Numerical Approximations of π

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Abstract

Python has allowed for π to be approximated in many easier ways than before. Using an algorithm where an n number of points are placed and the areas of a square and a circle are compared, π can be approximated to varying degrees of accuracy. However, the algorithm is a very brute-force method, and can heavily use system resources. In general, it would be easier to symbollicaly or numerically solve a Gaussian integral.

1 Introduction and Trial

π can be numerically approximated using many different methods. However, in this case, we are using a recursive Monte-carlo-like method in Python to essentially average all the areas of the circle. To compare, the Gaussian integral $f(x) = \left(\int_{-\infty}^{\infty} e^{-x^2}\, dx\right)^2 = \pi$ is used. It can be noted that e is an irrational number itself. However, e has a few clear definitions such as $e = \sum_{k=0}^{\infty} \frac{1}{k!}$.

It can be noted that the accuracy in which the algorithm measures π at increases signifigantly increases with each decimal place. You can see based on the table of trials below. A total of 10 trials were done for each of the values of n.

$n = 10$	$n = 100$	$n = 1000$	$n = 10000$	$n = 100000$
2.8	3.12	3.18	3.1104	3.147
3.2	3.16	3.124	3.146	3.14876
3.6	3.24	3.168	3.1332	3.15028
3.6	2.8	3.164	3.1544	3.1456
3.2	3.12	3.192	3.1496	3.1436
1.6	3.08	3.056	3.0976	3.135
2.4	3.08	3.132	3.1508	3.14416
3.2	3.0	3.08	3.1352	3.14248
2.8	3.44	3.048	3.1476	3.13816
2.4	3.16	3.14	3.1372	3.14184

Table 1: Values of π put through the approximater.

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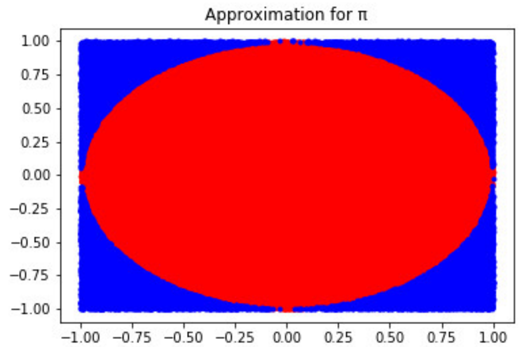


Figure 1: π approximated to $n = 100000$ places.

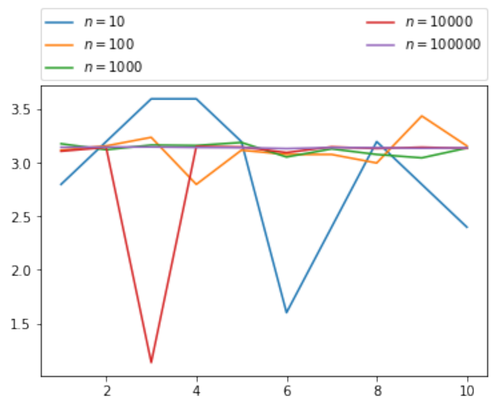


Figure 2: Values of the approximation for π.

$n = 10$	$n = 100$	$n = 1000$	$n = 10000$	$n = 100000$
0.58788	0.15492	0.04874	0.60126	0.00442

Table 2: Standard deviation for each value of n.

[1]

[1] $\sigma = \sum_i \sqrt{\frac{(x_i - \bar{x})^2}{n-2}}$

2 Conclusions

As can be seen from the graph, the graph allows for the circle to go outside of what a circle is. This can mess up the values of π since the area is different than that of a *true* cirle's area. However, the actual size of the 'circle' is rather similar to a true *circle*. Yet, it is accurate provided enough points are given. The main fault of this can simply be tossed away if one were to use the Gaussian integral

