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Maze Running Algorithm

Definitions:

First, we define a step in the maze running process as a single movement of an agent to a tile different from the agent's original tile, or a wait operation. We then define an initial iteration relative to the implemented star algorithm as a series of steps in which the agent checks the orthogonally adjacent (adjacent) tiles, waits once while deciding on a direction and moves to the tile in the chosen direction. Given an agent on a tile with k adjacent walls for $0 \leq k \leq 3$, a single initial iteration takes $10 - k$ steps. We define every subsequent iteration simply as an iteration, which removes the tile that the agent previously resided on. A single iteration takes $8 - k$ steps.

Algorithm Description:

First, we describe the implemented maze running algorithm as a star-pattern iteration algorithm (star algorithm) that checks the agent's adjacent tiles before deciding on a direction to move. The agent checks each adjacent tile in the order of up, right, down and left, before waiting and checking the priority of the visited tiles, eventually choosing the tile of the highest priority. Priority is initialized as a constant and decreased when the agent enters a tile and leaves a tile. In choosing a direction based on priority given equal priority numbers across all adjacent tiles, the directions are chosen in the order of up, down, left then right. Known wall tiles are given a priority of an extreme negative to prevent consideration. Furthermore, the agent will not check the tile that it resided on in the previous iteration in the star-pattern check.

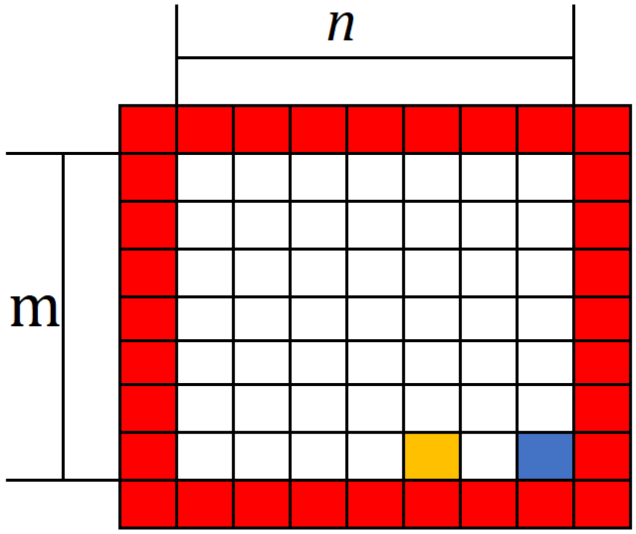

Figure 1: $n \times m$ maze with the artificially intelligent (yellow) and the end portal (blue).

<u>Worst-case Analysis:</u>

Step 1: We first consider a base maze as a given $n \times m$ rectangular maze for $0 < n, m$ and the worst-case for a fully random algorithm as infinity. We then consider a worst-case maze for the star algorithm as a maze similar to that of the one in Figure 1. while following the same restrictions on the maze as a base maze with the agent's starting position being $(0,0)$. The agent begins two tiles away from the portal. First, the agent will move towards the tile $(0, m)$ adjacent to the top wall as shown in Figure 1. In the initial and subsequent iterations of the star algorithm, the agent will first perform 9 steps with an extra 8 steps each subsequent iteration until the tile directly adjacent to the top wall. Upon reaching the tile adjacent to the top wall, the agent will perform 7 steps while choosing left based on the priority of the directions. We note that at this point, the priority of all the tiles between $(0,0)$ ... $(0, m)$ inclusive have reduced priority as they are previously visited tiles. In the traversal of a single stretch of tiles of m length, $8(m-2) + 16$ steps were taken. For subsequent traversal of columns, $8(m-2) + 14$ steps are taken since

the initial iteration takes two extra steps than normal iterations. Given a traversal of a column directly next to a wall, a corner tile takes 6 steps while a tile next to a wall takes 7 steps. In total, a column next to a wall takes $7(m-2)+12$ steps. Thus, in the traversal of the agent from its initial position of $(0,0)$ to $(-n-4, m)$ takes

$$(n-4)(8(m-2)+14)+(8(m-2)+16)+7(m-2)+12$$
$$= 8mn - 17m - 2n + 6$$

steps when simplified (see Appendix A).

