Probability class 12

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Outline

Problem statement

1. if $y = \sqrt{X}$ and X is an exponential random variable, show that Y represents a Rayleigh random variable

Solution

X has a probability density function:

$$f_X(x) = \frac{1}{\alpha} e^{-\frac{x}{\alpha}} \tag{1}$$

The transformation $Y = g(X) = \sqrt{X}$ is a 1-1 transformation from $X = \{x|x > 0\}$ to $Y = \{y|y > 0\}$ with inverse $X = g^{-1}(Y) = Y^2$ and jacobian $\frac{dX}{dY} = 2Y$

Therefore by the transformation technique, the probability density function of Y is:

$$f_Y(y) = f_X\left(g^{-1}(y)\right)\left|\frac{dX}{dY}\right| \tag{2}$$

$$= \frac{1}{\alpha} e^{-\frac{y^2}{\alpha}} |2y| \tag{3}$$

$$= \frac{2y}{\alpha} e^{-\frac{y^2}{\alpha}} \ (y > 0) \tag{4}$$

which is the probability density function of a Rayleigh random variable

Hence Proved