Supplementary Information for "Causal motifs and existence of endogenous cascades in directed networks with application to company defaults"

I. TWO-EDGE VARIANCE CALCULATION NEEDS A CHECK

$$E[C^{(1,2)}] = N \sum_{k_i,k_o} P(k_i,k_o) c^{(1,2)}_{k_i,k_o}(t) \tag{1}$$

$$V[C^{(1,2)}] = N \sum_{k_i,k_o} P(k_i,k_o) \Big[\left(c^{(1,2)}_{k_i,k_o}(t) - \frac{1}{N} E[C^{(1,2)}(t)] \right)^2 + c^{(1,2)COV}_{k_i,k_o}(t) \Big] \tag{2}$$

$$E[\tilde{C}^{(1,2)}(t)] = N \sum_{k_i,k_o} P(k_i,k_o) \tilde{c}^{(1,2)}_{k_i,k_o}(t) \tag{3}$$

$$V[\tilde{C}^{(1,2)}(t)] = N \sum_{k_i,k_o} P(k_i,k_o) \Big[\left(\tilde{c}^{(1,2)}_{k_i,k_o}(t) - \frac{1}{N} E[\tilde{C}^{(1,2)}(t)] \right)^2 + \tilde{c}^{(1,2)COV}_{k_i,k_o}(t) \Big] \tag{4}$$

One-edge:

$$c^{(1)COV}_{k_i,k_o}(t) = 0 \tag{5}$$

$$\tilde{c}^{(1)COV}_{k_i,k_o}(t) = 0 \tag{6}$$

Two-edge:

$$\begin{aligned} c^{(2)COV}_{k_i,k_o}(t) &= 2 \cdot (c^{VI}_{k_i,k_o}(t) + c^{\Lambda I}_{k_i,k_o}(t) + c^{V\Lambda}_{k_i,k_o}(t)) + c^{VV}_{k_i,k_o}(t)) + c^{\Lambda\Lambda}_{k_i,k_o}(t)) + c^{VVI}_{k_i,k_o}(t)) + c^{\Lambda\Lambda I}_{k_i,k_o}(t)) \\ &\quad + c^{I}_{k_i,k_o}(t)) + c^{V}_{k_i,k_o}(t)) + c^{\Lambda}_{k_i,k_o}(t)) \\ &= \sum_{z_i=0}^{k_i} \sum_{z_o=0}^{k_o} \Big[2 \cdot \left(z_i \binom{z_o}{2} + \binom{z_i}{2}\binom{z_o}{2} + \binom{z_i}{2} z_o \right) + \binom{z_i}{3} + \binom{z_i}{4} + \binom{z_o}{3} + \binom{z_o}{4} \\ &\quad + \binom{z_i}{3} z_o + z_i \binom{z_o}{3} + z_i z_o + \binom{z_i}{2} + \binom{z_o}{2} \Big] \times i^{m}_{k_i,k_o,z_i,z_o}(t) \end{aligned} \tag{7}$$

$$\begin{aligned} \tilde{c}^{(2)COV}_{k_i,k_o}(t) &= 2 \cdot (\tilde{c}^{VI}_{k_i,k_o}(t) + \tilde{c}^{\Lambda I}_{k_i,k_o}(t) + \tilde{c}^{V\Lambda}_{k_i,k_o}(t)) + \tilde{c}^{VV}_{k_i,k_o}(t)) + \tilde{c}^{\Lambda\Lambda}_{k_i,k_o}(t)) + \tilde{c}^{VVI}_{k_i,k_o}(t)) + \tilde{c}^{\Lambda\Lambda I}_{k_i,k_o}(t)) \\ &\quad + \tilde{c}^{I}_{k_i,k_o}(t)) + \tilde{c}^{V}_{k_i,k_o}(t)) + \tilde{c}^{\Lambda}_{k_i,k_o}(t)) \\ &= \sum_{m_i=0}^{k_i} \sum_{m_o=0}^{k_o} \Big[2 \cdot \left(\frac{1}{12} m_i \binom{m_o}{2} + \frac{1}{30} \binom{m_i}{2}\binom{m_o}{2} + \frac{1}{12} \binom{m_i}{2} m_o \right) \\ &\quad + \frac{1}{4}\binom{m_i}{3} + \frac{1}{5}\binom{m_i}{4} + \frac{1}{4}\binom{m_o}{3} + \frac{1}{5}\binom{m_o}{4} \\ &\quad + \frac{1}{20}\binom{m_i}{3} m_o + \frac{1}{20} m_i \binom{m_o}{3} + \frac{1}{6} m_i m_o + \frac{1}{3}\binom{m_i}{2} + \frac{1}{3}\binom{m_o}{2} \Big] \times i^{z}_{k_i,k_o,m_i,m_o}(t) \end{aligned} \tag{8}$$

II. TEMPORAL EVOLUTION

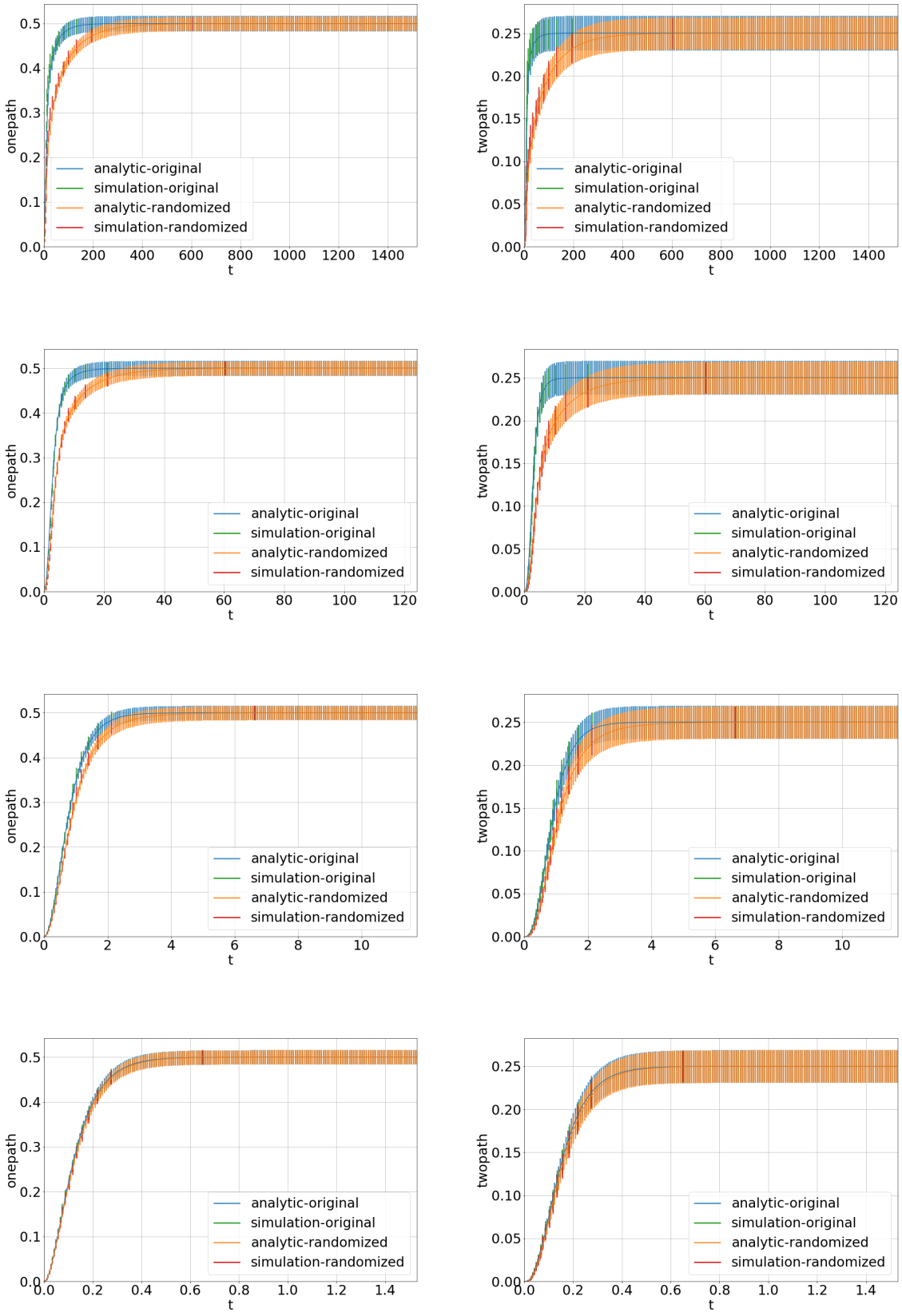


FIG. 1: $\langle k \rangle = 2$, $\zeta = 0.01, 0.1, 1, 10$, SI

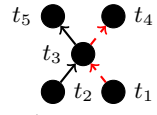
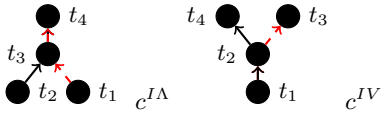
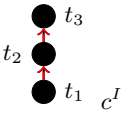
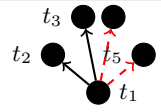
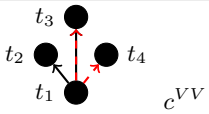
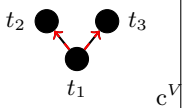
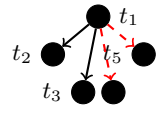
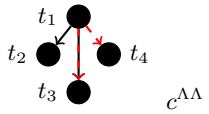
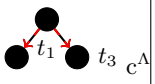
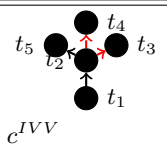
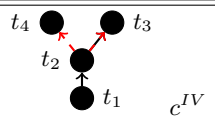
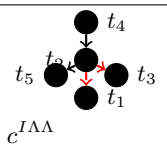
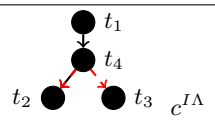
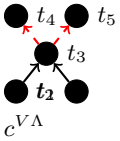
Overlap	One node	Two nodes	Three nodes
I-I			
V-V			
$\Lambda-\Lambda$			
I-V			
I-Λ			
V-Λ			

TABLE I: Table with all possible casual motifs up to order 3.

