Probability and random variables assignment

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1 Q3

1.1. If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$
then prove that $a + b + c = abc$

Solution:
we are given:

$$\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi \tag{1.1.1}$$

We know that

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \tag{1.1.2}$$

$$\tan^{-1} x + \cot^{-1} y = \frac{\pi}{2} \tag{1.1.3}$$

$$\tan^{-1} x = \cot^{-1}\frac{1}{x}\,(x \neq 0) \tag{1.1.4}$$

(1.1.1) can be re - written as:

$$\tan^{-1} a = \left(\frac{\pi}{2} - \tan^{-1} b\right) + \left(\frac{\pi}{2} - \tan^{-1} c\right) \tag{1.1.5}$$

using (1.1.2) ,(1.1.3) and (1.1.4) :

$$\therefore \tan^{-1} a = \cot^{-1} b + \cot^{-1} c \tag{1.1.6}$$

$$= \tan^{-1}\left(\frac{1}{b}\right) + \tan^{-1}\left(\frac{1}{c}\right) \tag{1.1.7}$$

$$= \tan^{-1}\left(\frac{\frac{1}{b}+\frac{1}{c}}{1-\frac{1}{bc}}\right) \tag{1.1.8}$$

$$\therefore a = \frac{\frac{1}{b}+\frac{1}{c}}{1-\frac{1}{bc}} \tag{1.1.9}$$

$$= \frac{\frac{b+c}{bc}}{\frac{bc-1}{bc}} \tag{1.1.10}$$

$$= \frac{b+c}{bc-1} \tag{1.1.11}$$

Now, cross multiplying

$$a(bc-1) = b + c \tag{1.1.12}$$

$$\implies abc - a = b + c \tag{1.1.13}$$

$$\implies abc = a + b + c \tag{1.1.14}$$

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