Mathematical modeling of thermodynamic effects in well bore zone of gas formation under hydraulic fracturing conditions

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Abstract

With the use of ideas of quasiconformal mapping and procedure of gradual recording of medium and process characteristic the methodology of mathematical modeling of filtration and mass-exchanging processes in well bore zone, upon conditions of occurrence of gas formation hydraulic fractures has been suggested. The influence of thermodynamic effects, caused by gas throttling through microfractures has been taken into consideration, when filtration process is described by specially modified Darcy's law with critical pressure gradient. Numerical algorithm to solve correspondent boundary tasks, build flow pattern, temperature field, filtration properties etc. has been developed.

Key words: *Joule–Thompson effect, non-linear filtration, numerical methods, quasiconformal mapping, reservoir hydraulic fracturing, shale sedimentary rock.*

Current conditions of insufficient resource potential and low level of own natural gas production in Ukraine demand effective development and further development of gas fields in complex geological conditions with low permeability reservoirs as well as in shale sedimentary rocks. Structural features of shale rocks, low permeability and conditions of natural gas occurrence demand considering of regularities of gas drainage from the systems of fractures and microfractures to wells. This process, as a result of conversion of work, performed by gas while transition from fractures to microfractures and vice versa is accompanied by heat emission and causes unbalanced gas expansion. Apart from thermodynamic effects, caused by such movement, it is reasonable also to consider non-linear effects in reservoir well bore zones, caused by overbalance of pressure gradient of its certain critical value [1]. Investigation of the above mentioned effects upon condition of reservoir hydraulic fractures existence is of significant interest for the theory and practice of production of gas fields with weakly structured and low permeability sedimentary rocks.

The article summarizes the methodology, suggested in [1] on the basis of application of concepts of quasiconformal mapping numerical method [2] and procedure of gradual recording of medium and process characteristics. This methodology comprises mathematical modeling of filtration and mass-exchanging processes in well bore zone, considering thermodynamic effects in case of existence of reservoir hydraulic fractures when the process of displacement is described by specially modified Darcy's law with critical gradient.

Let's have a look at model task for research of influence of fractures of reservoir hydraulic fracturing upon thermodynamic characteristics of wellbore zone G_z of reservoir (Figure 1) with considering of gas throttling through shale rock microfractures. For mathematical formulation of task of gas filtration through porous medium let's write down the continuity equation in the form of [1– 3]

$$div\left(\frac{\rho(p,T)k(x,y)\chi(I,I_{kr})}{\mu(p,T)}\,\boldsymbol{grad}\ p\right)=0\,, \qquad (1)$$

under appropriate conditions on reservoir boundaries $p|_{L_*}=p_*$, $p|_{L^*}=p^*$ ($p_*>p^*$) and conditions of flow continuity and pressure on reservoir hydraulic fractures boundaries.

Here

$$I=I(x,y)=|\,\boldsymbol{grad}\ p(x,y)\,|=\sqrt{(\partial p/\partial x)^2+(\partial p/\partial y)^2}$$

is the value of pressure gradient; $L_*=\{z:\ f_*(x,y)=0\}$, $L^*=\{z:\ f^*(x,y)=0\}$ are the external boundary and well boundary correspondingly; $\rho=\rho(p,T)$, $\mu=\mu(p,T)$ are the gas density and viscosity; T is the temperature; $k(x,y)$ is the coefficient of absolute permeability of medium

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$$k(x,y)=\begin{cases}k_\alpha, \ (x,y)\in D_\alpha,\\ k_*, \ (x,y)\in G_z\setminus\bigcup\limits_\alpha D_\alpha;\end{cases}$$

D_α is the reservoir compartment that corresponds to α – fracture with permeability k_α ($\alpha\in N$). Coefficient χ characterizes the dependence of sedimentary rock permeability upon pressure gradient value in complex geological filtration conditions (for which k/μ is a small quantity) and is determined by the following

$$\chi(I,I_{cr})=\begin{cases}1+F(I-I_{cr}), & \text{if } I>I_{cr};\\ 1, & \text{if } I\le I_{cr},\end{cases} \quad (2)$$

where F is the specified steadily increasing function; I_{cr} is the critical value of initial gradient.