III. Z-SCORE

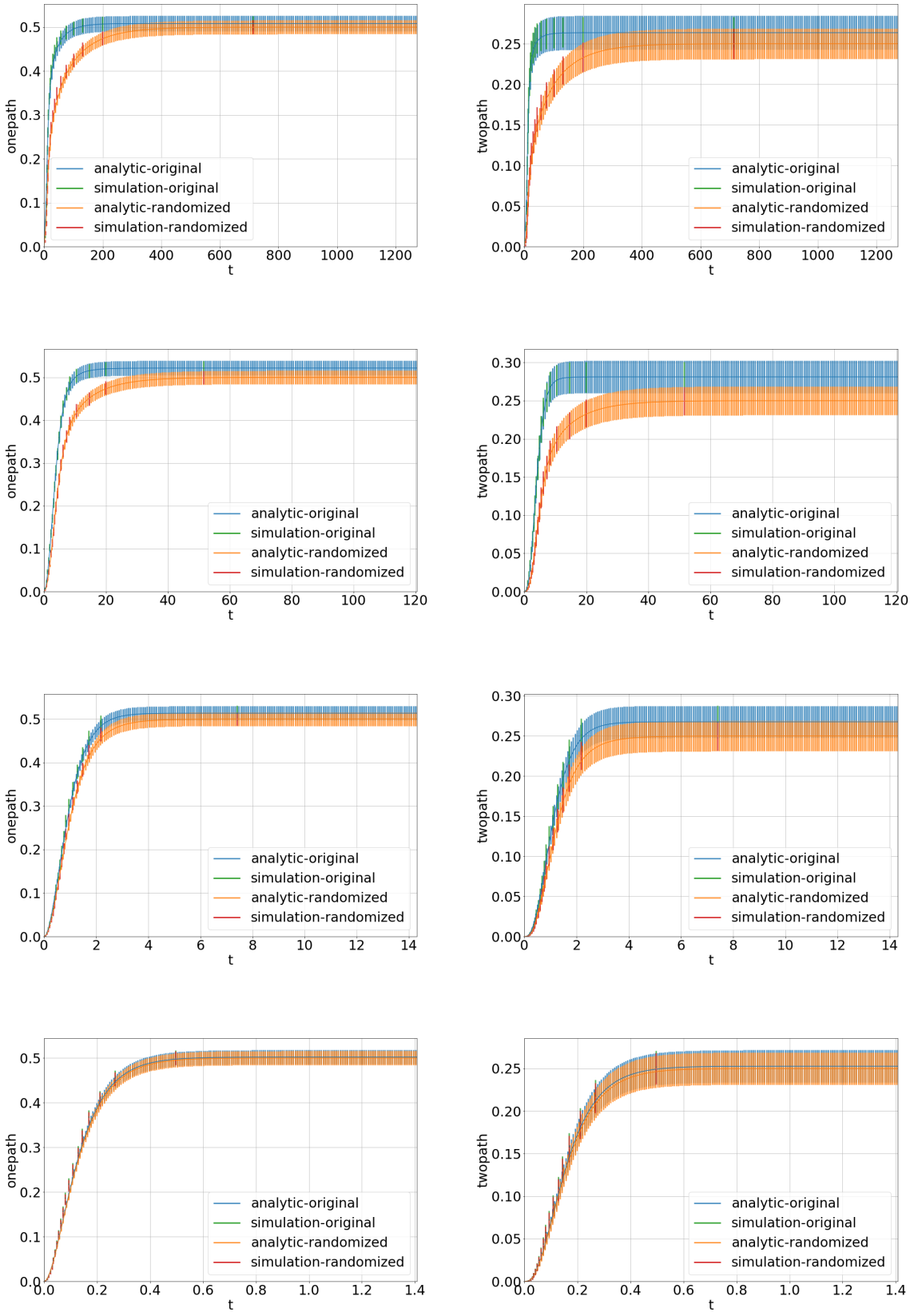


FIG. 2: $\langle k \rangle = 2$, $\zeta = 0.01, 0.1, 1, 10$, VM

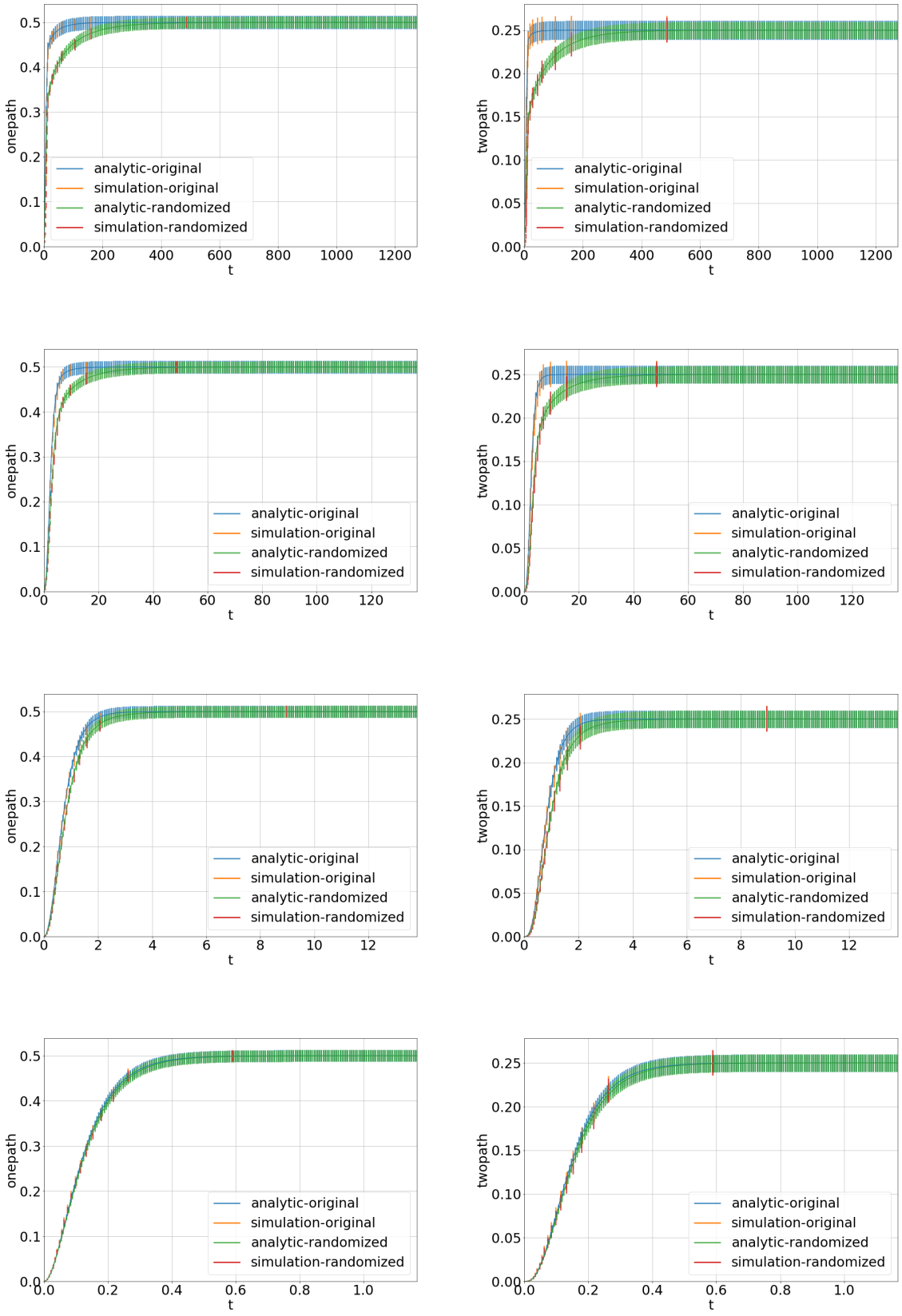


FIG. 3: $\langle k \rangle = 3$, $\zeta = 0.01, 0.1, 1, 10$, SI

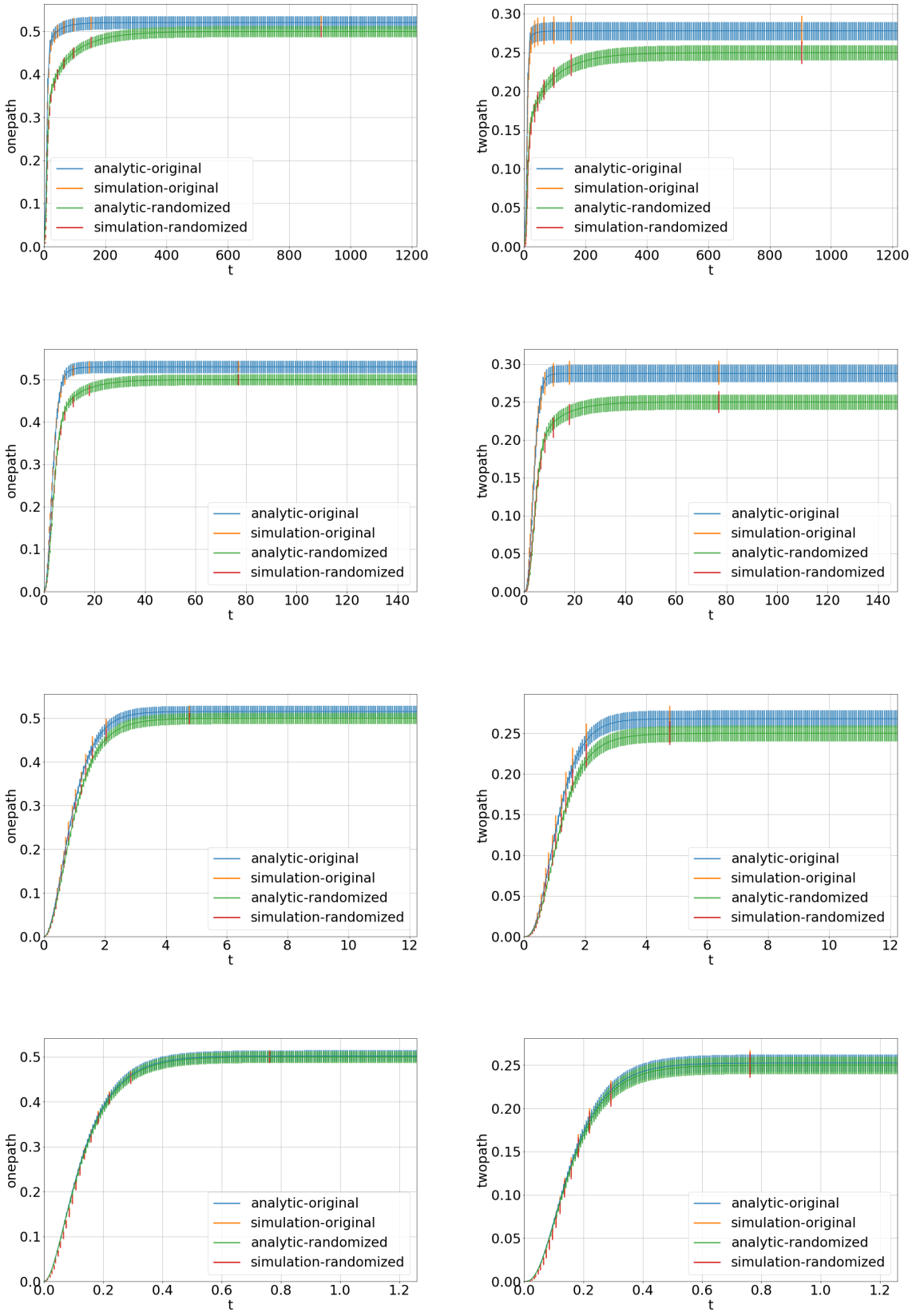


FIG. 4: $\langle k \rangle = 3$, $\zeta = 0.01, 0.1, 1, 10$, voter model

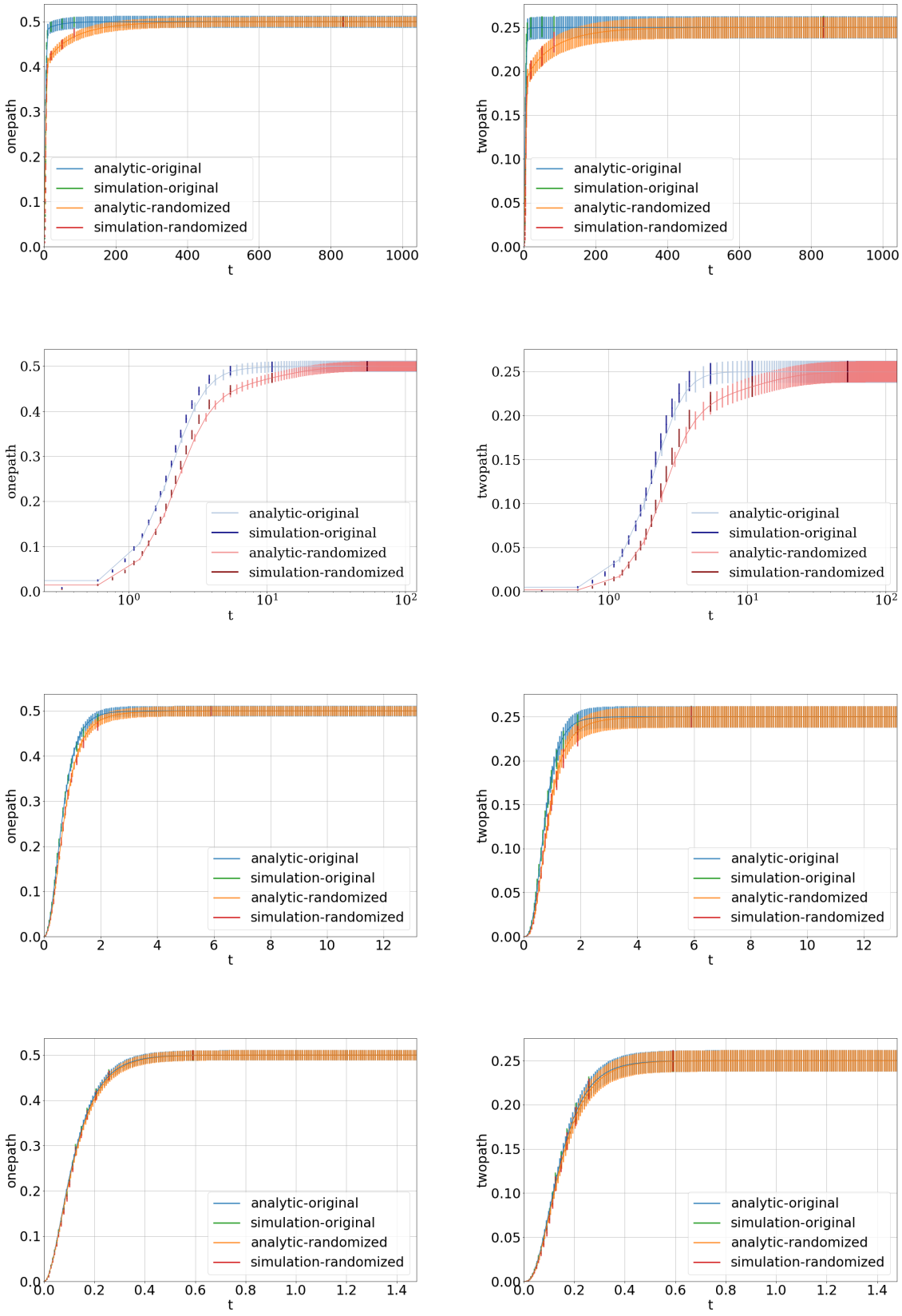


FIG. 5: $\langle k \rangle = 4$, $\zeta = 0.01, 0.1, 1, 10$, SI

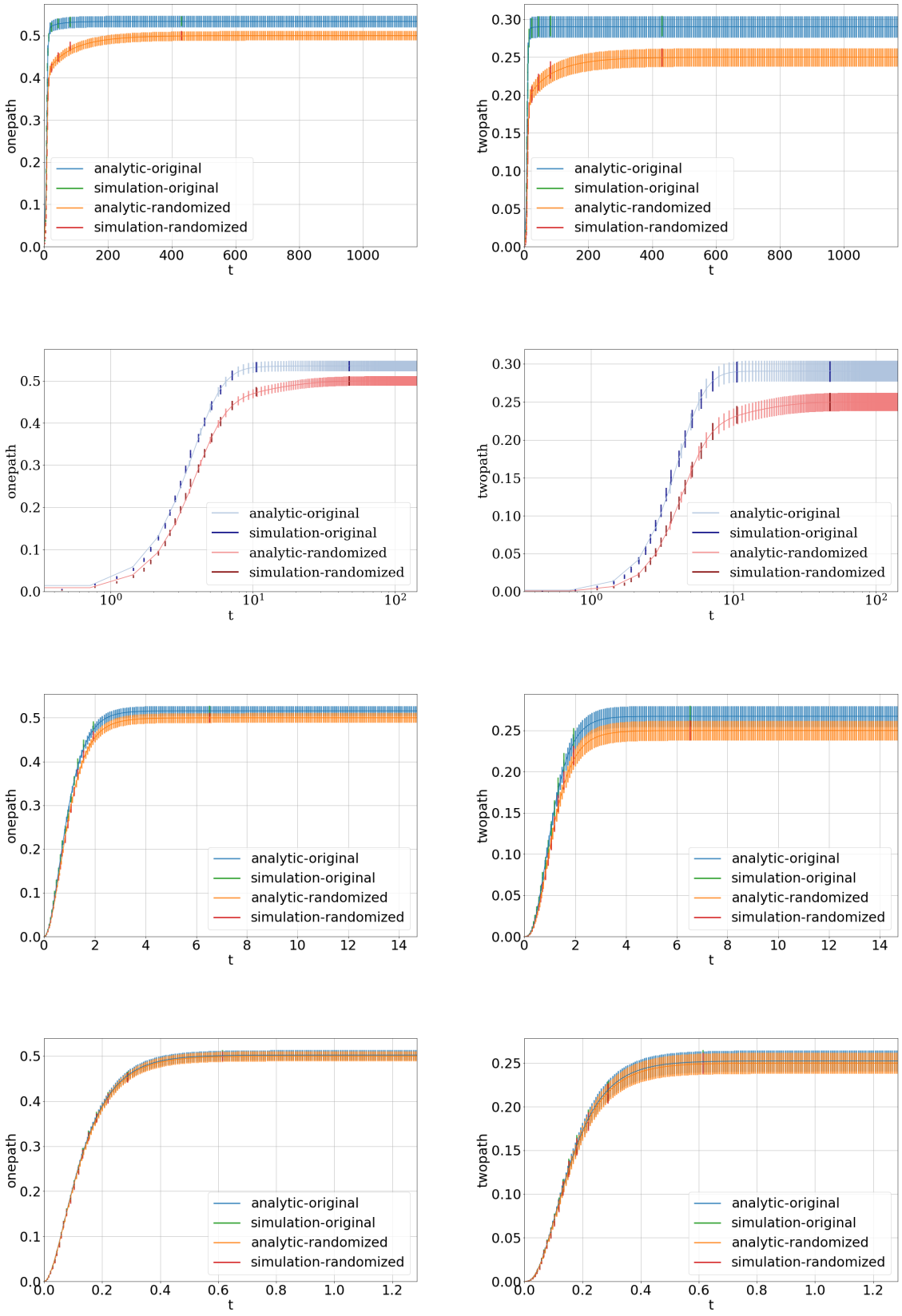


FIG. 6: $\langle k \rangle = 4$, $\zeta = 0.01, 0.1, 1, 10$, voter model

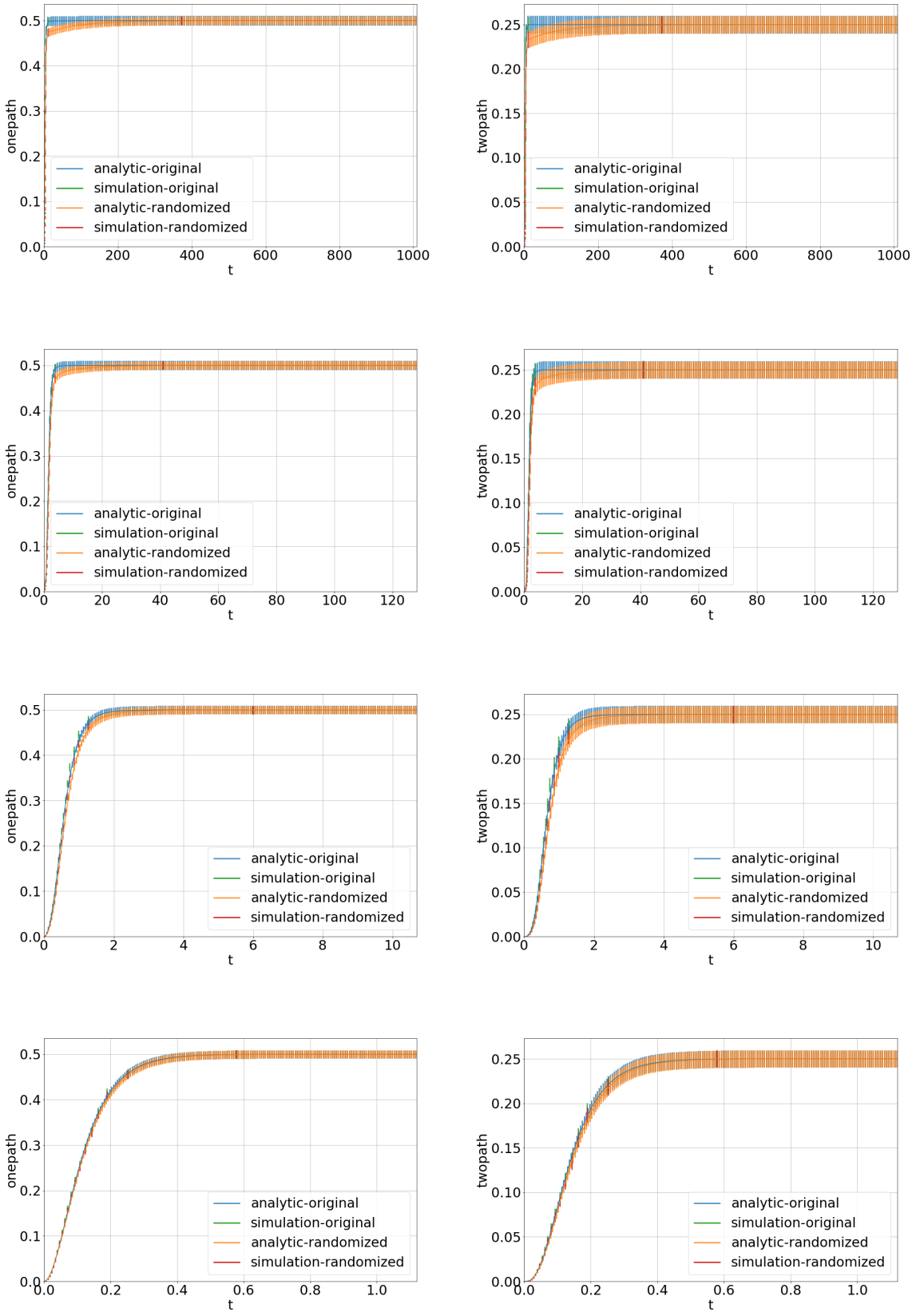


FIG. 7: $\langle k \rangle = 6$, $\zeta = 0.01, 0.1, 1, 10$, SI

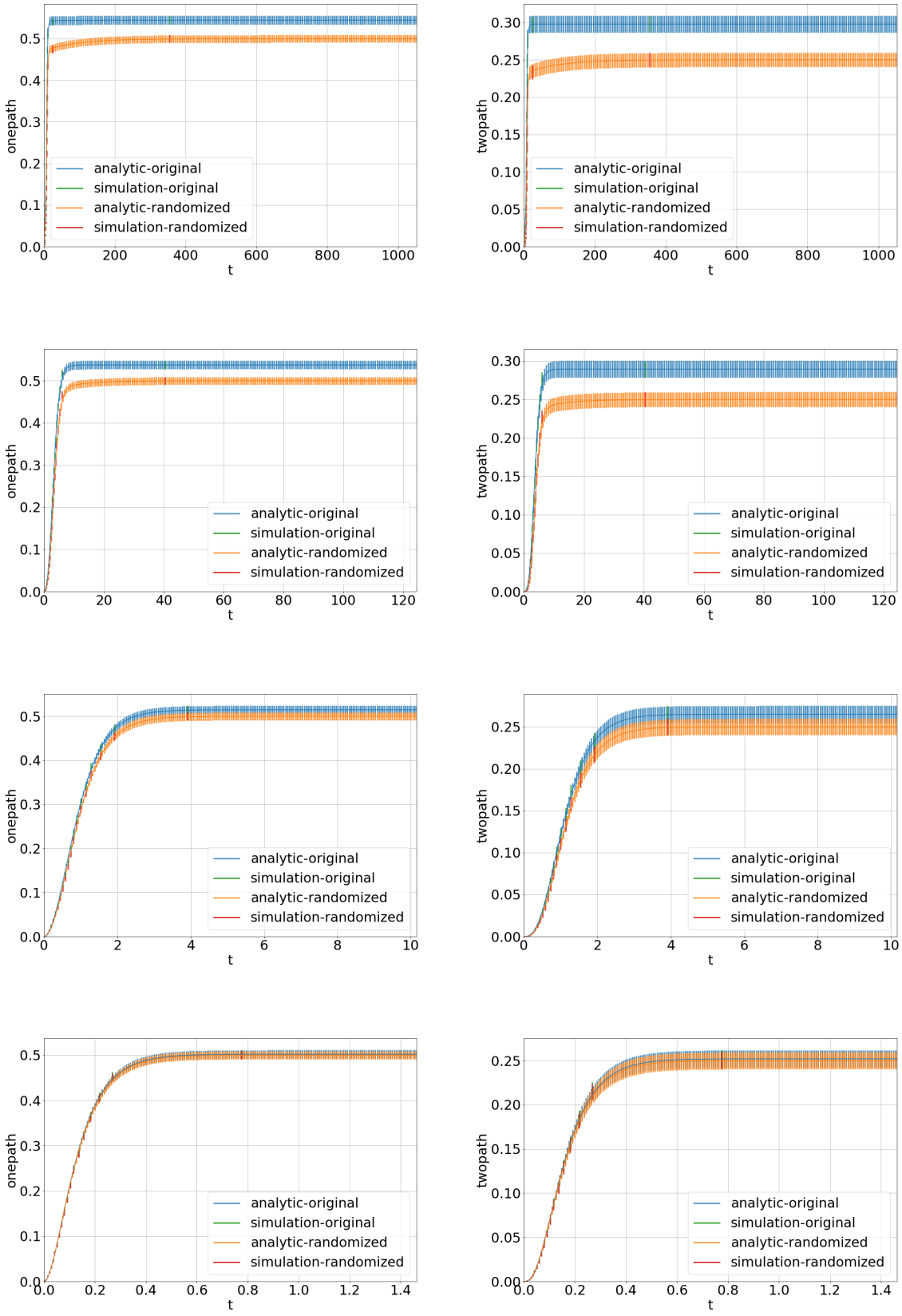


FIG. 8: $\langle k \rangle = 6$, $\zeta = 0.01, 0.1, 1, 10$, voter model

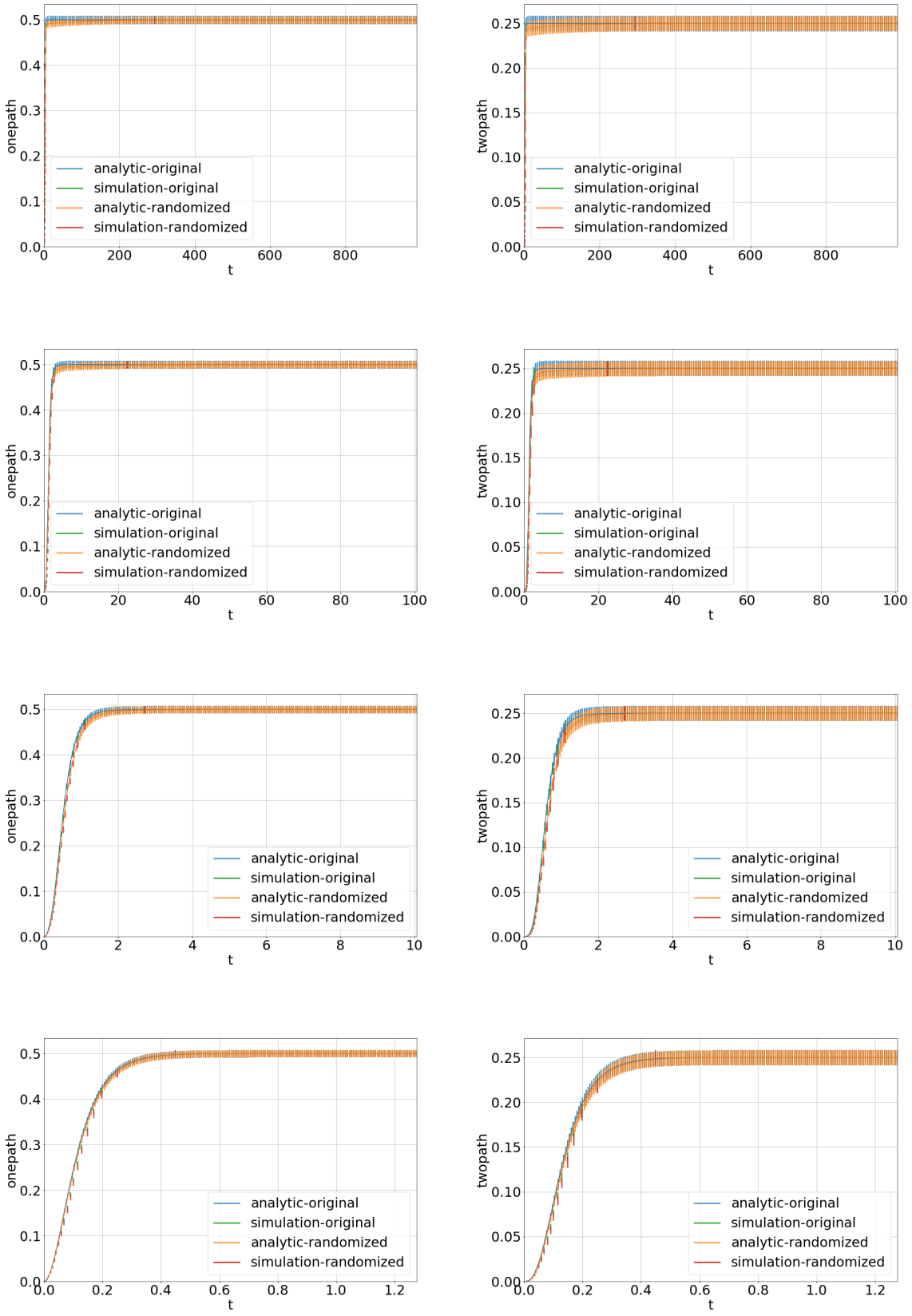


FIG. 9: $\langle k \rangle = 8$, $\zeta = 0.01, 0.1, 1, 10$, SI

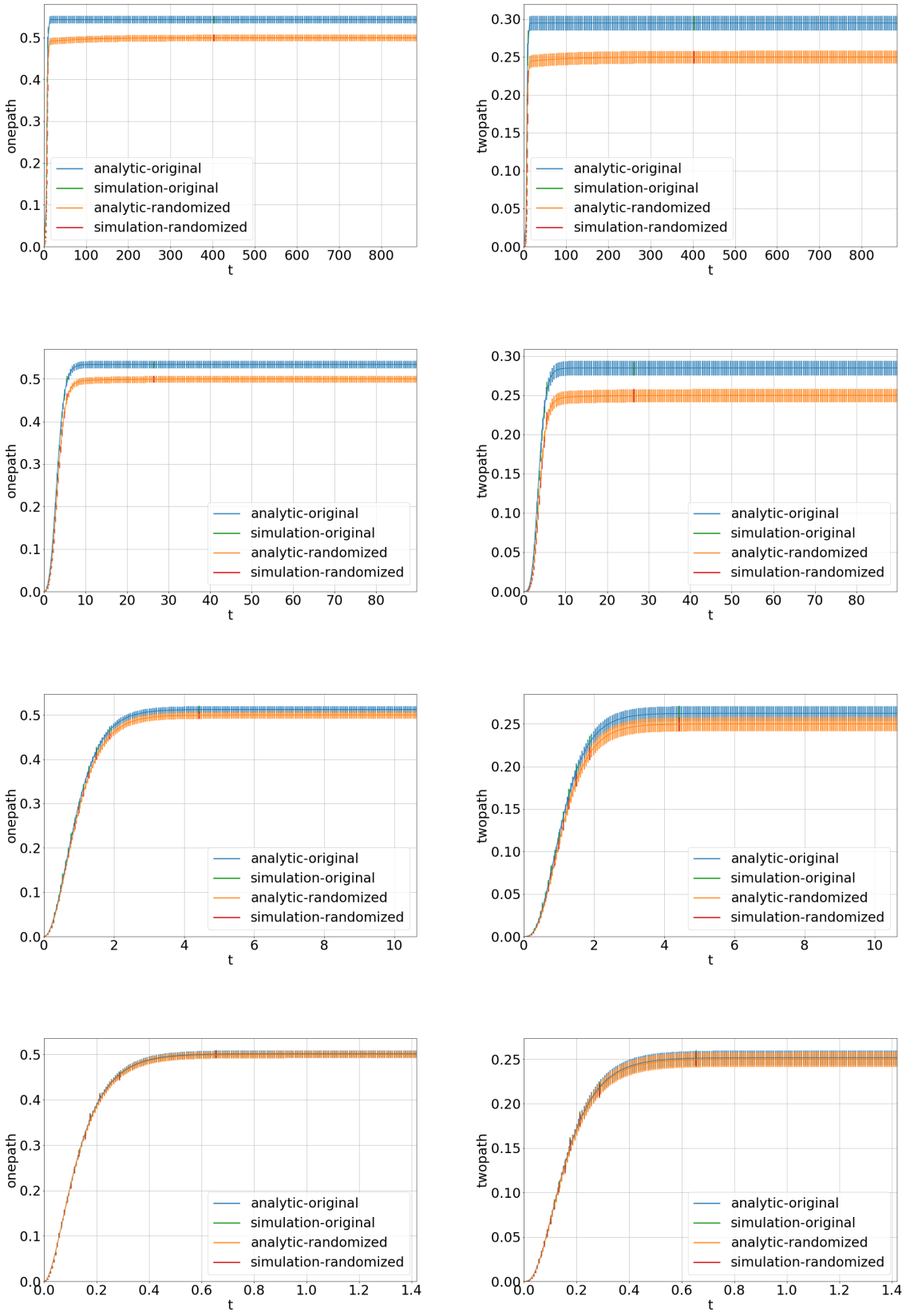


FIG. 10: $\langle k \rangle = 8$, $\zeta = 0.01, 0.1, 1, 10$, voter model

	2.0	3.0	4.0	6.0	8.0
5.0	4.11 (1.22)	4.66 (1.09)	5.01 (1.04)	5.12 (0.99)	5.14 (0.92)
10.0	6.35 (1.22)	7.0 (1.07)	7.15 (1.04)	7.01 (0.99)	6.9 (0.95)
15.0	7.77 (1.22)	8.5 (1.06)	8.58 (1.05)	8.04 (1.03)	7.63 (1.01)
20.0	8.8 (1.25)	9.45 (1.16)	9.41 (1.12)	8.55 (1.06)	8.14 (1.03)
25.0	9.49 (1.31)	10.11 (1.17)	9.89 (1.12)	8.88 (1.07)	8.25 (1.07)
30.0	9.9 (1.36)	10.6 (1.22)	10.09 (1.15)	9.04 (1.11)	8.39 (1.05)
35.0	10.05 (1.38)	10.95 (1.22)	10.21 (1.13)	9.08 (1.11)	8.37 (1.1)
40.0	9.93 (1.36)	11.12 (1.25)	10.28 (1.15)	9.06 (1.13)	8.23 (1.11)
45.0	9.72 (1.39)	11.15 (1.23)	10.35 (1.18)	9.06 (1.14)	8.12 (1.09)
50.0	9.4 (1.38)	11.16 (1.27)	10.16 (1.13)	8.87 (1.11)	7.83 (1.12)
