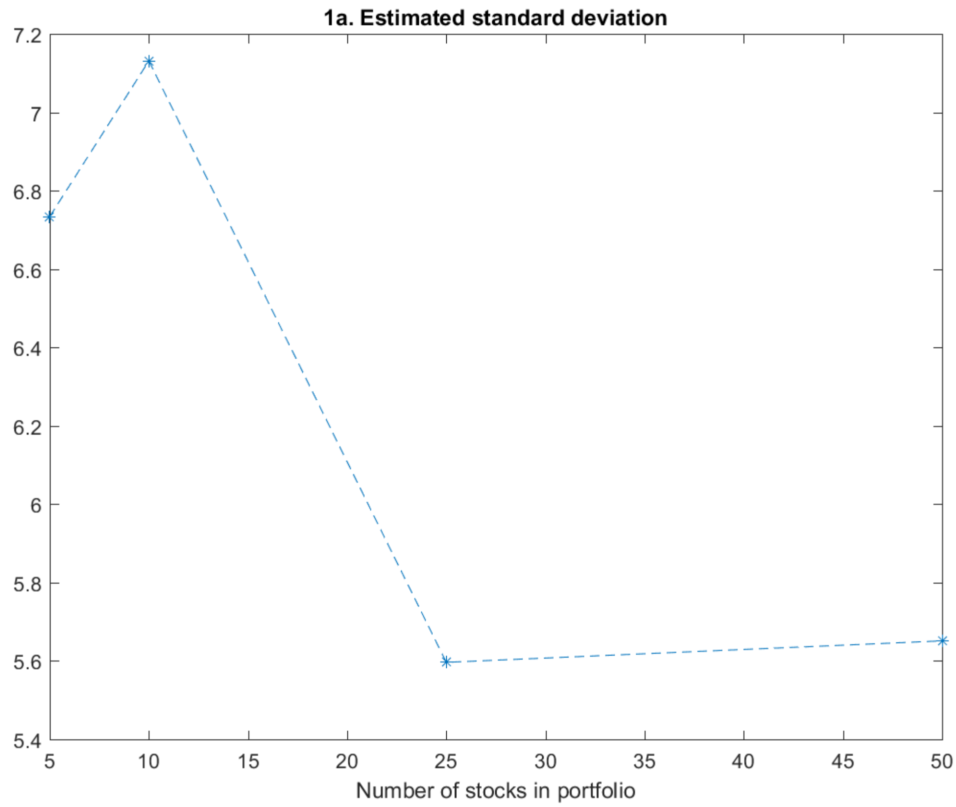
# Problem Set 1: Suggested Solutions

Part1:

a) Here is the standard deviation of monthly returns, for equal-weight portfolios with various numbers of stocks:

| N | 5 | 10 | 25 | 50 |
|---|---|---|---|---|
| Mean | 0.719 | 1.036 | 1.123 | 1.161 |
| Stdev | 6.734 | 7.131 | 5.596 | 5.651 |

The standard deviation of an equally-weighted portfolio decreases sharply from 10 to 25 stocks, but from 25 to 50 stocks, there does not seem to be any decrease in standard deviation. The below graph suggests that the standard deviation tends to a finite value around 5.6%. Consistent with theory (see b).




b) We can decompose the variance, $\sigma^2(R_p)$, of an equally-weighted portfolio $p$ as follows

$$\sigma_p^2 = \sum_{i=1}^{N} w_{ip}^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} w_{ip} w_{jp} \sigma_{ij}$$

