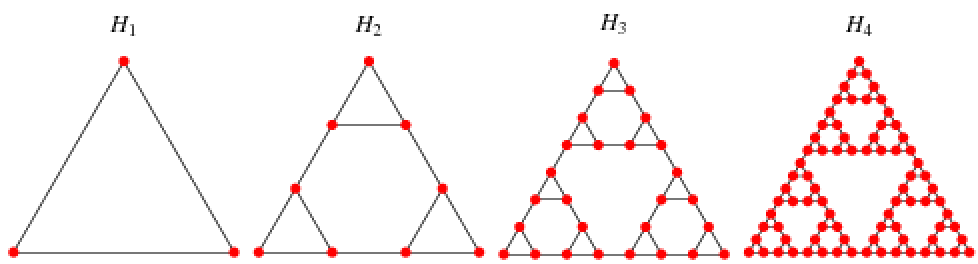
CSE611: DISCRETE MATHEMATICS AND ALGORITHMS

The Tower of Hanoi (TOH) Graph

The Hanoi graph will be shown and discussed in class. You can look for a picture on the web in the wolfram.com site (Link given below). It is constructed recursively, defined inductively and analyzed. It gives us a blueprint of the computation for ToH. Note that a solution to ToH is a path through this graph.

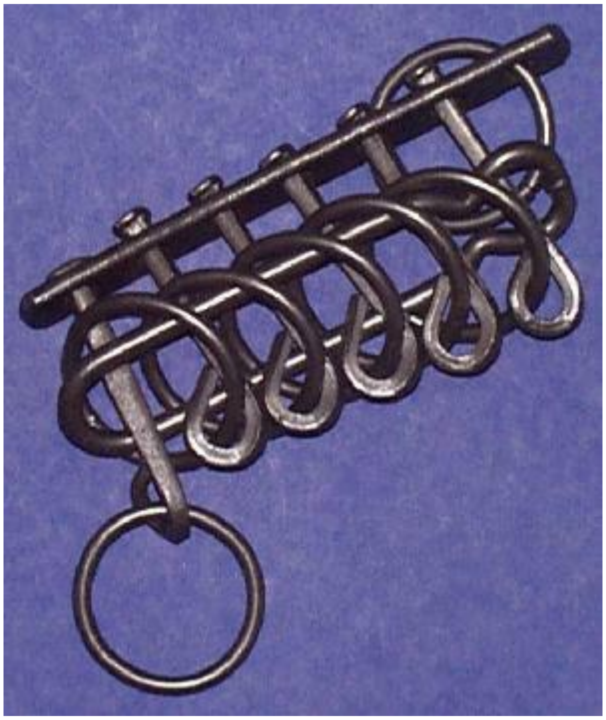
http://mathworld.wolfram.com/HanoiGraph.html

http://mathworld.wolfram.com/TowerofHanoi.html



The Hanoi graph H_n corresponding to the allowed moves in the tower of Hanoi problem. The above figure shows the Hanoi graphs for small n.

The Chinese Rings or Patience Puzzle



A Recursive Method to Remove Rings and Unlock the Puzzle

To Remove the n rings:

Reduce the puzzle to an n-1 ring puzzle.

Remove the leftmost n-1 rings.

End

To Reduce the puzzle to an n-1 ring puzzle:

Remove the leftmost n-2 rings.

Remove the n^{th} ring.

Replace the leftmost n-2 rings.

End

Resulting Recurrence Equation

$T(n) = 1 + T(n-1) + 2T(n-2)$

$T(1) = 1;\ \ T(2) = 2$

Analysis and Solution

We guessed (looking at n versus T(n)) the following recurrence relation and proved it by induction:

When n is even: $T(n) = 2T(n-1)$

When n is odd: $T(n) = 2T(n-1) + 1$

Now we can use repeated substitution to get:

$T(n) = 4T(n-2) + 2$, when n is even.

$T(n) = 4T(n-2) + 1$, when n is odd.

Continuing our substitutions gives:

$T(n) = 2/3\ (2^n - 1)$, when n is even.

$T(n) = 1/3\ (2^{n+1} - 1)$, when n is odd.

The Chinese Ring Puzzle motivates:

1. An Understanding of Recursion.
2. Natural proofs by induction.
3. Construction, analysis and solution of recurrence equations.
4. Binary Gray Codes.
5. Experimenting and Guessing.