smart 2 set tabular form

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SET THEORY

REPRESENTATION OF A SET

- tabular form $A = \{1, 2, 3, 4\}$
- set-builder form $A = \{x \mid P(x)\}$

such that condition

$A = \{2, 4, 6, \ldots\}$

$A = \{x \mid x = 2n$, n being a natural number$\}$

$A = \{1, 8, 27, 64, \ldots\}$

$A = \{x \mid x = n^3$, n being a positive integer$\}$

null set
empty set ϕ

finite set
infinite set

$n(A)$
$|A|$
order of a set
cardinal number
cardinality

Subset $\subseteq$

proper subset $\subset$

comparable sets $A \subseteq B$ / $B \subseteq A$

equality of sets $A \subseteq B \wedge B \subseteq A$ $(A = B)$

disjoint set

$A \cap B = \phi$

difference between sets

$A - B$, $A \setminus B = \{x \mid x \in A$, but $x \notin B\}$

thm if $A \subseteq B$ and $B \subseteq C$ $\Rightarrow$ $A \subseteq C$

thm if $A \subseteq B$ and $B \subseteq A$ $\Rightarrow$ $A = B$

thm $\phi \not\subset \phi$, but $\phi \subset \{\ldots\}$

POWER SET

a set formed of all subsets of a set S (as its elements).

$S = \{a, b, c\}$

$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

$\phi \in P(S)$ $\quad |P(S)| = 2^{|S|}$

$S \in P(S)$

universal set U $\quad S' = \{x \mid x \in U \text{ and } x \notin S\}$

complement of a set S' S^c $\bar{S}$ $\quad$ (venn-euler diagram)

union or join	intersection or meet
$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$	$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
$A \cup A = A$	$A \cap A = A$
$A \cup U = U$	$A \cap U = A$
if $A \subset B \Rightarrow A \cup B = B$	if $A \subseteq B \Rightarrow A \cap B = A$
$A \cup B = B \cup A$	$A \cap B = B \cap A$
$A \cup \phi = A$	$A \cap \phi = \phi$
$A \cup A' = U$	$A \cap A' = \phi$

LAWS OF ALGEBRA ON SETS

commutative laws	associative laws	idempotent laws
$A \cup B = B \cup A$	$A \cup (B \cup C) = (A \cup B) \cup C$	$A \cup A = A$
$A \cap B = B \cap A$	$A \cap (B \cap C) = (A \cap B) \cap C$	$A \cap A = A$

distributive laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

de morgan's laws

$A - B = A \cap B'$

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

$A - (B \cup C) = (A - B) \cap (A - C)$

$A - (B \cap C) = (A - B) \cup (A - C)$

SYMMETRIC DIFFERENCE

$A \Delta B = (A - B) \cup (B - A)$

$= \{x \mid (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}$

$A \Delta \phi = A$

$A \Delta A = \phi$

$A \Delta B = \phi \Rightarrow A = B$

CARTESIAN PRODUCT OF SETS

$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$

$A = \{a, b, c\}$ $B = \{m, n\}$

$A \times B = \{(a,m), (a,n), (b,m), (b,n), (c,m), (c,n)\}$

$|A \times B| = |A| \times |B|$

$A \times B \neq B \times A$

THE INCLUSION-EXCLUSION PRINCIPLE

let $A_1, A_2, \ldots A_n$ be n finite sets

$$|\cup_i A_i| = \sum_i |A_i| - \sum_{ij} |A_i \cap A_j| + \sum_{ijk} |A_i \cap A_j \cap A_k|$$

$$\ldots + (-1)^{n+1} |\cap_i A_i|$$

$n=2$ $\quad |A \cup B| = |A| + |B| - |A \cap B|$

$n=3$ $\quad |A \cup B \cup C| = |A| + |B| + |C|$ ~~[illegible]~~
$- |A \cap B| - |A \cap C| - |B \cap C|$
$+ |A \cap B \cap C|$

ex find the number of +ve ints ≤ 2076 and divisible by neither 4 or 5.

$A = \{x \mid x \leq 2076$ and divisible by $4\}$
$B = \{x \mid x \leq 2076$ and divisible by $5\}$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$= \left\lfloor \frac{2076}{4} \right\rfloor + \left\lfloor \frac{2076}{5} \right\rfloor - \left\lfloor \frac{2076}{4 \times 5} \right\rfloor$$

$$= 519 + 415 - 103 = 831$$

$\therefore$ ints NOT divisible by 4 or 5 $= 2076 - 831$

$$= 1245$$

PRIME NUMBER THEOREM

$\pi(x)$: no. of primes $\leq x$

$\pi(x) \to \dfrac{x}{\ln x}$ when $x \to \infty$

let $p_1, p_2, \ldots p_t$ be primes $\leq \sqrt{n}$

$$\pi(n) = n - 1 + \overset{t}{\pi(\sqrt{n})} - \sum_i \left\lfloor \frac{n}{p_i} \right\rfloor + \sum_{i<j} \left\lfloor \frac{n}{p_i p_j} \right\rfloor$$

$$\ldots + (-1)^t \left\lfloor \frac{n}{p_1 p_2 \ldots p_t} \right\rfloor$$

ex find the number of primes ≤ 100

the primality of numbers ≤ 100 can be checked by trying till 10. ($\sqrt{100}$)

primes till 10 = 2, 3, 5, 7

$$\pi(100) = 100 - 1 + 4 - \left\lfloor \frac{100}{2} \right\rfloor - \left\lfloor \frac{100}{3} \right\rfloor - \left\lfloor \frac{100}{5} \right\rfloor - \left\lfloor \frac{100}{7} \right\rfloor$$

$$+ \left\lfloor \frac{100}{6} \right\rfloor + \left\lfloor \frac{100}{10} \right\rfloor + \left\lfloor \frac{100}{14} \right\rfloor + \left\lfloor \frac{100}{15} \right\rfloor + \left\lfloor \frac{100}{21} \right\rfloor$$

$$+ \left\lfloor \frac{100}{35} \right\rfloor - \left\lfloor \frac{100}{30} \right\rfloor - \left\lfloor \frac{100}{70} \right\rfloor - \left\lfloor \frac{100}{105} \right\rfloor$$

$$+ \left\lfloor \frac{100}{210} \right\rfloor$$

$= 100 \cancel{-1} \cancel{+4} - 50 \; \overset{-31}{\cancel{-33}} \; \cancel{-20} \; \overset{2}{\cancel{-14}}$

$\cancel{+16} \cancel{+10} \cancel{+7} + \cancel{6} + \cancel{4} \cancel{+2}$

$\cancel{-3} \cancel{-1} - 0 + 0$

$= \cancel{19}\ 25$

ex prove that $(A-B)$ and $(A \cap B)$ are disjoint.

$(A-B) \cap (A \cap B)$

$= (A \cap B') \cap (A \cap B)$ using de morgan's law

$= A \cap (B' \cap B) \cap A$ using associative, commutative law

$= \phi$