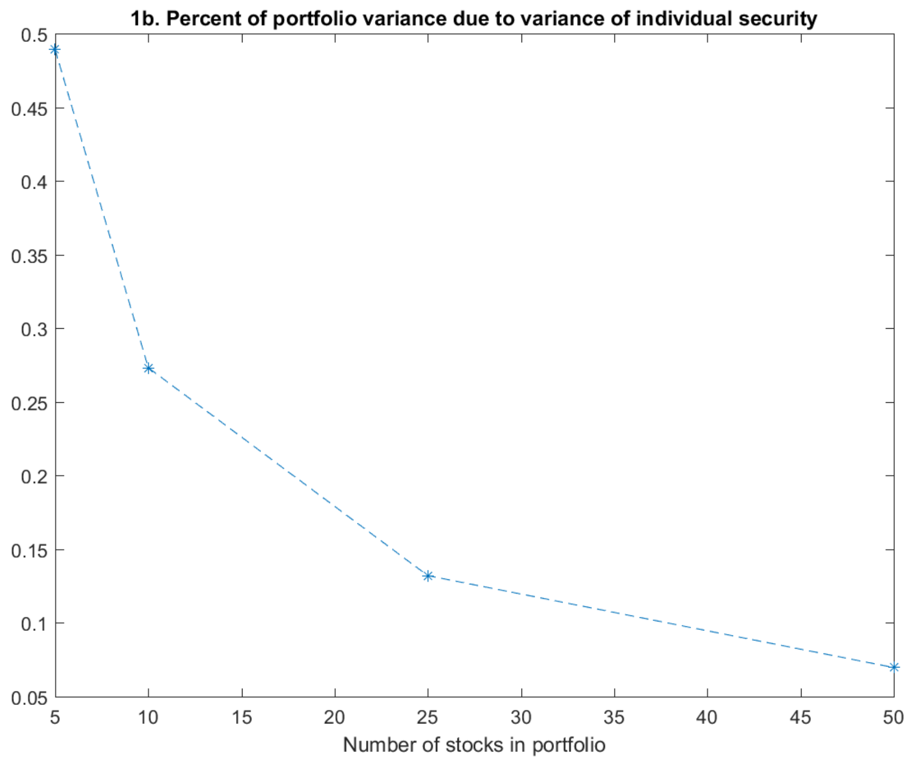
$$= \frac{1}{N}\overline{\sigma_i^2} + \frac{N-1}{N}\overline{\sigma_{ij}},$$

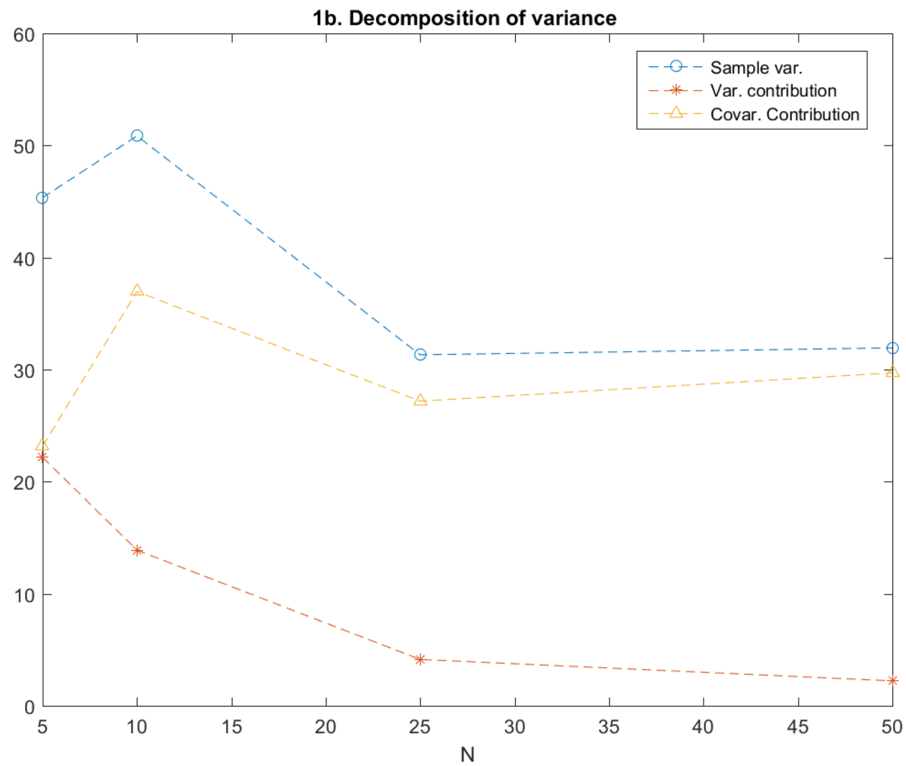
where $N$ is the number of stocks, and $\overline{\sigma^2}$ and $\overline{\sigma_{ij}}$ are respectively the average variance and the average covariance of the stocks in the portfolio.

