustering is an optimization problem in which to solve the problem it is necessary to reach an absolute minimum. In spite of the computational cost, we use equivalent algorithms with less complexity. In contrast, like many other types of numerical minimizations, the solution often depends on the starting points. It is possible to reach a local minimum for any clustering algorithm, where reassigning any one point to a new cluster would increase the total sum of point-to-centroid distances, but where a better solution does exist. However, the algorithm can be replicated several times to overcome that problem, without increasing the computational cost. Fig. 2 displays an example where there is a local minimum at 5 replicates.

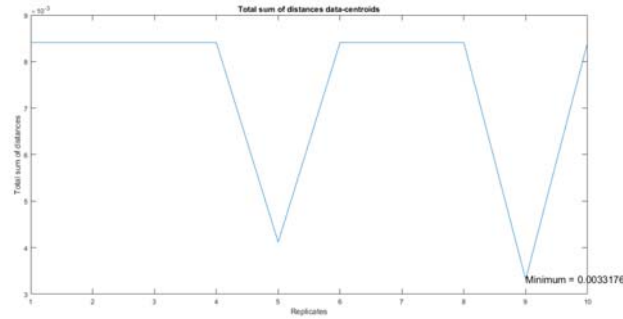


Fig. 2. Total sum of data-centroid distances based on the number of replicates

### 3.2 Optimal number of clusters

One of the most difficult tasks in any clustering problem is to find the best number of clusters,  $k$ . For the proposed clustering classifications over AECG signals, the desirable number of clusters is three and it should be also the minimum. Despite this, sometimes the signals will not be correctly classified, needing more than three groups. To solve this problem it is necessary to use different criteria indicating the optimum number of clusters. Basically the minimum average distance to centroid trying differ-

ent groups is looked for. When the best  $k$  is found, the average falls rapidly. There are two criteria which work well with our dataset, Davies-Bouldin criterion [16] and silhouette value [17]. Note that it is necessary to delimit these criteria in order to optimally solve our problem. In conclusion, it is not suitable to use these criteria for  $k > 8$ .

### 3.3 Clustering algorithms for fetal RS-peak detection

The clustering algorithms evaluated for fetal RS-peak detection are k-means, k-medoids, fuzzy C-means and hierarchical. About metrics, we have chosen squared euclidean (sqe) and cityblock (cb) distances. When using an unidimensional amplitude vector, cityblock distances are equal to Euclidean and Mahalanobis.

#### 3.3.1 K-means algorithm

This algorithm begins with initialization of the centroids. For this task we have selected k-means++ instead of Lloyd's k-means. Using a simulation study for several cluster orientations, Arthur and Vassilvitskii [18] have demonstrated that k-means++ achieves faster convergence to a lower sum of within-cluster, sum-of-squares point-to-cluster-centroid distances than Lloyd's algorithm. Then, the algorithm assigns each data to the cluster that has the closest centroid. When all data have been assigned, it recalculates the positions of the centroids. Assigning data and recalculating centroids are repeated until the centroids no longer move. Fig. 3 shows the data classification for this algorithm using abdomen-4 r01 recording from PhysioNet database [19]. The blue points correspond to maternal QRS complexes, green points to fetal QRS complexes and brown points to noise or other waves. The centroids are shown using red marks.

Comparing these classification results with database annotations, there is only one false-detected fetal QRS complex (FP) and one non-detected (FN). The FN occurs because the fetal QRS complex is masked by a maternal complex. Remember that false positive and negative corrections proposed in [7] are applied to improve the results.

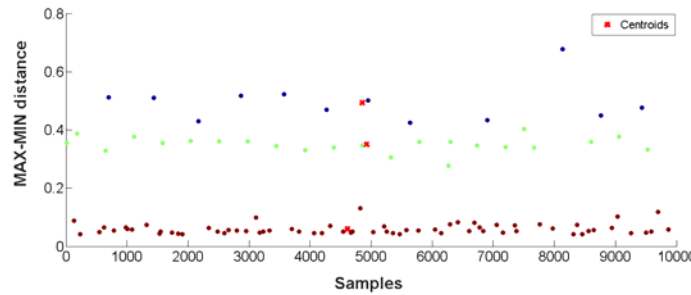


Fig. 3. K-means classification (squared Euclidean distance) for 10-seconds ab4 r01 recording

### 3.3.2 K-medoids algorithm

This algorithm is similar to k-means, thus its goal is to divide a set of measurements into  $k$  subsets or clusters so that the subsets minimize the sum of distances between a measurement and a center of the measurement's cluster. In the k-means algorithm, the center of the subset is the mean of measurements in the subset, often called a centroid. In the k-medoids algorithm, the center of the subset is a member of the subset, called a medoid [20]. Fig. 4 shows k-medoids classification for 10-seconds abdomen-2 r01 recording, where green points corresponds to maternal QRS complexes, brown to fetal QRS complexes and blue to noise or other waves.