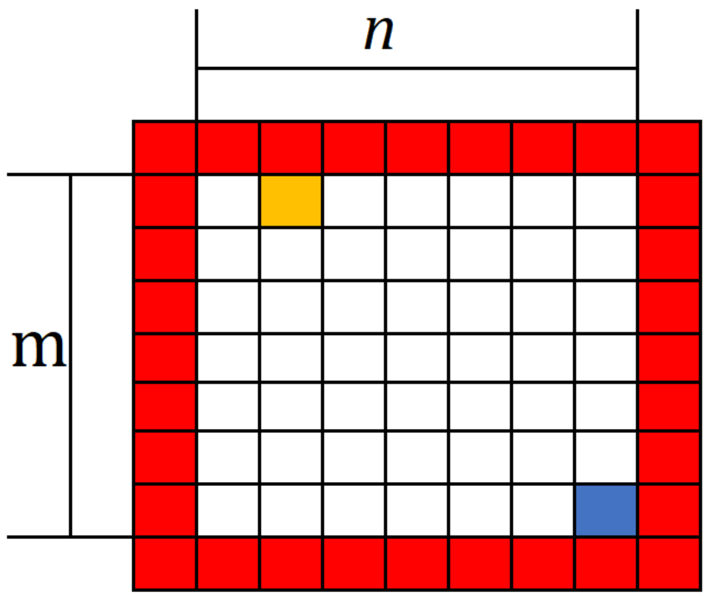

Figure 2: $n \times m$ maze with the artificially intelligent after Step 1. (yellow) and the end portal (blue).

Step 2: After reaching the tile $(-n-3, m)$ as shown in Figure 2., the agent will continue right until reaching a column it has not traversed by considering priority. To reach a new column, the agent repeatedly considers nearby priorities until finding an unexplored area with the default priority value. By the order of direction priorities, the agent in the position of Figure 2. will consider the tile to its right before looping back through the corner to its current position to change the priorities. This order of getting back to the same tile by going down, left, up and right takes 28 steps. Then, the algorithm progresses by repeatedly going right until it arrives at the column it has not traversed yet – the column between the agent's starting position and the portal (column δ). The agent's repeated movement towards column δ takes $n-3$ iterations, which is $7(n-3)$ steps.

Step 3: Once the agent reaches column δ, the agent continues down due to the directional priority until it eventually checks the portal tile at position $(2,0)$. The traversal through column δ before the tile at $(0,1)$ takes $8(m-2)+7$ steps. Upon reaching tile $(0,1)$, the agent only needs to check right in the star-pattern check, adding only a single step to reach the portal to $8(m-2)+8$ steps.

Total: In this worst-case input, we consider steps 1, 2 and 3 to reach a total of

$$8mn-17m-2n+6+7(n-3)+28+8(m-2)+8$$

$$=8mn-17m-2n+6+7n-21+28+8m-16+8$$

$$=8mn-9m+5n+5 \text{ steps.}$$

This is asymptotically linear as well as significantly less than infinity as a worst-case, suggesting a strict improvement compared to a fully random algorithm.

<u>Note on Parity of Maze Size:</u>

We further acknowledge that the parity of n determines the corner that the agent ends up in after traversing the $n-3$ columns. The addition of another column and the change in the corner tends to increase the number of steps relative to the number of tiles added, but we note this as a countable constant to the worst case and mention its triviality.

<u>Further Room Sizes:</u>

We acknowledge that given a maze of multiple rectangular rooms with doorways connecting the rooms, it takes $8mn-9m+5n+5$ steps to traverse a room plus a constant to reach the doorway of a given room and to traverse through the doorway. Given k rooms, we can count

$k(8mn - 9m + 5n + 5) + c$ steps alongside the variability of the values of m, n by the different room sizes to traverse multiple rooms to find a portal depending on the starting position and the layout of the room. Nevertheless, it would still be asymptotically linear and countably less than infinite.

Appendix A.

$$(n-4)(8m-16+14)+(8m-16+16)+(7m-14+12)$$

$$=(n-4)(8m-2)+(8m)+(7m-2)$$

$$=(n-4)(8m-2)+8m+7m-2$$

$$=(8mn-2n-32m+8)+(8m+7m-2)$$

$$=(8mn-17m-2n+6)$$