| N | 5 | 10 | 25 | 50 |
|---|---|---|---|---|
| Sample variance | 45.34 | 50.85 | 31.32 | 31.94 |
| var contribution | 22.17 | 13.89 | 4.14 | 2.23 |
| covar contribution | 23.17 | 36.96 | 27.18 | 29.71 |

Thus, the contribution given by the variance of stocks decreases as $N$ increases, and the variance of the portfolio tends to the average covariance among the stocks in the portfolio.

The below graph reports the contribution of the variances and covariance terms, as well as the total variances of the portfolios considered. While the contribution of the stocks' variances decreases sharply with $N$, the covariance term does not vary much with $N$. These results are consistent with what we would expect theoretically.






c) Value-weighting tilts a portfolio towards large stocks. Thus, we have to consider how differences in stock variances and covariances are related to their size. Larger stocks typically have lower standard deviation than small ones (e.g. since multiple business segments or in “mature” industries). For portfolios with few stocks (the “average-variance-term” dominates) value-weighted portfolios will therefore tend to have a lower total portfolio variance than the equally- weighted one. For covariances, if correlations are similar between large and small stocks, since the standard deviations of both firms enter in the covariance, smaller covariances will also have a higher weight in value-weighted portfolios. Thus, for portfolios with many stocks (the “average-covariance-term” dominates) the total portfolio variance will also be smaller for value-weighted portfolios.

d) The table with statistics is below:

| N | 5 | 10 | 25 | 50 |
|---|---|---|---|---|
| t statistic | 1.4280 | 1.9433 | 2.6859 | 2.7475 |
| 2-tailed p-value | 0.1550 | 0.0535 | 0.0079 | 0.0066 |

These test statistics follow a t distribution with 179 degrees of freedom. These t-statistics show that at the 95 percent confidence level (using a two tailed hypothesis test), the hypothesis that the mean of the portfolio is equal to zero, can be rejected for portfolios of 25 and 50 stocks.

e) The sample statistics are reported in the table below for both CTL, and the equal weighted portfolio.

| | CTL | 50-stocks | Market |
|---|---|---|---|
| Max | 41.2844 | 22.2131 | 11.3982 |
| Min | -34.0067 | -25.3390 | -18.4603 |
| Range | 75.2911 | 47.5521 | 29.8585 |
| Stdev | 8.0931 | 5.6670 | 4.6089 |
| Stu Range | 9.3031 | 8.3910 | 6.4785 |
| Skewness | -0.1438 | -0.3927 | -0.6633 |
| Kurtosis | 8.7292 | 6.0732 | 4.0787 |
| JB p-value | 0.0010 | 0.0010 | 0.0021 |

For a Normal Distribution, a skewness of 0 and a kurtosis of 3 is expected. The studentized range upper critical value for 180 observations at the 5% level is between 6.39 (T=150) and 6.59 (T=200) [Fama table 1.9]. Thus, using both the studentized range, and the Jarques-Berra ( L ) statistic, it does not appear that either the monthly returns of CTL, the Equal Weight Portfolio or the market follow a Normal Distribution.

f)