55.0	9.01 (1.38)	11.09 (1.3)	9.99 (1.16)	8.56 (1.09)	7.53 (1.09)
60.0	8.5 (1.34)	10.61 (1.21)	9.69 (1.18)	8.27 (1.08)	7.27 (1.08)
65.0	7.91 (1.28)	10.12 (1.21)	9.25 (1.13)	7.93 (1.09)	6.92 (1.07)
70.0	7.2 (1.21)	9.26 (1.34)	8.77 (1.15)	7.44 (1.05)	6.5 (1.06)
75.0	6.33 (1.2)	8.05 (1.28)	8.11 (1.09)	6.85 (1.01)	6.02 (1.07)
80.0	5.33 (1.14)	6.71 (1.17)	7.32 (1.09)	6.22 (1.03)	5.46 (1.05)
85.0	4.15 (1.11)	5.29 (1.13)	5.98 (1.2)	5.29 (1.01)	4.69 (1.05)
90.0	2.85 (1.07)	3.72 (1.11)	4.16 (1.16)	4.2 (1.02)	3.7 (1.03)
95.0	1.44 (1.04)	1.94 (1.06)	2.16 (1.14)	2.57 (1.07)	2.38 (1.03)
100.0	-0.08 (1.02)	0.06 (1.04)	-0.03 (1.1)	-0.14 (1.05)	-0.05 (1.04)

TABLE II: $\zeta = 0.01$, one-link

	2.0	3.0	4.0	6.0	8.0
5.0	1.87 (1.31)	2.2 (1.24)	2.49 (1.22)	2.8 (1.17)	2.96 (1.15)
10.0	3.25 (1.27)	3.71 (1.18)	4.1 (1.19)	4.42 (1.07)	4.49 (1.09)
15.0	4.39 (1.23)	4.88 (1.18)	5.24 (1.12)	5.46 (1.09)	5.52 (1.07)
20.0	5.23 (1.24)	5.85 (1.15)	6.06 (1.15)	6.17 (1.11)	6.13 (1.08)
25.0	5.84 (1.22)	6.44 (1.16)	6.59 (1.16)	6.64 (1.1)	6.47 (1.06)
30.0	6.4 (1.22)	6.98 (1.18)	7.08 (1.15)	7.05 (1.14)	6.69 (1.07)
35.0	6.87 (1.16)	7.43 (1.18)	7.52 (1.18)	7.24 (1.14)	6.78 (1.07)
40.0	7.15 (1.19)	7.74 (1.15)	7.71 (1.16)	7.39 (1.12)	6.89 (1.07)
45.0	7.36 (1.18)	7.85 (1.15)	7.8 (1.15)	7.45 (1.11)	6.86 (1.04)
50.0	7.4 (1.17)	7.96 (1.19)	7.82 (1.11)	7.4 (1.13)	6.7 (1.07)
55.0	7.33 (1.16)	7.92 (1.18)	7.74 (1.11)	7.31 (1.05)	6.56 (1.06)
60.0	7.24 (1.14)	7.78 (1.15)	7.64 (1.15)	7.13 (1.04)	6.41 (1.05)
65.0	6.99 (1.2)	7.59 (1.17)	7.36 (1.07)	6.86 (1.04)	6.16 (1.07)
70.0	6.51 (1.15)	7.19 (1.11)	7.03 (1.07)	6.56 (1.04)	5.83 (1.04)
75.0	5.93 (1.17)	6.73 (1.08)	6.64 (1.08)	6.12 (1.08)	5.41 (1.03)
