

Probability and random variables assignment

Maharshi Kadeval

1 Q8 c)

1.1. Using ruler and compass only, construct a $\triangle ABC$ such that $BC = 5$ cm and $AB = 6.5$ cm and $\angle ABC = 120^\circ$

(i) Construct a circum-circle of $\triangle ABC$

(ii) Construct a cyclic quadrilateral $ABCD$, such that D is equidistant from AB and BC .

Solution: The parameters for constructing the figure are given in the table below:

TABLE 1.1.1

Symbol	Value	Description
a	5	BC
c	6.5	AB
θ	$\frac{\pi}{3}$	$\pi - \angle ABC$
A	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	origin
B	$\begin{pmatrix} c \\ 0 \end{pmatrix}$	point of triangle

Deriving the coordinates of C,D,O(centre of circumcircle) and its radius r:

X-coordinate of any point is the perpendicular distance(*algebraic*) of point from Y-axis

Y-coordinate of any point is the perpendicular distance(*algebraic*) of point from X-axis

Let foot of perpendicular from C to X-axis be F and $C = \begin{pmatrix} X_c \\ Y_c \end{pmatrix}$

$$X_c = AB + BF \text{ and } Y_c = CF$$

from trigonometry $BF = BC \cos \theta = a \cos \theta$
and $CF = BC \sin \theta = a \sin \theta$

$$X_c = c + a \cos \theta \text{ and } Y_c = a \sin \theta$$

$$\therefore C = \begin{pmatrix} c + a \cos \theta \\ a \sin \theta \end{pmatrix}$$

Using Sine rule in $\triangle ABC$, we find α

Let $O = \begin{pmatrix} X_o \\ Y_o \end{pmatrix}$ and foot of perpendicular from O to X-axis be G .

Since O is the circumcentre, $OA=OB=OC$

$$\therefore \|O - A\| = \|O - B\| = \|O - C\| \quad (1.1.1)$$

$$\Rightarrow \|O - A\|^2 = \|O - B\|^2 = \|O - C\|^2 \quad (1.1.2)$$

$$\Rightarrow \|O^2\| = \|O^2\| - 2B^T O + \|B^2\| \quad (1.1.3)$$

$$\text{and } \|O^2\| = \|O^2\| - 2C^T O + \|C^2\| \quad (1.1.4)$$

$$\Rightarrow \|B^2\| = 2B^T O, \|C^2\| = 2C^T O \quad (1.1.5)$$

$$(1.1.6)$$

Taking $O = \begin{pmatrix} X_o \\ Y_o \end{pmatrix}$, putting values of B, C and solving, we get,

$$O = \begin{pmatrix} c/2 \\ (c/2) \cot \alpha \end{pmatrix}$$

Using O , we get radius $r = \|O\|$

$$r = \frac{c}{2 \sin \alpha}$$

l is found by applying sine rule in $\triangle ADB$

Let $D = \begin{pmatrix} X_D \\ Y_D \end{pmatrix}$ and foot of perpendicular from D to X-axis be H , and given that $AD=l$

From geometry $\angle DAB = 2\theta - \alpha$, Using trigonometry in $\triangle ADH$, $AH = X_D = l \cos(2\theta - \alpha)$ and $DH = Y_D = l \sin(2\theta - \alpha)$

$$\therefore D = l \begin{pmatrix} \cos(2\theta - \alpha) \\ \sin(2\theta - \alpha) \end{pmatrix}$$

Table for Output parameters:

Steps of construction:

- The point A is taken as origin and a line segment $AB = 6.5$ cm is drawn along positive x-axis.
- Draw a line segment emerging from B at $\angle 120^\circ$ in anticlockwise direction from BA

