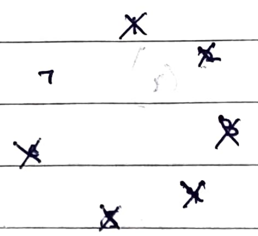
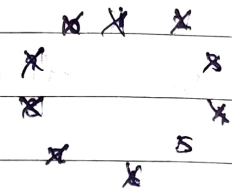
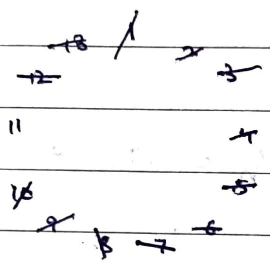
JOSEPHUS PROBLEM



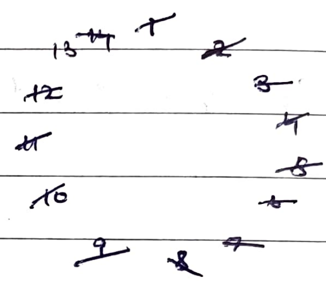
$J(7) = 7$



$J(10) = 5$



$J(13) = 11$?



$J(14) = 13$

Killer josephus number.

	n	J(n)		n	J(n)
2^0 group	1	1		13	11
	2	1		14	13
2^1 group	3	3	2^3 group	15	15
	4	1		16	1
	5	3			
	6	5			
2^2 group	7	7			
	8	1			
	9	3			
	10	5			
	11	7			
	12	9			

even $J(2n) = 2J(n) - 1$

odd $J(2n+1) = 2J(n) + 1$

guess the solution, and prove it inductively.

it appears $\underline{\underline{J(2^m+n) = 2n+1}}$

let us assume that this is true for all values $\leq n$

let $n = 2^m + p$

$$\underbrace{J(2^m + p)}_{\text{even}} = 2J\left(2^{m-1} + \frac{p}{2}\right) - 1$$

based on strong inductive hypothesis (SIH)

$$= 2\left(2 \cdot \frac{p}{2} + 1\right) - 1$$

$$= 2(p+1) - 1$$

$$= 2p + 1$$

$\therefore$ true for all even n.

case: <u>n is odd</u> $2^m + p$ $(p = 2k+1)$

base case $n = 1$ $J(2^0 + 0) = 2 \times 0 + 1 = \underline{1}$ ✓

let us assume that the conjecture is true for $< n$

let $n = 2^m + p$ ⇒ p odd $p = 2k+1$ $k = \frac{p-1}{2}$

$J(n) = J(2^m + p)$

$= J(2^m + 2k + 1)$

$= \cancel{2J\left(2^{m-1} + \frac{p}{2}\right)} + 1$

using based on ~~strong~~ inductive hypothesis (SIH)

$= \cancel{2\left(2 \cdot \frac{p}{2} + 1\right) + 1}$

$= 2J(2^{m-1} + k) + 1$

based on strong inductive hypothesis (SIH)

$= 2(2k+1) + 1$

$= 4k + 2 + 1$

$= \underline{\underline{2p + 1}}$

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left cycle shift

$a \xrightarrow{LCS} b$ is the josephus no.

	n	J(n)	
1	1	1	1
10	2	1	1
11	3	3	11
100	4	1	1
101	5	3	11
110	6	5	101
111	7	7	111
1000	8	1	1
1001	9	3	11
1010	10	5	101
1011	11	7	111
1100	12	9	1001
1101	13	11	1011
1110	14	13	1101
1111	15	15	1111
10000	16	1	

or basically $\underline{\underline{2p+1}}$