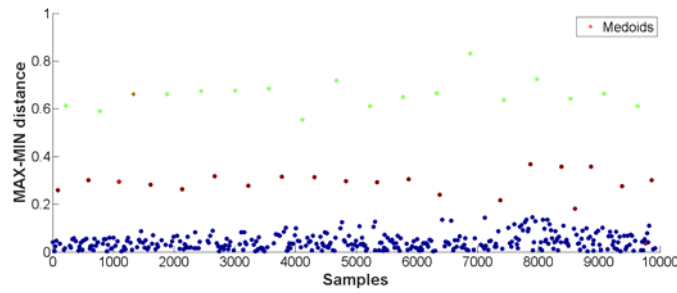


Fig. 4. K-medoids classification (squared Euclidean distance) for 10-seconds ab2 r10 recording

### 3.3.3 Fuzzy C-means algorithm

Fuzzy C-means [21] is a clustering method allowing each data to belong to multiple clusters with varying degrees of membership. For data classification we have chosen the greatest probability of every data, assigning it the group with a higher probability. Fig. 5 shows the data set along to their probability mass function.

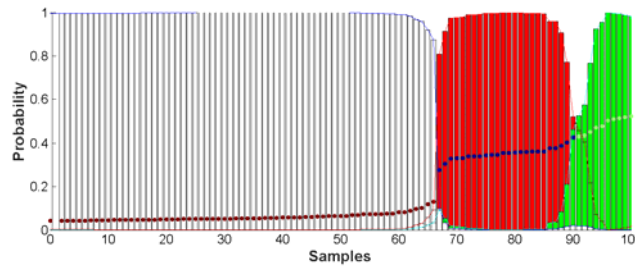


Fig. 5. Fuzzy C-means probability density function for 10-seconds ab4 r01 recording

### 3.3.4 Hierarchical algorithm

Hierarchical algorithm groups data over a variety of scales by creating a cluster tree or dendrogram. The tree is not a single set of clusters, but rather a multilevel

hierarchy, where clusters at one level are joined as clusters at the next level. Once obtained the dendrogram, criteria Davies-Bouldin and silhouette criterion value help us to determine the final number of clusters. Fig. 6 shows the dendrogram obtained for hierarchical classification of 10-seconds ab4 r01 recording.

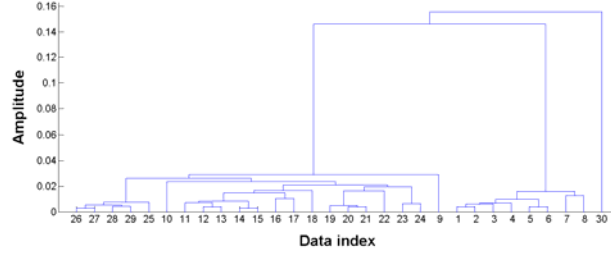


Fig. 6. Dendrogram for hierarchical classification of 10-seconds ab4 r01 recording

## 4 Results

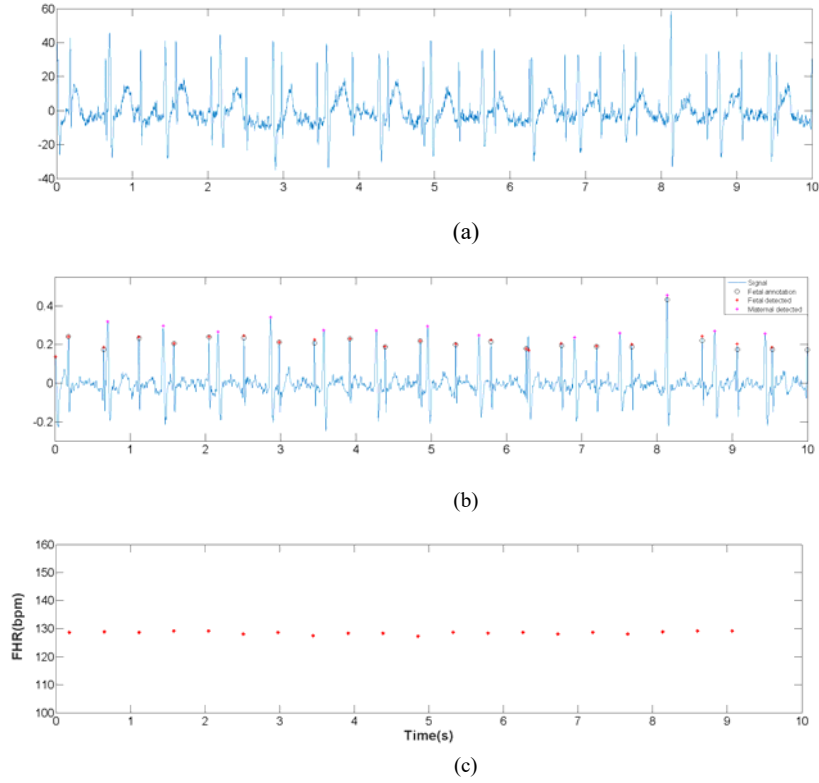
Recordings from the *Abdominal and Direct Fetal Electrocardiogram Database* [19] have been used for the training and the validation of the proposed method. Each recording comprises four differential signals acquired from maternal abdomen and the reference direct fetal electrocardiogram registered from the fetus head. The fetal R-wave locations were automatically determined in the direct FECG signal by means of on-line analysis applied in the KOMPOREL system [19]. The recordings, sampled with 16-bit resolution at 1ksp/s, are 5-minute long and the signal bandwidth is 1Hz–150Hz. Fig. 7 shows an example for the application of the proposed method. First subplot shows the original signal, second subplot shows the denoised signal including the detected maternal and fetal R-peaks using k-means algorithm and squared Euclidean distance, and finally, last subplot shows the FHR monitoring.

