Part - 1

\# **Solution 1:** $\beta = 2$, $m = 4$, $e_{min} = -3$, $e_{max} = 6$

1. convention - 1 : $(0.1111)_2 \times 2^6$
   " - 2 : $(1.1111)_2 \times 2^6$
   " - 3 : $(0.11111)_2 \times 2^6$
2. " - 1 : $(0.1000)_2 \times 2^{-3}$
   " - 2 : $(1.0000)_2 \times 2^{-3}$
   " - 3 : $(0.10000)_2 \times 2^{-3}$
3. " - 1 : $2^3 \times 2^{10}$
   " - 2 : $2^4 \times 2^{10}$
   " - 3 : $2^4 \times 2^{10}$
4. $(0.1000)_2 \times 2^{-3}$
5. largest $= (0.1111)_2 \times 2^6$
   smallest $= -(0.1111)_2 \times 2^6$

for $e = 0$

(6) for $e = -1$

$$\overset{-1\,-2\,-3\,-4}{(0.1000)} \times 2^{-1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$(0.1001) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^4}\right) \times \frac{1}{2} = \frac{9}{32}$$

$$(0.1010) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^3}\right) \times \frac{1}{2} = \frac{5}{16}$$

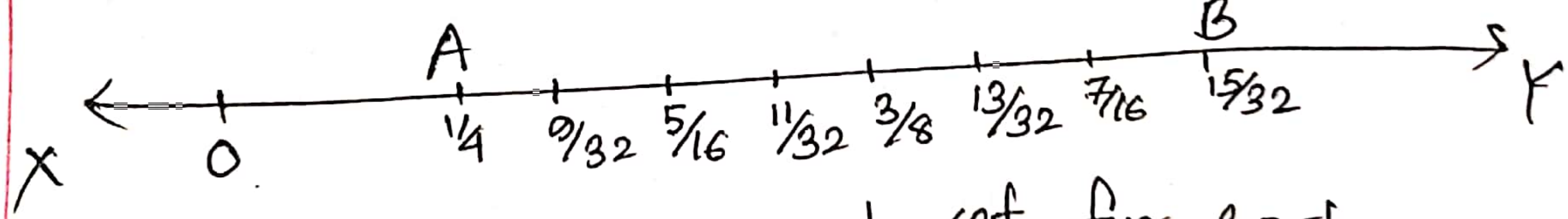
$$(0.1011) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^4}\right) \times \frac{1}{2} = \frac{11}{32}$$

$$(0.1100) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^2}\right) \times \frac{1}{2} = \frac{3}{8}$$

$$(0.1101) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^4}\right) \times \frac{1}{2} = \frac{13}{32}$$

$$(0.1110) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3}\right) \times \frac{1}{2} = \frac{7}{16}$$

$$(0.1111) \times 2^{-1} = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4}\right) \times \frac{1}{2} = \frac{15}{32}$$



AB is equally spaced set, for $e = -1$

But the number line XY is not equally spaced.

$\underline{\text{Sol}^n\ 2:}$ $\beta=2, m=4, e_{min}=-1, e_{max}=2$

1. $\min|x| = (1.0000)_\beta \cdot \beta^e$
$= \beta^0 \times \beta^e$
$= \beta^e$

2. $|fl(x)-x| = \frac{1}{2}\times\beta^{m}\times\beta^e$
$\epsilon_M = \frac{\frac{1}{2}\times\beta^{-m}\times\beta^e}{\beta^e}$
$= \frac{1}{2}\beta^{-m}$
$= \frac{1}{2}\cdot\beta^{-4} = \frac{1}{2}\times\frac{1}{2^4}$
$\therefore \epsilon_M = \frac{1}{32}$

3. As we can see β^e is our exponent and it cancel out for ϵ_M. So, machine epsilon does not depend on exponent.

4. $|fl(x)-x| = \frac{1}{2}\times\beta^{-(m+1)}\cdot\beta^e$; $\min|x| = \beta^{-1}\times\beta^e$
$\therefore \epsilon_M = \frac{\frac{1}{2}\cdot\beta^{-m}\cdot\beta^{-1}\cdot\beta^e}{\beta^{-1}\cdot\beta^e} = \frac{1}{2}\beta^{-m} = \frac{1}{2}\times\frac{1}{2^4} = \frac{1}{32}$

5. $\delta_{max} = \epsilon_M = \dfrac{|fl(x) - x|}{|x|}$

$= \dfrac{\frac{1}{2} \cdot \beta^{-m} \cdot \beta^{e}}{\beta^{-1} \times \beta^{e}}$

$= \frac{1}{2} \beta^{1-m}$

$= \frac{1}{2} \beta^{1-4}$

$= \frac{1}{2} \times \frac{1}{2^3}$

$= \frac{1}{16}$.

$\min |x| = (0.1 \cdots)_\beta \cdot \beta^{e}$

$= \beta^{-1} \times \beta^{e}$

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Part-2

Sol^n 1: $f(x) = e^x \cdot x$

1. $e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$

$f(x) = x + x^2 + \frac{1}{2!}x^3 + \frac{1}{3!}x^4 + \cdots$

2. $f(x) = x + x^2 + \frac{1}{2!}x^3 + \frac{1}{3!}x^4 + \cdots$

$P_3(x) = x^0 \times 0 + 1 \times x^1 + 1 \times x^2 + \frac{1}{2!}x^3$

$P_n(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$

Comparing,

$a_0 = 0,\ a_1 = 1,\ a_2 = 1,\ a_3 = \frac{1}{2}$

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3. $f(0.1) = e^{(0.1)} \times (0.1)$

$= 1.10517090 \times 0.1 = 0.1105171$

$P_3(x) = x + x^2 + \frac{x^3}{2}$

$P_3(0.1) = 0.1105000$

4. $\text{error} = \frac{|f(x) - P(x)|}{f(x)} \times 100\%$

$= \frac{|0.1105171 - 0.1105000|}{0.1105171} \times 100\%$

$= 0.015\%$

Sol^n 2:

1. $x_0 = -1, x_1 = 0, x_2 = 1$; $f(x) = xe^x$

So, $P_2(x_0) = a_0 + a_1x_0 + a_2x_0^2 = f(x_0) = -0.36788$

$P_2(x_1) = a_0 + a_1x_1 + a_2x_1^2 = f(x_1) = 0$

$P_2(x_2) = a_0 + a_1x_2 + a_2x_2^2 = f(x_2) = 2.71828$

Now, $a_0 - a_1 + a = -0.36788$

$a_0 = 0$

$a_0 + a_1 + a_2 = 2.71828$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -0.36788 \\ 0 \\ 2.71828 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

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2. $\det(V) = 1\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$

$= 2.$

3. $V^{-1} = \frac{1}{\det(V)} \times \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} 0 & 2 & 0 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$

$V^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \end{bmatrix}$

4. $\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -1 & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} -0.36788 \\ 0 \\ 2.71828 \end{bmatrix}$

$a_0 = 0$

$a_1 = 1.54308$

$a_2 = 1.1752$

5. $P_2(x) = a_0 + a_1 x + a_2 x^2$

$P_2(0.25) = \quad 0.45922$

$f(0.25) = 0.25 \times e^{0.25} = 0.321001$

6. Percent error $= \frac{|f(x) - P(x)|}{f(x)} \times 100\%$

$= 43.055\%$