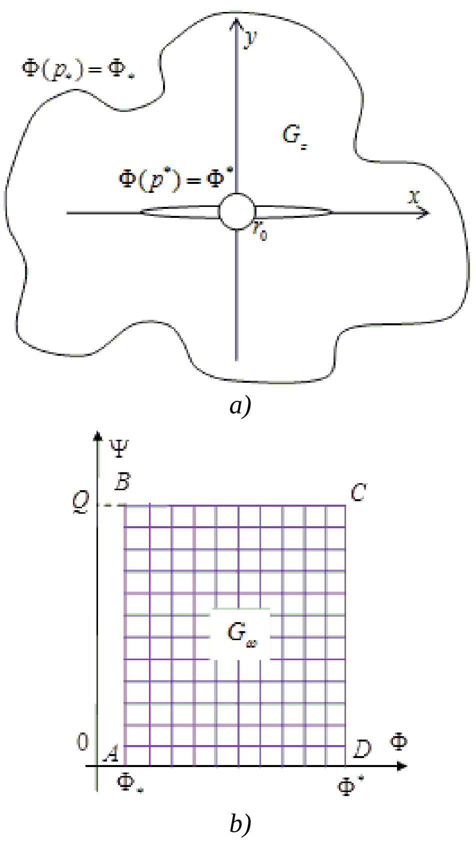


Figure 1 – Schematic layout of reservoir with hydraulic fracture (*a*) and correspondent complex potential area (*b*)

Similarly [1, 4], in disregard for heat conduction and adiabatic effect, temperature field is found as the solution of the following differential equation

$$\frac{\partial T(x,y,t)}{\partial t}+\boldsymbol{u}(x,y)\left(\boldsymbol{grad}\ T(x,y,t)+\varepsilon\ \boldsymbol{grad}\ p(x,y)\right)=0 \quad (3)$$

under correspondent initial $T(x,y,0)=T_0(x,y)$ and critical $T(x,y,t)\big|_{L_*}=T_*$ conditions. Here ε is the Joule–Thompson coefficient; $\boldsymbol{u}(x,y,t)=-\dfrac{k\chi(I,I_{cr})\rho c}{\mu c_f}\boldsymbol{grad}\ p$ is the reservoir heat convection velocity; c, c_f are the specific heat capacity of gas and correspondingly reservoir, saturated with gas. It must be mentioned that all the values, introduced in the article are measured in SI system units.

To solve the task velocity potential must be entered in the form of Laybenson function [5]

$$\Phi(p)=\Phi_*+\int\limits_p^{p_*}\frac{\rho(\tilde p,T)}{\mu(\tilde p,T)}d\tilde p$$

and task must be rewritten in accordance with it (1), taking into account (2):

$$\begin{cases}div\left(k(x,y)\chi(\tilde I,I_{cr})\boldsymbol{grad}\ \Phi\right)=0,\\ \boldsymbol{\upsilon}=\dfrac{k(x,y)\chi(\tilde I,I_{cr})}{\tilde\rho(\Phi)}\boldsymbol{grad}\ \Phi,\\ \Phi\big|_{L_*}=\Phi(p_*)=\Phi_*,\ \Phi\big|_{L^*}=\Phi(p^*)=\Phi^*,\\ [\Phi]\big|_{\partial D_\alpha}=0,\ [\upsilon_n]\big|_{\partial D_\alpha}=0,\end{cases} \quad (4)$$

where $\tilde\rho(\Phi)=\rho(p(\Phi))$; $\tilde I=\dfrac{\mu}{\tilde\rho(\Phi)}\sqrt{\Phi_x^2+\Phi_y^2}$; $\upsilon(x,y)$ is the filtration velocity vector; $\Phi_*<\Phi^*$; $[\cdot]\big|_{\partial D_\alpha}$ are the correspondent function jump to ∂D_α.

