PROOFS AND LOGIC

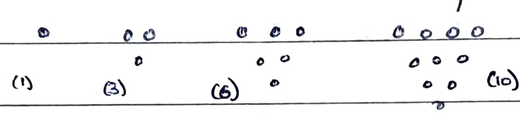
INTRODUCTION

connection between things that are apparently not connected,

counting → no. of pairs of students

bowling alley → pin counting

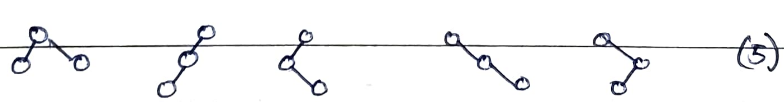
similar pattern



how many?

looks like a bowling alley problem.

q. how many binary trees possible with 3 nodes?



q. construct minimum spanning tree from graph. how many?

order of n^4

q. linear algebra:

multiplying 3 matrices $A_1 A_2 A_3$

hard to see but they are related

$((A_1 A_2) A_3)$ $(A_1 (A_2 A_3))$ ②

how many 2 node binary trees can you construct

2 $\leftrightarrow$ 2 node BT^3

5 $\leftrightarrow$ 3 node BT^3

$\leftrightarrow$ 4 node BT^3

PROOFS

✓ thm $\sqrt{2}$ is irrational

(aristotle) - by contradiction.

assume on the contrary, $\sqrt{2}$ is rational

rational no. - ratio of 2 natural numbers

$\sqrt{2} = \frac{a}{b}$ $a, b \in$ integers, $b \neq 0$

no common factors for a, b.

$2 = \frac{a^2}{b^2}$ $a \rightarrow b$ / $\sim b \rightarrow \sim a$

✓ lemma: helper theorm

$a^2 = 2b^2$ if n^2 is even, then n is even

(using contrapositive argument)

a^2 is even (proof by contrapositive)

a is even

let $a = 2k$, some integer k

$2b^2 = (2k)^2$

$b^2 = 2k^2$

b^2 is even

b is even

but, a & b have no common factors < contradiction >

$\rightarrow \sqrt{2}$ is irrational.

NUMBER OF PRIMES ARE INFINITE

(euclid -> founder of western mathematics)
(euclid's elements)

assume no. of primes are finite

2, 3, 5, ..., 31

$(2 \times 3 \times 5 \times \ldots \times 31) + 1$ —— prime

or has factors other than given

is this divisible by any prime no. ?

inductive learning from examples $\longrightarrow$ related to recursion

recurrence relation

n

$+ (n-1)$

$+ (n-2)$

$+ \vdots$

$+ (3)$

$+ (2)$

$+ (1)$

possible handshakes

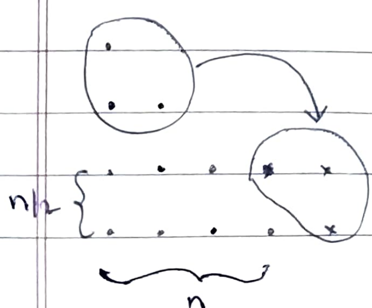
possible pairs

TRIANGLE NUMBER

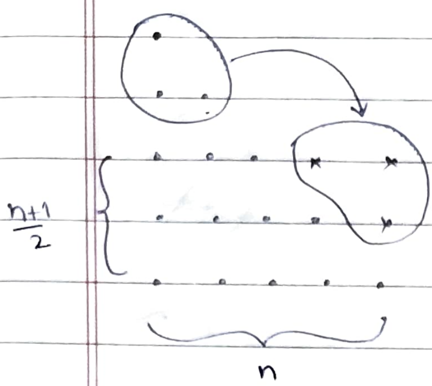
T_1	1	
T_2	3	
T_3	6	
T_4	10	

GEOMETRIC METHOD

n = even



dots = $(n+1)\left(\frac{n}{2}\right)$



dots $= (n)\left(\frac{n+1}{2}\right)$

$$T_n = 1 + 2 + 3 + \dots + n$$

$$= \frac{n(n+1)}{2}$$

GAUSS

$$1 + 2 + 3 + \dots + 98 + 99 + 100$$

101

101

101

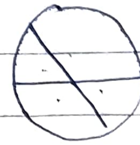
$101 \times \frac{100}{2}$

sum

pairs

class teacher problem 5 min.

CUTTING PLANES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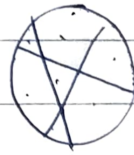
cuts	no. of pieces.
0	1
1	2
2	4
3	7
4	11

$2 \xrightarrow{+2} 4 \xrightarrow{+3} 7 \xrightarrow{+4} 11$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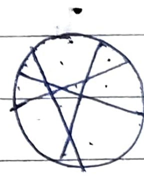
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$P_n = T_n + 1$

$P_n = P_{n-1} + n$ — observation

prove $P_n = T_n + 1$

proof by induction

base case $n=1$, $P_1 = T_1 + 1 = 1 + 1 = 2$

inductive assumption, suppose $P_k = T_k + 1$ is true

need to prove $P_{k+1} = T_{k+1} + 1$

$P_{k+1} = P_k + (k+1)$ (from observation)

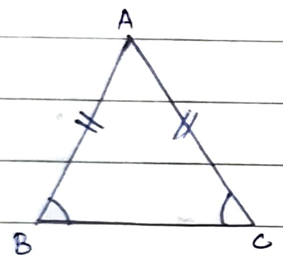
$= (T_k + 1) + (k+1)$ (from assumption)

$= [T_k + (k+1)] + 1$

$= T_{k+1} + 1$ ✓ (from T_k definition)

LOGIC

foundation for designing circuits
theorm provers (automatic)



ΔABC and ΔACB are congruent
∴ angles ... same

AI also customer for logic

~~var~~

VARIABLES AND CONNECTIVES

$R \vee W$	OR	$+$	binary
$R \wedge W$	AND	$\cdot$	binary
$\neg R$	NOT	$-$	unary

precedence order
- unary
- and
- or

$R \rightarrow W$ implication / conditional
$R \leftrightarrow W$ biconditional.

COMPLETE SET

minimal set of connectives that will generate all the rest.

TRUTH TABLE

A	B	A ∨ B	A ∧ B	A → B
0	0	0	0	1
0	1	1	0	1
1	0	1	0	0
1	1	1	1	1

applications in circuits
3-sat algo.

ex list out all possible function between 2 variables.
give names. (2^{2^n})

W ∧

0	and	¬(A→B)	A	¬(B→A)	B	A⊕B	or
0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1
0	0	1	1	0	0	1	1
0	1	0	1	0	1	0	1

nor	¬(A⊕B)	¬B	B→A	¬A	A→B	nand	1
1	1	1	1	1	1	1	1
0	0	0	0	1	1	1	1
0	0	1	1	0	0	1	1
0	1	0	1	0	1	0	1