80.0	5.1 (1.12)	6.03 (1.1)	5.99 (1.06)	5.59 (1.03)	4.96 (1.05)
85.0	4.05 (1.1)	5.02 (1.11)	5.18 (1.05)	4.83 (1.0)	4.26 (0.99)
90.0	2.85 (1.05)	3.6 (1.06)	4.0 (1.07)	3.84 (0.99)	3.43 (0.98)
95.0	1.49 (1.02)	1.91 (1.04)	2.22 (1.07)	2.4 (1.0)	2.21 (0.97)
100.0	-0.05 (1.0)	0.04 (1.0)	0.06 (1.05)	-0.05 (1.01)	0.01 (1.01)

TABLE III: $\zeta = 0.1$, one-link

IV. KS-TEST

	2.0	3.0	4.0	6.0	8.0
5.0	0.4 (1.15)	0.55 (1.2)	0.6 (1.15)	0.71 (1.15)	0.82 (1.15)
10.0	0.78 (1.11)	1.02 (1.17)	1.13 (1.12)	1.33 (1.12)	1.46 (1.12)
15.0	1.11 (1.13)	1.43 (1.16)	1.55 (1.11)	1.86 (1.08)	1.99 (1.1)
20.0	1.49 (1.13)	1.75 (1.14)	1.93 (1.09)	2.23 (1.1)	2.43 (1.07)
25.0	1.77 (1.07)	2.05 (1.11)	2.27 (1.1)	2.57 (1.12)	2.83 (1.05)
30.0	2.05 (1.07)	2.34 (1.1)	2.55 (1.1)	2.82 (1.1)	3.1 (1.08)
35.0	2.26 (1.1)	2.61 (1.1)	2.86 (1.12)	3.07 (1.05)	3.36 (1.05)
40.0	2.45 (1.09)	2.79 (1.1)	3.1 (1.12)	3.27 (1.09)	3.52 (1.06)
45.0	2.68 (1.08)	3.0 (1.08)	3.24 (1.08)	3.45 (1.08)	3.62 (1.05)
50.0	2.88 (1.04)	3.13 (1.08)	3.4 (1.08)	3.56 (1.04)	3.72 (1.04)
55.0	2.96 (1.04)	3.26 (1.08)	3.47 (1.07)	3.65 (1.02)	3.75 (1.06)
60.0	3.02 (1.07)	3.3 (1.07)	3.48 (1.07)	3.67 (1.05)	3.7 (1.04)
65.0	3.02 (1.05)	3.32 (1.05)	3.45 (1.05)	3.68 (1.05)	3.73 (1.03)
70.0	2.98 (1.04)	3.26 (1.02)	3.42 (1.04)	3.59 (1.05)	3.61 (1.05)
75.0	2.89 (1.06)	3.15 (1.03)	3.33 (1.06)	3.43 (1.05)	3.48 (1.06)
80.0	2.68 (1.03)	2.96 (1.02)	3.13 (1.05)	3.21 (1.05)	3.23 (1.04)
85.0	2.36 (1.04)	2.61 (1.03)	2.77 (1.04)	2.84 (1.03)	2.87 (1.05)
90.0	1.9 (1.03)	2.1 (1.02)	2.23 (1.02)	2.35 (1.02)	2.35 (1.03)
95.0	1.18 (1.01)	1.33 (1.03)	1.44 (1.03)	1.56 (1.02)	1.58 (1.04)
100.0	0.02 (1.03)	-0.02 (1.04)	-0.04 (1.01)	0.05 (0.99)	0.07 (1.03)