In this case, heat convection task (3) will be as follows:

$$\begin{cases}\dfrac{\partial T(x,y,t)}{\partial t}+\\ +\boldsymbol{u}(x,y)\left(\boldsymbol{grad}\ T(x,y,t)-\dfrac{\varepsilon\mu}{\tilde\rho(\Phi)}\boldsymbol{grad}\ \Phi(x,y)\right)=0,\\ T(x,y,0)=T_0(x,y),\ T(x,y,t)\big|_{L_*}=T_*,\end{cases} \quad (5)$$

where $\boldsymbol{u}(x,y)=k(x,y)\chi(\tilde I,I_{cr})\dfrac{c}{c_f}\boldsymbol{grad}\ \Phi(x,y)$.

To solve formulated task (4) and (5) we can use numerical methods of complex analysis, in order to do this, by analogy with [2], let's input flow function Ψ, complex conjugate with Φ and carry out conditional cross section L of area G_z along flow line [2, 3]. Meanwhile, task to build flow pattern, find seepage discharge and other specific filtration parameters on the basis of the found (fixed at a certain moment t) temperature field, accounting for (5), is narrowed down to quasiconformal mapping task $\omega=\omega(z)=$ $=\Phi(x,y)+\mathrm{i}\Psi(x,y)$ of area $G_z\setminus L$ for correspondent complex quasipotential area G_ω:

$$\begin{cases}\dfrac{k(x,y)\chi(\tilde I,I_{kr})}{\tilde\rho(\Phi)}\dfrac{\partial\Phi}{\partial x}=\dfrac{\partial\Psi}{\partial y},\\ \dfrac{k(x,y)\chi(\tilde I,I_{kr})}{\tilde\rho(\Phi)}\dfrac{\partial\Phi}{\partial y}=-\dfrac{\partial\Psi}{\partial x},\\ \Phi\big|_{L_*}=\Phi_*,\ \Phi\big|_{L^*}=\Phi^*,\ [\Phi]\big|_{\partial D_\lambda}=0,\\ [\upsilon_n]\big|_{\partial D_\lambda}=0,\ \Psi\big|_{L_-}=0,\ \Psi\big|_{L_+}=Q;\end{cases} \quad (6)$$

where L_-, L_+ are the lower and upper banks of L; $G_\omega = \{\omega : \Phi_* < \Phi < \Phi^*,\ 0 < \Psi < Q\}$; $Q = \oint_{L_*} -\upsilon_y dx + \upsilon_x dy$ are the unknown well production rate.

Reciprocal to (6) boundary task for quasiconformal mapping $z = z(\omega) = x(\Phi,\Psi) + iy(\Phi,\Psi)$ of G_ω area on $G_z \setminus L$ R, and, as a result, equation for real $x = x(\Phi,\Psi)$ and imaginary $y = y(\Phi,\Psi)$ part of characteristic flow function can be written as:

$$\frac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\frac{\partial y}{\partial \Psi} = \frac{\partial x}{\partial \Phi},\ \frac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\frac{\partial x}{\partial \Psi} = -\frac{\partial y}{\partial \Phi}, \quad (7)$$

$$f_*(x(\Phi_*,\Psi), y(\Phi_*,\Psi)) = 0,$$

$$f^*(x(\Phi^*,\Psi), y(\Phi^*,\Psi)) = 0,\ 0 \le \Psi \le Q,$$

$$x(\Phi,0) = x(\Phi,Q),\ y(\Phi,0) = y(\Phi,Q),\ \Phi_* \le \Phi \le \Phi^*,$$

$$\left[\frac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi) J}\sqrt{\left(\frac{\partial y}{\partial \Psi}\right)^2 + \left(\frac{\partial x}{\partial \Psi}\right)^2}\cos(\boldsymbol{\upsilon}, \boldsymbol{n})\right]\Bigg|_{\partial D_\alpha} = 0,$$

$$\left[x(\Phi,\Psi)\right]\Big|_{\partial D_\alpha} = \left[y(\Phi,\Psi)\right]\Big|_{\partial D_\alpha} = 0,$$

$$\begin{cases} \dfrac{\partial}{\partial \Psi}\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\dfrac{\partial x}{\partial \Psi}\right) + \dfrac{\partial}{\partial \Phi}\left(\dfrac{\tilde{\rho}(\Phi)}{k\chi(\tilde{I}, I_{cr})}\dfrac{\partial x}{\partial \Phi}\right) = 0, \\ \dfrac{\partial}{\partial \Psi}\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\dfrac{\partial y}{\partial \Psi}\right) + \dfrac{\partial}{\partial \Phi}\left(\dfrac{\tilde{\rho}(\Phi)}{k\chi(\tilde{I}, I_{cr})}\dfrac{\partial y}{\partial \Phi}\right) = 0. \end{cases} \quad (8)$$