1. The table of slopes, intercepts, and $R^2$ of the regressions for all ten of the stocks is below:

| Stocks | CTL | T | CSCO | FCX | XL |
|---|---|---|---|---|---|
| R-squared | 0.171 | 0.127 | 0.437 | 0.308 | 0.272 |
| Alpha | 0.188 | 0.163 | -0.474 | 0.796 | 0.094 |
| Beta | 0.727 | 0.529 | 1.531 | 1.570 | 1.439 |
| Beta, no int | 0.731 | 0.533 | 1.520 | 1.588 | 1.441 |

| Stocks | IVZ | AMT | WHR | IR | WFT |
|---|---|---|---|---|---|
| R-squared | 0.645 | 0.125 | 0.339 | 0.435 | 0.308 |
| Alpha | 0.192 | 1.160 | 0.745 | 0.526 | 0.335 |
| Beta | 2.036 | 1.352 | 1.442 | 1.461 | 1.514 |
| Beta, no int | 2.041 | 1.379 | 1.459 | 1.473 | 1.521 |

The slope coefficients (beta) in these regressions are equivalent to covariances divided by variances. Specifically, for instance, for CTL,

$$\beta_{CNE,M} = \frac{Cov(R_{CNE}, R_M)}{Var(R_M)}$$

The interpretation of this slope coefficient is for a one unit movement in the market return (1 percent), the return of CTL is expected to increase by 0.727 percent.

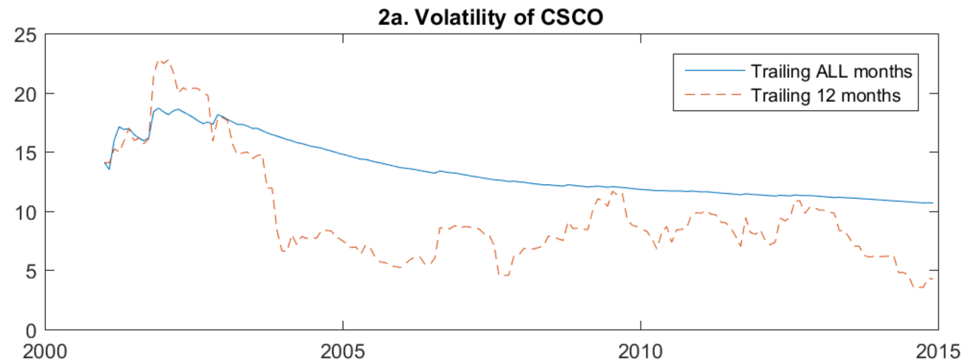
2. The interpretation of the intercepts (alpha) in these regressions, with respect to the market return, is that, for instance for CTL, when the market has zero percent return for a given month or if we take away the effect of the market return, the expected return of CTL for that same month is 0.188 percent.

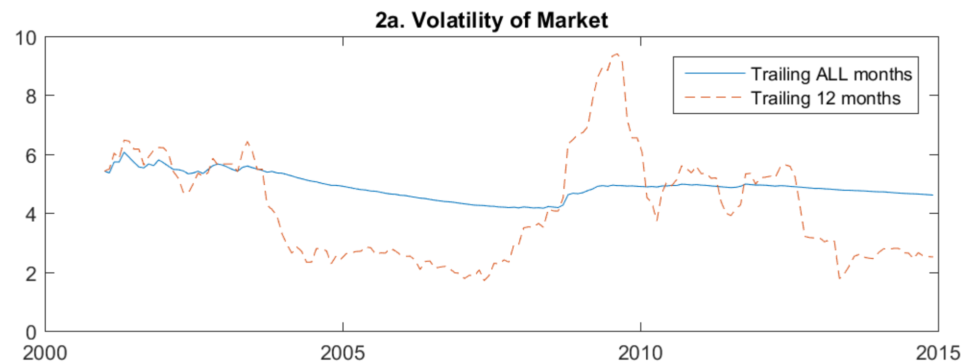
3. The $R^2$ is a measure of the amount of variation of the dependent variable explained by the independent variables, $R^2 = 1 - \frac{e'e}{y'y - n\bar{y}^2}$. Thus, for instance, for CTL, the amount of variation in the monthly return of CTL that is explained by the variation in the monthly market return. The relatively low $R^2$ for most of these regressions indicates that there is a

large amount of variation in the monthly return of these securities that is not explained by the variation in the monthly return of the market.