TABLE IV: $\zeta = 1.0$, one-link

	2.0	3.0	4.0	6.0	8.0
5.0	0.0 (1.03)	0.11 (1.04)	0.03 (1.06)	0.09 (1.01)	0.07 (1.01)
10.0	0.02 (1.03)	0.16 (1.04)	0.13 (1.04)	0.21 (1.07)	0.2 (1.05)
15.0	0.09 (1.06)	0.21 (1.05)	0.17 (1.06)	0.28 (1.05)	0.29 (1.04)
20.0	0.17 (1.04)	0.27 (1.01)	0.26 (1.05)	0.35 (1.05)	0.38 (1.04)
25.0	0.19 (1.07)	0.32 (1.02)	0.31 (1.05)	0.44 (1.01)	0.48 (1.05)
30.0	0.25 (1.06)	0.36 (1.02)	0.39 (1.05)	0.5 (1.0)	0.55 (1.08)
35.0	0.29 (1.05)	0.44 (1.0)	0.45 (1.03)	0.58 (1.02)	0.61 (1.06)
40.0	0.34 (1.06)	0.46 (1.0)	0.5 (1.03)	0.62 (1.05)	0.66 (1.03)
45.0	0.36 (1.06)	0.48 (1.0)	0.54 (1.02)	0.68 (1.04)	0.71 (1.03)
50.0	0.38 (1.09)	0.51 (1.01)	0.6 (1.03)	0.69 (1.03)	0.78 (1.02)
55.0	0.4 (1.07)	0.54 (0.99)	0.64 (1.02)	0.75 (1.03)	0.83 (1.0)
60.0	0.43 (1.06)	0.59 (0.99)	0.67 (1.04)	0.77 (1.01)	0.87 (1.0)
65.0	0.45 (1.04)	0.61 (0.98)	0.67 (1.01)	0.79 (1.03)	0.88 (0.99)
70.0	0.44 (1.01)	0.64 (0.99)	0.7 (1.01)	0.82 (1.04)	0.92 (1.0)
75.0	0.43 (1.0)	0.63 (0.97)	0.7 (0.99)	0.79 (1.02)	0.92 (1.02)
80.0	0.41 (1.0)	0.61 (0.99)	0.67 (1.01)	0.75 (1.02)	0.87 (1.0)
85.0	0.35 (1.0)	0.54 (1.0)	0.63 (1.02)	0.68 (1.02)	0.79 (1.0)
90.0	0.3 (1.0)	0.47 (1.0)	0.55 (1.03)	0.57 (1.02)	0.68 (1.02)
95.0	0.18 (1.01)	0.31 (1.01)	0.39 (1.01)	0.37 (1.03)	0.45 (1.01)
100.0	-0.09 (1.01)	-0.01 (1.01)	0.03 (1.04)	-0.06 (1.04)	-0.02 (1.01)

TABLE V: $\zeta = 10.0$, one-link

	2.0	3.0	4.0	6.0	8.0
5.0	-0.02 (1.01)	0.04 (1.04)	0.06 (1.01)	0.03 (1.03)	0.02 (1.01)
10.0	-0.0 (1.01)	0.07 (1.04)	0.07 (1.0)	0.03 (1.06)	0.01 (1.04)
15.0	-0.02 (1.05)	0.09 (1.04)	0.07 (1.04)	0.04 (1.02)	0.05 (1.03)
20.0	-0.02 (1.06)	0.1 (1.06)	0.07 (1.08)	0.05 (1.04)	0.08 (1.02)
25.0	-0.03 (1.04)	0.09 (1.05)	0.06 (1.08)	0.05 (1.04)	0.11 (1.02)
30.0	-0.01 (1.02)	0.11 (1.06)	0.07 (1.05)	0.07 (1.03)	0.14 (1.01)
35.0	0.0 (1.01)	0.1 (1.04)	0.08 (1.05)	0.1 (1.0)	0.15 (0.99)
40.0	0.0 (1.0)	0.09 (1.02)	0.07 (1.06)	0.11 (1.02)	0.15 (1.0)
45.0	0.0 (1.0)	0.11 (1.03)	0.05 (1.03)	0.11 (1.05)	0.13 (1.0)
50.0	0.02 (1.02)	0.1 (1.01)	0.07 (1.04)	0.11 (1.02)	0.14 (1.02)
55.0	0.04 (1.02)	0.11 (0.99)	0.08 (1.02)	0.12 (1.0)	0.14 (1.03)
60.0	0.05 (1.03)	0.1 (1.0)	0.08 (1.0)	0.1 (0.99)	0.14 (1.02)
65.0	0.04 (1.03)	0.1 (1.02)	0.08 (1.04)	0.1 (1.01)	0.13 (1.04)
70.0	0.06 (1.05)	0.1 (1.03)	0.08 (1.02)	0.11 (1.03)	0.13 (1.04)
75.0	0.05 (1.05)	0.1 (1.04)	0.07 (1.0)	0.12 (1.02)	0.12 (1.04)
80.0	0.05 (1.07)	0.11 (1.05)	0.06 (1.0)	0.12 (1.01)	0.12 (1.02)
85.0	0.04 (1.06)	0.1 (1.03)	0.05 (1.02)	0.08 (1.02)	0.11 (1.03)
90.0	0.03 (1.05)	0.07 (1.01)	0.05 (1.0)	0.08 (1.03)	0.07 (1.02)
95.0	0.03 (1.05)	0.04 (1.01)	0.01 (1.01)	0.07 (1.02)	0.04 (1.03)
100.0	0.01 (1.04)	0.03 (1.04)	-0.03 (1.01)	0.02 (1.03)	-0.03 (1.01)

TABLE VI: $\zeta = 100.0$, one-link

	2.0	3.0	4.0	6.0	8.0
5.0	4.56 (2.12)	5.14 (1.9)	5.46 (1.86)	5.53 (1.75)	5.52 (1.62)
10.0	7.09 (2.06)	7.75 (1.81)	7.77 (1.76)	7.5 (1.64)	7.43 (1.5)
15.0	8.62 (1.98)	9.33 (1.73)	9.21 (1.64)	8.47 (1.58)	8.1 (1.51)
20.0	9.64 (1.87)	10.19 (1.72)	9.9 (1.6)	9.04 (1.53)	8.53 (1.44)
25.0	10.24 (1.88)	10.7 (1.7)	10.36 (1.56)	9.2 (1.5)	8.61 (1.42)
30.0	10.58 (1.86)	11.06 (1.67)	10.47 (1.57)	9.29 (1.46)	8.68 (1.38)
35.0	10.52 (1.83)	11.25 (1.62)	10.43 (1.49)	9.25 (1.44)	8.51 (1.35)
40.0	10.25 (1.78)	11.17 (1.64)	10.33 (1.48)	9.1 (1.41)	8.3 (1.35)
45.0	9.76 (1.77)	10.95 (1.56)	10.18 (1.47)	8.97 (1.41)	8.11 (1.29)
50.0	9.08 (1.68)	10.63 (1.57)	9.87 (1.38)	8.7 (1.32)	7.74 (1.3)
55.0	8.35 (1.57)	10.3 (1.6)	9.54 (1.41)	8.28 (1.32)	7.37 (1.28)
60.0	7.49 (1.49)	9.5 (1.45)	9.06 (1.39)	7.86 (1.25)	7.04 (1.23)
65.0	6.63 (1.42)	8.66 (1.42)	8.44 (1.33)	7.45 (1.25)	6.63 (1.23)
70.0	5.71 (1.32)	7.61 (1.46)	7.71 (1.35)	6.84 (1.17)	6.14 (1.19)
75.0	4.73 (1.28)	6.34 (1.38)	6.79 (1.26)	6.14 (1.13)	5.58 (1.19)
80.0	3.75 (1.19)	5.02 (1.28)	5.73 (1.24)	5.36 (1.13)	4.93 (1.16)
85.0	2.75 (1.14)	3.76 (1.22)	4.36 (1.28)	4.38 (1.11)	4.11 (1.14)
90.0	1.77 (1.1)	2.52 (1.17)	2.85 (1.22)	3.23 (1.1)	3.08 (1.11)
95.0	0.81 (1.05)	1.27 (1.11)	1.39 (1.16)	1.71 (1.1)	1.79 (1.09)
100.0	-0.11 (1.01)	0.08 (1.07)	-0.02 (1.12)	-0.11 (1.05)	-0.06 (1.06)

TABLE VII: $\zeta = 0.01$, two-link

	2.0	3.0	4.0	6.0	8.0
5.0	1.54 (1.98)	1.89 (1.95)	2.25 (1.89)	2.66 (1.86)	2.85 (1.8)
10.0	2.93 (1.86)	3.47 (1.82)	3.9 (1.78)	4.34 (1.67)	4.47 (1.62)
15.0	4.03 (1.81)	4.67 (1.76)	5.01 (1.67)	5.35 (1.61)	5.47 (1.53)
20.0	4.93 (1.75)	5.63 (1.65)	5.87 (1.57)	6.09 (1.56)	6.1 (1.46)
25.0	5.54 (1.67)	6.2 (1.61)	6.43 (1.57)	6.52 (1.49)	6.43 (1.38)
30.0	6.05 (1.66)	6.67 (1.56)	6.87 (1.53)	6.87 (1.49)	6.62 (1.37)
35.0	6.48 (1.57)	7.05 (1.52)	7.29 (1.52)	7.04 (1.46)	6.68 (1.34)
40.0	6.65 (1.54)	7.24 (1.5)	7.43 (1.48)	7.16 (1.38)	6.75 (1.3)
45.0	6.77 (1.48)	7.29 (1.47)	7.43 (1.4)	7.14 (1.37)	6.69 (1.24)
50.0	6.74 (1.43)	7.28 (1.44)	7.39 (1.33)	7.09 (1.36)	6.48 (1.25)
55.0	6.55 (1.41)	7.18 (1.42)	7.21 (1.3)	6.93 (1.25)	6.3 (1.24)
60.0	6.31 (1.36)	6.92 (1.35)	7.02 (1.34)	6.7 (1.2)	6.1 (1.2)
65.0	5.89 (1.33)	6.61 (1.35)	6.67 (1.23)	6.36 (1.18)	5.79 (1.19)
70.0	5.3 (1.28)	6.11 (1.29)	6.23 (1.21)	5.98 (1.16)	5.42 (1.15)
75.0	4.61 (1.25)	5.51 (1.24)	5.72 (1.19)	5.48 (1.16)	4.97 (1.13)
80.0	3.78 (1.2)	4.72 (1.22)	4.97 (1.14)	4.87 (1.11)	4.45 (1.12)
85.0	2.83 (1.13)	3.69 (1.19)	4.06 (1.12)	4.06 (1.07)	3.74 (1.07)
90.0	1.86 (1.07)	2.49 (1.13)	2.88 (1.09)	3.05 (1.05)	2.87 (1.03)
95.0	0.92 (1.05)	1.24 (1.08)	1.47 (1.05)	1.71 (1.03)	1.7 (1.0)
100.0	-0.02 (1.01)	0.05 (1.04)	0.05 (1.01)	-0.02 (1.0)	-0.01 (1.0)

TABLE VIII: $\zeta = 0.1$,two-link

	2.0	3.0	4.0	6.0	8.0
5.0	0.17 (1.39)	0.32 (1.46)	0.39 (1.46)	0.55 (1.41)	0.64 (1.39)
10.0	0.48 (1.34)	0.69 (1.34)	0.8 (1.35)	1.07 (1.36)	1.25 (1.33)
15.0	0.74 (1.3)	1.09 (1.34)	1.22 (1.33)	1.58 (1.3)	1.76 (1.31)
20.0	1.07 (1.32)	1.37 (1.34)	1.58 (1.29)	1.98 (1.31)	2.19 (1.27)
25.0	1.31 (1.27)	1.66 (1.29)	1.93 (1.27)	2.32 (1.34)	2.57 (1.21)
30.0	1.6 (1.29)	1.94 (1.28)	2.21 (1.28)	2.56 (1.3)	2.87 (1.24)
35.0	1.77 (1.28)	2.21 (1.29)	2.53 (1.28)	2.8 (1.24)	3.12 (1.19)
40.0	1.98 (1.25)	2.4 (1.29)	2.76 (1.29)	2.98 (1.25)	3.3 (1.2)
45.0	2.15 (1.21)	2.61 (1.25)	2.9 (1.25)	3.15 (1.23)	3.4 (1.17)
50.0	2.31 (1.15)	2.72 (1.24)	3.03 (1.23)	3.25 (1.19)	3.5 (1.16)
55.0	2.39 (1.15)	2.83 (1.21)	3.09 (1.2)	3.32 (1.17)	3.54 (1.16)
60.0	2.42 (1.16)	2.85 (1.18)	3.09 (1.18)	3.35 (1.17)	3.49 (1.14)
65.0	2.43 (1.16)	2.85 (1.16)	3.05 (1.15)	3.34 (1.16)	3.51 (1.12)
70.0	2.38 (1.14)	2.79 (1.12)	3.0 (1.14)	3.23 (1.14)	3.36 (1.12)
75.0	2.26 (1.13)	2.67 (1.11)	2.88 (1.14)	3.07 (1.13)	3.22 (1.12)
80.0	2.06 (1.1)	2.47 (1.1)	2.66 (1.11)	2.84 (1.13)	2.94 (1.1)
85.0	1.78 (1.08)	2.15 (1.09)	2.31 (1.09)	2.48 (1.09)	2.59 (1.09)
90.0	1.37 (1.07)	1.68 (1.07)	1.8 (1.05)	2.0 (1.08)	2.07 (1.06)
95.0	0.78 (1.03)	1.01 (1.05)	1.08 (1.03)	1.25 (1.05)	1.33 (1.06)
100.0	-0.01 (1.01)	0.03 (1.03)	-0.03 (0.99)	0.04 (1.01)	0.07 (1.03)