Having used correspondent formulas of transition [2], condition (7) and formulas for calculating of bulk velocity components, temperature task (5) is rewritten as:

$$\frac{\partial T}{\partial t} + \frac{c\left(\tilde{\rho}(\Phi)\boldsymbol{\upsilon}\right)^2}{k\chi(\tilde{I}, I_{cr})c_f}\frac{\partial T}{\partial \Phi} = \frac{c\mu\tilde{\rho}(\Phi)\upsilon^2}{k\chi(\tilde{I}, I_{cr})c_f}, \quad (9)$$

$$T(x(\Phi_*,\Psi), y(\Phi_*,\Psi), t) = T_*,\ 0 \le \Psi \le Q,$$

$$T(x(\Phi,\Psi), y(\Phi,\Psi), 0) = T_0(x(\Phi,\Psi), y(\Phi,\Psi)),$$

$$\Phi_* \le \Phi \le \Phi^*,\ 0 \le \Psi \le Q,$$

where equation (9) is as a matter of fact single-dimensional because variable ψ here functions as a parameter.

To build difference analogue of task let's input in G_ω area regular orthogonal grid with grid points (Φ_i, Ψ_j):

$$\Phi_i = \Phi_* + i\,\Delta\Phi,\ \Psi_j = j\,\Delta\Psi,$$

where $\Delta\Phi = (\Phi^* - \Phi_*)/n$; $\Delta\Psi = Q/m$, $i = \overline{1,n}$, $j = \overline{1,m}$, $n, m \in N$. Equation (8) is approximated, using method of finite volumes [2] as follows:

$$\begin{cases} x_{i,j} = (a_n x_{i,j+1} + a_s x_{i,j-1} + a_e x_{i-1,j} + a_s x_{i+1,j})/a_p, \\ y_{i,j} = (a_n y_{i,j+1} + a_s y_{i,j-1} + a_e y_{i-1,j} + a_s y_{i+1,j})/a_p, \end{cases} \quad (10)$$

where $x_{i,j} = x(\Phi_i, \Psi_j)$, $y_{i,j} = y(\Phi_i, \Psi_j)$,

$$a_p = a_n + a_s + a_e + a_w,$$

$$a_n = \frac{2\Delta\Phi\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j}\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j+1}}{\left(\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j} + \left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j+1}\right)\Delta\Psi},$$

$$a_s = \frac{2\Delta\Phi\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j}\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j-1}}{\left(\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j} + \left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j-1}\right)\Delta\Psi},$$

$$a_e = \frac{2\Delta\Phi\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j}\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i-1,j}}{\left(\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j} + \left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i-1,j}\right)\Delta\Psi},$$

$$a_w = \frac{2\Delta\Phi\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j}\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i+1,j}}{\left(\left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i,j} + \left(\dfrac{k\chi(\tilde{I}, I_{cr})}{\tilde{\rho}(\Phi)}\right)_{i+1,j}\right)\Delta\Psi}.$$

Approximations of boundary conditions can be written as:

$$\begin{cases} f_*(x_{0,j}, y_{0,j}) = 0,\ f^*(x_{n,j}, y_{n,j}) = 0,\ j = \overline{0,m}, \\ x_{i,0} = x_{i,m},\ y_{i,0} = y_{i,m},\ i = \overline{0,n}. \end{cases} \quad (11)$$