$$f(x) = \left(\int_{-\infty}^{\infty} e^{-x^2} \, dx \right)^2 = \pi \, .$$

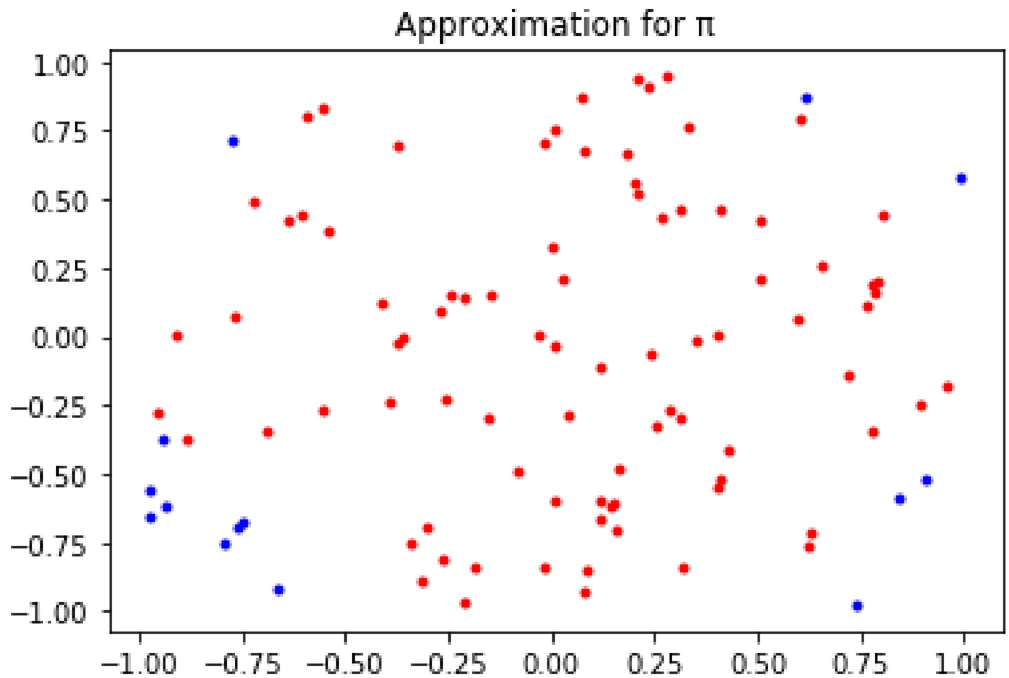
The use of this integral would serve to be much simpler on calculations and on computer resources. However, problems arise if one does not have a library or program for any sort of symbolic computation (i.e. SymPy, Maxima, or Mathematica™). Yet, the resources used by this algorithm is far more taxing on system resources—negating this solution for lower-performing computers.

3 Code

```
In [1]: ### PI APPROXIMATER ###
        %matplotlib inline

        import matplotlib.pyplot as plt
        import numpy as np
        from numpy import random
        from sympy import *

        # Sympy approximation

        X = symbols('X')

        f = exp(-(X**2))

        Pi_1 = N((integrate(f, (X, -oo, oo)))**2)

        # ----

        # Numpy approximation
        ## Credit to Andrew Dotson with his video "How to Estimate Pi Numerically in Pyt

        n = input("Input the number of points. n = ") # Number of points
        # print('Input your amount of points. n = ')

        circlex = []
        circley = []
```

```python
squarex = []
squarey = []

i = 1

# Approximation for pi
while i<=int(n):
    x = random.uniform(-1, 1)
    y = random.uniform(-1, 1)
    if (x**2 + y**2 <= 1):
        circlex.append(x)
        circley.append(y)
    else:
        squarex.append(x)
        squarey.append(y)
    i+=1


Pi_2 = 4*len(circlex)/float(n)

plt.plot(circlex,circley,'r.')
plt.plot(squarex,squarey,'b.')
plt.grid
plt.title('Approximation for π')
# Plot the approximation

# ----

print("According to sympy, π is equal to {0}. However, the circle approximated i
```

