## EXTENDED MASTER'S THEORY

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

$$a = b = 2 \quad \text{if } (n) = \left(\frac{n}{2}\right)$$

$$T(n) = a T\left(\frac{n}{b}\right) + f(n)$$
  $T(i) = d$ 

with 
$$\alpha$$
 (-1 then  $T(n) = \beta \left(n^{\log_a b}\right)$ 

(2) of 
$$f(n) = \theta\left(n \log^{\alpha} \left(\log_{b} n\right)^{-1}\right)$$
 here  $-1$ 

(3) if 
$$f(n) = \theta\left(n \log_{b} \theta\left(\log_{b} n\right)^{\alpha}\right)$$
; with  $\alpha > -1$ 

then 
$$\tau(n) = \Theta\left(n \log_b a + \log_b a\right)$$

(4) if  $f(n) = \Omega\left(n \log_b a + e\right)$  for some  $e > 0$ 

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{gn}$$

$$a = b = 2$$
  $f(n) = \frac{n}{\lg n}$ 

(i) 
$$\longleftrightarrow$$
 3 think about how case (i) (i)  $\longleftrightarrow$  includes case (i) (is included in (i)

$$T(n) = \theta(n, \lg \lg n)$$

68e 2

ex 
$$T(n) = 2T\left(\frac{n}{2}\right) + n\left(\frac{lgn}{2}\right)^2$$

ex 
$$T(n) = 8T(\frac{h}{2}) + n(lgn)^2$$

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CHANGE OF VARIABLES

$$S(n)-1 = 2(S(n-1)-1)+1$$

3(n) -1 = 2 (28(n-1)-2+1

$$\Rightarrow$$
  $s(n) = 2^n$ 

$$S(n) = T(n) + 1$$

$$2^n = T(n) + 1$$

$$T(n) = 2^n - 1$$

ignoring rounding off issues

let 
$$m = \log_2 n \Rightarrow n = 2^m$$

$$T(2^m) = ZT(2^{m/2}) + m$$

$$7 = 2S\left(\frac{m}{3}\right) + m$$

$$S(m) = O\left(m \log m\right)$$

$$\Rightarrow T(n) = T\left(2^{m}\right) = S(m) = O\left(m \log m\right)$$

ex  $T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + 1.5n$ 

supposed to be a linear fime algorithm.

our guess 
$$T(n) = O(n)$$
assame  $T(n) \le kn$ 

$$T\left(\frac{n}{5}\right) \leq 12n$$

$$T\left(\frac{7n}{10}\right) \leq 42n$$

$$\frac{\left(\frac{7n}{10}\right)}{\left(\frac{7n}{10}\right)} \leftarrow \frac{42n}{10}$$

$$\frac{4(n)}{5} < \frac{k}{5}$$

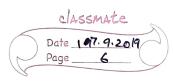
K= 150

$$T\left(\frac{7n}{5}\right) < \frac{7\kappa n}{10}$$

$$\frac{7\eta}{10}$$
  $\frac{7\kappa\eta}{10}$ 

$$\frac{1}{5}$$
  $\frac{1}{5}$   $\frac{7}{10}$   $\frac{1}{5}$   $\frac{15}{10}$ 

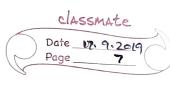
( 9 k + 15 )



RECURRENCE RELATION AND FIBONACCI SERIES

$$\begin{bmatrix}
1 & 1 & F(1) \\
1 & 0 & F(0)
\end{bmatrix} = F(n+1)$$

$$V_n = A^n V_o \longrightarrow$$



such that CTAC is	a coag. Matrix
Λ	0
Anxn	Cnun
9, 9, In	baxa
,"	
AC = CD (M)	
iff h, are e-values of	f A
9: is corresponding e-v	ector,
- i sima t	The second second
AC = A 9, 2 2,	- (4
1601	A in the second
(exity)	100
= AV, AV2	. Ag
	s. %. ~••
CD = 10, 12 V	27 2
	3.
	2 2 1
	1
2 1	
λυ, λν <sub>2</sub>	AVn
	<u> </u>

Ax = 2X
eigen value
eigen vector

th Anxon is diag. iff
it has a linearly independent eigen vectors

A = CDC-1 - full rank matrix of order in all columns are linearly independent.

f(A) = f(Zi)

 $\begin{array}{c|c} & & & \\ & & & \\ & & & \\ & & & \\ \end{array}$ 

A 0 = 2 0

(A - \(\chi I) \(\forall = 0\)

det (A-27)=0

... bla bla bla

 $\frac{1}{15} \left( \frac{1}{1} + \frac{1}{15} \right)^{n} = \left( \frac{1 - \frac{1}{15}}{2} \right)^{n}$ 

$$f(n) = \frac{1}{\sqrt{5}} \left( \frac{1+\sqrt{5}}{2} \right)^n + \left( \frac{1-\sqrt{5}}{2} \right)^n$$

$$f(0) = \frac{1}{\sqrt{5}} \left( 1 - 1 \right) = 0$$

$$f(1) = \frac{1}{15} \left( \frac{1+15}{2} - \frac{1-15}{2} \right) = 1$$

Solved

$$f(n) = \frac{1}{\sqrt{5}} \left( \frac{1+\sqrt{5}}{2} \right)^n - \left( \frac{1-\sqrt{5}}{2} \right)^n$$