TABLE 1.1.2

Symbol	Value	Description
α	$\cot^{-1} \frac{11\sqrt{3}}{13}$	$\angle ACB$
l	$\frac{6.5\sqrt{3}}{2\sin\alpha}$	AD
C	$\begin{pmatrix} c + a \cos \theta \\ a \sin \theta \end{pmatrix}$	point of triangle
E	$\begin{pmatrix} c/2 \\ (c/2) \cot \alpha \end{pmatrix}$	centre of circumcircle of $\triangle ABC$.
r	$\frac{c}{2\sin\alpha}$	radius of circumcircle of $\triangle ABC$.
D	$l \begin{pmatrix} \cos(2\theta - \alpha) \\ \sin(2\theta - \alpha) \end{pmatrix}$	intersection point of angle bisector of AB and BC and circumcircle

of length 5 cm.

c) Name the other endpoint of the line segment as C.

d) Join AC. This completes the $\triangle ABC$.

e) Now take the perpendicular bisector of any two sides, mark their point of intersection as E (centre of circumcircle).

f) Taking E as centre and $EA=EB=EC$ as radius draw a circle (circumcircle).

g) Take internal angle bisector of AB and BC, let its point of intersection with the circumcircle be D.

h) Join AD and CD.

(i) 1.1.1

center of the circumcircle is the point of intersection of the perpendicular bisectors of AB and BC.

(ii) 1.1.2

the point D of the cyclic quadrilateral ABCD is the point of intersection of the angle bisectors of AB and BC and the circumcircle.

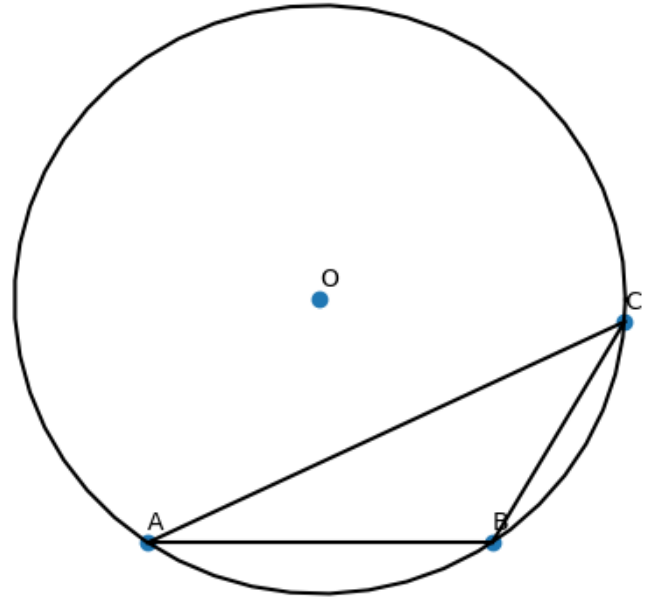


Fig. 1.1.1.

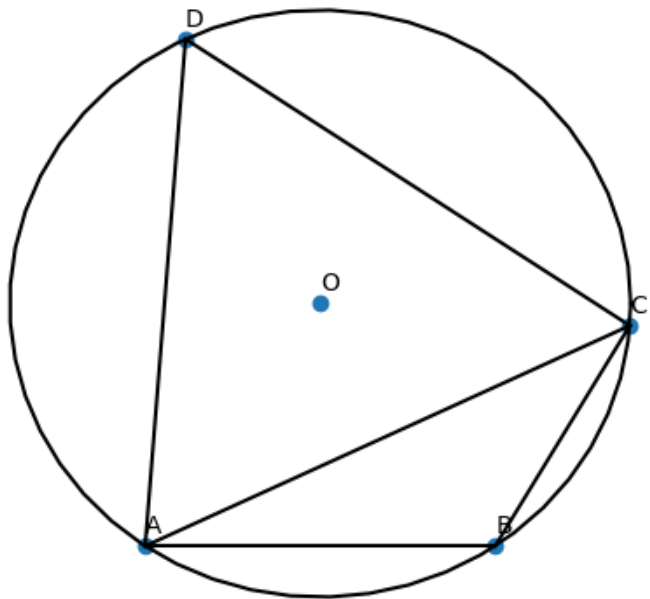


Fig. 1.1.2.