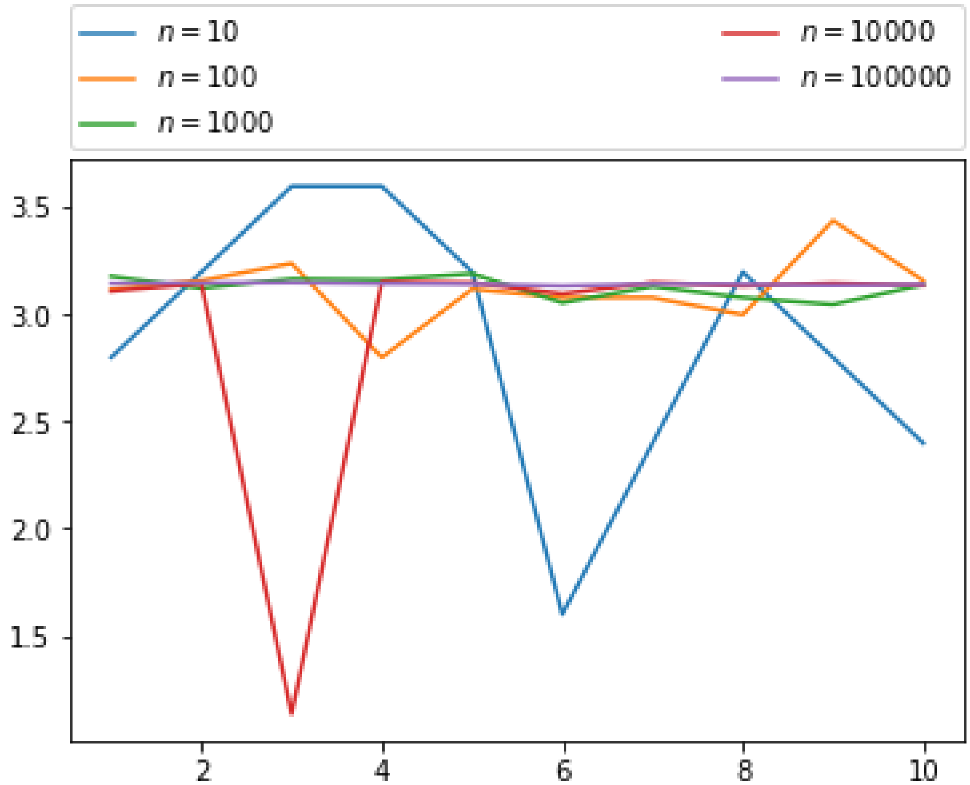
```
According to sympy, π is equal to 3.14159265358979. However, the circle approximated it
```



In [2]:
```
### STANDARD DEVIATION OF THE POINTS FROM THE APPROXIMATER ###

import matplotlib.pyplot as plt
import numpy as np
from numpy import random
from sympy import *

A = [2.8, 3.2, 3.6, 3.6, 3.2, 1.6, 2.4, 3.2, 2.8, 2.4]
B = [3.12, 3.16, 3.24, 2.8, 3.12, 3.08, 3.08, 3.0, 3.44, 3.16]
C = [3.18, 3.124, 3.168, 3.164, 3.192, 3.056, 3.132, 3.08, 3.048, 3.14]
D = [3.1104, 3.146, 1.1332, 3.1544, 3.1496, 3.0976, 3.1508, 3.1352, 3.1476, 3.13
E = [3.147, 3.14876, 3.15028, 3.1456, 3.1436, 3.135, 3.14416, 3.14248, 3.13816,

dev_A = np.round(np.std(A),5)
dev_B = np.round(np.std(B),5)
dev_C = np.round(np.std(C),5)
dev_D = np.round(np.std(D),5)
dev_E = np.round(np.std(E),5)

print("|{0}|{1}|{2}|{3}|{4}|".format(dev_A, dev_B, dev_C, dev_D, dev_E))
```

```
|0.58788|0.15492|0.04874|0.60126|0.00442|
```

In [3]:
```python
### PLOT OF THE POINTS FOR THE APPROXIMATER ###
%matplotlib inline

import matplotlib.pyplot as plt
import numpy as np
from numpy import random
from sympy import *

A = [2.8, 3.2, 3.6, 3.6, 3.2, 1.6, 2.4, 3.2, 2.8, 2.4]
B = [3.12, 3.16, 3.24, 2.8, 3.12, 3.08, 3.08, 3.0, 3.44, 3.16]
C = [3.18, 3.124, 3.168, 3.164, 3.192, 3.056, 3.132, 3.08, 3.048, 3.14]
D = [3.1104, 3.146, 1.1332, 3.1544, 3.1496, 3.0976, 3.1508, 3.1352, 3.1476, 3.13
E = [3.147, 3.14876, 3.15028, 3.1456, 3.1436, 3.135, 3.14416, 3.14248, 3.13816,
Number = [1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0]
#plt.subplot(2,1,1)
plt.plot(Number, A, label="$n = 10$")

#plt.subplot(1,2,1)
plt.plot(Number, B, label="$n = 100$")

#plt.subplot(1,1,1)
plt.plot(Number, C, label="$n = 1000$")

#plt.subplot(1,1,2)
plt.plot(Number, D, label="$n = 10000$")

#plt.subplot(2,2,2)
plt.plot(Number, E, label="$n = 100000$")
#plt.plot[B,'b.']

plt.legend(bbox_to_anchor=(0., 1.02, 1., .102), loc=3,
           ncol=2, mode="expand", borderaxespad=0.)

plt.savefig('pi_pictures/values.png')
# Used index.jpg instead since it actually features **THE WHOLE PLOT.**

plt.show()
```



$n = 10$
$n = 100$
$n = 1000$
$n = 10000$
$n = 100000$
3.5
3.0
2.5
2.0
1.5
2
4
6
8
10