To assess the performance of the proposed FHR extraction method, the accuracy parameter can be studied [7]:

$$Acc = \frac{(TD)}{(TD+FP+FN)}$$

where TD are the true-detected fetal QRS complexes, FN are false negatives and FP are false positives. Table 2 shows the accuracies obtained for 1-minute for recordings of the database (4<sup>th</sup> minute s) using the algorithms and distances detailed in section 3. Annotations of database were used in order to detect the FPs and the FNs.  $Acc_1$  indicates the accuracy obtained after clustering classification step while  $Acc_2$  indicates the accuracy of the proposed method, after the FP and FN correction step. The average accuracies manifests that K-means cityblock and K-medoids squared Euclidean and cityblock achieve the best results. Fig. 8 and Fig. 9 display the FHR monitoring obtained using k-means algorithm and squared Euclidean distance for fetal QRS extraction in two different recordings.

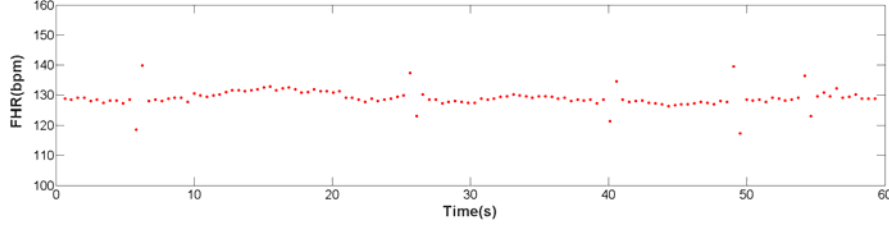




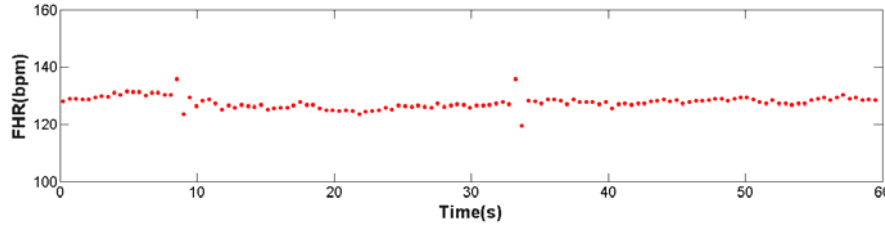
**Fig. 7.** 10-seconds ab4 r01 recording a) Original signal b) Denoised signal including detected maternal and fetal R-peaks (k-means and squared Euclidean distance) c) FHR monitoring

Recording	K-means sqe		K-means cb		K-medoids sqe		K-medoids cb		Fuzzy c-means		Hierarchical	
	Acc <sub>1</sub>	Acc <sub>2</sub>	Acc <sub>1</sub>	Acc <sub>2</sub>	Acc <sub>1</sub>	Acc <sub>2</sub>	Acc <sub>1</sub>	Acc <sub>2</sub>	Acc <sub>1</sub>	Acc <sub>2</sub>	Acc <sub>1</sub>	Acc <sub>2</sub>
<b>Ab1 r01</b>	88,3	93,8	88,3	93,8	88,3	93,8	88,3	93,8	88,3	88,3	57,2	57,2
<b>Ab4 r01</b>	83,1	88,0	63,1	70,2	86,9	90,8	85,3	90,8	63,5	66,0	63,2	66,0
<b>Ab2 r04</b>	86,2	92,3	87,0	90,8	86,2	90,8	86,9	90,8	86,2	88,0	46,4	46,5
<b>Ab3 r07</b>	87,5	93,7	86,2	91,5	87,5	93,1	86,2	91,5	86,8	92,2	87,5	93,7
<b>Ab4 r07</b>	54,0	58,6	83,8	94,4	83,8	94,4	83,8	94,4	83,8	94,4	83,8	94,4
<b>Ab1 r08</b>	95,3	99,3	95,3	99,3	95,3	99,3	95,3	99,3	95,3	99,3	95,3	99,3
<b>Ab1 r10</b>	90,9	99,4	90,9	99,4	90,9	99,4	90,9	99,4	90,9	90,9	27,9	29,4
<b>Ab4 r10</b>	87,0	94,4	87,0	94,4	87,0	94,4	87,0	94,4	87,0	94,4	87,0	94,4
<b>Average</b>	84,04	89,9	85,2	<b>94,8</b>	88,2	<b>94,5</b>	88,0	<b>94,3</b>	85,2	93,7	68,5	87,8

