Solution: Makeup Quiz # 1

1. Given that we have a polynomial function of $P_5(a)$. What will be the basis of the polynomial:
   a. $\{1,x,x^2,x^3,x^4,x^5\}$
   b. $\{1,a,a^2,a^3,a^4,a^5\}$✅
   c. $\{x,x^2,x^3,x^4,x^5\}$
   d. $\{a,a^2,a^3,a^4,a^5\}$

2. Suppose we have an equation of $P_n(x) = 5x^4+3x^2+5x+8$. Which of the following is true:
   a. The degree of the equation is 4 and the basis is $\{1,x,x^2,x^3,x^4,x^5\}$
   b. The degree of the equation is 5 and the basis is $\{1,x,x^2,x^4,x^5\}$
   c. The degree of the equation is 4 and the basis is $\{1,x,x^2,x^3,x^4\}$✅
   d. The degree of the equation is 5 and the basis is $\{1,x,x^2,x^4\}$

3. While finding the machine epsilon, we use:
   a. The maximum value of |x|
   b. The minimum value of |x|✅
   c. The minimum value of |fl(x)-x|
   d. The maximum value of |fl(x)-x|✅

4. According to the IEEE Standard for Floating-Point Arithmetic (IEEE 754), the number of bits assigned to sign, exponent and mantissa for Double Precision are:
   a. s=1,e=11,m=52✅
   b. s=1,e=8, m=23
   c. s=1,e=5,m=10
   d. None of the above

5. The value of 2^{-1022} is stored for
   a. $\pm\infty$
   b. ± 0✅
   c. $-\infty$
   d. $+\infty$

6. It is given that $\beta=2$, $m=3$ and $e \in [-2,2]$. What will be the highest possible value that can be generated using the normalized form of the floating point representation?
   a. 3.75
   b. 31/4
   c. 1.75
   d. None of the above✅

7. Which of the following statement is true:
    a. The scale invariant error can approach infinity if we divide two large numbers
    b. The scale invariant error can approach infinity if we subtract two close numbers✅
    c. The scale invariant error can approach infinity if we add two large Numbers.
    d. None of the above

8. It is given that $\beta=2$, $m=3$ and $e \in [-2,2]$. What will be the Machine Epsilon value for the denormalized form?
    a. 0.0625✅
    b. 1/64
    c. 1/16✅
    d. 0.03125

9. If we are given 6 nodes, what should be the degree of the polynomial that is used to find the value of the coefficients using the Hermite Interpolation method?
    a. 13
    b. 5
    c. 11✅
    d. None of the above

10. Which of the following statement is true:
    a. Using Hermite interpolation, it is easier to incorporate new nodes when compared to Lagrange interpolation technique.
    b. Using Newton's divided difference, it is easier to incorporate new nodes when compared to Lagrange interpolation technique.✅
    c. Using the Vandermonde matrix technique, it is easier to incorporate new nodes when compared to Lagrange interpolation technique.
    d. None of the above

11. Which of the following statement is true:
    a. For the runge function, the relative error decreases with the increase in degree of the polynomial
    b. For the runge function, the most efficient choice is Chebyshev nodes.✅
    c. For the runge function, the most efficient choice is equally spaced nodes.
    d. None of the above.

12. For given nodes of (x_0,y_0), (x_1,y_1) and (x_2,y_2), the Lagrange basis, $l_0(x)$ would be:
    a. $(x-x_1)*(x-x_2)/(x_0-x_1)*(x_0-x_2)$✅
    b. $(x-x_1)*(y-y_1)/(x_0-x_1)*(y_0-y_2)$
    c. $(y-y_1)*(y-y_2)/(y_0-y_1)*(y_0-y_2)$
    d. None of the above

13. Which of the following is the correct expression for the Forward Difference method?
    a. (f(x)−f(x−h)/2h)
    b. (f(x+h)−f(x−h)/h)
    c. (f(x+h)−f(x)/h)✅
    d. None of the above

14. The truncation error of the central difference method varies according to:
    a. Truncation error is directly proportional to −h
    b. Truncation error is directly proportional to h^2✅
    c. Truncation error is inversely proportional to h
    d. Truncation error is directly proportional to h^4

15. Let f(x) = cos(x). The value of f'(0.5) when h=0.01, using the forward difference is:
    a. -0.095884
    b. 0.19177
    c. -0.48381✅
    d. None of the above

16. Let f(x) = cos(x). The value of f'(0.5) when h=0.001, using the backward difference is:
    a. -0.47899✅
    b. -0.47986
    c. -0.00015246
    d. None of the above

17. Let f(x) = cos(x). The value of $D^{(1)}_h$ using the Richardson Extrapolation formula with h=0.01 and h/2=0.005 is:
    a. -0.00015246
    b. 0.19177
    c. -0.095884
    d. None of the above✅