PROOFS AND LOGIC

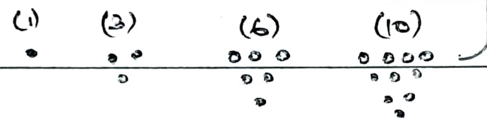
INTRODUCTION

Connection between things that are apparently not connected,

counting → no. of pairs of students

bowling alley → pin counting

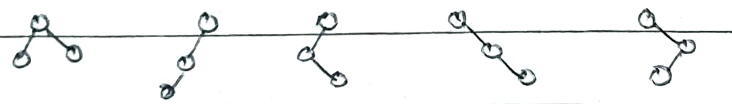
similar pattern



how many?

looks like a bowling alley problem.

q. how many binary trees possible with 3 nodes?



(5)

q. construct minimum spanning tree from graph. how many?

order of n^4

q. linear algebra:

multiplying 3 matrices $A_1 A_2 A_3$.

hard to see but they are related

$(A_1 A_2) A_3$ $\quad$ $A_1 (A_2 A_3)$ $\quad$ (2)

how many 2 node binary trees can you construct

2 $\longleftrightarrow$ 2 node BT^3

5 $\longleftrightarrow$ 3 node BT^3

$\longleftrightarrow$ 4 node BT^3

PROOFS

thm $\sqrt{2}$ is irrational.

(aristotle) - by contradiction.

assume on the contrary, $\sqrt{2}$ is rational

rational no. - ratio of 2 natural numbers

$\sqrt{2} = \frac{a}{b}$ $\quad$ $a, b \in$ integers, $b \neq 0$

no common factors for a, b

$2 = \frac{a^2}{b^2}$ $\quad$ $a \to b$ / $\sim b \to \sim a$

lemma: helper theorm

$a^2 = 2b^2$ $\quad$ if n^2 is even, then n is even

(using contrapositive argument)

(proof by contrapositive)

a^2 is even
$\rightarrow a$ is even

let $a = 2k$, some integer k

$(2k)^2 = 2b^2$

$4k^2 = 2b^2$

$b^2 = 2k^2$

b^2 is even
$\rightarrow b$ is even

but, a & b have no common factors <contradiction>

$\rightarrow \sqrt{2}$ is irrational (by mr. aristotle).

NUMBER OF PRIMES ARE INFINITE

(mr. euclid $\rightarrow$ founder of western mathematics)
(euclid's elements)

assume no. of primes are finite.
2, 3, 5, ..., 31

$(2 \times 3 \times 5 \times \ldots 31) + 1$ —— prime

or has factors other than given

is this divisible by any prime no.?

