Makeup Assignment Solution

1 (a)

$$\begin{bmatrix} 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix}$$

(b)

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix}^{-1} \begin{bmatrix} 30 \\ 50 \\ 70 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -15 & 6 \\ -4.5 & 8 & -3.5 \\ 0.5 & -1.0 & .5 \end{bmatrix} \begin{bmatrix} 30 \\ 50 \\ 70 \end{bmatrix}$$

$$= \begin{bmatrix} -30 \\ 20 \\ 0 \end{bmatrix}$$

(c)

$$P_2 = -30 + 20x$$

$$f(4.5) = -30 + 20 \times 4.5$$

$$= 60 \text{ Ans:}$$

(2) (a) $l_0 = \frac{(x - x_1)}{(x_0 - x_1)} = \frac{x - 2}{-3 - 2}$

$= \frac{x - 2}{-5}$

$l_1 = \frac{(x - x_0)}{(x_1 - x_0)} = \frac{x + 3}{5}$

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(b) $P_3(x) = h_0(x) f(x_0) + h_1(x) f(x_1)$

$+ \hat{h}_0(x) f'(x_0) + \hat{h}_1(x) f'(x_1)$

$= h_1(x) f(x_1) + \hat{h}_0(x) f'(x_1)$

$h_1(x) \Rightarrow$

$l_1(x) = \frac{x+3}{5}$

$l_1'(x) = \frac{1}{5}$

$h_1(x) = \{1 - 2(x - x_1) l_1'(x_1)\} [l_1(x)]^2$

$= \left[1 - \frac{2}{5}(x-2)\right] \left\{\frac{1}{5}(x+3)\right\}^2$

$\hat{h}_0(x) \Rightarrow l_0(x) \Rightarrow -\frac{1}{5}(x-2)$

$\hat{h}_0(x) = (x+3)\left\{-\frac{1}{5}(x-2)\right\}^2$

$= (x+3)\left\{\frac{1}{5}(x-2)\right\}^2$

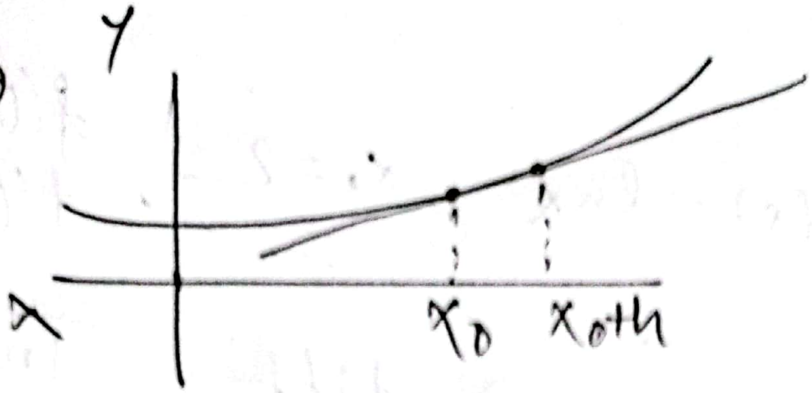
$\cancel{P_3(x) = \hat{h}_0(x) + h_1}$

$$P_3(x) = 2\left[1 - \frac{2}{5}(x-2)\right]\left\{\frac{1}{5}(x+3)\right\}^2$$

$$+ (-3)(x+3)\left\{\frac{1}{5}(x-2)\right\}^2$$

$$P_3(-1.7) =$$

(3) (a)



Let's take the Lagrange form of this.

$$P_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

$$f(x) = \frac{x - x_0}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1) + \frac{f''(\xi)}{2}(x - x_0)(x - x_1)$$

$$f'(x) = \frac{1}{x_0 - x_1} f(x_0) + \frac{1}{x_1 - x_0} f(x_1) + \frac{f'''(\xi)}{2}\frac{d\xi}{dx}(x - x_0)(x - x_1) + \frac{f''(\xi)}{2}(2x - x_0 - x_1)$$

$$f'(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} + \frac{f''(\xi)}{2}\underline{(x_0 - x_1)}$$

$$= \boxed{\frac{f(x_0 + h) - f(x_0)}{h}} + \boxed{\frac{f''(\xi)}{2}(-h)}$$

Error $\propto h$

Forward difference formula.

(b) $f(x) = \ln x \quad x_0 = 2, \quad f'(x) = \frac{1}{x}$

$f'(2) = \frac{1}{2} = 0.5$

h	Forward diff	Truncation Error
1	$\frac{\ln(2+1) - \ln(2)}{1} = 0.4055$	0.0945
0.1	0.4879	0.0121
0.01	0.4988	0.001245

(c) As the stepsize decreases the Truncation Error get reduced as well.

So, it is consistent.

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(9) (a) central difference method

Error $\propto h^2$

while the others (forward, Backward) difference method

Error $\propto h$

~~the bigger the~~ thats why for a given stepsize the error is more lesser than the others.

* they can show an example if the students want with a function computing the error and illustrating the differences between two method.

(b) $f(x) = x^2 + 2x$

<u>CDD</u>

$$f'(x) = \frac{f(x+h) - f(x-h)}{\cancel{d}\,2h}$$

$$= \frac{f(3+1) - f(3-1)}{2\times1}$$

$$= \frac{f(4) - f(2)}{2}$$

$$= \frac{24 - 8}{2} = 8$$

$f'(x) = 2x + 2$

$f'(3) = 2\times3 + 2 = 8$