

Let s_1, s_2, \dots, s_k be all the sources and t_1, t_2, \dots, t_l be all the targets in G (these can be identified in $O(|V| + |E|)$ time by Part (a)). We convert G to a new DAG G' whose vertex set contains two additional vertices s and t . We add the edges (s, s_i) for all $i = 1, 2, \dots, k$ and also the edges (t_j, t) for all $j = 1, 2, \dots, l$. G' is a DAG with a unique source s and a unique target t . Moreover, the count of all (s_i, t_j) paths (for all i, j) in G is the same as the count of all (s, t) paths in G' . The size of G' continues to remain $O(|V| + |E|)$. We make a topological sorting of the vertices in G' . This can be done in $O(|V| + |E|)$ time. Let the listing be $s = v_0, v_1, v_2, \dots, v_n, t = v_{n+1}$. We use an array C indexed by the vertices in G' to store the count of paths from s to the vertices.

Initialize $C[v_0] = 1$ and $C[v_i] = 0$ for all $i = 1, 2, 3, \dots, n+1$.

For $i = 0, 1, 2, \dots, n$ {

For all edges (v_i, v_j) in G' , set $C[v_j] = C[v_j] + C[v_i]$.

}

Return $C[v_{n+1}]$.

Since there are no back edges (that is, edges (v_i, v_j) with $i > j$), the for loop does not miss a path from s to t . With the adjacency list representation of G' , this phase can again be finished in $O(|V| + |E|)$ time. The introduction of the new vertices s, t could have been avoided. In that case, we start by setting $C[s_i] = 1$ for all the sources s_i in G . At the end, we return $C[t_1] + C[t_2] + \dots + C[t_l]$. However, a topological sorting of G is necessary for the correctness of this algorithm.

Topological Sorting

```
#include <stdio.h>
```

```
int main(){
    int i,j,k,n,a[10][10],index[10],flag[10],count=0;

    printf("Enter the no of vertices:\n");
    scanf("%d",&n);

    printf("Enter the adjacency matrix:\n");
    for(i=0;i<n;i++){
        printf("Enter row %d\n",i+1);
        for(j=0;j<n;j++)
            scanf("%d",&a[i][j]);
    }
```

```
        for(i=0;i<n;i++){
            index[i]=0;
            flag[i]=0;
        }

        for(i=0;i<n;i++)
            for(j=0;j<n;j++)
                index[i]=index[i]+a[j][i];

        printf("\nThe topological order is:");

        while(count<n){
            for(k=0;k<n;k++){
                if((index[k]==0) && (flag[k]==0)){
                    printf("%d ",(k+1));
                    flag [k]=1;
                }

                for(i=0;i<n;i++){
                    if(a[i][k]==1)
                        index[k]--;
                }
            }

            count++;
        }

        return 0;
    }
```

Input and Output

Enter the no of vertices:

4

Enter the adjacency matrix:

