Discrete Mathematics and Algorithms (CSE611)

Assignment-2: Relations, Functions, Cardinality, Computable Functions, Rates of Growth

Total Marks: 100

Deadline: September 03, 2019 (Tuesday), 5:00 pm

Submission Instructions:

Note: If found copying both, the copier and from whom it was copied, will be given ZERO!!

Please start each question at the top of a page.

Please submit the assignment in hard copy stating the following at the top:

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Submitted by XYZ, Roll No.

Submitted on

General Note: *Relations* is assigned as *self-study* assignment. Please look at slides for Relations of Prof. A. K. Das as additional resource on the portal.

Q1. Let the relation R be defined by ${}_{(a,b)}R_{(c,d)}$ if and only if $ad = bc, \forall a, b, c, d \in \mathcal{Z}$, where $\mathcal{Z}$ is the set of all integers. Test whether R is an equivalence relation on the set $\mathcal{Z} \times \mathcal{Z}$.

Q2. If R is a relation in the set of integers $\mathcal{Z}$ defined by ${}_xR_y = \{(x, y) \in \mathcal{Z} \times \mathcal{Z} : (x - y)\text{is divisible by } 7\}$, then find all the distinct equivalence classes of the relation R.

Q3. Find the smallest relation containing the relation $R = \{(1, 3), (1, 4), (2, 2), (4, 1)\}$ on $\{1, 2, 3, 4\}$ that is (a). reflexive on $\{1, 2, 3, 4\}$ and symmetric (b). symmetric and transitive (c). reflexive on $\{1, 2, 3, 4\}$ and transitive (d). an equivalence relation on $\{1, 2, 3, 4\}$.

Q4 Prove that power set of natural numbers, $\mathcal{P}(\mathcal{N})$, is not countable using *diagonalization* argument.

Q5 Determine whether or not the following set is countable: the set $A = \{a^2 \mid a \in \mathcal{N}\}$ where $\mathcal{N}$ is the set of natural numbers.

Q6 Show that there are functions that are *not computable*, that is there exist *uncomputable functions*. Hint: Formally complete all the steps we gave in the class, starting from proving that there are countably infinite number of programs in any programming language and finally leading to a diagonal argument for showing that there exist uncomputable functions. See exercises 37, 38, 39 in Ch 2.5 in Rosen 7^{th} Edition.

Q7 (A). Let $f : A \to B$ and $g : B \to C$ be two functions. Prove that if the composite function $g \circ f : A \to C$ is injective, then f is injective.

Q7 (B). Find a function g such that $h = g \circ f$ and $h(x) = 10x + 10$, $f(x) = 2x + 1$, all the functions are defined over the set $\mathcal{R}$ of real numbers, where $g \circ f$ is the composite function.

Q8 A set S is said to be *infinite* if there is a one-to-one correspondence between S and a proper subset of S. Using this definition, prove that the set of real numbers $\mathcal{R}$ is *infinite*.

Q9 You know that if $f(x)$ and $g(x)$ are functions from the $\mathcal{R} \to \mathcal{R}$ (the set of real numbers), then $f(x)$ is $\Theta(g(x))$ if and only if there are positive constants k, C_1, and C_2 such that $C_1 g(x) \leq f(x) \leq C_2 g(x)$ whenever $x > k$. Now show that $3x^2 + x + 1$ is $\Theta(3x^2)$ by directly finding the constants k, C_1, and C_2. Express this Θ relationship using a picture showing the functions $3x^2 + x + 1$, $C_1 \cdot (3x^2)$, and $C_2 \cdot (3x^2)$, and the constant k on the x-axis, where C_1,C_2, and k are the constants found earlier to show that $3x^2 + x + 1$ is $\Theta(3x^2)$.

Q10 (A). Arrange the functions $(1.5)^n$, n^{100}, $(logn)^3$, $\sqrt{n}\, logn$, 10^n,$(n!)^2$, and $n^{99} + n^{98}$ in a list so that each function is $big - O$ of the next function. Give brief justification.

Q10(B). Give a big-O estimate of the product of the first n odd positive integers.