- Derivatives.
- Errors.
- Higher order derivatives.

Derivatives

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- Derivatives of known functions can always be calculated analytically. So there is less of a need to calculate it numerically.
- There are significant practical problems with numerical derivatives. As a result they are used less often than numerical intergration.

Forward and Backward differences

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There is also backward difference:

$$\frac{df}{dx} \simeq \lim_{h \to 0} \frac{f(x) - f(x-h)}{h}$$

Errors

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- Round off errors.
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Consider the forward difference. We can write the Taylor expansion of $f(x+h)$ as

$$f(x+h) = f(x) + hf'(x) + \frac{1}{2}h^2 f''(x) + \dots$$
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But because we are subtracting numbers very close to each other (if h is small), then the round off error also comes into play!

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- If $|f(x)|$ and $|f''(x)|$ are order 1, $h \sim \sqrt{C} \sim 10^{-8}$.

$$\frac{df}{dx} \simeq \frac{f(x+h/2) - f(x-h/2)}{h}$$

The error analysis gives:

$$h = \left(24C\left|\frac{f(x)}{f'''(x)}\right|\right)^{1/3}$$

If $f(x)$ and $f'''(x)$ are of order 1, then $h \sim C^{1/3} \sim 10^{-5}$.

Similar ideas can be extended to higher order derivatives.