TABLE IX: $\zeta = 1.0$, two-link

	2.0	3.0	4.0	6.0	8.0
5.0	nan (nan)	0.08 (1.21)	0.02 (1.07)	0.09 (1.16)	0.04 (0.99)
10.0	0.04 (1.07)	0.13 (1.14)	0.12 (1.08)	0.18 (1.12)	0.14 (1.05)
15.0	0.08 (1.09)	0.16 (1.11)	0.16 (1.05)	0.26 (1.11)	0.22 (1.09)
20.0	0.1 (1.11)	0.22 (1.09)	0.23 (1.07)	0.31 (1.1)	0.32 (1.06)
25.0	0.13 (1.07)	0.24 (1.07)	0.28 (1.08)	0.4 (1.06)	0.42 (1.09)
30.0	0.18 (1.06)	0.27 (1.05)	0.32 (1.07)	0.45 (1.03)	0.48 (1.1)
35.0	0.23 (1.04)	0.35 (1.04)	0.38 (1.06)	0.53 (1.04)	0.57 (1.09)
40.0	0.28 (1.05)	0.39 (1.06)	0.43 (1.06)	0.56 (1.06)	0.61 (1.06)
45.0	0.29 (1.09)	0.41 (1.04)	0.47 (1.06)	0.62 (1.06)	0.65 (1.05)
50.0	0.31 (1.11)	0.44 (1.04)	0.53 (1.07)	0.63 (1.07)	0.73 (1.04)
55.0	0.33 (1.06)	0.47 (1.01)	0.57 (1.06)	0.69 (1.06)	0.77 (1.04)
60.0	0.37 (1.06)	0.52 (1.01)	0.59 (1.08)	0.72 (1.03)	0.82 (1.02)
65.0	0.39 (1.06)	0.54 (1.0)	0.6 (1.06)	0.73 (1.06)	0.82 (1.0)
70.0	0.38 (1.02)	0.56 (0.99)	0.62 (1.05)	0.77 (1.07)	0.86 (1.01)
75.0	0.36 (1.03)	0.55 (0.98)	0.61 (1.03)	0.73 (1.06)	0.86 (1.04)
80.0	0.35 (1.03)	0.52 (0.99)	0.58 (1.05)	0.7 (1.04)	0.81 (1.02)
85.0	0.31 (1.02)	0.46 (1.0)	0.54 (1.04)	0.64 (1.05)	0.73 (1.0)
90.0	0.25 (1.01)	0.4 (1.0)	0.46 (1.03)	0.53 (1.04)	0.61 (1.02)
95.0	0.14 (1.01)	0.25 (1.0)	0.32 (1.01)	0.35 (1.05)	0.4 (1.01)
100.0	-0.08 (1.01)	-0.03 (0.99)	0.0 (1.02)	-0.03 (1.05)	-0.03 (1.01)

TABLE X: $\zeta = 10.0$, two-link

	2.0	3.0	4.0	6.0	8.0
5.0	nan (nan)	0.07 (1.17)	-0.04 (0.96)	0.02 (1.08)	0.05 (1.08)
10.0	0.02 (1.03)	0.07 (1.16)	0.04 (1.02)	0.05 (1.03)	0.03 (1.08)
15.0	0.01 (1.06)	0.05 (1.06)	0.06 (1.05)	0.03 (1.02)	0.07 (1.05)
20.0	-0.02 (1.07)	0.08 (1.08)	0.07 (1.1)	0.05 (1.04)	0.09 (1.04)
25.0	-0.02 (1.05)	0.08 (1.06)	0.06 (1.09)	0.05 (1.02)	0.09 (1.02)
30.0	-0.01 (1.04)	0.1 (1.07)	0.07 (1.07)	0.06 (1.02)	0.12 (1.02)
35.0	-0.01 (1.01)	0.1 (1.04)	0.06 (1.05)	0.08 (0.98)	0.13 (1.0)
40.0	-0.0 (1.01)	0.09 (1.03)	0.04 (1.08)	0.1 (1.01)	0.13 (1.0)
45.0	-0.01 (1.03)	0.11 (1.02)	0.01 (1.05)	0.09 (1.04)	0.11 (1.01)
50.0	0.01 (1.06)	0.1 (1.0)	0.04 (1.05)	0.09 (1.01)	0.12 (1.04)
55.0	-0.0 (1.03)	0.11 (0.99)	0.04 (1.03)	0.1 (1.01)	0.12 (1.05)
60.0	0.01 (1.04)	0.1 (1.0)	0.06 (1.02)	0.08 (1.0)	0.13 (1.03)
65.0	0.0 (1.04)	0.08 (1.02)	0.06 (1.07)	0.09 (1.02)	0.12 (1.03)
70.0	0.01 (1.05)	0.09 (1.05)	0.05 (1.04)	0.1 (1.05)	0.12 (1.04)
75.0	0.02 (1.04)	0.08 (1.05)	0.04 (1.01)	0.12 (1.05)	0.12 (1.03)
80.0	0.02 (1.07)	0.09 (1.05)	0.04 (1.0)	0.11 (1.02)	0.11 (1.02)
85.0	0.01 (1.04)	0.08 (1.02)	0.02 (1.02)	0.07 (1.01)	0.1 (1.03)
90.0	0.01 (1.03)	0.04 (1.01)	0.03 (1.01)	0.07 (1.03)	0.06 (1.01)
95.0	0.01 (1.04)	0.02 (1.01)	-0.01 (1.01)	0.07 (1.03)	0.02 (1.02)
100.0	-0.02 (1.04)	0.0 (1.02)	-0.06 (1.0)	0.02 (1.03)	-0.04 (1.01)

TABLE XI: $\zeta = 100.0$, two-link

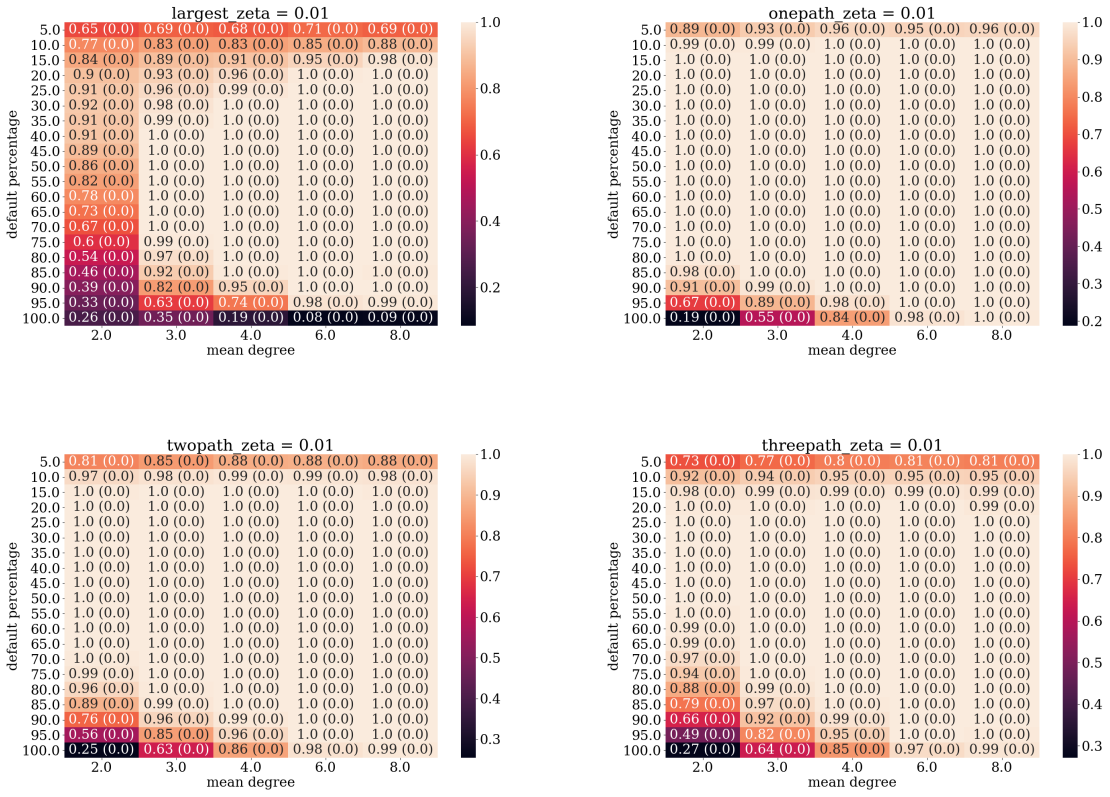


FIG. 11: KS-test of the results for a voter-model process with $\zeta = 0.01$ on a network of $N = 1000$ vertices.

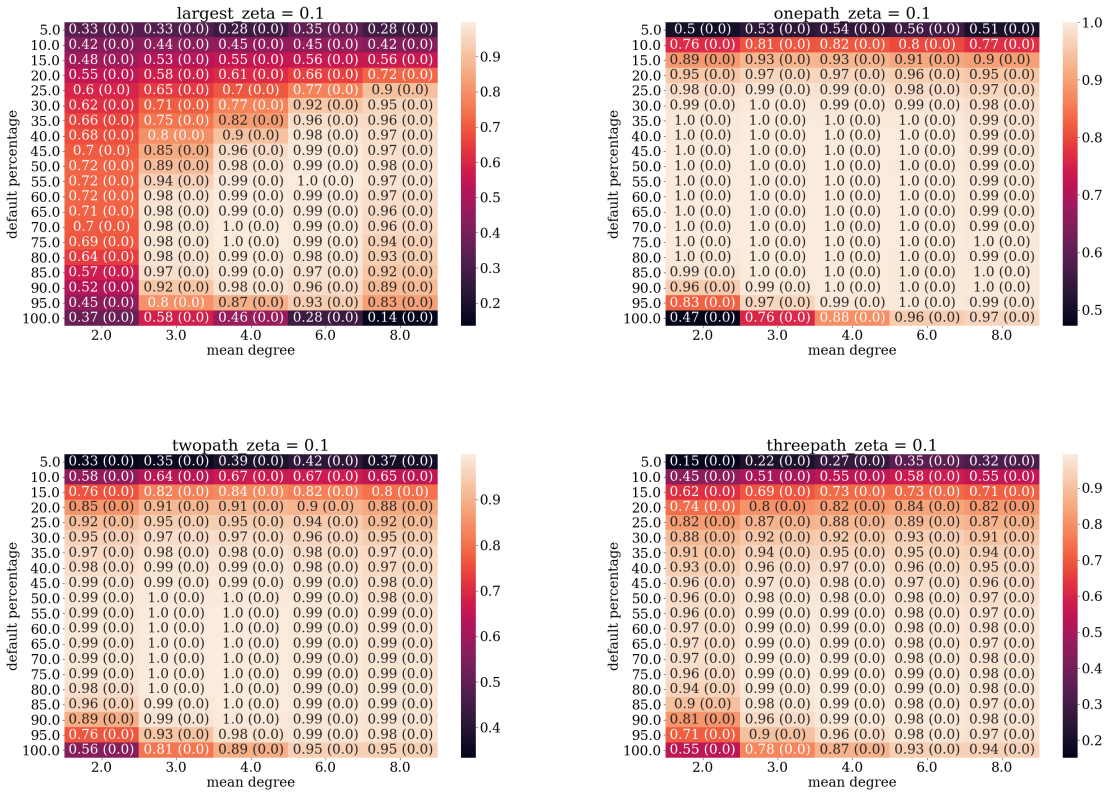


FIG. 12: KS-test of the results for a voter-model process with $\zeta = 0.1$ on a network of $N = 1000$ vertices.

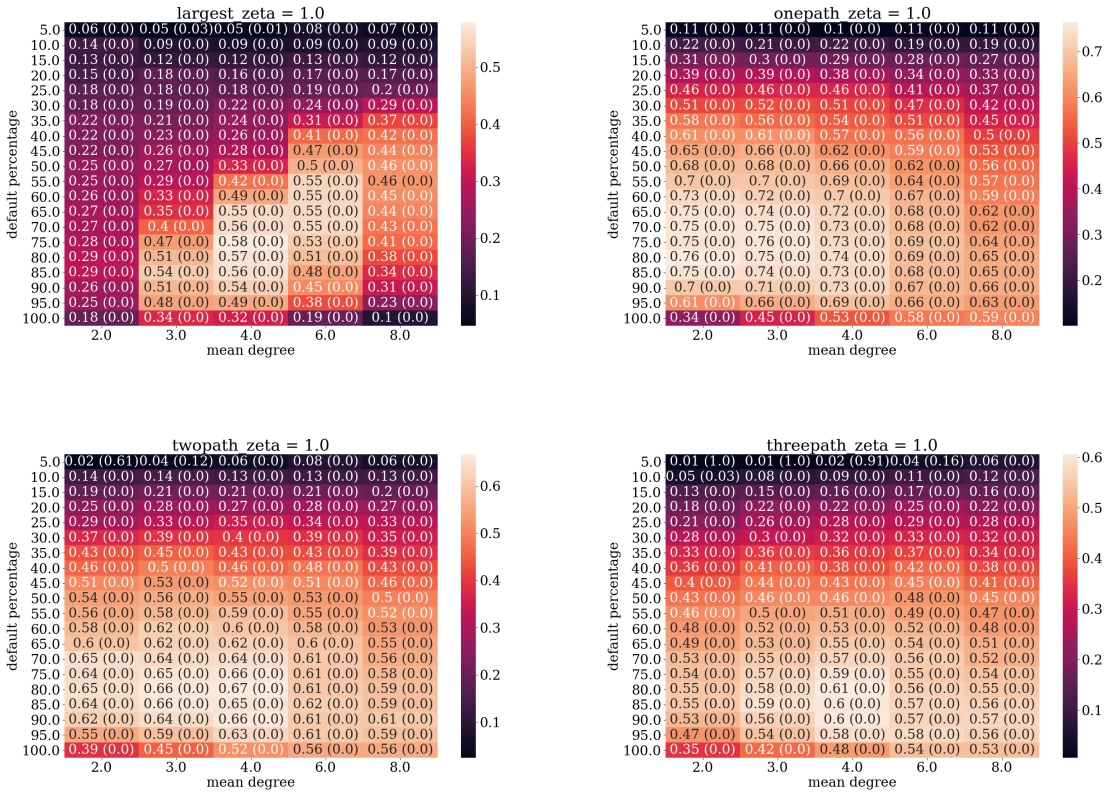


FIG. 13: KS-test of the results for a voter-model process with $\zeta = 1$ on a network of $N = 1000$ vertices.

