Probability class 12

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Outline

Problem statement

1. . In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Solution

Let $X \in (0, 20)$ and $X \in Z$ be the random variable representing the number of questions answered correctly

This question requires use of binomial distribution with number of trials n $=20$:

Let probability of success be p and that of failure be q such that $p + q = 1$

suppose head appears on the toss with probability $\frac{1}{2}$, then he answers true, which maybe correct with probability $\frac{1}{2}$

hence probability will become $\frac{1}{4}$

similarly, for toss being tale probability of being correct is $\frac{1}{4}$

$$\therefore p = \frac{1}{2} \tag{1}$$

$$\therefore q = \frac{1}{2} \tag{2}$$

Solution

Required probability is $\Sigma_{i=12}^{20} P\left(X=i\right)$
we know that,

$$P\left(X=i\right)=\binom{n}{i}\times p^{i}\times q^{n-i} \tag{3}$$

$$P\left(X=i\right)=\binom{n}{i}\times\left(\frac{1}{2}\right)^{i}\times\left(\frac{1}{2}\right)^{n-i} \tag{4}$$

$$P\left(X=i\right)=\binom{n}{i}\times\left(\frac{1}{2}\right)^{n} \tag{5}$$

$$\Sigma_{i=12}^{20} P\left(X=i\right)=\left(\Sigma_{i=12}^{20}\binom{n}{i}\right)\times\left(\frac{1}{2}\right)^{n} \tag{6}$$

$$\Sigma_{i=12}^{20} P\left(X=i\right)=\frac{\binom{20}{12}+\binom{20}{13}+....+\binom{20}{19}+\binom{20}{20}}{2^{20}} \tag{7}$$