Answer of question 1:

Newton's Method:

$f(x) = 7x^3 - 8x + 4$

$\therefore f'(x) = 21x^2 - 8$

$$\therefore x_{k+1} = x_k - \frac{7x_k^3 - 8x_k + 4}{21x_k^2 - 8}$$

Starting point: $x_0 = -1$ $(k = 0)$

k	x_k	$f(x_k)$	If $f(x_k) < 10^{-6}$
0	-1	5	NO
1	-1.3846154	-3.5047797	NO
2	-1.2759749	-0.3342106	NO
3	-1.2632141	-0.0043496	NO
4	-1.2630436	-0.0000009	Yes

$f(x_4) \approx 0$

$\therefore x_* = -1.2630436$

Aitken Acceleration

k	x_k	$f(x_k)$	If $\lvert f(x_k)\rvert < 10^{-6}$
0	-1	5	NO
1	-1.3846154	-3.5047797	NO
2	-1.2759749	-0.3342106	NO
$\hat{2}$	-1.2999032	-0.9763392	NO
3	-1.2663802	-0.0341328	NO
4	-1.2630454	-0.0000468	NO
$\hat{4}$	-1.2629933	0.0012817	NO
5	-1.2630436	-0.0000009	Yes

$f(x_5) \approx 0$

$\therefore x_* = -1.2630436$

Answer of question 2:

a

(1)

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$$

Augmented matrix:-

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & -3 \end{array}\right] \xrightarrow[r_3' = r_3 - \frac{3}{1} \times r_1]{r_2' = r_2 - \frac{2}{1} \times r_1} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -1 & -1 & -1 \\ 0 & -4 & -1 & -6 \end{array}\right]$$

$$\xrightarrow{r_3'' = r_3' - \frac{-4}{-1} \times r_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -1 & -1 & -1 \\ 0 & 0 & 3 & -2 \end{array}\right]$$

Using forward substitution:-

$3x_3 = -2$

$x_3 = -2/3 = -0.667$

(2) finding x_2:-

$(-1)x_2 + (-1)x_3 = -1$

$\Rightarrow -x_2 + 0.667 = -1$

$\Rightarrow x_2 = 1 + 0.667 = 1.6667$

(3) finding x_1:-

$1(x_1) + (1)x_2 + 0 \cdot x_3 = 1$

$x_1 + x_2 = 1$

$x_1 = 1 - x_2 = 1 - 1.6667 = -0.667$

b

(4)

Upper triangular matrix, U will be the final matrix from gaussian elimination

$$U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

c

(5)

Lower triangular matrix will be the multipliers used.

$$\begin{bmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$$

Since $m_{21} = \frac{2}{1} = 2$

$m_{31} = \frac{3}{1} = 3$

$m_{32} = \frac{-4}{-1} = 4$

Answer of question 3:

$$\boxed{\therefore x_1 = -\frac{11}{9}}$$

$P_2(x) = a_0 + a_1 x + a_2 x^2 \Rightarrow \begin{cases} P_2(x_0) = a_0 + a_1 x_0 + a_2 x_0^2 = f(x_0) \\ P_2(x_1) = a_0 + a_1 x_1 + a_2 x_1^2 = f(x_1) \\ P_2(x_2) = a_0 + a_1 x_2 + a_2 x_2^2 = f(x_2) \\ P_2(x_3) = a_0 + a_1 x_3 + a_2 x_3^2 = f(x_3) \end{cases}$

(a) $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \end{pmatrix}$,

$b = \begin{pmatrix} 3 \\ 7 \\ -2 \\ -4 \end{pmatrix}$, $A^T = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 1 & 1 & 9 \end{pmatrix}$

③ (a) $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \end{pmatrix}$, $b = \begin{pmatrix} 3 \\ 7 \\ -2 \\ -4 \end{pmatrix}$

$u_1 \quad u_2 \quad u_3$

(b) $P_1 = u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $|P_1| = \sqrt{1^2+1^2+1^2+1^2} = \sqrt{4} = 2$

