classmate

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RECURRENCE RELATIONS

Chinese 6 ring puzzle

geometric series

tower of hanoi usage.

GEOMETRIC SERIES

$$X = 1 + 2 + 2^2 + \ldots + 2^{n-1}$$

$$2X = 2 + 2^2 + \ldots 2^{n-1} + 2^n$$

$$X = 2^n - 1$$

Zeno's paradox

$$X = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$$

$$\frac{1}{2} X = \frac{1}{4} + \frac{1}{8} + \ldots$$

$$\frac{X}{2} = \frac{1}{2} \Rightarrow X = 1$$

$$X = \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \ldots$$

$$\frac{X}{10} = \frac{1}{10^2} + \frac{1}{10^3} + \ldots$$

$$\frac{9X}{10} = \frac{1}{10}$$

$$X = \frac{1}{9}$$

$$x = \frac{1}{10} + \frac{2}{10^2} + \frac{3}{10^3} + \dots$$

$$\frac{x}{10} = \frac{1}{10^2} + \frac{2}{10^3} + \dots$$

$$\frac{9x}{10} = \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \dots$$

$$\frac{9x}{10} = \frac{1}{9}$$

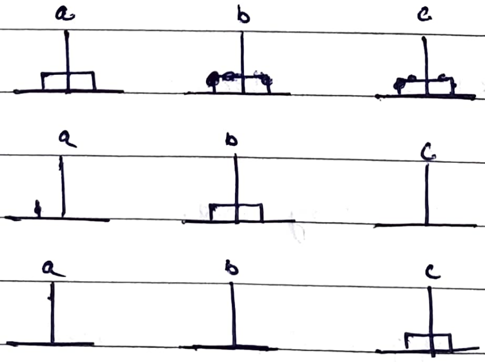
$$x = \frac{10}{81}$$

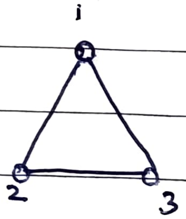
ex $x = \frac{1}{10} + \frac{2^2}{10^2} + \frac{3^2}{10^3} + \frac{4^2}{10^4} + \dots$

try

GRAHPICAL REPRESENTATION OF TOWER OF HANOI

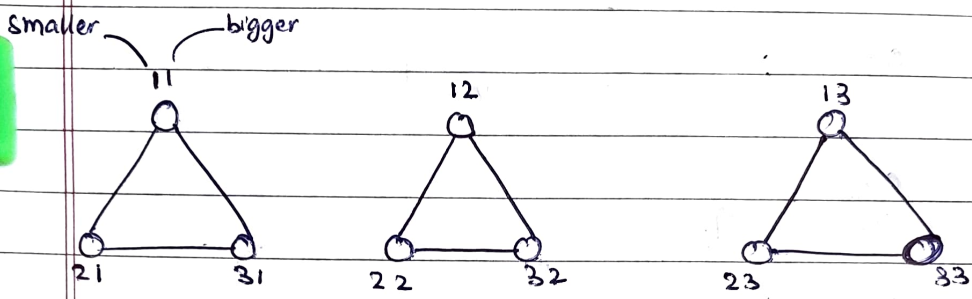
1 disk

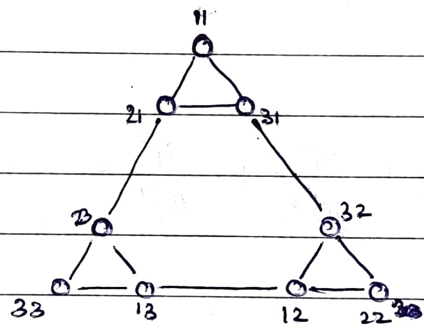




2 disk - 9 possibilities (3) x (3)

n disk - 3^n possibilities.





