Probability class 12

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Outline

1. Problem statement

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Problem statement

1. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

Solution

Let $X_1 \in \{1, 2, 3\}$ and $X_2 \in \{0, 1\}$ be random variables which have the following meanings associated with them :

description of events

$X_1 = 1$: Bag 1 is selected
$X_1 = 2$: Bag 2 is selected
$X_1 = 3$: Bag 3 is selected
$X_2 = 0$: gold coin is selected
$X_2 = 1$: silver coin is selected

Therefore,the required probability is $P(X_1 = 1 | X_2 = 0)$.

By Baye's Theorem and total probability,

$$P\left(X_1 = 1 | X_2 = 0\right) = \frac{P\left(X_1 = 1\right) P\left(X_2 = 0 | X_1 = 1\right)}{\sum_{i=1}^{3} P\left(X_1 = i\right) P\left(X_2 = 0 | X_1 = i\right)} \tag{1}$$

$$= \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times 0 + \frac{1}{3} \times \frac{1}{2}} \tag{2}$$

$$= \frac{2}{3} \tag{3}$$

$$\therefore P\left(X_1 = 1 | X_2 = 0\right) = \frac{2}{3} \tag{4}$$