Here, as well as in [2, 3], complex contingency of functions $x_{i,j} = x(\Phi_i, \Psi_j)$, $y_{i,j} = y(\Phi_i, \Psi_j)$ is ensured by conditions of orthogonality of near-border normal vectors to correspondent tangent lines along boundary of $G_z \setminus L$ area. Their difference analogues are as follows:

$$\begin{cases} (3x_{n,j} + x_{n-2,j} - 4x_{n-1,j})(x_{n,j+1} - x_{n,j-1}) + \\ \quad + (3y_{n,j} + y_{n-2,j} - 4y_{n-1,j})(y_{n,j+1} - y_{n,j-1}) = 0, \\ (4x_{1,j} - 3x_{0,j} - x_{2,j})(x_{0,j+1} - x_{0,j-1}) + \\ \quad + (4y_{1,j} - 3y_{0,j} - y_{2,j})(y_{0,j+1} - y_{0,j-1}) = 0,\ j = \overline{0,m}. \end{cases}$$

Unknown well production rate Q is being searched for in the process of iterations according to the formula $Q = m\Delta\Psi$, where $\Delta\Psi = \dfrac{\Delta\Phi}{\gamma}$, and γ is obtained on the basis of condition of «quasiconformal similarity in the small» of correspondent elementary rectangular of the two areas:

$$\gamma = \frac{1}{mn}\sum_{i=0,\,j=0}^{n-1,\,m-1}\left(\frac{a_{i,j} + a_{i,j+1}}{b_{i,j} + b_{i+1,j}}\right)\left(\frac{\tilde{\rho}(\Phi)}{k\chi(\tilde{I}, I_{cr})}\right)_{i+1/2,\,j+1/2}, \quad (12)$$

where $a_{i,j} = \sqrt{\left(x_{i+1,j} - x_{i,j}\right)^2 + \left(y_{i+1,j} - y_{i,j}\right)^2}$,

$b_{i,j} = \sqrt{\left(x_{i,j+1} - x_{i,j}\right)^2 + \left(y_{i,j+1} - y_{i,j}\right)^2}$.

Equation (9) is approximated with the help of difference scheme «against the flow» [2] in the following way:

$$\hat{T}_{i,j} = T_{i,j} - \left(\frac{c\left(\tilde{\rho}(\Phi)\boldsymbol{\upsilon}\right)^2}{k\chi(\tilde{I}, I_{cr})c_f} \right)_{i,j} \frac{\left(T_{i,j} - T_{i-1,j}\right)}{\Delta\Phi} + \left(\frac{c\mu\tilde{\rho}(\Phi)\upsilon^2}{k\chi(\tilde{I}, I_{cr})c_f} \right)_{i,j}, \tag{13}$$

where $j = \overline{1,m}$, $i = \overline{1,n}$; τ is the time step; $T_{i,j}$, $\hat{T}_{i,j}$ are temperature at the correspondent periods of time; $\upsilon_{i,j}$ is the velocity (it can be obtained as in [2]). Critical and initial conditions for temperature in grid area can be written as:

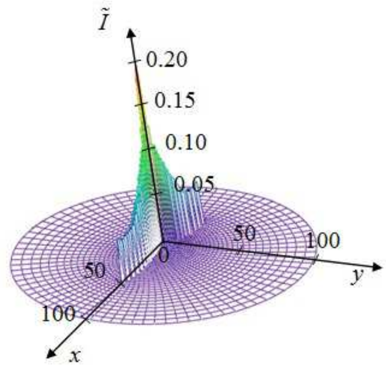
$$T_{0,j} = T_*, \; j = \overline{1,m};$$

$$T(x_{i,j}, y_{i,j}, 0) = T_0(x_{i,j}, y_{i,j}), \; i = \overline{1,n}, \; j = \overline{1,m}.$$