**Table 2.** Evaluation results



**Fig. 8.** FHR monitoring for 1-minute ab4 r01 recording



**Fig. 9** FHR monitoring for 1-minute ab1 r08 recording

#### 4.1 Graphical User Interface

A graphical user interface (GUI) has been developed as a platform for evaluating the results of the clustering algorithms proposed in this work. Fig. 10 shows a capture of this GUI. The user can select between the following input parameters:

- Signal parameters: AECG signal, channel, window length, start and final points.
- Cluster parameters: minimum number of groups ( $k = 3$ ) and maximum (advisable to use  $k \leq 8$ ), to calculate the number of clusters which makes the best classification, according to Davies-Bouldin criterion and silhouette value.

After the classification of the data, the principal table of this GUI (displayed in Fig. 10) shows the following results: used algorithm and metrics, number of clusters, computational cost, *Acc*, *Se* and *PDV* values [7]. *FPs and FNs* section shows the extraction evaluation when the correction over the algorithm with the best *Acc* is computed. *Fetal Heart Rate* section shows the instantaneous FHR after the FP and FN correction step. Finally, *Average results* section displays the average *Acc* of every cluster algorithm. Attending to GUI graphs shown in Fig. 10, upper-left-corner graph represents the AECG signal with the threshold algorithm proposed in [7] including a fix threshold, where fetal QRS complexes marked in pink, and the annotated complexes are marked in black (in every graph). Upper-right-corner graph displays the AECG signal with red marks that corresponds to the fetal QRS complexes extracted from the clustering algorithm with the highest *Acc*. Lower-left-corner graph shows the data groups (data are shorted by amplitude) accordingly with the clustering algorithm which has the highest *Acc*. Lower-right-corner graph represents the dendrogram

of hierarchical clustering with the highest *Acc.* Finally, the GUI also displays an additional graph which is not shown in Fig 10, with the fetal QRS complexes marked after FPs and FNs corrections.

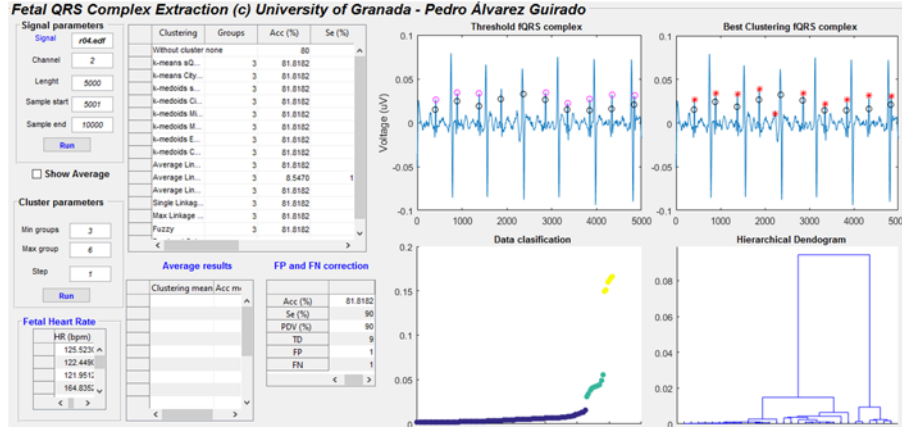


Fig. 10. Capture of the Graphical User Interface

## 5 Conclusion

This paper presents a new method for FHR extraction from AECG signals. It introduces a novel clustering procedure for the data classification and subsequent determination of the fetal QRS complexes. It consists of extracting the amplitude of the located maxima followed by minimum as main feature, which helps in the identification of RS-peaks of fetal QRS complexes. The obtained amplitudes are classified into three clusters corresponding to maternal QRS complexes, fetal QRS complexes and noise or other waves. This paper also shows the study of different clustering algorithms and related parameters meeting the best classification and thus the best accuracy in the fetal QRS complex extraction. From the extracted fetal QRS complexes, FHR monitoring is carried out. The proposed clustering-based presented was validated for real AECG signals obtaining high accuracy.

## 6 Acknowledgments

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