RECURRESION; INDUCTION; RECURRENCE RELATION

compound interest; tower of hanoi

$m_0 = 100$

interest $= 10\%$

$m_1 = 110$

$m_2 = 121$

$\vdots$

$m_i = 1.1\, M_{i-1}$ $\quad (M_{i-1} + 0.1\, M_{i-1})$

$M_i = 1.1\, M_{i-1}$ recurrence relation

(i) brute force
repeated substitution

$M_{i-1} = 1.1\, M_{i-2}$

$\vdots$

to the base case

we are looking for closed form solution for this recurrence relation.

$M_i = (1.1)^2 M_{i-2}$

... do this r times (go backwards r steps)

$M_i = 1.1^r M_{i-r}$

$i - r = 0$

$\Rightarrow i = r$

$\therefore M_r = 1.1^r M_0$

$M_i = (1.1)^i M_0$ closed form solution
don't have to compute telescopically.

ex binary search : (number guess game)
20q (object guess game) y/n

$T(n) = 1 + T\left(\frac{n}{2}\right)$

closed form Solution

$T\left(\frac{n}{2}\right) = 1 + T\left(\frac{n}{4}\right)$

$T(n) = 2 + T\left(\frac{n}{2^2}\right)$

$T(n) = r + T\left(\frac{n}{2^r}\right)$

classmate

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$\frac{T(n)}{2^r} < 1$ $\quad 2^r > T(n)$ $\quad r > \log T(n)$

base case :

$T(1) = 0$

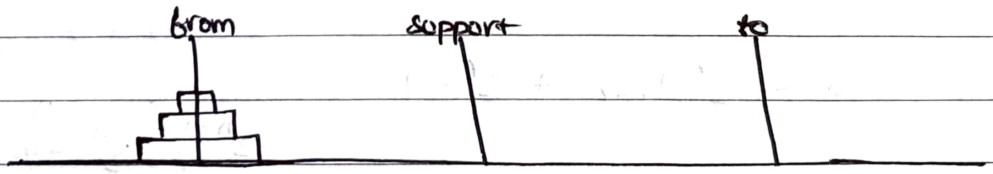
$r = \log_2 n$

$\therefore T_n = \log_2 n + \cancel{T_1}^{\,0}$

$\therefore T_n = \log_2 n$ RR

TOWER OF HANOI

buddhist elites 64 blocks enlightenment



3 disk problem $\longrightarrow$ 2 disk problem

```
TOH (n, from, to, supp)
 if n>0 {
     TOH (n-1, from, supp, to)
     display ('move disk from 'to' to)
     TOH (n-1, supp, to, from)
 }
```

$T_0 = 0$

$T_1 = 1$

$T_n = 2T_{n-1} + 1$ RR

↪ recursive calls

$T_{n-1} = 2(T_{n-2}) + 1$

$T_n = 2(2T_{n-2} + 1) + 1$

$T_n = 2^2 T_{n-2} + 2 + 1$

$= 2^3 T_{n-3} + \underbrace{4 + 2 + 1}_{2^3 - 1}$

$= 2^3 T_{n-3} + 2^3 - 1$

$T_n = 2^r$ $T_n = 0$

$T_n = 2^r T_{n-r} + 2^r - 1$

$\frac{2^{n-1}\left(\frac{1}{2}^n - 1\right)}{\frac{1}{2} - 1}$

$r = n$

$T_n = \cancel{2^n T_0}^{\,0} + 2^n - 1$

$2^n\left(1 - \frac{1}{2}^n\right)$

$T_n = 2^n - 1$

$\frac{a\, 2^n - 1}{\frac{1}{2}^{n-1}}$