Enter row 1

0 0 0 1

Enter row 2

0 0 0 1

Enter row 3

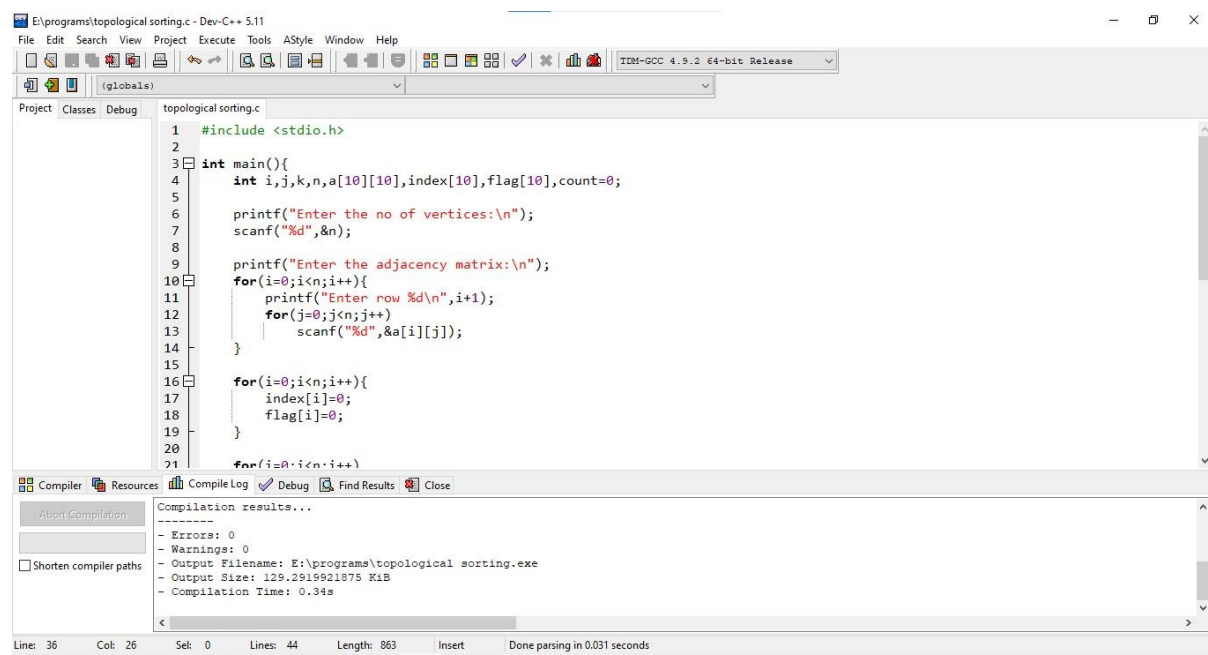
0 0 0 0

Enter row 4

0 1 1 0

The topological order is:1 2 3 4

Screenshots



```
1 #include <stdio.h>
2
3 int main(){
4     int i,j,k,n,a[10][10],index[10],flag[10],count=0;
5
6     printf("Enter the no of vertices:\n");
7     scanf("%d",&n);
8
9     printf("Enter the adjacency matrix:\n");
10    for(i=0;i<n;i++){
11        printf("Enter row %d\n",i+1);
12        for(j=0;j<n;j++){
13            scanf("%d",&a[i][j]);
14        }
15    }
16
17    for(i=0;i<n;i++){
18        index[i]=0;
19        flag[i]=0;
20    }
21
22    for(i=0;i<n;i++){
```

Compilation results...

- Errors: 0
- Warnings: 0
- Output Filename: E:\programs\topological sorting.exe
- Output Size: 129.2919521875 KiB
- Compilation Time: 0.34s

Line: 36 Col: 26 Sel: 0 Lines: 44 Length: 863 Insert Done parsing in 0.031 seconds

E:\programs\topological sorting.c - Dev-C++ 5.11

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TM-GCC 4.9.2 64-bit Release

(globals)

Project Classes Debug topological sorting.c

```
19 }
20
21 for(i=0;i<n;i++)
22     for(j=0;j<n;j++)
23         index[i]=index[i]+a[j][i];
24
25 printf("\nThe topological order is:");
26
27 while(count<n){
28     for(k=0;k<n;k++){
29         if((index[k]==0) && (flag[k]==0)){
30             printf("%d ",(k+1));
31             flag[k]=1;
32         }
33     }
34     for(i=0;i<n;i++){
35         if(a[i][k]==1)
36             index[k]--;
37     }
38 }
39 }
```

Compiler Resources Compile Log Debug Find Results Close

About Compilation

Shorten compiler paths

Compilation results...

```
-----
- Errors: 0
- Warnings: 0
- Output Filename: E:\programs\topological sorting.exe
- Output Size: 129.2919921875 KiB
- Compilation Time: 0.34s
```

Line: 36 Col: 32 Sel: 0 Lines: 44 Length: 863 Insert Done parsing in 0.031 seconds

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TM-GCC 4.9.2 64-bit Release

(globals)

Project Classes Debug topological sorting.c

```
25 printf("\nThe topological order is:");
26
27 while(count<n){
28     for(k=0;k<n;k++){
29         if((index[k]==0) && (flag[k]==0)){
30             printf("%d ",(k+1));
31             flag[k]=1;
32         }
33     }
34     for(i=0;i<n;i++){
35         if(a[i][k]==1)
36             index[k]--;
37     }
38 }
39
40 count++;
41 }
42
43 return 0;
44 }
```

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