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preimage, image

(a, b) range

domain codomain

$$2^n = \binom{n}{0} + \binom{n}{1} + \dots \binom{n}{n}$$ } by induction

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$ combinatorial arg.

$$\binom{n}{0}$$

dist. of m similar objects to n groups $= \binom{m+n-1}{n-1}$

$$\binom{n+1}{0} + \binom{n+1}{1} + \dots \binom{n+1}{n+1}$$

$$= \binom{n}{0} + \binom{n}{0} + \binom{n}{1} + \binom{n}{1} \quad \binom{n}{n}$$

$$= 2\left\{\binom{n}{0}\right\} = 2 \cdot 2^n$$

$$\binom{2n}{2} = 2\binom{n}{2} + n^2$$

$$= \binom{2n}{2} = \binom{n}{2} + \binom{n}{2} + \binom{n}{1}\cdot\binom{n}{1}$$

$$10^n - 9^n - \binom{n}{1} 9^{n-1} - \binom{n}{2} 9^{n-2}$$

$$= 10^n - \sum_{i=0}^{2} \binom{n}{i} 9^{n-i}$$

$$P(n_1, n_2, n_k) = \frac{n!}{n_1!\, n_2! \dots n_k!}$$

dist. of m similar objects into n groups

$$= \binom{m+n-1}{n-1}$$

$x_1 \quad x_2 \quad x_3 \geq 4$

$$\binom{4+2}{2} = \binom{6}{2} = \frac{6\times 5}{2} = 15$$

$x_1 + x_2 = 1 \quad \binom{\cancel{8}9}{1} = 9$

$$\frac{9!}{3!3!3!} - (3a's \cup 3b's \cup 3c's)$$

$$= |A| + |B| + |C| - |A\cap B| - |B\cap C| - |C\cap A| + |A\cap B\cap C|$$

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$$\frac{9!}{3!\,3!\,3!} - \left(\frac{7!}{3!\,3!} \times 3 - \frac{5!}{3!} \times 3 - 3!\right)$$

$$= n! - \left(P_1 \cup P_2 \cup P_3 \ldots P_n\right)$$

$$= n! - \left(\binom{n}{1}(n-1)! - \binom{n}{2}(n-2!) \ldots \binom{n}{n}\right)$$

$$= n! - \left(n! - \frac{n!}{7!} + \frac{n!}{3!} - \ldots \frac{n!}{n!}\right)$$

$$= n!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \ldots (-1)^{n+1}\frac{1}{n!}\right)$$

$$\frac{D_n}{n!} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \ldots (-1)^{n+1}\frac{1}{n!}$$

$$\frac{D_n}{n!} \to \frac{1}{e} \approx 0.368 \quad \text{as } n \to \infty. \quad \underline{36\%}.$$

$$\frac{\cancel{2}.\,n.(n-1)}{\cancel{2}!} \quad \frac{2!}{2n.(2n-1)} \quad \left(\frac{n-1}{2n-1}\right.$$

$$C_n = \frac{\binom{2n}{n}}{n+1}$$

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$$C_n = \frac{\binom{2n}{n}}{n+1} =$$

$$C_n = C_0 C_{n-1} + C_1 C_{n-2} + \cdots \quad C_{n-1} C_0$$

$$C_n = \sum_{i=0}^{n-1} C_i C_{n-i-1}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$\cancel{P(A|B) =}$ $P($

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

$$= \frac{P(A|B)\,P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})}$$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} \quad P(A|B)P(B) + (P(A|\bar{B})P(\bar{B}))$$

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$t_1 + t_2 + t_3 = 8$

$2 \le t_i \le 4$

$(x^2 + x^3 + x^4)^3$ coeff of x^8. $\boxed{?}\ x^8$.

Σ ex $\frac{1}{1-x} = 1 + x + x^2 + \dots$

$\frac{1}{(1-x)^2} = (1 + x + x^2 + \dots)(1 + x + x^2 + \dots)$

$\therefore B_n = \sum_{i=0}^{n} A_i A_{n-i} = 1 + 2$

$\frac{1}{(1-x^4)} \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3$

$\frac{1}{(1-x)^4} = \frac{1}{(1-x)^2} \frac{1}{(1-x)^2}$

$= (1 + 2x + 3x^2 + 4x^3 + \dots)(1 + 2x + 3x^2 + 4x^3 + \dots)$

$\boxed{?}\ x^5$

$= 1 \cdot 6x^5 + 2x \cdot 5x^4 + 3x^2 \cdot 4x^3 + 4x^3 \cdot 3x^2 + 5x^4 \cdot 2x + 6x^5 \cdot 1$

$= 6 + 10 + \underbrace{12 + 12}_{24} + 10 + 6$

$=$

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$$\frac{1}{(1-x)^n} = \sum_{k=0}^{\infty} \binom{n+k-1}{k} x^k.$$

$$\frac{1}{(1-x)^n} = \binom{n+k-1}{k} x^k$$

$$\frac{x}{(1-x)^3} = x + 3x^2 + 6x^3 + 10x^4 + 15x^5 + \dots$$

$$\frac{x}{1-x} = \frac{1}{1-x} - 1 \quad \frac{1-(1-x)}{1-x} = \frac{x}{1-x},$$

$$a_n = 9a_{n-1} + [10^{n-1} - a_{n-1}]$$

$$= 10^{n-1} + 8a_{n-1} \qquad a_0 = 1$$

$$a_1 = 9$$

$$g - 8xg = 1 + \frac{x}{1-10x} \quad G(x) - 1 = 8x\,G(x) + \frac{x}{1-10x}$$

$$g - 1 = 8xg + \frac{x}{1-10x}$$

$$\frac{x}{1-10x}$$

$$(g-1)(1- \quad 8xg - g = 1 - \frac{x}{1-10x}$$