Probability and random variables assignment

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1 Q8 C)

1.1. Using ruler and compass only, construct a $\triangle$ABC such that BC = 5 cm and AB = 6.5 cm and $\angle$ABC = 120°
(i) Construct a circum-circle of $\triangle$ABC
(ii) Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC.

Solution: The parameters for constructing the figure are given in the table below:

TABLE 1.1.1

Symbol	Value	Description
a	5	BC
c	6.5	AB
θ	$\frac{\pi}{3}$	$\pi - \angle ABC$
A	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	origin
B	$\begin{pmatrix} c \\ 0 \end{pmatrix}$	point of triangle

Deriving the coordinates of C,D,O(centre of circumcircle) and its radius r:

X-coordinate of any point is the perpendicular distance($algebraic$) of point from Y-axis
Y-coordinate of any point is the perpendicular distance($algebraic$) of point from X-axis

Let foot of perpendicular from C to X-axis be F and C = $\begin{pmatrix} X_c \\ Y_c \end{pmatrix}$
$X_c = AB + BF$ and $Y_c = CF$
from trigonometry $BF = BC\cos\theta = a\cos\theta$ and $CF = BC\sin\theta = a\sin\theta$
$X_c = c + a\cos\theta$ and $Y_c = a\sin\theta$
$\therefore C = \begin{pmatrix} c + a\cos\theta \\ a\sin\theta \end{pmatrix}$
Using Sine rule in $\triangle$ABC, we find α
Let O = $\begin{pmatrix} X_O \\ Y_O \end{pmatrix}$ and foot of perpendicular from O to X-axis be G.
Since O is the circumcentre, OA=OB=OC

$$\therefore \|\mathbf{O} - \mathbf{A}\| = \|\mathbf{O} - \mathbf{B}\| = \|\mathbf{O} - \mathbf{C}\| \tag{1.1.1}$$

$$\implies \|\mathbf{O} - \mathbf{A}\|^2 = \|\mathbf{O} - \mathbf{B}\|^2 = \|\mathbf{O} - \mathbf{C}\|^2 \tag{1.1.2}$$

$$\implies \left\|\mathbf{O}^2\right\| = \left\|\mathbf{O}^2\right\| - 2\mathbf{B}^T\mathbf{O} + \left\|\mathbf{B}^2\right\| \tag{1.1.3}$$

$$and \left\|\mathbf{O}^2\right\| = \left\|\mathbf{O}^2\right\| - 2\mathbf{C}^T\mathbf{O} + \left\|\mathbf{C}^2\right\| \tag{1.1.4}$$

$$\implies \left\|\mathbf{B}^2\right\| = 2\mathbf{B}^T\mathbf{O}, \left\|\mathbf{C}^2\right\| = 2\mathbf{C}^T\mathbf{O} \tag{1.1.5}$$

Taking $\mathbf{O} = \begin{pmatrix} X_O \\ Y_O \end{pmatrix}$, putting values of $\mathbf{B}$,$\mathbf{C}$ and solving,we get,
$\mathbf{O} = \begin{pmatrix} c/2 \\ (c/2)\cot\alpha \end{pmatrix}$
Using $\mathbf{O}$, we get radius r = $\|\mathbf{O}\|$
r = $\frac{c}{2\sin\alpha}$

l is found by applying sine rule in $\triangle$ADB
Let D = $\begin{pmatrix} X_D \\ Y_D \end{pmatrix}$ and foot of perpendicular from D to X-axis be H, and given that AD=l
From geometry $\angle DAB = 2\theta - \alpha$, Using trigonometry in $\triangle$ADH, $AH = X_D = l\cos(2\theta - \alpha)$and $DH = Y_D = l\sin(2\theta - \alpha)$
$\therefore D = l\begin{pmatrix} \cos(2\theta - \alpha) \\ \sin(2\theta - \alpha) \end{pmatrix}$

Table for Output parameters:
Steps of construction:

a) The point A is taken as origin and a line segment AB = 6.5 cm is drawn along positive x-axis.

b) Draw a line segment emerging from B at $\angle$120° in anticlockwise direction from BA of length 5 cm.

