Probability and random variables assignment

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1 Q8 C)

1.1. Using ruler and compass only, construct a △ABC such that BC = 5 cm and AB = 6.5 cm and ∠ABC = 120°
(i) Construct a circum-circle of △ABC
(ii) Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC.

Solution: The parameters for constructing the figure are given in the table below:

TABLE 1.1.1

Symbol	Value	Description
a	5	BC
c	6.5	AB
θ	$\frac{\pi}{3}$	$\pi - \angle ABC$
A	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	origin
B	$\begin{pmatrix} c \\ 0 \end{pmatrix}$	point of triangle

Deriving the coordinates of C,D,E and the radius r:

X-coordinate of any point is the perpendicular distance($algebraic$) of point from Y-axis
Y-coordinate of any point is the perpendicular distance($algebraic$) of point from X-axis

Let foot of perpendicular from C to X-axis be F and C = $\begin{pmatrix} X_c \\ Y_c \end{pmatrix}$
$X_c = AB + BF$ and $Y_c = CF$
from trigonometry $BF = BC\cos\theta = a * \cos\theta$ and $CF = BC\sin\theta = a * \sin\theta$
$X_c = c + a * \cos\theta$ and $Y_c = a * \sin\theta$
$\therefore C = \begin{pmatrix} c + a\cos\theta \\ a\sin\theta \end{pmatrix}$
Using Sine rule in △ABC, we find α
Let E = $\begin{pmatrix} X_E \\ Y_E \end{pmatrix}$ and foot of perpendicular from E to X-axis be G. Using the fact that angle subtended by a chord at any point of the circle is half of that subtended at the centre, $\angle AEB = 2\alpha$
Since E is centre of circumcircle, EA = EB and hence △AEB is isoceles by which we conclude that $\angle AEG = \angle BEG = \alpha$
Using trigonometry in △AEG $X_E = c/2$, $\cot\alpha = \frac{Y_E}{X_E}$ and $\operatorname{cosec}\alpha = \frac{r}{X_E}$,
$Y_E = (c/2)\cot\alpha$
$\therefore E = \begin{pmatrix} c/2 \\ (c/2)\cot\alpha \end{pmatrix}$ and $r = \frac{c}{2\sin\alpha}$
l is found by applying sine rule in △ADB
Let D = $\begin{pmatrix} X_D \\ Y_D \end{pmatrix}$ and foot of perpendicular from D to X-axis be H, and given that AD=l
From geometry $\angle DAB = 2\theta - \alpha$, Using trigonometry in △ADH, $AH = X_D = l\cos(2\theta - \alpha)$and $DH = Y_D = l\sin(2\theta - \alpha)$
$\therefore D = l * \begin{pmatrix} \cos(2\theta - \alpha) \\ \sin(2\theta - \alpha) \end{pmatrix}$

Table for Output parameters:

TABLE 1.1.2

Symbol	Value	Description
α	$\cot^{-1}\frac{11*\sqrt{3}}{13}$	$\angle ACB$
l	$\frac{6.5*\sqrt{3}}{2*\sin\alpha}$	AD
C	$\begin{pmatrix} c + a * \cos\theta \\ a * \sin\theta \end{pmatrix}$	point of triangle
E	$\begin{pmatrix} c/2 \\ (c/2) * \cot\alpha \end{pmatrix}$	centre of circumcircle of △ABC.
r	$\frac{c}{2\sin\alpha}$	radius of circumcircle of △ABC.
D	$l * \begin{pmatrix} \cos(2\theta - \alpha) \\ \sin(2\theta - \alpha) \end{pmatrix}$	intersection point of angle bisector of AB and BC and circumcircle

Steps of construction:

1. The point A is taken as origin and a line segment AB = 6.5 cm is drawn along positive x-axis.

2. Draw a line segment emerging from B at ∠120° in anticlockwise direction from BA of length 5 cm.

3. Name the other endpoint of the line segment as C.

4. Join AC. This completes the △ABC.

5. Now take the perpendicular bisector of any two sides, mark their point of intersection as E(centre of circumcircle).

6. Taking E as centre and EA=EB=EC as radius draw a circle(circumcircle).

7. Take internal angle bisector of AB and BC, let its point of intersection with the circumcircle be D.

8. Join AD and CD.

(i)1.1.1
center of the circumcircle is the point of intersection of the perpendicular bisectors of AB and BC.

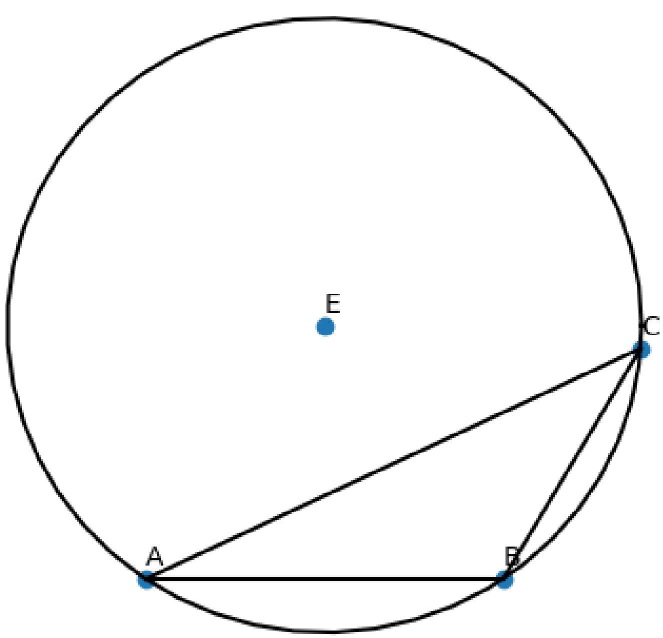

Fig. 1.1.1.

(ii)1.1.2
the point D of the cyclic quadrilateral ABCD is the point of intersection of the angle bisectors of AB and BC and the circumcircle.

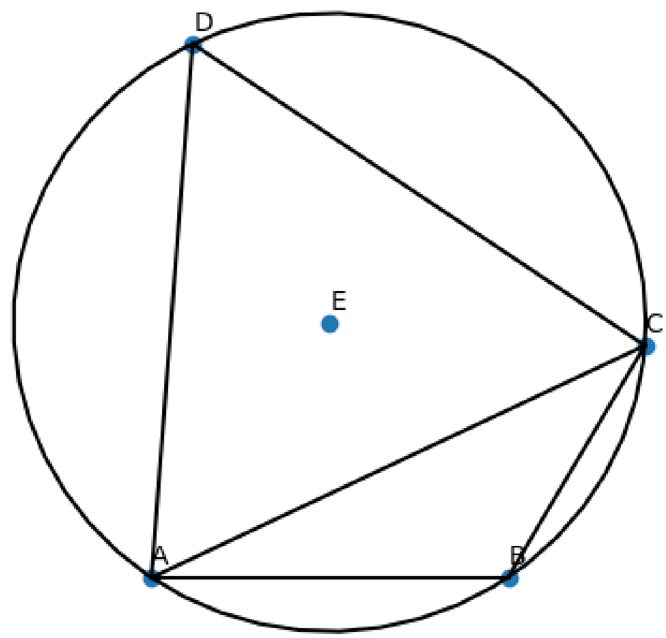

Fig. 1.1.2.