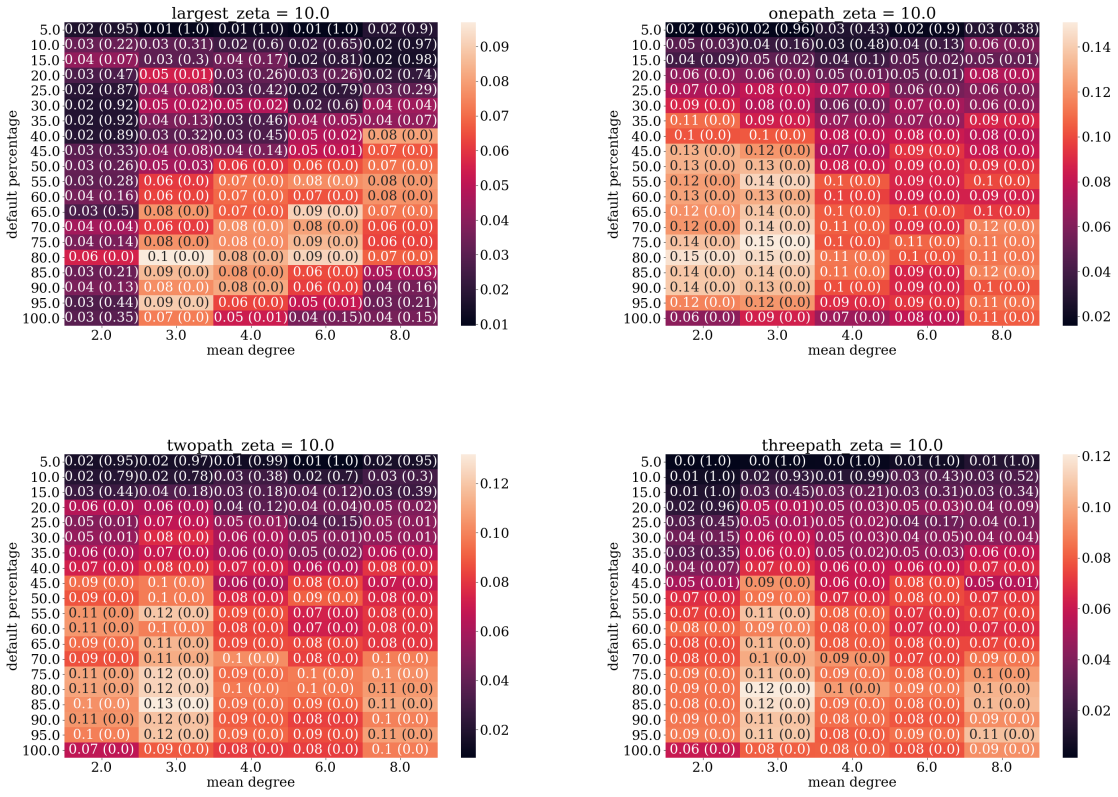


FIG. 14: KS-test of the results for a voter-model process with $\zeta = 10$ on a network of $N = 1000$ vertices.

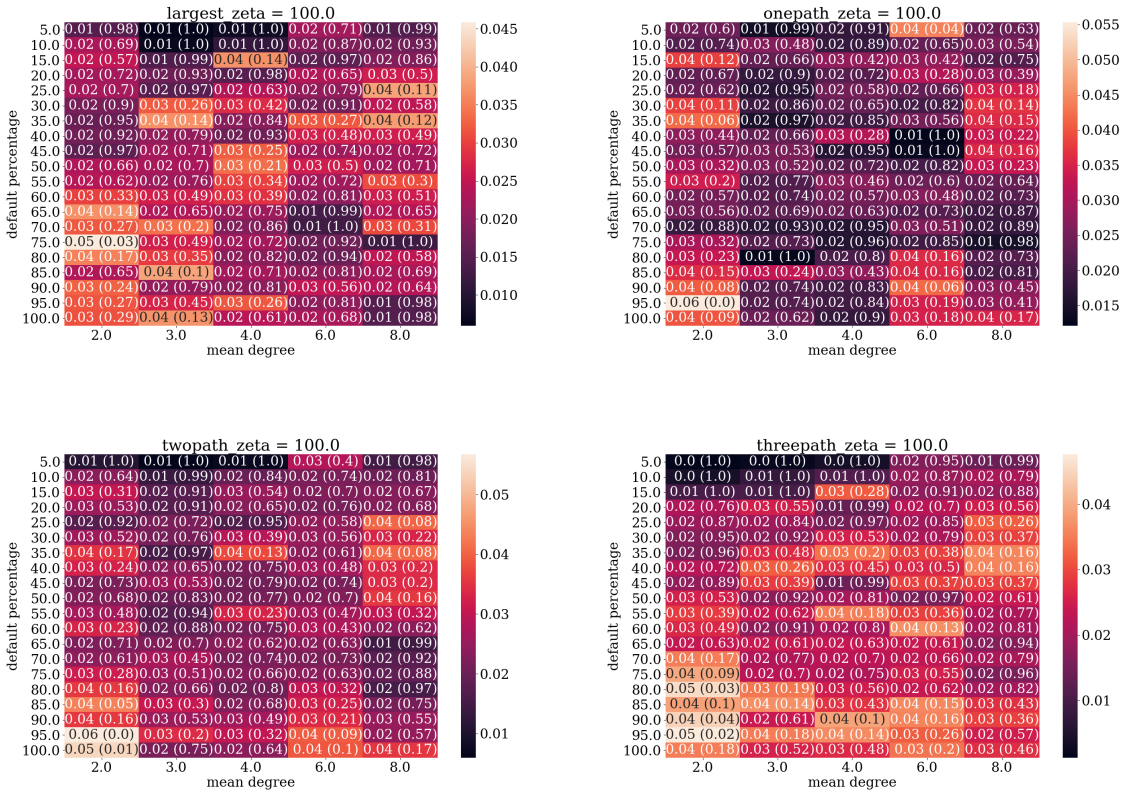


FIG. 15: KS-test of the results for a voter-model process with $\zeta = 100$ on a network of $N = 1000$ vertices.

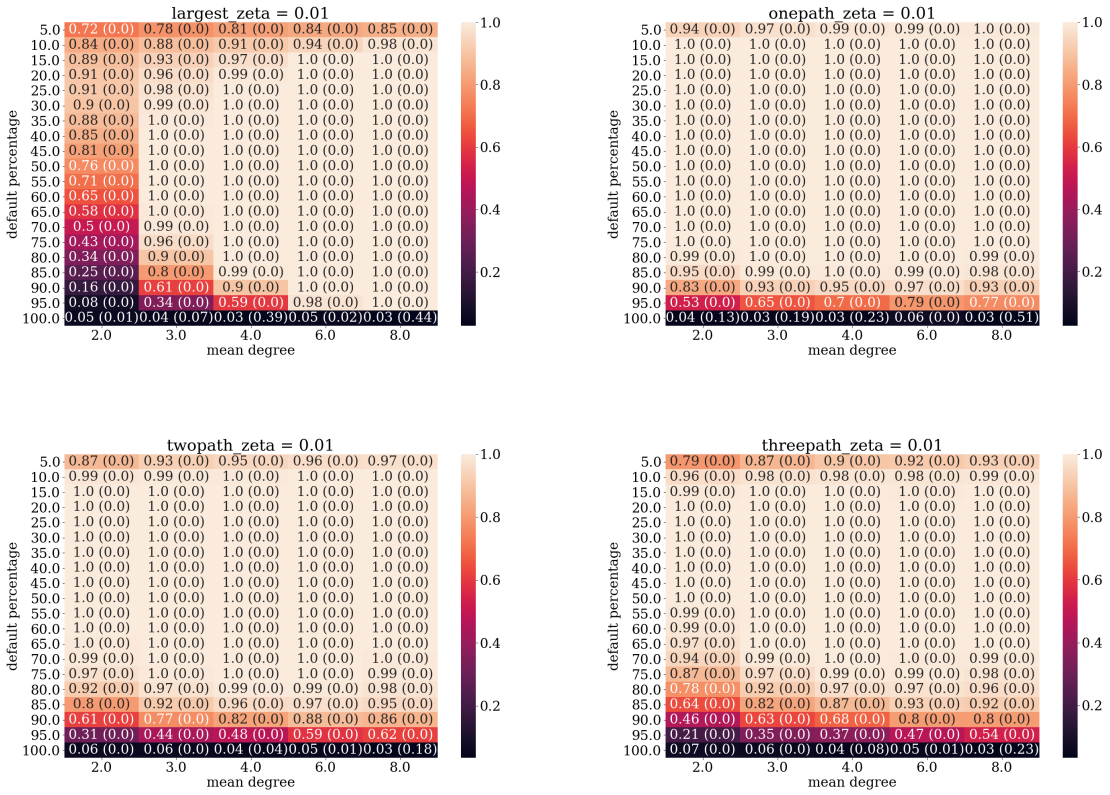


FIG. 16: KS-test of the results for a SI process with $\zeta = 0.01$ on a network of $N = 1000$ vertices.

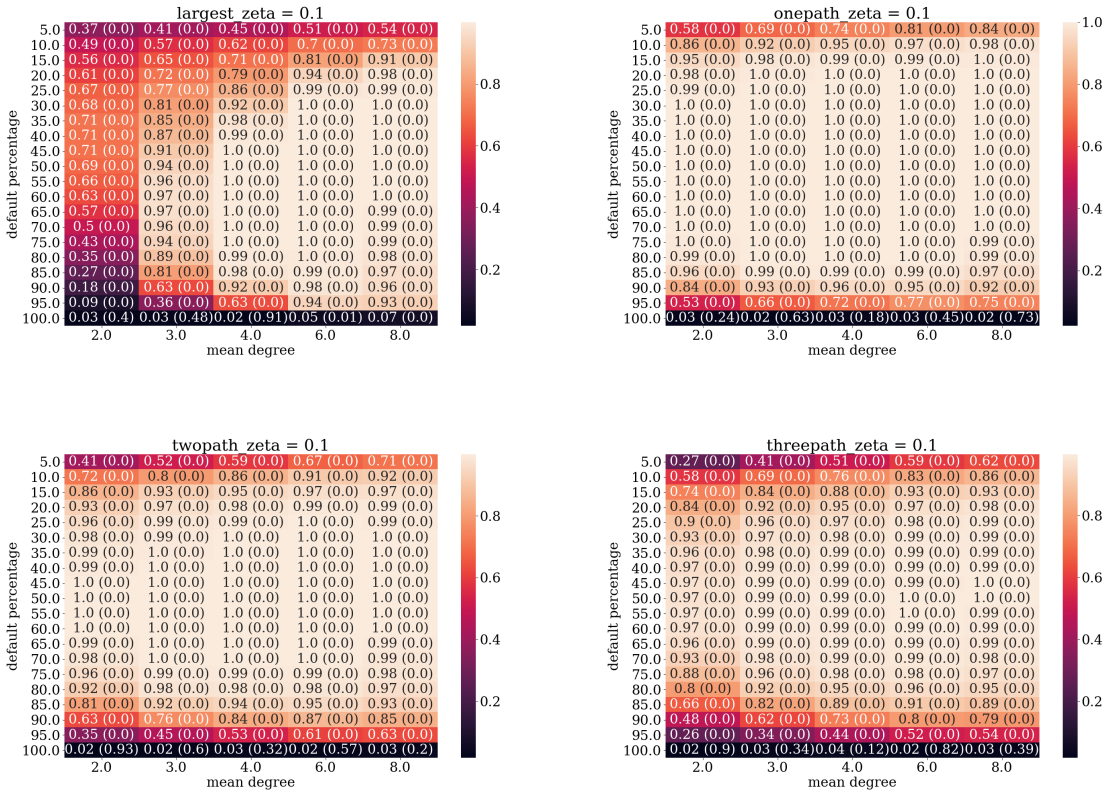


FIG. 17: KS-test of the results for a SI process with $\zeta = 0.1$ on a network of $N = 1000$ vertices.

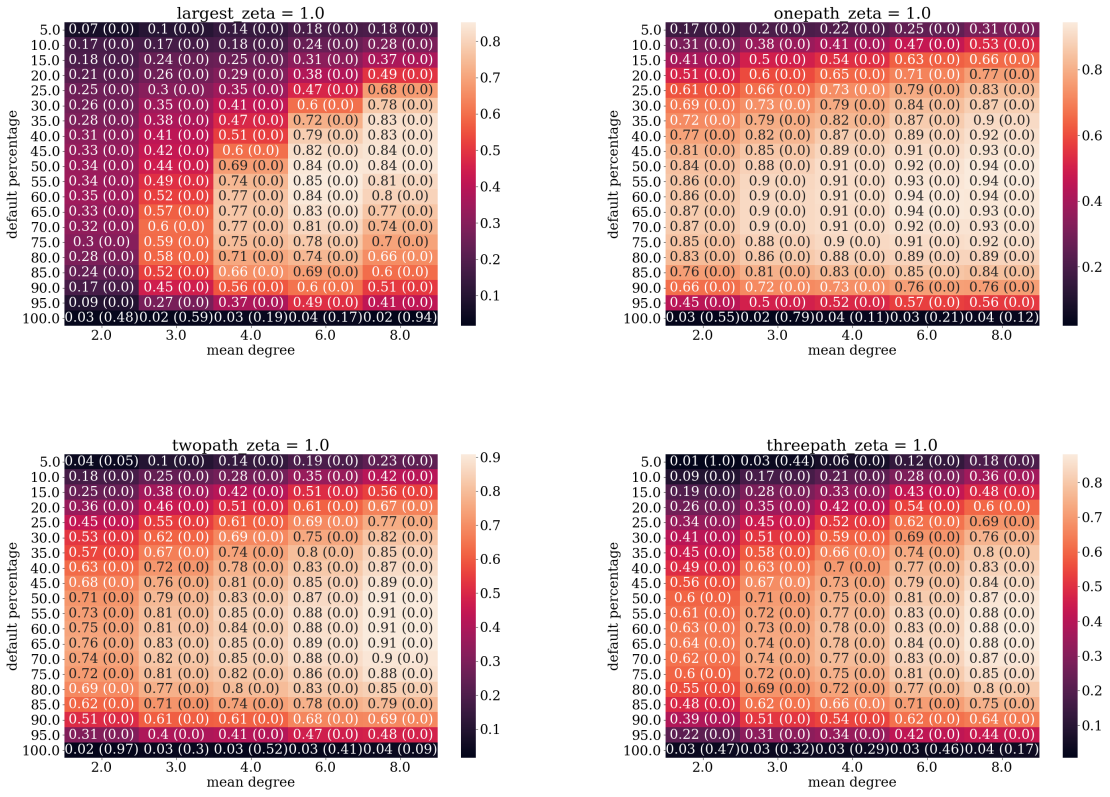


FIG. 18: KS-test of the results for a SI process with $\zeta = 1$ on a network of $N = 1000$ vertices.

