1(a) $n \geq \dfrac{\log(|b_0 - a_0|) - \log(\delta)}{\log(2)}$

$$\geq \frac{\log(|-2.8-(-6.2)|) - \log(1\times10^{-3})}{\log(2)}$$

$$\geq 11.7$$

$\therefore$ minimum number of iteration required would be 12.

1(b)

k	a_k	m_k	b_k	$f(a_k)$	$f(m_k)$	$f(b_k)$	$x_* \in [\ ,\ ]$
0	-6.2	-4.5	-2.8	-196.568	-49.875	13.008	[-4.5, -2.8]
1	-4.5	-3.65	-2.8	-49.875	-9.7996	13.008	[-3.65, -2.8]
2	-3.65	-3.225	-2.8	-9.7996	3.5324	13.008	[-3.65, -3.225]
3	-3.65	-3.4375	-3.225	-9.7996	-2.6228	3.5324	[-3.4375, -3.225]
4	-3.4375	-3.3313	-3.225	-2.6228	0.5774	3.5324	[-3.4375, -3.3313]
5	-3.4375	-3.3844	-3.3313	-2.6228	-0.9912	0.5774	[-3.3844, -3.3313]
6	-3.3844	-3.3579	-3.3313	-0.9912	-0.2006	0.5774	[-3.3579, -3.3313]
7	-3.3579	-3.3446	-3.3313	-0.2006	0.1904	0.5774	[-3.3579, -3.3446]
8	-3.3579	-3.3513	-3.3446	-0.2006	-6×10^{-3}	0.1904	[-3.3513, -3.3446]
9	-3.3513	-3.3480	-3.3446	-6×10^{-3}	0.0908	0.1904	[-3.3513, -3.3480]
10	-3.3513	-3.3497	-3.3480	-6×10^{-3}	0.0409	0.0908	[-3.3513, -3.3497]
11	-3.3513	-3.3505	-3.3497	-6×10^{-3}	0.0174	0.0409	[-3.3513, -3.3505]
12	-3.3513	-3.3509	-3.3505	-6×10^{-3}	5×10^{-3}	0.0174	[-3.3513, -3.3509]
13	-3.3513	-3.3511	-3.3509	-6×10^{-3}	$\underbrace{-1.9\times10^{-4}}_{<10^{-3}}$	5×10^{-3}	

$\therefore \boxed{x_* = -3.351}$ (upto 3 d.p./ 1×10^{-3}/ machine epsilon)

②

$$f(x) = x^2 e^{-x} - 0.5 \quad , \quad x_0 = 0.2$$

$$f'(x) = (x^2)(e^{-x})(-1) + (e^{-x})(2x)$$

$$= -x^2 e^{-x} + 2x e^{-x}$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k - \frac{x_k^2 e^{-x_k} - 0.5}{-x_k^2 e^{-x_k} + 2x_k e^{-x_k}}$$

k	x_k	$f(x_k)$	is $f(x_k) < 1\times10^{-4}$
0	0.2	-0.46725	NO
1	1.78528	0.03466	NO
2	1.24633	-0.05332	NO
3	1.44375	-8×10^{-3}	NO
4	1.48592	-3.5×10^{-4}	NO
5	1.48796	-3.5×10^{-7}	YES

Ans: $x_* = 1.4880$ (upto 4 d.p. / 1×10^{-4} / machine epsilon)

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3(a) $f(x) = x^6 - x^3 - 1 = 0$

let $a = x^3$

$f(x) = a^2 - a - 1 = 0$

$$a = \frac{+1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$a = \frac{1+\sqrt{5}}{2}$

$= 1.618$

$x^3 = 1.618$

$x_* = 1.174$

$a = \frac{1-\sqrt{5}}{2}$

$= -0.618$

$x^3 = -0.618$

$x_* = -0.852$

(b) $x^6 - x^3 - 1 = 0$

$x = \underbrace{(x^6 - 1)}_{g(x)}^{1/3}$

$g(x) = (x^6 - 1)^{1/3}$

$g'(x) = \frac{1}{3}(x^6 - 1)^{-2/3}(6x^5)$

$\lambda = |g'(x_*)| = \begin{cases} 3.236 & \text{at } x_* = 1.174 \\ -1.238 & \text{at } x_* = -0.852 \end{cases}$

$\therefore g(x) = (x^6 - 1)^{1/3}$ will not converge to any roots.

$x^6 - x^3 - 1 = 0$

$x = \underbrace{(x^3 + 1)}_{g(x)}^{1/6}$

$g(x) = (x^3 + 1)^{1/6}$

$g'(x) = \frac{1}{6}(x^3 + 1)^{-5/6}(3x^2)$

$\lambda = |g'(x_*)| = \begin{cases} 0.309 & \text{at } x_* = 1.174 \\ 0.810 & \text{at } x_* = -0.852 \end{cases}$

$\therefore g(x) = (x^3 + 1)^{1/6}$ might converge to [redacted] both roots.

(c) $x_0 = 60$

$g(x) = (x^3+1)^{1 \div 6}$

$g(60) = 7.746$

$g(7.746) = 2.784$

$g(2.784) = 1.681$

$g(1.681) = 1.338$

$g(1.338) = 1.226$

$g(1.226) = 1.190$

$g(1.190) = 1.179$

$g(1.179) = 1.176$

$g(1.176) = 1.175$

$g(1.175) = 1.174$

$g(1.174) = 1.174$

(4)

$$f(x) = 2x^3 + 7x^2 - 14x + 5$$

$$x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}$$

$$= x_k - \frac{(2x_k^3 + 7x_k^2 - 14x_k + 5)(x_k - x_{k-1})}{(2x_k^3 + 7x_k^2 - 14x_k + 5) - (2x_{k-1}^3 + 7x_{k-1}^2 - 14x_{k-1} + 5)}$$

k	x_k	$f(x_k)$
0	-5.5	-3.9
1	-4.5	27.5
2	-4.9135	5.5382
3	-5.0178	-1.1821
4	-4.9995	0.0330
5	-5.0000	0

(5) $f(x) = xe^{x} - 1$

Fixed point iteration formula:

$g(x) = e^{-x}$

Aitken Acceleration formula:

$$\hat{x}_{k+2} = x_k - \frac{(x_{k+1} - x_k)^2}{x_{k+2} - 2x_{k+1} + x_k}$$

$\boxed{x_0 = 0}$

k	x_k	$f(x_k)$	is $\lvert f(x_k)\rvert < 10^{-5}$?
0	0	-1	No
1	1	1.718281	No
2	0.367879	-0.468537	No
$\hat{2}$	0.612700	0.130681	No
3	0.541885	-0.068367	No
4	0.581650	0.040565	No
$\hat{4}$	0.567350	5.7×10^{-4}	No
5	0.567026	-3.2×10^{-4}	No
6	0.567210	1.8×10^{-4}	No
$\hat{6}$	0.567143	-8×10^{-7}	Yes

$x_* = 0.56714$ (upto 5 d.p./ 10^{-5}/ machine epsilon)