Discrete Mathematics and Algorithms (CSE611)

Assignment-1: Proof Methods, Logic, Set Theory, , Principle of Incl-Excl

Total Marks: 100

Deadline: August 27, 2019 (Tuesday), 5:00 pm

Submission Instructions:

Note: If found copying both, the copier and from whom it was copied, will be given ZERO!!

Please start each question at the top of a page.

Please submit the assignment in hard copy stating the following at the top:

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Submitted by XYZ, Roll No.

Submitted on

Q1. Prove, without using the Venn diagram, that $A - B$, $B - A$ and $A \cap B$ are pairwise disjoint, where A and B are two sets. [General Hint: Use Prof. A. K. Das' Set theory slides on the portal as additional study material]

Q2. If A, B and C are any three non-empty sets, then prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

Q3. Let x be a positive real number and $\pi(x)$ denote the number of primes $\leq x$. Find the number of primes $\leq$ 50, using the *inclusion-exclusion principle* for prime numbers.

Q4. IIIT-H has 400 students, and 289 of them are taking computer science courses and 187 are taking natural science courses. If 100 are taking both computer science and natural science courses, how many are taking neither?

Q5. If a post office sells only ₹ 5 and ₹ 9 stamps, show that any postage of ₹ 35 or more can be paid using only these two stamps. [Hint: Try *proof by induction*]

Q6. Prove that n is an odd integer if and only if n^2 is an odd integer.

Q7. Let $a, b,$ and c be integers such that $a^2 + b^2 = c^2$. Prove that at least one of a and b is even. [Hint: Try *proof by contradiction*]

Q8. Because every Boolean function can be represented using the boolean operators *product* $(\cdot)$, *sum* $(+)$, and *negation* ($\neg$) we say that the set $\{\ \cdot,\ +,\ \neg\ \}$ is *functionally complete*. We can find a smaller set of functionally complete operators. This can be done if one of the three operators of this set can be expressed in terms of the other two. Now, show that the set $\{\ \cdot,\ \neg\ \}$ is functionally complete.

Q9 (A). The exclusive-or operator $\oplus$, is defined by the rule that $a \oplus b$ is true whenever a or b is true but not both. Now write CNF (conjunctive normal form) and DNF (disjunctive normal form) expressions for $x \oplus y$.

Q9 (B). The exclusive-or operator is not *functionally complete*. Consider three operators $\{\wedge, \vee, \neg\}$. Which ones, if any, of the three can be combined with exclusive-or to make a complete set. Justify your answer.

Q10 (A). Prove by formal logic (remember we discussed in class the connection between set theory and logic) the following: (i) The complement of the intersection of two sets equals the union of the complements. (ii) $(B - A) \cup (C - A) = (B \cup C) - A$.

Q10 (B) We gave De Morgan's laws for two sets in class. Generalize these for n sets and prove the laws by induction.