- Discrete Cosine Transform.
- Fast Fourier Transform.
- Convolution.
- Power spectrum.

- If the function $f(x)$ is even (i.e. symmetric) about the midpoint $(x = \frac{L}{2})$ then one can write the cosine series:

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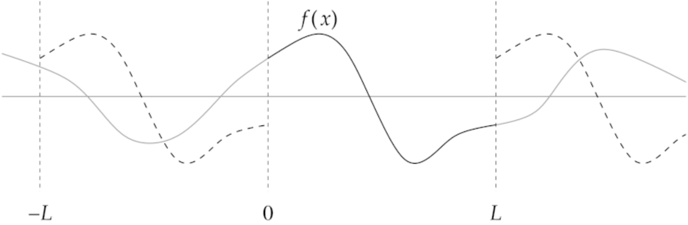
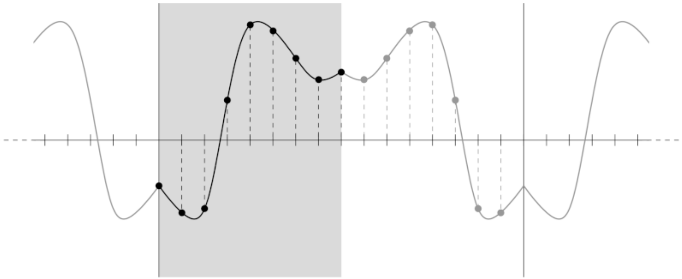
- This might seem like a big limitation making the whole cosine transform virtually useless – but this is not the case.

- We had made any function periodic – say if we are only interested in a portion of this non periodic function over a finite interval, 0 to L, we can just take that portion and repeat it to create a periodic function.



- If we are interested in the function in a finite region, we can make it symmetric by adding to it a mirror image of itself and then repeating it endlessly!



Discrete Cosine Transform – even functions

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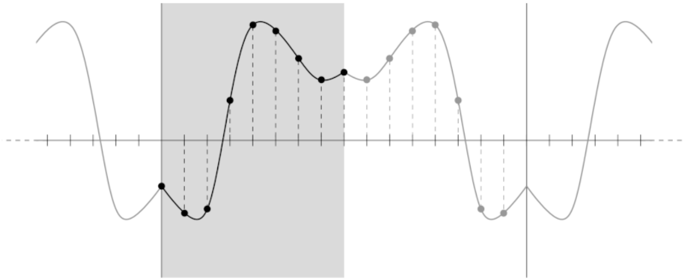
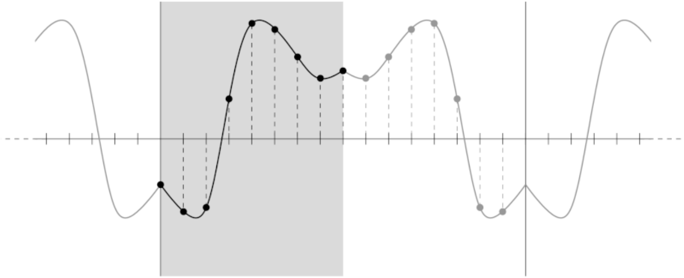
- In practice, this is how the cosine transform is always used.



- If we are interested in the function in a finite region, we can make it symmetric by adding to it a mirror image of itself and then repeating it endlessly!
- In practice, this is how the cosine transform is always used.
- This also implies that the number of samples in the transform is always even.



Discrete Cosine Transform (DCT)

Discrete Cosine Transform is a special case of Discrete Fourier Transform.

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Because the function is symmetric $y_0 = y_N, y_1 = y_{N-1}, \ldots$ and $e^{i2\pi k} = 1$ for all $k \in \mathbb{Z}$

Changing variables $N - n \to n$ in the right hand expression:

$$c_k = \sum_{n=0}^{\frac{1}{2}N} y_n \exp\left(-i\frac{2\pi kn}{N}\right) + \sum_{n=1}^{\frac{1}{2}N-1} y_n \exp\left(i\frac{2\pi kn}{N}\right)$$

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Normally the cosine transform is applied to real samples, which implies that the coefficients c_k will all be real as well (as they are sums of real terms).

Discrete Cosine Transform (DCT)

As y_n and c_k are real, $c_{N-r} = c_r^* = c_r$. The inverse/ transform:

$$y_n = \frac{1}{N}\left[\sum_{k=0}^{N-1} c_k \exp\left(i\frac{2\pi kn}{N}\right)\right]$$

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- A nice feature of this DCT is that unlike DFT, it does not assume that the samples are periodic.
- This is much better suited for non periodic functions as there is no discontinuity introduced.
- In principle, the discrete sine transform can also be computed. However, the requirement of anti-symmetry forces the function to be zero at either end of the range. This does not happen often in real-world applications...