TABLE 1.1.2

Symbol	Value	Description
α	$\cot^{-1}\frac{11\sqrt{3}}{13}$	$\angle ACB$
l	$\frac{6.5\sqrt{3}}{2\sin\alpha}$	AD
C	$\begin{pmatrix} c+a\cos\theta \\ a\sin\theta \end{pmatrix}$	point of triangle
E	$\begin{pmatrix} c/2 \\ (c/2)\cot\alpha \end{pmatrix}$	centre of circumcircle of △ABC.
r	$\frac{c}{2\sin\alpha}$	radius of circumcircle of △ABC.
D	$l\begin{pmatrix} \cos(2\theta-\alpha) \\ \sin(2\theta-\alpha) \end{pmatrix}$	intersection point of angle bisector of AB and BC and circumcircle

c) Name the other endpoint of the line segment as C.

d) Join AC. This completes the △ABC.

e) Now take the perpendicular bisector of any two sides, mark their point of intersection as E(centre of circumcircle).

f) Taking E as centre and EA=EB=EC as radius draw a circle(circumcircle).

g) Take internal angle bisector of AB and BC, let its point of intersection with the circumcircle be D.

h) Join AD and CD.

(i)**??**
center of the circumcircle is the point of intersection of the perpendicular bisectors of AB and BC.

(ii)**??**
the point D of the cyclic quadrilateral ABCD is the point of intersection of the angle bisectors of AB and BC and the circumcircle.

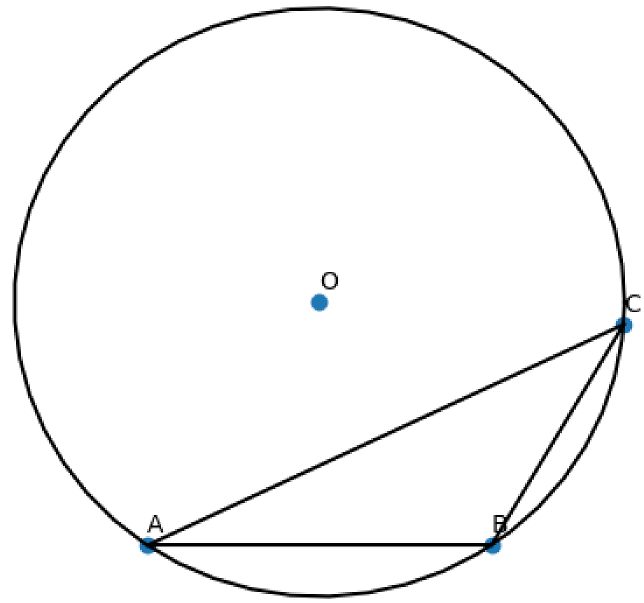

Fig. 1.1.1.

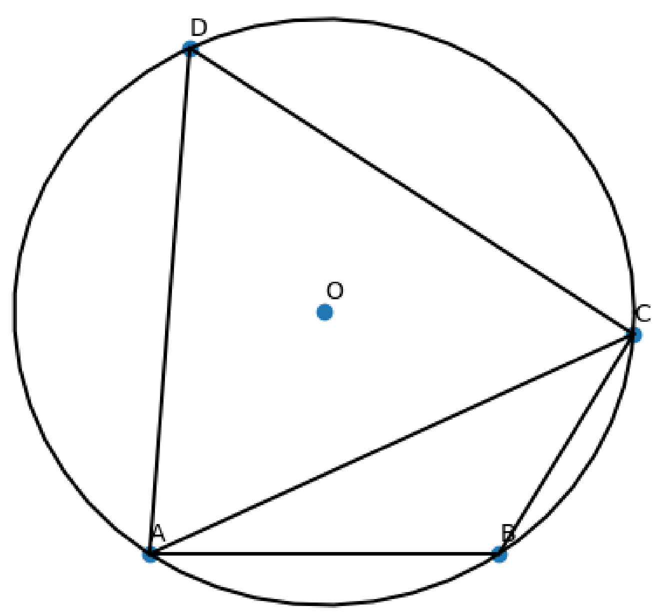

Fig. 1.1.2.