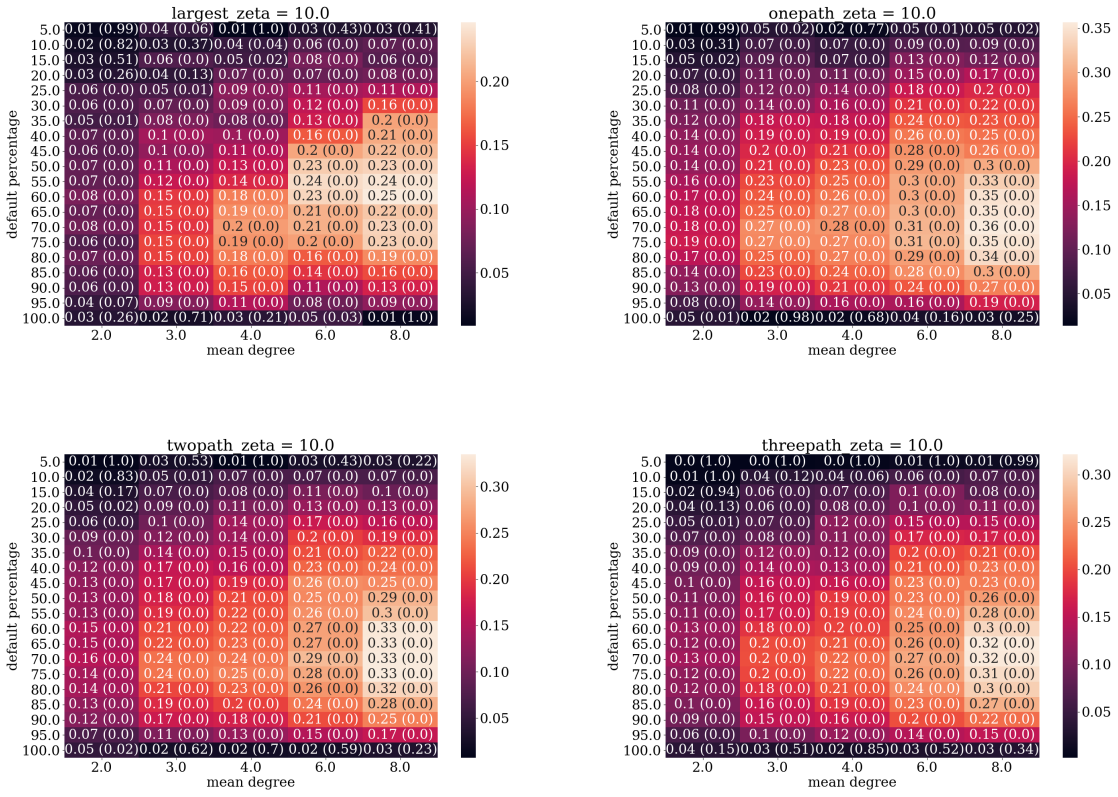


FIG. 19: KS-test of the results for a SI process with $\zeta = 10$ on a network of $N = 1000$ vertices.

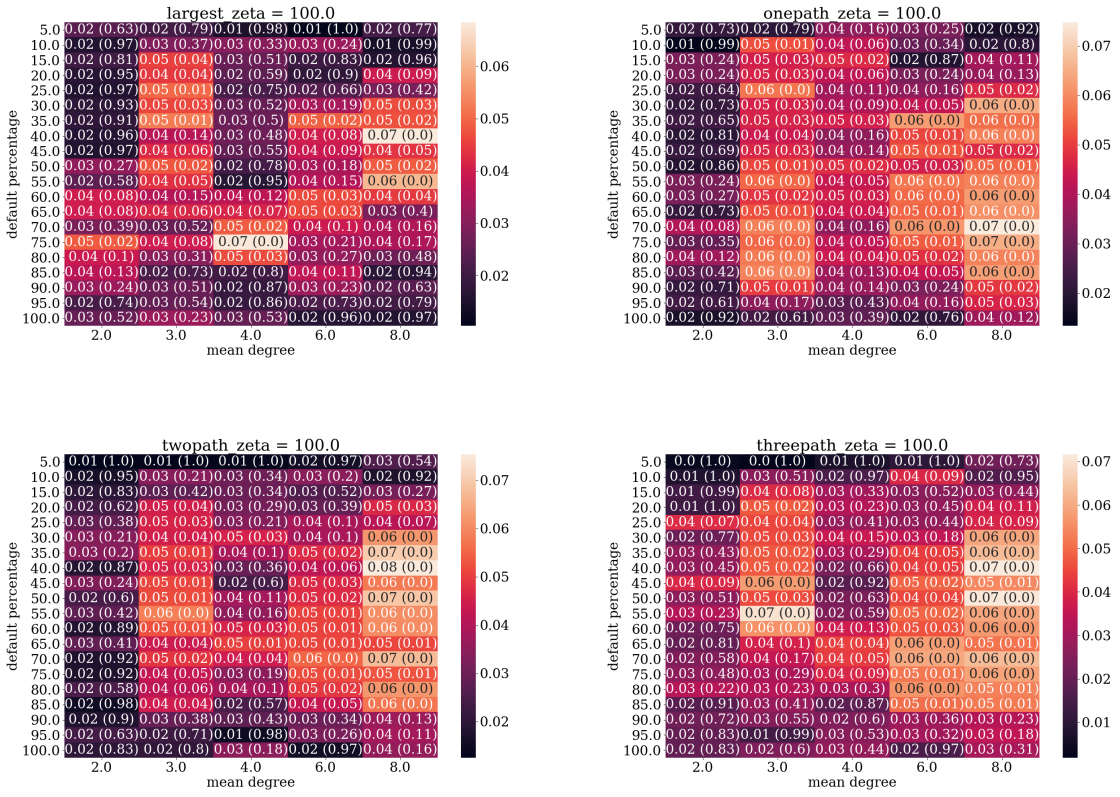


FIG. 20: KS-test of the results for a SI process with $\zeta = 100$ on a network of $N = 1000$ vertices.

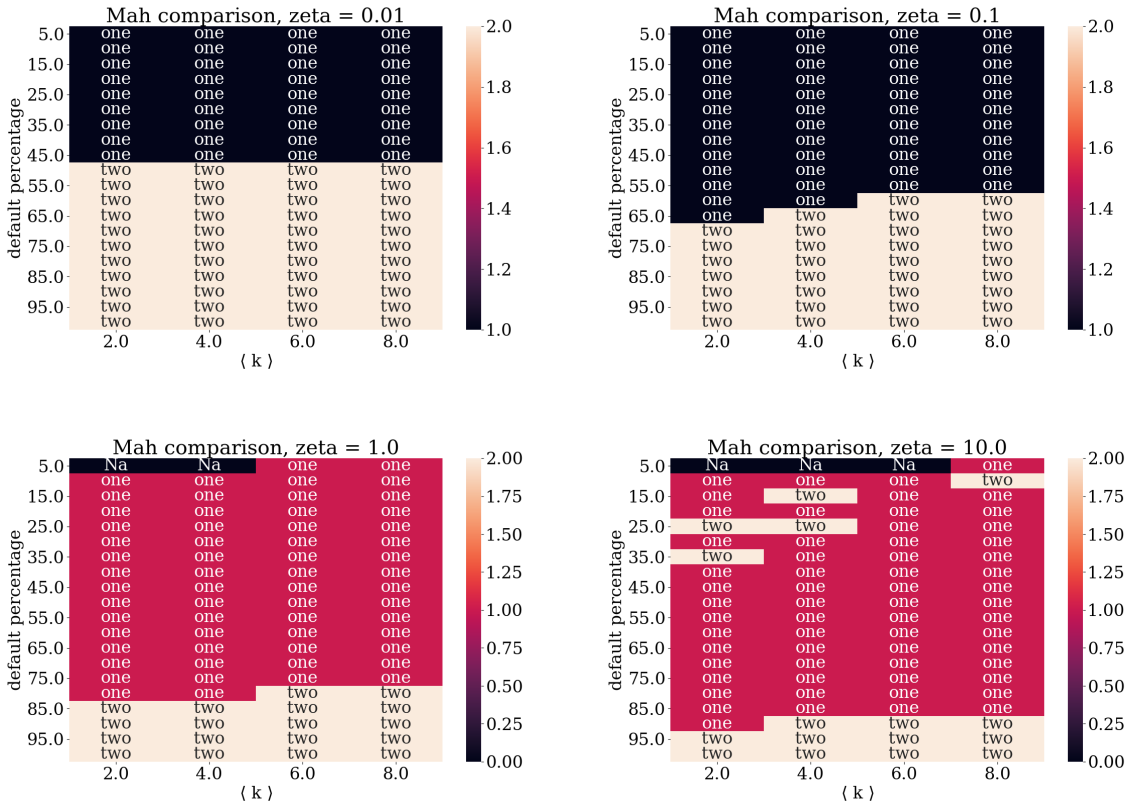


FIG. 21: Comparison of Mahalanobis distance of the results for an SI process on a network of $N = 1000$ vertices. The best statistically significant statistic for a given percentage, ζ and $\langle k \rangle$ is stated as "lc" for largest component, "one" for one-edge, "two" for two-edge, etc. A z-score greater than 1 is considered to be statistically significant, otherwise an "Na" value is put in the table.

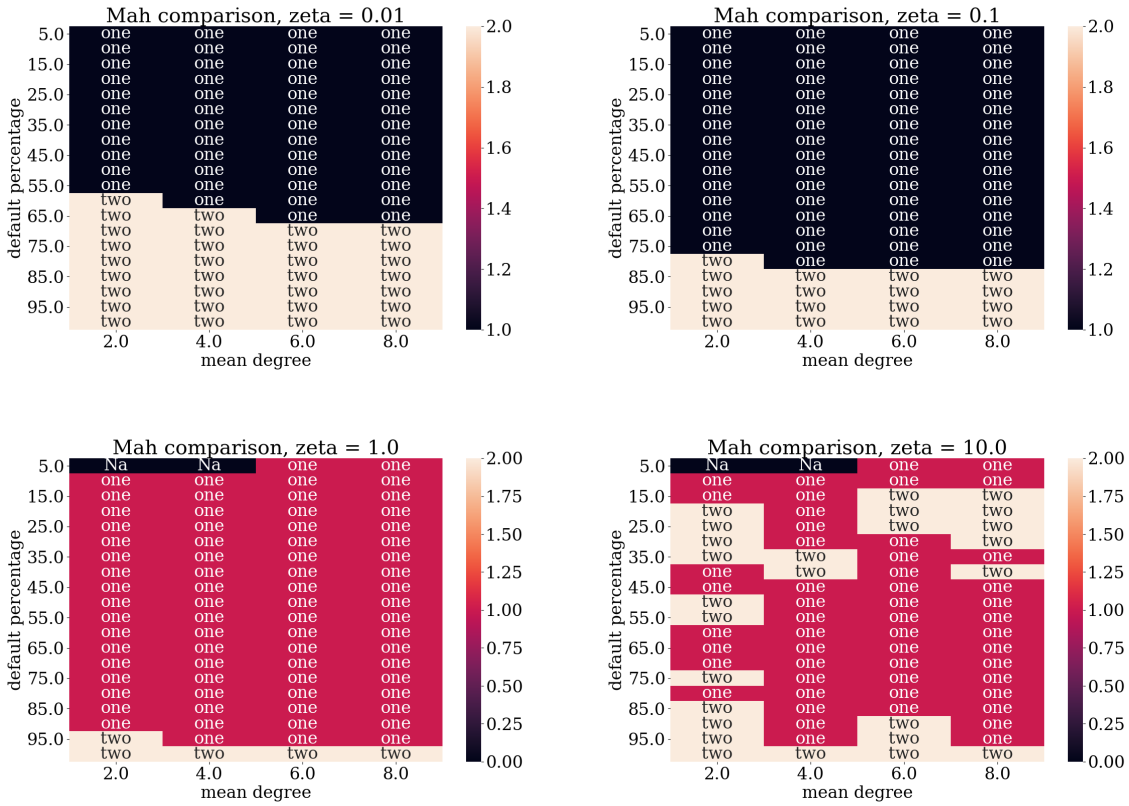


FIG. 22: Comparison of Mahalanobis distance of the results for a voter model process on a network of $N = 1000$ vertices. The best statistically significant statistic for a given percentage, ζ and $\langle k \rangle$ is stated as "lc" for largest component, "one" for one-edge, "two" for two-edge, etc. A z-score greater than 1 is considered to be statistically significant, otherwise an "Na" value is put in the table.