$q_1 = \frac{P_1}{|P_1|} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$

$$P_2 = u_2 - (u_2^T \cdot q_1) q_1$$

$$= \begin{pmatrix} 0 \\ -1 \\ 1 \\ 3 \end{pmatrix} - \left[\underset{1\times 4}{\begin{pmatrix} 0 & -1 & 1 & 3 \end{pmatrix}} \times \underset{4\times 1}{\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}} \right] \times \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} -3/4 \\ -7/4 \\ \cdot 1/4 \\ 9/4 \end{pmatrix}$$

$$|P_2| = \sqrt{(-3/4)^2 + (-7/4)^2 + (-1/4)^2 + (9/4)^2}$$

$$= \frac{\sqrt{35}}{2}$$

$$\therefore q_2 = \left(\frac{\sqrt{35}}{2}\right)^{-1} \begin{pmatrix} -3/4 \\ -7/4 \\ -1/4 \\ 9/4 \end{pmatrix} = \begin{pmatrix} -\frac{3\sqrt{35}}{70} \\ -\sqrt{35}/10 \\ \sqrt{35}/70 \\ 9\sqrt{35}/70 \end{pmatrix}$$

$$P_3 = u_3 - (u_2^T \cdot q_1) \cdot q_1 - (u_3^T \cdot q_2) \cdot q_2$$

$$= \begin{pmatrix} \cancel{43/2} \\ \cancel{45/2} \\ \cancel{45/2} \\ \cancel{61/2} \end{pmatrix} = \begin{pmatrix} -8/7 \\ 2 \\ -16/7 \\ 10/7 \end{pmatrix}$$

$$|p_3'| = \frac{2\sqrt{154}}{7}$$

$$q_3 = \begin{pmatrix} -\frac{2\sqrt{154}}{7} \\ \frac{\sqrt{154}}{22} \\ -\frac{4\sqrt{154}}{77} \\ \frac{5\sqrt{154}}{154} \end{pmatrix}$$

$$Q = \begin{pmatrix} 1/2 & -\frac{3\sqrt{35}}{70} & -2\sqrt{154}/7 \\ 1/2 & -\sqrt{35}/70 & \frac{\sqrt{154}}{22} \\ 1/2 & \frac{\sqrt{35}}{70} & \frac{-4\sqrt{154}}{77} \\ 1/2 & 3\sqrt{35}/70 & \frac{5\sqrt{154}}{154} \end{pmatrix}$$

(c) $R = \begin{pmatrix} u_1^T q_1 & u_2^T q_1 & u_3^T q_1 \\ 0 & u_2^T q_2 & u_3^T q_2 \\ 0 & 0 & u_3^T q_3 \end{pmatrix}$

$= \begin{pmatrix} 2 & 3/2 & 11/2 \\ 0 & \sqrt{35}/2 & \frac{15\sqrt{35}}{14} \\ 0 & 0 & \frac{2\sqrt{154}}{7} \end{pmatrix}$

(d) $Rx = \begin{pmatrix} 2 & 3/2 & 11/2 \\ 0 & \sqrt{35}/2 & (15\sqrt{35})/14 \\ 0 & 0 & 2\sqrt{154}/7 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$

$= \begin{pmatrix} 2a_0 + \frac{3}{2} a_1 + \frac{11}{2} a_2 \\ \frac{\sqrt{35}}{2} a_1 + \frac{15\sqrt{35}}{14} a_2 \\ \frac{2\sqrt{154}}{7} a_2 \end{pmatrix}$

$Q^T \cdot b = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{-3\sqrt{35}}{70} & \frac{-\sqrt{35}}{10} & \frac{\sqrt{35}}{70} & \frac{9\sqrt{35}}{70} \\ \frac{-2\sqrt{154}}{7} & \frac{\sqrt{154}}{22} & \frac{-4\sqrt{154}}{77} & \frac{5\sqrt{154}}{154} \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 7 \\ -2 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -8.11314 \\ -7.0103 \end{pmatrix}$

(e) $Rx = Q^T b$

$\Rightarrow \begin{pmatrix} 2a_0 + \frac{3}{2} a_1 + \frac{11}{2} a_2 \\ \frac{\sqrt{35}}{2} a_1 + \frac{15\sqrt{35}}{14} a_2 \\ \frac{2\sqrt{154}}{7} a_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -8.11314 \\ -7.0103 \end{pmatrix}$

$$x_3 \Rightarrow a_2 = -7.0103 \times \frac{7}{\sqrt{154} \times 2} \therefore \boxed{a_2 = 1.977}$$

$$x_2 \Rightarrow \frac{\sqrt{35}}{2} a_1 + 12.5326 = -8.11314$$

$$\boxed{\therefore a_1 = -6.9795}$$

$$x_1 \Rightarrow 2a_0 + 10.46925 + 10.8735 = 2$$

$$\boxed{\therefore a_0 = -9.671375}$$

$$\therefore P_2 = -9.671375 - 6.9795x + 1.977x^2.$$

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