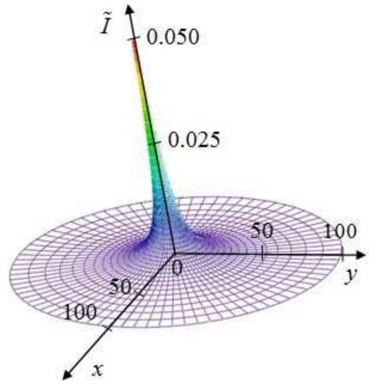
Having assigned step τ, splitting parameters n, m field G_ω (they are sorted out as in [2, 3]) and algorithm accuracy ε_1, ε_2, initial approximations of coordinates of boundary points $x_{i,j}^{(0)}$, $y_{i,j}^{(0)}$ (to comply with the conditions of (11)) and initial approximations of coordinates of internal points ($x_{i,j}^{(0)}, y_{i,j}^{(0)}$) can be found in accordance with the formulas (12) of γ value approximation. Hereafter the coordinates of internal points of flow pattern are clarified by solving (10) with respect to $x_{i,j}$ and $y_{i,j}$. After that as well as in [2], boundary points are readjusted under conditions of fixation of external boundary and near-boundary ones, using the conditions of orthogonality and approximated values of Q, $\tilde{I}$, $\chi(\tilde{I}, I_{cr})$ are obtained. To complete flow pattern building algorithm (apart from tracing of unknown filtration parameters, velocity field in particular) expenses Q ($\left|Q^{(\kappa+1)} - Q^{(\kappa)}\right| < \varepsilon_1$), boundary points ($\max\limits_{i,j} \sqrt{(x_{i,j}^{(\kappa)} - x_{i,j}^{(\kappa-1)})^2 + (y_{i,j}^{(\kappa)} - y_{i,j}^{(\kappa-1)})^2} < \varepsilon_2$) etc. must be stabilized at the given iteration stage. If any of these conditions isn't fulfilled, violation of quasiconformality must be indicated on the dynamic grid of the area.

Using built velocity field $\upsilon_{i,j}$ and temperature field $T_{i,j}$ from the previous iteration step (accounting for boundary conditions) we find temperature distribution in the formation at the given time stage in accordance with (13), after that velocity and potential fields are recalculated.

The described algorithm of numerical solution of the assigned task is realized in the form of software application for IBM PC/AT. Fig. 2 shows flow pattern with $\Phi = idem$ and $\Psi = idem$ in $x0y$ and potential gradient distribution in a radial formation with a hydraulic fracture (*a*) under condition that fracture is simulated by field $D_1 = \left\{(x,y): \; x^2/50 + y^2/0.2 \le 1\right\}$ with permeability coefficient of $k_1 = 1 \cdot 10^{-14}\,\text{m}^2$, and without it (*b*) if input data are from [1]. If reservoir hydraulic fracturing is not taken into consideration, then obtained numerical results comply with results of [1].



a)



b)

Figure 2 – Potential gradient distribution in radial formation with a hydraulic fracture (*a*) and without it (*b*)

Thus, using the ideas of numerical method of quasiconformal mapping and procedure of gradual recording of medium and process characteristics, the methodology of mathematical modeling of filtration and mass-exchanging processes in well bore zone, upon conditions of occurrence of hydraulic fractures and consideration of influence of thermodynamic effects, caused by gas throttling through shale rock microfractures when filtration process is described by specially modified Darcy's law with critical pressure gradient has been suggested. Numerical algorithm to solve correspondent boundary tasks, build flow pattern, temperature field, filtration properties etc. has been developed.

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Математичне моделювання термодинамічних ефектів у присвердловинній зоні газового пласта за умов гідравлічного розриву

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З використанням ідей квазіконформного відображення та процедури поетапної фіксації характеристик середовища і процесу запропоновано методологію математичного моделювання фільтраційно-масообмінних процесів у присвердловинній зоні за умов існування в ній тріщин гідравлічного розриву газового пласта. Враховано вплив термодинамічних ефектів, що виникають внаслідок дроселювання газу через мікротріщини в породі, коли процес фільтрації описується спеціальним чином модифікованим законом Дарсі з критичним градієнтом тиску. Розроблено числовий алгоритм розв’язування відповідних крайових задач на побудову гідродинамічної сітки, температурного поля, фільтраційних характеристик тощо.

Ключові слова: *гідравлічний розрив пласта, ефект Джоуля–Томсона, квазіконформне відображення, нелінійна фільтрація, сланцева осадова порода, числові методи.*