Part 2:

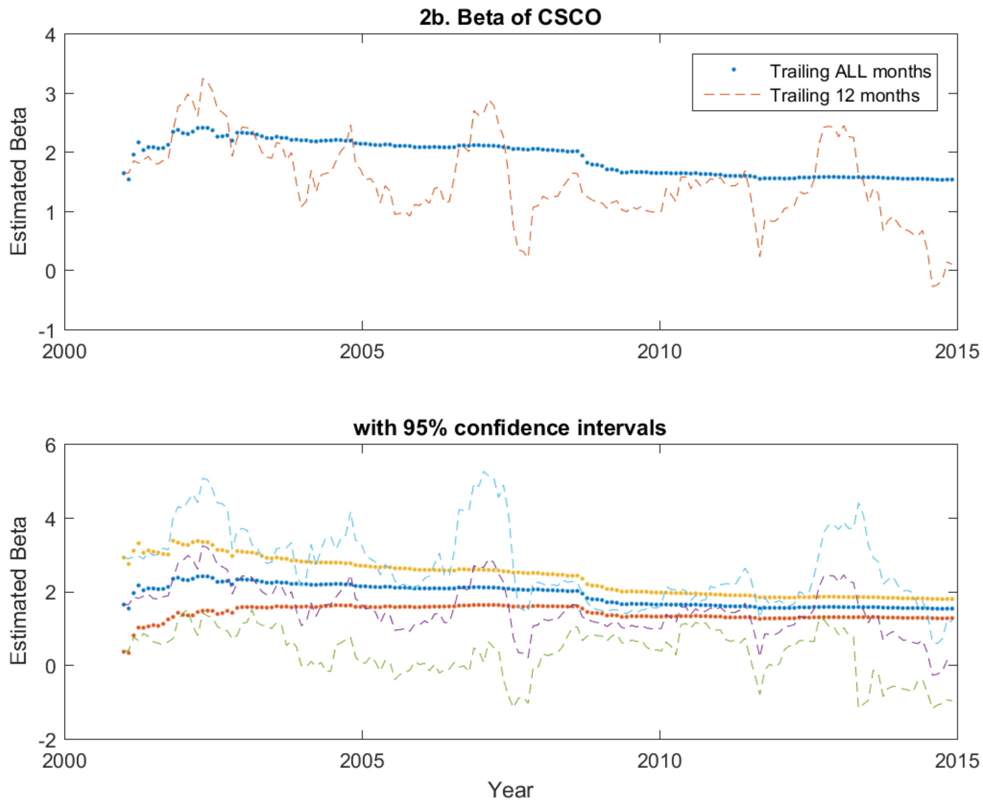
a) Here is a plot of the estimate of the standard deviations of returns for CSCO and the Market using all past data, and only data from the past year.




The estimate of standard deviation of returns is more volatile using only a year's data for two reasons. The main reason is that we believe volatility varies over time—it is not constant—so when we measure over a year long duration, we correctly perceive some of these changes in volatility (in other words, using all data will reduce our ability to detect changes in volatility). This is why both the volatility of CSCO and the market seem to move together—because there are market-wide variations in volatility. A second minor reason for some of the variation is that there is more noise when using less data, so confidence intervals will generally be wider for the estimates with only one year of data.

b) Here is a plot of the estimated Beta for CSCO:




This is consistent with the betas moving through time, but it is also consistent with just lots of noise. If we look at the confidence intervals for the beta we find that with just a few exceptions, the full-time-series beta is within the confidence interval of the annual betas, which would be consistent with noise driving much (but not all) of the results.