Fast Fourier Transform

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- Gauss came up with a trick to reduce the number of operations. Often the FFT is attributed to Cooley and Tukey – but Gauss used it in 1805 (when he was 28 yrs).

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- Divide the sum into two equal sized groups – first group containing terms where n is even and second group containing terms where n is odd.

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&\equiv E_k + e^{-i2\pi k/N} O_k
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However, thanks to the periodicity of the DFT:

$$E_{k+\frac{1}{2}N} = E_k \quad O_{k+\frac{1}{2}N} = O_k$$

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Therefore, we can write:

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However we also know:

$$\begin{aligned} e^{-i2\pi(k+\frac{1}{2}N)/N} &= e^{-i2\pi k/N - i\pi} \\ &= e^{-i\pi} e^{-i2\pi k/N} \\ &= -e^{-i2\pi k/N} \end{aligned}$$

Then for $0 \leq k < \frac{1}{2}N$:

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- From the original N^2 multiplications, now one has $2(\frac{N}{2})^2 = \frac{N^2}{2}$ multiplications.
- This procedure can be recursively repeated – leading to a scaling of $\mathcal{O}(N \log_2 N)$.

- $N \to 2 \times (N/2)^2 + N = N^2/2 + N$ operations.

Fast Fourier Transform

- $N \to 2 \times (N/2)^2 + N = N^2/2 + N$ operations.
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- $\ldots$
- $N \to N^2/2^m + mN$ where $m = \log_2 N$.
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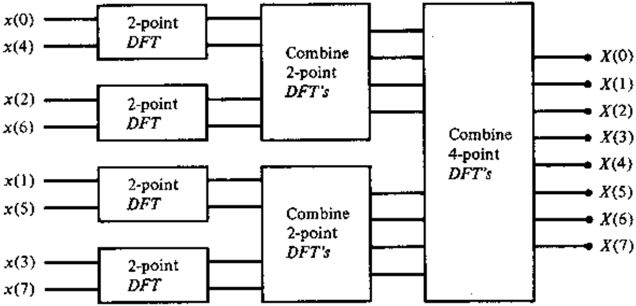
Fast Fourier Transform



Discrete Fourier Transform

```
from numpy import zeros
from cmath import exp, pi

def dft(y):
    N = len(y)
    c = zeros(N//2 +1, complex)
    for k in range(N//2+1):
        for n in range(N):
            c[k] += y[n]*exp(-2j*pi*k*n/N)
return c
```

Discrete Fourier Transform

```python
import numpy as np
def DFT_slow(x):
    """Compute the discrete Fourier Transform"""
    x = np.asarray(x, dtype=float)
    N = x.shape[0]
    n = np.arange(N)
    k = n.reshape((N, 1))
    M = np.exp(-2j * np.pi * k * n / N)
    return np.dot(M, x)
```

Fast Fourier Transform

```
def FFT(x):
    """A recursive implementation of the 1D Cooley-Tukey FFT"""
    x = np.asarray(x, dtype=float)
    N = x.shape[0]

    if N % 2 > 0:
        raise ValueError("size of x must be a power of 2")
    elif N <= 4:  # this cutoff should be optimized
        return DFT_slow(x)
    else:
        X_even = FFT(x[::2])
        X_odd = FFT(x[1::2])
        factor = np.exp(-2j * np.pi * np.arange(N) / N)
        return np.concatenate([X_even + factor[:N / 2] * X_odd,
                               X_even + factor[N / 2:] * X_odd])
```

Convolution

$$
\begin{aligned}
(f * g)(t) &= \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau \\
&= \int_{-\infty}^{\infty} f(t-\tau)\, g(\tau)\, d\tau
\end{aligned}
$$

The convolution theorem states that the Fourier transform of a convolution of two functions is the pointwise product of their Fourier transforms.

$$
\begin{aligned}
&\mathcal{F}\{f * g\} = \mathcal{F}\{f\} \cdot \mathcal{F}\{g\} \\
\implies\ & f * g = \mathcal{F}^{-1}\big\{\mathcal{F}\{f\} \cdot \mathcal{F}\{g\}\big\}
\end{aligned}
$$

Power Spectrum

- For a given signal, the power spectrum gives a plot of the portion of a signal's power (energy per unit time) falling within given frequency bins.

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- One method for estimating power spectral densities is based on using a function called the periodogram. The periodogram of an N-point sequence y_n is defined to be

$$I[k] = \frac{1}{N}\left|c_k\right|^2$$

Power Spectrum

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- Parseval's theorem tells us:

$$\sum_{n=0}^{N-1} |y_n|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |c_k|^2$$