enumerates all possible moves

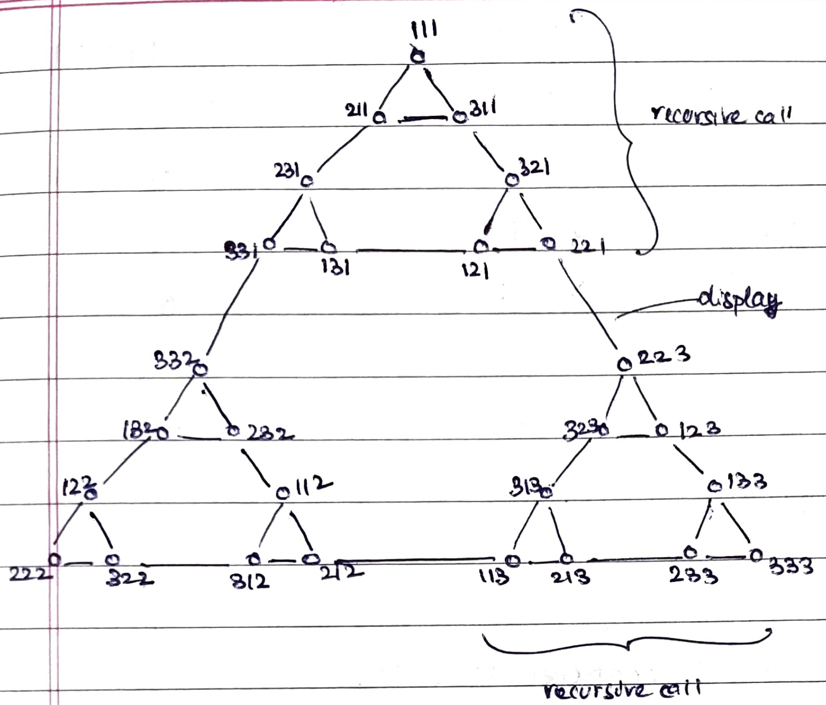
properties of graph.

degree of node > no. of edges coming out of graph

edges (corners) = 2 others = 3

recursive call + display move.

for 3 disk configuration, copy above 3 times.



7 step

optimal soln.

at edges (corners)

2 possibilities

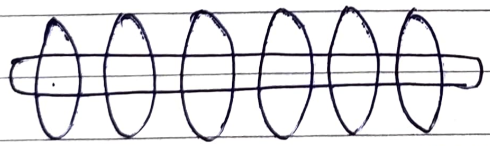
others

3 possibilities

self similar patterns

typically discussed on fractals - design principle.

CR (Chinese ring puzzle 6 rings)



harder ... +2 easier

111 111

all rings on rod,

111101

111111

111 110

RULE

- start at right end
- go until you hit the first "1" bit.
- ring left of this can come off.

101101 → 101111
  ↳ 101100

101100 → 101110
  ↳ 101101

111111
↓
011111
↓
001111
↓
000111
↓
...

RECURSIVE PROCEDURE FOR CR

CR (n)

- get the harder string off
  - CR (n-2) 4 rings taken off
  - take out the hardest rings
  - reverse CR (n-2) - CR (n-1)

RECURRENCE RELATION

$T(n) = 2T(n-2) + T(n-1) + 1$

$T(n-1) = 2T(n-3) + T(n-2) + 1$

$\therefore T(n) = 2T(n-2) + 2T(n-3) + T(n-2) + 2$

$= 3T(n-2) + 2T(n-3) + 2$

× repeated substitution not a good idea

n	T(n)

base case $T(1) = 1$

$T(2) = 2$

11 × 11 ✓

10 01

00 00

111 111

011 111

110000

010000

reverse cn

011111

$T(3) = 2T(1) + T(2) + 1$

$= 2 + 2 + 1 = 5$

$T(4) = 2T(2) + T(3) + 1$

$= 4 + 5 + 1 = 10$

$T(5) = 2T(3) + T(4) + 1$

$= 10 + 10 + 1 = 21$

n	T(n)
1	1
2	2
3	5
4	10
5	21
6	42
7	85

n odd
T(n) =

n even
T(n) =

our guesswork
our conjectures
this behaviour can be proved by induction

PROVE BY INDUCTION

inductive assumption, it is true for n.

$$\cancel{T(n)}\ T(n-1) = \begin{cases} 2T(n-2) & n-1 \text{ even} \\ 2T(n-2)+1 & n-1 \text{ odd.} \end{cases}$$

strong induction vs weak induction

case n is even - then (n-1) is odd
so by inductive assumption. $T(n-1) = 2T(n-2)+1$

$$T(n) = 2T(2T(n-2)+1)$$

$$= 4T(n-2)+2$$

$$= 2T(n-1)$$

$T(n) = 2T(n-2)+T(n-1)+1$

proved even case.

case, n = odd,   n-1 = even

$T(n-1) = 2T(n-2)$

$T(n-1) = T(n-1) + T(n-1) + 1 = 2T(n-1) + 1$

n even: $T(n) = \frac{2}{3}(2^n - 1)$ 42 stops for 6 ring CR

n odd: $T(n) = \frac{1}{3}(2^{n+1} - 1)$