inductive learning from examples → related to recursion recurrence relation

n
$+ (n-1)$
$+ (n-2)$
$+ \vdots$
1

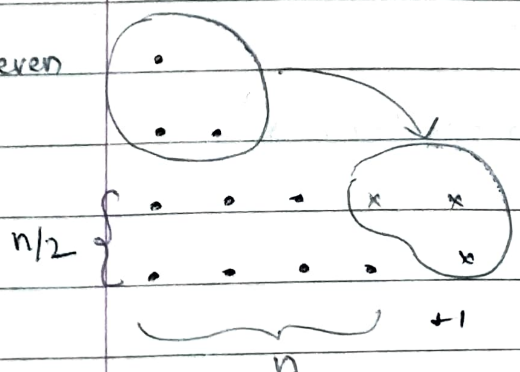
possible handshakes
possible pairs

TRIANGLE NUMBER

T_1	1	.
T_2	3	∴
T_3	6	
T_4	10	

GEOMETRIC METHOD

n = even

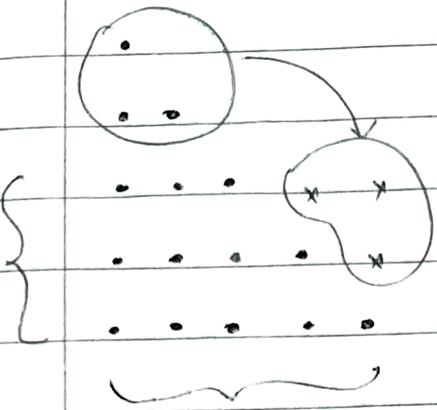


dots = $(n+1)\left(\frac{n}{2}\right)$

n=odd

dots = $(n)\left(\frac{n+1}{2}\right)$

$\frac{n+1}{2}$



n

$T_n = 1 + 2 + 3 + \ldots n = \frac{n(n+1)}{2}$

GAUSS

1 + 2 + 3 + . . . 98 + 99 + 100

101

101

101

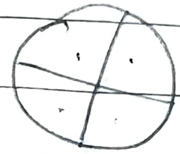
$101 \times \frac{100}{2}$

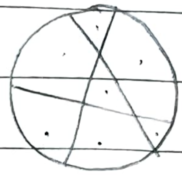
sum   pairs

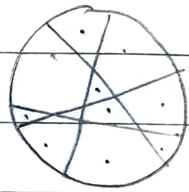
class teacher problem — 5 min

CUTTING PLANES

cuts	no. of pieces	
0	1	
		1
1	2	
		2
2	4	
		3
3	7	
		4
4	11	

2  (4)

3  (7)

4  (11)

$P_n = P_{n-1} + n$ (observation)

prove $P_n = T_n + 1$

proof by induction

base case $n=1$: $P_1 = T_1 + 1 = 2$ ✓

inductive assumption:

suppose $P_K = T_K + 1$ is true

need to prove $P_{K+1} = T_{K+1} + 1$

$P_{K+1} = P_K + (K+1)$ from observation

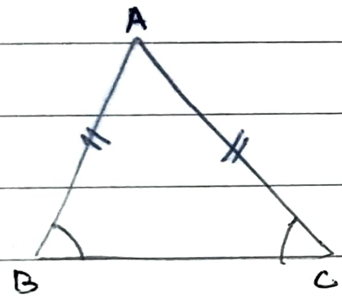
$P_{K+1} = (T_K + 1) + (K+1)$ from assumption

$P_{k+1} = (T_k + (k+1)) + 1$

$P_{k+1} = T_{k+1} + 1$ ✓ from definition of T_n.

LOGIC

foundation for designing circuits
theorm provers (automatic)



ΔABC and ΔACB are congruent
$\therefore$ angles are same

AI also customer for logic.

VARIABLES AND CONNECTIVES

$R \vee W$	OR	$+$	binary
$R \wedge W$	AND	$\cdot$	binary
$\neg R$	NOT	$-$	unary

precedence order
- unary
- and
- or

$R \rightarrow W$ implication / conditional
$R \leftrightarrow W$ biconditional

COMPLETE SET

minimal set of connectives that will generate all the rest.

TRUTH TABLE

A	B	A ∨ B	A ∧ B	A → B
0	0	0	0	
0	1	1	0	1
1	0	1	0	0
1	1	1	1	1

applications in circuits (3-sat algo).

ex. list out all possible functions between 2 variables. give names

0	and	¬(A→B)	A	¬(B→A)	B	xor	or
0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1
0	0	1	1	0	0	1	1
0	1	0	1	0	1	0	1

2^n

↓

2^{2^n}

nor	xnor	¬B	B→A	¬A	A→B	nand	1
1	1	1	1	1	1	1	1
0	0	0	0	1	1	1	1
0	0	1	1	0	0	1	1
0	1	0	1	0	1	0	1

∴ 2^{2n} functions

LOGIC REDUCTION

$(R \vee W) \wedge \overline{(R \wedge W)} = (\bar{R} \wedge W) \vee (R \wedge \bar{W})$ (xor)

and-form

$(R \vee W) \wedge (\bar{R} \vee \bar{W})$

→ or-form

TRUTH TABLE

tautology } true

falasy
contradiction } false

LOGICAL EQUIVALENCE

$R \rightarrow W \equiv \neg R \vee W$

$\neg\neg R \equiv R$

$(R \vee W) \wedge S \equiv (R \wedge S) \vee (W \wedge S)$

$(R \wedge W) \vee S \equiv (R \vee S) \wedge (W \vee S)$

distributive laws

$\neg(A \vee B) \equiv \neg A \wedge \neg B$

$\neg(A \wedge B) \equiv \neg A \vee \neg B$

de morgan's laws

LHS = $(R \vee W) \wedge \overline{(R \wedge W)}$

$= \left(R \wedge \overline{(R \wedge W)}\right) \vee \left(W \wedge \overline{(R \wedge W)}\right)$ using distributive law

$= \left(R \wedge (\bar{R} \vee \bar{W})\right) \vee \left(W \wedge (\bar{R} \vee \bar{W})\right)$ using dermorgans law

$= (R \wedge \bar{W}) \vee (\bar{R} \wedge W)$ using commutative law

RHS = $(\bar{R} \wedge W) \vee (R \wedge \bar{W})$ ✓

COMPLETE SET OF CONNECTIVES, OPERATORS